DYNAMIC METEOROLOGY AND HYDROGRAPHY

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Part I. STATICS

BY

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CHAPTER I.

SYSTEM OF UNITS.

1. Meter-Ton-Second System.— In quantitative physical investigations the absolute units of the centimeter-gram-second system are now in general use. Sometimes these units are used directly. But, as one set of units can not have the proper magnitude for all sorts of measurements, special practical units are in many cases introduced which are derived from the corresponding fundamental units by the multiplication by suitable powers of 10. The choice of practical units is a question of great importance. It is of great advantage if they themselves form a connected system, or if they are at least in some simple relation to a connected system which can be used as a system of reference, the incessant troublesome return to the c.g.s. system being thus avoided.

For the purpose of dynamic meteorology and hydrography, the centimeter and gram are too small as units of length and mass. If for unit-length we choose the meter, and for unit-mass the metric ton, *i. e.*, the mass of a cubic meter of water at maximum density, great advantages are gained. The choice of a convenient unit of time unfortunately implies difficulties. Evidently the second is far too small a length of time for the measurement of changes in the state of the atmosphere and still more so in that of the sea. But the circumstance that the division of time is not decimal makes every change in the fundamental unit of time inadvisable. As fundamental units of reference we shall therefore use consistently

Meter = 10^2 centimeters. Metric ton = 10^6 grams. Second.

We shall refer to this system as the meter-ton-second system, or the m.t.s. system.

To the fundamental mechanical units we have to add, finally, the fundamental thermal unit. For this we shall choose the degree of the centigrade thermometer.

Unfortunately there is great confusion as to the units to which meteorological and hydrographical observations are referred. As we proceed we shall, therefore, give the tables required to derive from the observations recorded in the principal publications the results we wish to express in the units used in this treatise. These auxiliary tables, which would be superfluous if all observations were recorded in absolute units, are collected in the annexed "Appendix to meteorologic and hydrographic tables."

2. Simplest Derived Units. — From the values of the fundamental units those of the derived units are easily deduced. For completeness we shall add the dimensions of each derived quantity, expressed in the usual way in terms of length L, mass M, and time T.

The m.t.s. unit-velocity $[LT^{-1}]$ is the velocity of 1 meter per second, or 100 c.g.s. units of velocity.

The m.t.s. unit of momentum $[MLT^{-1}]$ is the momentum of the mass of a ton moving with the defined unit velocity. It is, therefore, equal to 100,000,000 c.g.s. units of momentum.

The m.t.s. unit of acceleration $[LT^{-2}]$ is the acceleration of 1 meter per second, or 100 c.g.s. units of acceleration. The acceleration of gravity is, therefore, in the m.t.s. system, equal to 9.8 approximately, or in rougher approximation equal to 10.

The m.t.s. unit of force $[MLT^{-2}]$ is the force which gives the mass of a ton the defined unit-acceleration. This unit of force is equal 100,000,000 c.g.s. units or dynes, *i. e.*, equal to 100 megadynes. Taking 10 for the acceleration of gravity, it will represent the weight of a tenth of a ton or of 100 kilograms.

The m.t.s. unit-impulse $[MLT^{-1}]$ is the change of momentum given by the defined unit-force during the time of 1 second. This unit-impulse is equal to 100,000,000 c.g.s. units of the same quantity. With respect to numerical value and dimensions this unit is identical with that of momentum.

The m.t.s. unit of force per unit-mass, sometimes called accelerating force $[LT^{-2}]$, is equal to the defined unit-force per ton of mass which is subject to the action of the force. It is equal, therefore, to 100 c.g.s. units of the same quantity. With respect to numerical value and dimensions the unit of force per unit-mass is identical with that of acceleration. The weight per unit-mass of a resting body is thus numerically equal to the acceleration which the body would take if it was free to fall. On account of this numerical accordance the expression "acceleration of gravity" is used to designate the intensity of gravity measured statically, *i. e.*, the weight per unit-mass of a heavy body.

The m.t.s. unit-work $[ML^2T^{-2}]$ is the work performed by the defined unit-force over the length of 1 meter. This unit-work is 10,000,000,000 c.g.s. units or ergs, *i. e.*, 10,000 meg-ergs. A unit of work in common use is the *joule*, which is equal to 10 meg-ergs. The m.t.s. unit of work, therefore, is a *kilojoule*. It represents approximately the work performed by lifting 1 ton 1 decimeter, or 100 kilogram-meters.

The m.t.s. unit of kinetic energy $[ML^2T^{-2}]$ is the kinetic energy of the mass of 1 ton moving with the unit-velocity defined above. The unit-increase of kinetic energy and also the unit-increase of potential energy $[ML^2T^{-2}]$ are obtained as equivalents for a unit of work performed. For this reason we can use numerically the kilojoule as a unit of kinetic and of potential energy as well as of work. When gravity is the acting force, unit-increase of potential energy is obtained by lifting 1 ton the approximate height of 1 decimeter.

The m.t.s. unit of activity $[ML^2T^{-3}]$ is the activity of 1 kilojoule per second. This is the kilowatt, an extensively used unit, introduced to replace the old unit of activity, the horsepower. The kilowatt is equal to 10,000,000,000 c.g.s. units of activity, and equal to 1.36 horsepower.

3. Units Used in Dynamics of Continuous Media. — In elementary dynamics definite masses are considered, to which the above-mentioned quantities are referred. In the dynamics of continuous media we have to deal with continuous distributions in space of mass, as well as of the quantities serving to define the static or the dynamic state of this distribution of mass. We then meet with the idea of *fields* of scalar as well as of vectorial quantities.

The purely kinematic quantities velocity and acceleration can be used at once for the description of fields in continuous media. But the quantities involving the idea of mass are not immediately serviceable. They must be referred either to *unit-mass* or to *unit-volume* of the medium.

The distribution of mass itself is described either by the volume per unit-mass or by the mass per unit-volume of the medium. The first of these quantities is the *specific volume* $[M^{-1}L^3]$, the second is the *density* $[ML^{-3}]$. They are reciprocal to each other, and the units in the m.t.s. system are the same as in the c.g.s. system.

Referring a mechanical quantity once to unit-mass and once to unit-volume of the medium, we arrive at two corresponding quantities. The passage from a quantity referred to unit-mass to the corresponding quantity referred to unit-volume involves the multiplication by a density, while the return involves the multiplication by a specific volume.

Most investigations in the dynamics of continuous media have been restricted to the case where the media are homogeneous. Then the fields of the corresponding quantities do not differ essentially from each other in their geometrical feature. This is the reason why the correspondence mentioned has attracted no greater attention hitherto. But in the problem now before us we shall have to treat the dynamics of essentially heterogeneous media. In this case the fields of corresponding quantities may differ widely from each other, and it is important to notice the analogies as well as the contrasts in these fields.

Momentum when referred to unit-mass leads back to the velocity, while momentum per unit-volume or specific momentum $[ML^{-2}T^{-1}]$ is the product of a velocity by a density. The m.t.s. unit of specific momentum is equal to 100 c.g.s. units of the same quantity, just as in the case of velocity. Velocity and specific momentum are the two corresponding quantities serving to describe the fields of motion in a continuous material medium.

Force when referred to unit-mass leads back to accelerating force, or acceleration, while force per unit-volume $[ML^{-2}T^{-2}]$ is equal to the product of an acceleration by a density. The m.t.s. unit of force per unit-volume is equal to 100 c.g.s. units of the same quantity, just as in the case of force per unit-mass. For the description of fields of force, the two defined kinds of force are theoretically equivalent to each other. The acceleration of gravity, used generally to describe the gravitational field of force, is a force per unit-mass. The gradient serving to describe the field of force due to a distribution of pressure in a fluid is a force per unit-volume. But for special reasons it may also be useful occasionally to describe the gravitational field by the force per unit-volume, and the field due to the pressure by the force per unit-mass of the medium.

The kinetic energy per unit-mass has the dimensions of the square of a velocity $[L^2T^{-2}]$. The kinetic energy per unit-volume is the square of the velocity multiplied by the density of the moving medium $[ML^{-1}T^{-2}]$. The units of each of these quantities in the m.t.s. system are equal to 10,000 of their c.g.s. units. They are perfectly equivalent to each other for the description of the field of kinetic energy in a moving medium. The work per unit-mass and per unit-volume have the same dimensions, respectively, as the kinetic energy per unit-mass and per unit-volume, and can be measured by the same units.

Activities referred either to unit-mass or to unit-volume come into consideration when processes of continuous transformations of energy are going on in the medium. The units of these quantities in the m.t.s. system are also equal to 10,000 of the corresponding c.g.s. units.

The gravity potential is a quantity which has the character of a work per unit-mass $[L^2T^{-2}]$, while a pressure is a quantity which has the character of a work per unit-volume $[ML^{-1}T^{-2}]$. The pressure is defined in a more elementary manner as a force per unit-area. But, however the definition be chosen, potential and pressure are closely related to each other from a theoretical point of view, and in a broader sense of the word they may be considered as corresponding quantities. Their dimensions differ by a quantity of the dimensions of a density, and their units in the m.t.s. system are equal to 10,000 of their c.g.s. units. As the units of these two quantities are of special importance to us, they will be discussed separately.

4. Units of Gravity Potential. — To every point in space we attribute a certain value of the gravity potential, defined numerically by this rule: It is equal to the potential energy relatively to sea-level possessed by a unit-mass situated in the point. The gravity potential of a point is therefore equal to the amount of work required to lift unit-mass from sea-level to the point against the action of gravity.

To unit-increase of gravity potential will therefore correspond, in any given locality, a definite increase of height, numerically equal to the reciprocal value of the acceleration of gravity. This increase of height will be slightly different in different localities, depending on the variations from place to place of the acceleration of gravity. But setting smaller variations aside, and taking 10 for the acceleration of gravity in the m.t.s. system, the height giving unit rise of potential will be equal to a decimeter.

To fix in our minds the approximate value of this height, we shall call the m.t.s. unit of gravity potential a *dynamic decimeter.* A ten times greater unit is the dynamic meter. Expressing gravity potentials in this latter unit, we gain the practical advantage that the number giving the gravity potential of a point will be very nearly equal to the number giving its height above or its depth below sea-level, expressed in common meters. This fortunate accordance makes it very convenient to use the dynamic meter as a technical unit of gravity potential. Values of the gravity potential expressed by an integer number of dynamic meters will be called *standard values*, and will be used very much as representatives for heights or depths.

But it should be emphasized that the dynamic meter and its subdivisions are units of gravity potential, not of length. In every given locality, however, they represent definite lengths measured along the plumb-line, and for this reason they can be used as full equivalents for the common length-measure when distances measured along the plumb-line are concerned.

5. **Units of Pressure.** — The unit-pressure of the m.t.s. system is the pressure of the unit-force defined above when it is exerted over the area of a square meter, and, as mentioned already, is equal therefore to 10,000 c.g.s. units of pressure, or 10,000 dynes per square centimeter. To avoid circumlocution, it will be necessary for us to have names for the employed units of pressure. The megadyne per square centimeter is approximately equal to the present practical unit, the atmosphere. It has often been proposed to introduce the megadyne per square centimeter as a practical unit of pressure, and to designate it by some name derived from the word "barometer." We shall choose the name *bar* as being the shortest, and designate the decimal parts of it as the decibar, centibar, and millibar. The m.t.s. unit of pressure will then be the *centibar*, while the c.g.s. unit will be the *microbar*.

Very simple rules are obtained for the columns of water exerting these pressures if we agree to have the heights of the water-columns represented by their values in dynamic meters, their multiples or subdivisions. Taking pure water at maximum of density, and neglecting its compressibility, we get these relations:

1 bar = pressure of 1 dynamic decameter of water.
1 decibar = pressure of 1 dynamic meter of water.
1 centibar = pressure of 1 dynamic decimeter of water.
1 millibar = pressure of 1 dynamic centimeter of water.

Finally, the c.g.s. unit, the microbar, is equal to the pressure of 10 dynamic microns of water.

Among these units we shall use often the decibar as a technical unit, on account of its correspondence to the dynamic meter as unit of gravity potential. Completing our terminology, we shall denote pressures represented by an integer number of decibars as *standard* pressures. In cases where we have to do with the relations between pressures and gravity potentials we shall often refer to these standard values of both quantities, using thus dynamic meter and decibar as connected units. But when the relation to other quantities comes in, we shall have to return to the m.t.s. units, the centibar, and the dynamic decimeter.

We shall also make frequent use of the millibar as that technical unit which is most convenient in reading the barometer. It will replace the present practical units, the millimeter or the inch of mercury. Using 13.59545 for the density of mercury at 0° C.,* and 9.80617 for the standard value of gravity (compare section 8 below), we find that 1 meter of mercury of 0° C. at a place where gravity has this standard value exerts the pressure of 1.333193 bars. Thus, a mercury

* THIESEN UND SCHEEL: Tätigkeitsbericht der Phys. Techn. Reichsanstalt, 1 Feb., 1897–31 Jan., 1898. Berlin, 1898.

barometer which gives, for standard value of gravity, direct readings in millibars is a barometer whose scale has its divisions at the mutual distance of 0.750079 mm. or 0.75 mm., practically, instead of at the distance of integer millimeters.

In meteorology it is common to give the barometric pressure either in millimeters or in inches of mercury. The millimetric division is not in the least more rational than the division into inches. Neither of them has anything to do with the system of absolute units. The consequences of this irrationality have not yet been seriously felt, because the barometric records have until now served for qualitative purposes mainly. But the further development of dynamic meteorology will compel us to introduce rational units sooner or later. Meanwhile we shall be obliged to change from the one system of units to the other by auxiliary tables.

The tables required for the direct passage from millimeters or inches of mercury to millibars are given in the Appendix. In many cases, however, it will be a saving of time and labor for a while to retain the units to which the original observations are referred, in order to carry out the transition to the rational units at a later stage of the work of computation, as will be developed in the proper places below.

CHAPTER II.

GRAVITY AND GRAVITY POTENTIAL.

6. Gravity. — The exterior force upon which the conditions of equilibrium and motion in the atmosphere and in the sea depend is gravity. By gravity without further specification we mean the force the intensity of which is found by the pendulum experiments. It is the resultant of two different actions — the attraction of the earth and the centrifugal force due to the earth's rotation. But in practical application we shall never make use of this decomposition of the force into the two components of different origin.

A first condition for the solution of concrete problems relating to the equilibrium and the motion of the air and the sea is therefore a knowledge of the intensity of gravity at every point of the space filled by these two media. This knowledge is founded on the actual measurements of the intensity of gravity at the earth's surface. But it is not necessary for us to take into consideration all the small irregularities in the variation of this force as they present themselves in geodetic investigations. Where no measured values of the intensity of gravity are at hand it will suffice to work with the "normal" values, as they can be calculated by the general formulæ of geodesy. They will give an approximation far closer than that by which we can find the values of any other force upon which the atmospheric or oceanic equilibrium or motion depends.

We shall therefore write down the formulæ necessary for the calculation of this normal intensity of gravity, and give a complete tabulation of these formulæ. According to the common terminology, we shall call the tabulated quantity the acceleration of gravity. But it should be remembered that it represents, as already mentioned (section 2), at the same time the intensity of gravity measured statically by the weight per unit-mass of the heavy body.

7. Normal Decrease of Gravity in the Atmosphere. — Let the numerical value g_1 of the acceleration of gravity be known at a point of the earth's surface. Its value g can then be calculated at any height z above this point from the decrease of the attraction with the increase of the distance from the attracting masses, and from the increasing influence of the centrifugal force with the increasing distance from the earth's axis. Setting aside quantities of the order of magnitude of the square of the ratio z/r, z being the height and r the radius of the earth, we find, according to Helmert,* as the best expression for the decrease of the gravity with the height,

$$(a) \qquad g = -(g_1 - 0.000003086z)$$

* Helmert: Ueber die Reduction der auf der physischen Erdoberfläche beobachteten Schweerebeschleunigungen auf ein gemeinsames Niveau, zweite Mitteilung. Sitzungsberichte der Akademie der Wissenschaften, Berlin, 1903, p. 650.

The sign − is used because the acceleration of gravity is directed downwards, while we take the direction upwards as positive. The value of the correction term $-0.000003086z$ is given in table 1 M of the Meteorological Tables.

8. Reduction to Sea-Level and Normal Value of Gravity at Sea-Level. — By sea-level we mean on the one hand the surface of the sea in the case of perfect equilibrium and on the other an ideal continuation of this surface below the continents, determined by the condition of being always at right angles to the plumb-line.

Values of the acceleration of gravity, which are found by pendulum experiments at the surface of the earth, are reduced to sea-level to make them intercomparable. The purpose of the reduction is to arrive as closely as possible to the theoretical value of the acceleration of gravity, which is a function only of the latitude and which depends upon the figure and the rotation of the earth, all irregularities of topography and of local mass distribution being neglected. There has been much discussion as to how this reduction should be performed properly. Two different views have been advanced, based upon physically different conceptions of the nature of the equilibrium of the earth's crust. According to the first view the equilibrium is that of a solid elastic body. The masses of the continents present above sea-level are considered as additional masses whose weight is carried by the stress produced in the solid crust of the earth. According to the second view, the earth's crust has sufficient stiffness only to carry the weight of local elevations above the main level of the land, while on a larger scale the equilibrium is of a hydrostatical nature. The elevation of the continents above sea-level are, then, due to their buoyancy, their density being smaller than the average density of the earth's crust. The average density would be attained if the masses of the continents present above sea-level were absorbed by the underlying masses.

These two views of the nature of the equilibrium of the earth's crust lead of course to two different principles for the reduction to sea-level. According to the first view, the continental masses present above sea-level represent a surplus of mass, the attraction of which must be subtracted if the reduction should lead to the required normal value. This leads to the reduction according to the formula of Bouguer, which until lately has been used almost universally. According to the second view the reduction is made as if the continental masses were absorbed by the earth's crust below the continents, no mass being present between the physical surface of the earth and sea-level. The reduction is, then, simply the same as in the free air.

According to the result of recent geodetic investigations * this simple reduction leads with much closer approximation to the normal value of gravity at sea-level than the reduction according to the formula of Bouguer. Thus the theory of the

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*G. R. Putnam: Results of a transcontinental series of gravity measurements. Phil. Soc. of Washington, February 2, 1895. Bulletin of the Society, vol. 13. Washington, D. C., 1900, p. 31.

G. K. Gilbert: Notes on the gravity determinations reported by Mr. G. R. Putnam. Phil. Soc. of Washington, March 16, 1895. Bulletin of the Society, vol. 13. Washington, D. C., 1900, p. 61.

R. v. Sterneck: Relative Schweerebestimmungen. Mitteilungen der Militär-Geographischen Institut. Wien, 1898, p. 100.

F. R. Helmert: Ueber die Reduction der auf der physischen Erdoberfläche beobachteten Schweerebeschleunigungen auf ein gemeinsames Niveau. Sitzungsberichte der Akademie der Wissenschaften. Berlin, 1902, p. 843; 1903, p. 650.

approximate hydrostatic equilibrium of the masses in the earth's crust is verified. More recently this verification has also been extended to the open sea by the measurements of the Nansen Expedition in the Polar Sea and those of Hecker on the Atlantic.* These results are very important for dynamic meteorology and hydrography, as they show that the gravitational field of force in atmosphere and sea is much more regular than originally supposed. The continental masses present above sea-level do not cause perturbations of the field. On the contrary, they make it more regular, because they compensate for subterranean mass defects. Neither does the sea, with its smaller density, complicate the field, because there are compensating excesses of mass below the sea-bottom. The only perturbations of the field are due to irregularities of local topography or of local mass distribution sufficiently small to be balanced by the elastic stresses which they produce in the earth's crust. We shall make no corrections for these local irregularities. The reduction to sea-level of the numerical value g_1 of the acceleration of gravity found by pendulum experiments at the earth's surface at the height z above sea-level will be given by the formula

$$g_0 = g_1 + 0.000003086z \tag{a}$$

the correction term being the same as that of formula section 7 (a), or of table 1 M of Meteorological Tables, but with the sign reversed. We shall use this reduction consistently in cases where we start with really measured values of the acceleration of gravity at the earth's surface. It will be convenient, as all heights are measured from sea-level, and the reduction will bring in no errors in the values of gravity calculated for the free air, as errors possibly introduced by the use of formula (a) for reductions downward will drop out again by the reduction upward, according to formula section 7 (a).

If no measurements of the acceleration of gravity are at hand, we shall start with the "normal" value of gravity at sea-level, and derive from it by formula section 7 (a) or table 1 M the value at the earth's surface or at any height above sea-level. The normal value g_0 of the acceleration of gravity at sea-level we shall consider as given by the formula of Helmert: †

$$g_0 = 9.80617\,(1 - 0.002644 \cos 2\phi + 0.000007 \cos^2 2\phi) \tag{b}$$

The values of g_0 are tabulated according to this formula in table 2 M of our Meteorological Tables.

9. Normal Increase of Gravity in the Sea. — Calculating the decrease of gravity in the atmosphere, we could simplify the problem by neglecting the mass of the air. But in view of the greater density of the water, the corresponding simplification will not be allowable for the case of the sea.

* O. E. SCHIÖTZ: Results of the pendulum observations. The Norwegian North Pole Expedition 1893–96, vol. II. Christiania, 1901.

O. HECKER: Bestimmung der Schwerkraft auf dem Atlantischen Ocean. Veröffentlichungen des preussischen geodätischen Instituts. Berlin, 1903.

† R. F. HELMERT: Der normale Teil der Schwerkraft im Meeresniveau. Sitzungsberichte der Akademie der Wissenschaften. Berlin, 1901, p. 328.

In order to calculate the correction in this case, we shall consider the earth as a sphere of radius r, and make use of the well-known theorem in the theory of attraction that a spherical shell of constant density does not exert any influence on a point inside it. In the depth z below sea-level we have therefore only to take into account the attraction of the mass contained within a sphere of radius $r - z$. M being the whole mass of the earth and m that of the shell, we have for the acceleration of gravity at sea-level

$$g_0 = k\frac{M}{r^2}$$

and at the depth z below sea-level

$$g = k\frac{M - m}{(r - z)^2}$$

Neglecting squares or products of the small ratios m/M and z/r, we conclude from these equations

$$g = g_0 + 2g_0\frac{z}{r} - g_0\frac{m}{M}$$

Denoting by ρ_m the mean density of the earth, and by ρ that of the spherical shell, we have

$$M = \tfrac{4}{3}\pi r^3 \rho_m \qquad m = 4\pi r^2 z \rho$$

and thus

$$(a) \qquad g = g_0 + 2\frac{g_0}{r}\left(1 - \frac{3}{2}\frac{\rho}{\rho_m}\right)z$$

For the factor $2g_0/r$ we have to use, according to Helmert, the value 0.000003086. For the density of the spherical shell we shall use as an average value $\rho = 1.05$, corresponding to the density of the sea-water at the depth of nearly 5000 meters (compare table 14 H). Choosing finally $\rho_m = 5.5$ as the probable value of the average density of the earth, we get the formula

$$(b) \qquad g = g_0 + 0.000002202z$$

by which we shall calculate the normal values of the acceleration of gravity in the sea.

The values of the correction term $0.000002202z$ are given in table 2 H of the Hydrographic Tables.

Of course the normal values of the acceleration of gravity, which we are thus able to calculate, will generally slightly differ from the real local values, as a consequence of the local distribution of mass. It must also be remembered that the spherical shell does not consist exclusively of water, but also contains the land-masses below the continents. For this reason we might have chosen a still greater value for the mean density of the shell. But this heterogeneity of the shell will have different effects near the coasts and in the middle of the open sea, and we therefore leave it out entirely, the more so as the "normal" value of gravity gives a precision amply sufficient for the discussion of the dynamics of the sea in the present state of development of this science.

10. Level Surfaces and Dynamic Height or Depth. — A surface everywhere perpendicular to the plumb-line is a level surface. The free surfaces of liquids in equilibrium always form level surfaces, and the surface of the sea, together with its continuation below the continents as referred to above, is the fundamental level surface, to which all differences of level are referred.

If gravity is the only acting force, no work is required to move a weight along a level surface. But in order to lift it from one level surface to another, a certain amount of work is required, and always the same amount, irrespective of where on the two surfaces the two extreme points of the path are situated. Otherwise perpetual motion could be realized by lifting the weight at the place where less work is required and letting it down at the place where more work is required. Any level surface is therefore specified without ambiguity by the amount of work required to lift a certain mass, say unit-mass, from sea-level to any point of the surface. Or, in other words, a level surface is a surface of equal gravity potential (section 4) and is perfectly specified by the gravity potential of any of its points.

The level surfaces must be carefully distinguished from the surfaces of equal height above or equal depth below sea-level. The intensity of gravity decreases from the pole to the equator. Consequently the unit-mass must be lifted higher at the equator than at the pole, if the same amount of work is to be performed, and thus the same level surface be attained. A surface of equal height above or of equal depth below sea-level must therefore cut through the system of level surfaces. The surface of equal height or depth is a slanting surface, which is not normal to the plumb-line, and on which equilibrium is not possible under the sole action of gravity. If the surfaces were hard and smooth a ball would remain in equilibrium on a level surface. But on a surface of equal height above sea-level it would roll in the direction from the pole to the equator; and on a surface of equal depth below sea-level it would roll in the direction from the equator to the pole.

This property at once shows that the surfaces of equal height or depth are not suitable as coördinate surfaces in problems relating to the statics or the dynamics of the atmosphere or the sea. For this purpose only level surfaces are found suitable.

The introduction of the level surfaces as coördinate surfaces involves the use of gravity potentials for the specification of heights and depths. With this application of gravity potentials in view, we have introduced the names dynamic meter, dynamic decimeter, etc., for units of this quantity. To standard values of the gravity potential in the sense defined (section 4) will correspond *standard equipotential surfaces.* These will serve us as coördinate surfaces.

We shall also use the expressions dynamic height and dynamic depth as synonymous with gravity potential, with the difference only that we take the dynamic depth in the sea as a positive quantity, while the corresponding values of the gravity potential are negative. By this mode of expression *the level surfaces are surfaces of equal dynamic height above or of equal dynamic depth below sea-level*, the height or depth of the standard surfaces being an integer number of dynamic meters. We shall as a rule prefer the expressions dynamic height or depth when we refer to the dynamic meter as unit, and the expression gravity potential when we use the m.t.s. unit, the dynamic decimeter.

Denoting the gravity potential by ϕ, and the dynamic heights and depths respectively by H and D, we have the relations

$$(a) \qquad \phi = 10H \qquad \phi = -10D$$

by which we return from the technical unit, the dynamic meter, to the m.t.s. unit, the dynamic decimeter.

11. Fundamental Formulæ for the Gravity Potential. — The difference of potential between any two points can be found if we know the value of the acceleration of gravity everywhere along a curve s leading from the one point to the other. Let g_s be the component of the acceleration of gravity in the direction tangential to the curve s. The work per unit-mass performed against the action of gravity, when a mass is displaced the length ds along the curve is then $-g_s ds$. That is, the elementary difference of potential between the end-points of the line element ds is $-g_s ds$, and the finite difference of potential $\phi_2 - \phi_1$ between any two points joined by the curve s is found by the integration

$$(a) \qquad \phi_2 - \phi_1 = -\int_1^2 g_s ds$$

If the curve s coincides with the plumb-line, the acceleration of gravity will always come in with its full value g. It the lengths measured along the plumb-line be denoted by z, and the heights of the points 1 and 2 above sea-level by z_1 and z_2, the expression (a) takes the form

$$(b) \qquad \phi_2 - \phi_1 = -\int_{z_1}^{z_2} g dz$$

If from (b) we pass to dynamic heights in the atmosphere, expressed in dynamic meters, we have

$$(c) \qquad H_2 - H_1 = -\tfrac{1}{10}\int_{z_1}^{z_2} g dz$$

Correspondingly for the difference ot dynamic depths in the sea we have

$$(d) \qquad D_2 - D_1 = \tfrac{1}{10}\int_{z_1}^{z_2} g_s dz$$

These formulæ serve to calculate the dynamic value of given geometric differences of height or depth.

12. Normal Relation between Geometric and Dynamic Heights.—Introducing the value (a), section 7, of the acceleration of gravity g in the integral 11 (c), and integrating from the initial height z_1 to any height z, we get for the corresponding difference of dynamic height

$$(a) \qquad H - H_1 = \frac{g_1}{10}(z - z_1) - 0.0000001543(z^2 - z_1^2)$$

By this formula we find the dynamic difference of height corresponding to any given geometric difference of height. It is to be noted that in the first approximation we can neglect the term containing $(z^2 - z_1^2)$, and instead of that use the approximate value 9.80 for the acceleration of gravity at sea-level. This gives the approximate relations

$$(a') \qquad H - H_1 = 0.98(z - z_1), \text{ or counted from sea-level, } H = 0.98z$$

$$(b') \qquad z - z_1 = 1.02(H - H_1), \text{ or counting from sea-level, } z = 1.02H$$

That is, the number expressing a height in dynamic meters is approximately 2 per cent smaller than the number expressing it in meters.

Supposing that the dynamic height be given, while the corresponding value of the geometric height should be found, we have to solve equation (a) with respect to $z - z_1$. To do this conveniently we first substitute from (b') the approximate values of z and z_1 in the correction term of equation (a), which is thus made linear in $z - z_1$. Solving and simplifying the correction term by the introduction of the approximate value 9.80 for the acceleration of gravity g_1, we get the equation

$$(b) \qquad z - z_1 = \frac{10}{g_1}(H - H_1) + 0.0000001637(H^2 - H_1^2)$$

by which the geometrical value of a given dynamic height can be calculated.

In practical application it will generally be most convenient to have all heights measured from sea-level. We then have $z_1 = 0$, $H_1 = 0$, $g_1 = g_0$, and formulæ (a) and (b) take the form

$$(a'') \qquad H = \frac{g_0}{10}z - 0.0000001543z^2$$

$$(b'') \qquad z = \frac{10}{g_0}H + 0.0000001637H^2$$

In order to tabulate conveniently these formulæ, we shall write them in a slightly modified form. In both the main term depends upon two variables, namely, g_0 and z or g_0 and H, respectively. But, thanks to the small variations of g_0, we can account for the influence of the variations of this quantity in a correction term, while the main term is made to depend upon one variable only. To attain this we shall write

$$(c) \qquad g_0 = 9.80\left(1 + \frac{g_0 - 9.80}{9.80}\right)$$

The fraction contained within the parentheses will have a value never exceeding 0.004. Neglecting squares of this quantity as well as products of it by quantities of its own order of magnitude, we bring the formulæ (a'') and (b'') to the forms

$$(a''') \qquad H = \{0.98z - 0.0000001543z^2\} + 0.1(g_0 - 9.80)z$$

$$(b''') \qquad z = \{1.020408H + 0.0000001637H^2\} - \frac{g_0 - 9.80}{9.60}H$$

The expressions inclosed within parentheses depend upon one variable only. Their values are given in tables 3 M and 5 M of Meteorological Tables. They give the relation between geometric and dynamic height for places where the acceleration of gravity at the sea-level height z_1 has the special value 9.80. The last term in each equation gives the correction for other values of g_0. The value of this correction is given in tables 4 M and 6 M of Meteorological Tables. These tables can thus be used to pass from geometric to dynamic heights and *vice versa*, the only supposition being that we know the value g_0 of the acceleration of gravity at sea-level, which is found either by table 2 M, or by reduction to sea-level of the value of the acceleration of gravity found by direct determinations at the earth's surface. Proceeding in this way, we find the dynamic heights above sea-level both of the ground and of points in the free atmosphere. The height of the ground will contain an uncertainty due to that of the reduction of g to sea-level. But the heights of the points in the free atmosphere above the ground will contain no error due to this reduction.

13. Normal Relation between Geometric and Dynamic Depths.— Introducing the value (*b*), section 9, of the acceleration of gravity below the integral sign of (*a*), section 11, and integrating from sea-level, where $D = z = 0$ to any depth z, we find the corresponding value of the dynamic depth D

(*a*) $$D = \frac{g_0}{10} z + 0.0000001101 z^2$$

This formula serves to calculate the dynamic depth D corresponding to any given geometric depth z.

From this formula we draw as a first approximation

(*a'*) $$D = 0.98 z$$

or, solving with respect to z,

(*b'*) $$z = 1.02 D$$

That is, in the case of the sea we have the same approximate difference as in the atmosphere between the figures representing the two kinds of depth amounting to about 2 per cent.

Solving (*a*) by the method employed for (*a*), section 12, we find the equation

(*b*) $$z = \frac{10}{g_0} D - 0.0000001168 D^2$$

by which the geometric value of a given dynamic depth is calculated.

To make the formulæ (*a*) and (*b*) suitable for tabulation, we use the same artifice as above. Introducing (*c*), section 12, and neglecting small quantities of the second order, we can write the formulæ

(*a''*) $$D = \{0.98 z + 0.0000001101 z^2\} + 0.1 (g_0 - 9.80) z$$

(*b''*) $$z = \{1.020408 D - 0.0000001168 D^2\} - \frac{g_0 - 9.80}{9.60} D$$

The expressions within the brackets depend on one variable only, and their values are given in tables 3 H and 5 H respectively of the Hydrographic Tables. They give the relation between geometric and dynamic depth in the special case that acceleration of gravity in sea-level has the value 9.80. The last term in each equation gives the correction for other values of g, and the numerical values of these corrections are given in tables 4 H and 6 H respectively of the Hydrographic Tables.

14. Gravity Potential of Points at the Earth's Surface. — According to the modern principles of geodesy, levelings of high precision should always be combined with determinations of the acceleration of gravity. This combination of leveling with gravity measurements gives all the data required for the determination of gravity potentials of points at the earth's surface.

Leveling consists in sighting along level surfaces and in measurements of heights normal to them. A curve consisting of successive horizontal and vertical parts is thus traced out. Forming for this curve the integral (a), section 11, we have to take into account the vertical parts only. Let their lengths be $z, z', z'', \cdots\cdots$, and let $g, g', g'', \cdots\cdots$ be the mean values of the acceleration of gravity along each of them. The integral then takes the form

$$(a) \qquad \phi_2 - \phi_1 = gz + g'z' + g''z'' + \cdots\cdots$$

The sum on the right side thus gives the difference of gravity potential between the end-points of the curve.

All the measurements required for the determination of gravity potentials are thus performed by modern geodetic work. But unfortunately the results are not worked out and published in this form. Attention is directed mainly to the sum

$$(b) \qquad Z = z + z' + z'' + \cdots\cdots$$

which is supposed to represent the difference of height between the two end-points. This Z is, however, no well-defined quantity, because the level surfaces are not parallel to each other. If the leveling be performed along another route, a slightly different sum Z will generally be found. The discrepancies caused by the lack of parallelism between the level surfaces may be diminished by suitable corrections, but no general method can be conceived which would make them disappear, and the real relation of the determined Z to the vertical distance of the one point from the level surface passing through the other will remain obscure.

The only quantity which can be determined without ambiguity is the gravity potential ϕ. The same will be the case if we pass to the other fundamental method for the determination of heights, the barometric method. As we shall have occasion to show later, this method also gives gravity potentials as its direct natural result, while the passage to heights brings uncertainties.

That under these circumstances gravity potentials, when wanted, must be found by recalculation from the published heights, is very unsatisfactory, so much the more so as it will probably presently become apparent that gravity potentials are what are really needed for scientific purposes, heights being only of secondary importance. Such at least is the case in meteorology, and will also be that of geology as soon as the question of the statics and dynamics of the earth's crust is taken up seriously. It would therefore be a great advance if gravity potentials were published as the main scientific result of geodetic work, and heights only as results computed from gravity potentials.

Provisionally we have to do the reverse. The problem to compute the most probable values of the gravity potential from the published heights is therefore of some importance. The method will mainly consist in removing the corrections originally introduced to pass from gravity potentials to heights, and will therefore turn out somewhat differently according as the barometric, the leveling, or trigonometric methods have been used. Further, it will differ with the different rules for the reduction used in each of these methods. Thus different methods would have to be used on different occasions, and the data determining the choice of method would not always be at hand. In this state of confusion the normal reduction, which we have developed in the case of points in the free atmosphere (section 12), seems to be the most worthy of recommendation, also for the determination of gravity potentials at points on the earth's surface.

15. Maps of Dynamic Topography. — When the gravity potential or the dynamic height is known for a sufficient number of points of the earth's surface we shall be enabled to draw a new kind of topographic maps, representing not the geometric but the dynamic heights of the country. The curves of these maps would be real level curves, which would represent the coast-lines if the country were partially submerged under the sea. The number of curves between two points would represent the amount of work per unit-mass which had to be performed against the action of gravity, if a body should be moved from the one point to the other. The maps would thus represent the height of a mountain, not by the vertical distance of its summit from sea-level, but by the work required to reach the summit. They would further directly give the amounts of potential energy possessed by the masses of water stored in the lakes and would show how this potential is given up during the flow of the water down the rivers.

The motion of the air is restricted by the condition of tangential contact with the earth's surface. The knowledge of the topography of the land is therefore indispensable for the study of this motion. Both the geometric and the dynamic topography must be known, but for evident reasons the dynamic topography is of first importance.

For the construction of these maps the close accordance of the common and the dynamic meter is of great practical value. Especially if the maps should represent large parts of the world on a moderate scale, there will be no visible difference between the course of two curves, one of which represents the height of a certain number of common meters, while the other represents the height of the same number of dynamic meters. To make such maps practically useful in meteorology it will be necessary to simplify the topography, smoothing out all the small irregularities. These maps of idealized topography, drawn on a moderate scale, can therefore, according to circumstances, be considered as representing both the geometric and the dynamic topography.

If the topography of the earth's surface is of importance for the motion of the air, that of the bottom of the sea is of still higher importance for the motion of the sea. As in the case of the air, the dynamic topography is of the greatest importance, the geometric being only of secondary interest. But for maps on moderate scales we can identify both kinds of topography. Near the coasts it will generally be necessary to simplify the course of the curves. But for greater distances from the coasts the bottom configuration is generally so regular, or our knowledge of it so incomplete, that artificial simplifications may be more or less dispensed with.

The topographical maps accompanying this work can be considered as representing both geometric and dynamic topography. On the map of the world, giving the topography of the earth's surface both above and below sea-level, the main curves are drawn for the interval of 1000 meters, which may be interpreted as geometric or dynamic meters according to circumstances. For the displacement from curve to curve of a unit-mass, we have a gain or loss of potential energy of 10,000 m.t.s. units.

16. Scalar Field. — It will be useful to refer here to some fundamental notions relating to scalar fields, and their variations from place to place in space. Let α be a scalar quantity which has a uniquely determined value in every point of space. To represent distinctly the distribution in space of these values, or, in other words, to represent the field of the scalar α, we can draw a set of *equiscalar surfaces*

$$\alpha = \alpha_0, \ \alpha = \alpha_1, \ \alpha = \alpha_2, \ \cdot \cdot \cdot \cdot \cdot$$

Each of these contains the points in space where the scalar has a certain constant value, α_0, α_1, α_2 $\cdot \cdot \cdot \cdot \cdot$, respectively. This is the well-known method of representing the distribution of potential by equipotential surfaces, that of pressure by isobaric surfaces, that of temperature by isothermic surfaces, and so on.

The sheet between two equiscalar surfaces α_0 and α_1 will be called an *equiscalar sheet*. The use of the word "equiscalar" in connection with a sheet must not be misunderstood. The scalar is not constant in the sheet, but it has limited variations, the limits being given by its values α_0 and α_1 on the boundary. The word "equiscalar" used for a sheet should remind us of this limitation of the variations, as well as of the possibility of defining an average value of the scalar, which is constant all along the sheet.

In most cases it will be found convenient to draw the equiscalar surfaces for unit differences of the scalar. These surfaces will then divide the space into a set of equiscalar *unit-sheets*. Choosing a unit of suitable magnitude, we can always be certain that the unit-sheets get a suitable thickness for a perspicuous distinct representation of the field. If sufficiently thin sheets are obtained we can always say that the difference between the values α_1 and α_0 of the scalar in two points of space 1 and 0 is equal to the number of unit-sheets contained between them.

This difference, $\alpha_1 - \alpha_0$, divided by the length s of any curve joining the points 0 and 1,

$$(a) \qquad \frac{\alpha_1 - \alpha_0}{s}$$

gives the *average* rate of variation of the scalar along the curve s.

Now let the curve s be a straight segment of line, and let its length diminish indefinitely. In this limiting case (a) gives the *local* rate of variation of the scalar α in the direction determined by the elementary segment of line s. This rate will vary with the direction of s. To examine this variation let us choose the unit of the scalar quantity so small that the thickness of the unit-sheets is small in comparison to the elementary length s. Further, let s have one end-point fixed and let it have a constant length, while it can have any direction. Within the spherical space of radius s the equiscalar surfaces separating the unit-sheets can be considered as parallel and equidistant. Then the number of unit-sheets cut by the segment s will evidently be proportional to the cosine of the angle which this segment forms with the normal n to the equiscalar surfaces. The rate of variation of the scalar being in direct proportion to the number of unit-sheets cutting the segment of line s, we get this result:

The rate of variation of a scalar quantity in any direction s is equal to its rate of variation along the normal to the equiscalar surfaces, multiplied by the cosine of the angle contained between this direction s and the normal n to the equiscalar surfaces.

17. Gradient and Ascendant. — In accordance with this result we can represent the main rate of variation of the scalar field by a vector directed along the normal to the equiscalar surfaces. The rate of variation along any direction is, then, represented by the component of the vector along this direction. The vector may be defined with the positive or with the negative sign, according as the rate of variation be interpreted as the rate of increase, or as the rate of decrease of the scalar quantity. The vector representing the rate of decrease is generally called the *gradient* and, more specially, potential gradient, pressure gradient, temperature gradient, etc., in accordance with the nature of the scalar quantity. To have a name for the vector representing the rate of increase of the scalar, we shall call it the *ascendant*. Generally the gradient has the most perspicuous physical sense. But still in some cases the use of the ascendant is to be preferred for practical reasons.

From what precedes it will be seen that the gradient G and the ascendant A of the scalar α may be defined by the equations

$$(a) \qquad G = -\frac{d\alpha}{dn}$$

$$(b) \qquad A = \frac{d\alpha}{dn}$$

n being the normal to the equiscalar surfaces, counted positive in the direction of increasing values of α. In the same way the components G_s and A_s of these vectors along any direction s are given by the rates of decrease or of increase respectively along the direction s

$$(c) \qquad G_s = -\frac{\partial\alpha}{\partial s}$$

$$(d) \qquad A_s = \frac{\partial\alpha}{\partial s}$$

The equiscalar surfaces or the unit-sheets ? representing the field of the scalar quantity α give at the same time a complete representation of the field of the vector G or A. From what is stated above we can immediately draw these conclusions:

(1) The direction of the vectors is that of the normal to the equiscalar surfaces.

(2) If a sufficiently small unit be used, the magnitude of the vector will be represented numerically by the number of unit-sheets per unit-length of the normal; or, what comes to the same thing, by the reciprocal thickness of the unit-sheet.

(3) The component of any of the vectors in any direction s is numerically equal to the number of unit-sheets per unit-length in this direction; or, in other words, it is equal to the reciprocal length of that segment of the line s which is contained in a unit-sheet.

If, finally, we add that the gradient points in the direction of decreasing and the ascendant in the direction of increasing values of the scalar, we see that the equiscalar surfaces and the unit-sheets give a full representation of the field of the gradient or of the ascendant. If greater units be used, so that the unit-sheets have greater thickness than supposed above, perfectly corresponding theorems may be formed for the average values of the vectors or their components referred to definite lengths of the segment s.

As we can pass by a process of differentiation from the field of a scalar to the field of its gradient or its ascendant, we can, *vice versa*, return by a process of integration from one of the latter fields to the first. To show this, say for the gradient, we can multiply equation (c) by the line element ds and integrate along the curve s from a point 0 to a point 1. This gives

$$(e) \qquad \int_0^1 G_s ds = -\int_0^1 \frac{\partial \alpha}{\partial s} ds = -\int_0^1 d\alpha = \alpha_0 - \alpha_1$$

α_0 and α_1 being the values of α at the points 0 and 1, respectively. The first member of this equation is the line-integral of the component of the vector G tangential to the curve s. As we shall usually have to take line-integrals only of the tangential vector-components, we may denote an integral of this nature simply as *the line-integral of the vector*. This line-integral of the gradient gives us the means of reconstructing the field of the scalar. For, knowing the field of the gradient and the value of the scalar quantity in one point of space, we can find the value of the scalar in any point by integrating the gradient along any curve leading from the first point to the second.

It will be useful, finally, to express in terms of the gradient the ratio (a), section 16, from which we derived originally the definition of this vector. Taking in the integral (e) the mean value $G_{s,m}$ of the tangential component of the gradient outside the integral sign, the integration can be performed, and gives the length s of the curve. Dividing by this s, we get

$$(f) \qquad G_{s,m} = \frac{\alpha_0 - \alpha_1}{s}$$

Thus, the mean value of the component of the gradient tangential to any curve s is equal to the difference of the values which the corresponding scalar quantity has in the end-points of s, divided by the length of s.

18. The Gravitational Field of Force. — The relation of gravity potential to the acceleration of gravity is that of a scalar quantity to its gradient. The gravity potential will therefore serve to give us not only a rational system of coördinates; it will also give us a full representation of the gravitational field of force.

To sum up the facts relating to this representation, we see that formula (a), section 11, which defined the gravity potential in terms of the acceleration of gravity, has exactly the form of the formula (e), section 17, which defines a scalar quantity in terms of the gradient. *Vice versa*, the acceleration of gravity can be represented by the rate of decrease of the gravity potential along the normal to the equipotential surfaces, *i. e.*, along the plumb-line z,

$$g = -\frac{d\phi}{dz} \tag{a}$$

In the same way the component of the acceleration of gravity along any direction s is given by the rate of decrease of the gravity potential along this direction

$$g_s = -\frac{\partial\phi}{\partial s} \tag{b}$$

Another form of expressing the facts contained in the formulæ (a) and (b) is the statement that the equipotential surfaces and the unit-sheets give a full representation of the gravitational field of force. First, the acceleration of gravity is directed along the normal to these surfaces, *i. e.*, along the plumb-line. Second, it is numerically equal to the reciprocal thickness of the unit-sheets. Thirdly, its component along any line s is numerically equal to the reciprocal length of the segment of this line which is contained in the unit-sheet. As we see, these statements are simply the reversal of the statements by which we defined originally our unit of gravity potential, the dynamic decimeter, in terms of the acceleration of gravity. Corresponding to (f), section 17, we get finally

$$g_{s,m} = \frac{\phi_0 - \phi_1}{s} \tag{c}$$

where $g_{s,m}$ is the average value of the component which the acceleration of gravity has tangentially to the curves, while ϕ_0 and ϕ_1 are the values of the gravity potential in the end-points of the curve.

The gravitational field of force is a field in space and thus a 3-dimensional field. The components of its field intensity tangentially to a surface will represent a 2-dimensional field of force. These 2-dimensional fields, which will be of great importance for us, are represented fully by a map giving the dynamical topography of the surface.

We can exemplify this by reference to our maps of dynamic topography. Formula (c) can be used to find the average value of the acceleration of gravity along any part of a curve contained in the surface represented by the map. Any such curve will be divided into segments s by the level curves of the maps, and to each such segment the formula (c) can be brought into application.

In this case it will, however, be inconvenient to measure the length of the curve in meters ; but the m.t.s. value of $g_{s,m}$ will come out correctly also when the length of the curve s is measured in kilometers and the difference of potential $\phi_0 - \phi_1$ for each 100 dynamic meters of height is taken for unity. The average value of the component of the accelération of gravity along a segment limited by two successive level curves on our map of the world will therefore be

$$(d) \qquad g_{s,m} = \frac{10}{s}$$

If the map represented the topography of a perfectly hard and smooth surface, and if the curve s be the path of a particle forced to slide on it with any initial velocity under the sole action of gravity, this formula would serve to find the average acceleration of the particle in any part of its path.

CHAPTER III.

SPECIFIC VOLUME AND DENSITY OF ATMOSPHERIC AIR AND SEA-WATER.

19. **Distribution of Mass.** — Every motion consists in the displacement of masses. Only in certain definite distributions of mass will the causes of motion cease to act. As introductory to the investigation of the conditions of equilibrium and motion of the atmosphere and the hydrosphere, we will therefore have to consider the distribution of mass in general, and the methods of finding and representing it.

For the numerical representation of the distribution of mass in a continuous medium, such as air or water, we have, as mentioned already (sec. 3), two methods: We can specify the volume occupied by the different unit-masses, or we can specify the masses present in the different units of volume. In the first case we register the *specific volume*, in the second the *density* of the medium. The number representing one of these quantities is the reciprocal of that representing the other. These quantities are completely equivalent in representing the distribution of mass. But which to choose is a question of importance, as it leads to one or the other of two different methods already referred to (section 3) of formulating the conditions of equilibrium and motion of the medium. Theoretically neither of these methods has any advantage over the other, but they supplement each other in a convenient manner. We shall therefore develop both side by side.

Specific volume or density of atmospheric air or of sea-water are as a rule not observed directly. Generally they will have to be calculated from other quantities, more easily observed with sufficient precision. These quantities are pressure, temperature, and humidity in the case of the atmosphere; depth, temperature, and salinity in the case of the sea.

20. **Equation of State of the Atmospheric Air.** — To calculate the specific volume of dry atmospheric air, we use the equation of Boyle-Gay-Lussac. As the letter t will be reserved for the most fundamental of all independent variables, time, and the letter v for the most important vector quantity related to the motion of the atmosphere or the sea, velocity, we shall represent the temperature according to the common centigrade scale by τ, and the corresponding temperature referred to the absolute zero by ϑ, thus

$$(a) \qquad \vartheta = \tau + 273$$

while we shall denote the specific volume by α. The equation connecting pressure, specific volume, and temperature of a perfect gas is then

$$(b) \qquad p\alpha = R\vartheta$$

The gas constant R of dry atmospheric air is 2153 when the pressure is expressed in millimeters of mercury, and 2870 when it is measured in m-bars.

If the air be more or less moist, an equation of the form (b) can still be used, only with a new value R' of the gas constant

$$p\alpha = R'\vartheta$$

If the unit-mass of moist air contains m parts of water-vapor, and consequently $1 - m$ parts of dry air, the laws for the mixtures of gases give for the constant R' the expression

$$R' = (1 - m)R + mR''$$

R being the gas constant of dry atmospheric air, and R'' that of water-vapor. Now the constants of two gases are in proportion to their specific volumes. For the case of water-vapor and dry atmospheric air this proportion has the well-known approximate value 8/5, which will give sufficient accuracy for our purposes. Consequently $R' = R(1 + 0.6m)$.

21. Virtual Temperature. — The gas constant R' of moist air is thus a variable quantity, changing with the variable mass m of water present. The second member of the equation for moist air will therefore contain two variable quantities, R' and ϑ. The first of these will be, however, variable only between narrow limits. We can therefore advantageously use a well-known artifice, considering the slightly variable quantity R' constant and equal to R, while we for compensation add a small correction to the widely variable quantity ϑ. Thus, introducing

$$(a) \qquad \vartheta_r = \vartheta(1 + 0.6m)$$

we can write the equation for moist air in the form

$$(b) \qquad p\alpha = R\vartheta_r$$

R being the gas constant for dry air, and ϑ_r a somewhat increased temperature, namely, the temperature which dry air ought to have in order to get the same specific volume as the assumed mass of moist air of temperature ϑ. With Guldberg and Mohn, who first introduced this useful auxiliary quantity, we shall call ϑ_r the virtual temperature.

As may prove most convenient, we shall count this virtual temperature either from the freezing-point of water or from the absolute zero, and denote it by τ_r and ϑ_r respectively, thus

$$(c) \qquad \tau_r = \tau + \varepsilon_r \qquad \vartheta_r = \vartheta + \varepsilon_r$$

where, according to (a), the correction ε_r has the value

$$d) \qquad \varepsilon_r = 0.6m\vartheta$$

m being the mass of water-vapor per unit-mass of atmospheric air.

22. Tables for the Virtual-Temperature Correction. — The formula (d) above can be used to calculate the virtual temperature when the mass m of water-vapor per unit-mass of the air is known. But this quantity m is never observed directly. What is generally determined is the relative humidity, *i. e.*, the proportion of the quantity of water-vapor really present to that which would be present if, at the same pressure and temperature, the air were saturated with moisture.

If f be the pressure of the saturated vapor at the temperature considered, p the total pressure, and r the relative humidity, rf will represent the pressure of the vapor and $p - rf$ that of the dry air in the mixture. The partial pressures of each constituent in a mixture are in the same ratio as their volumes were before mixing. The specific volumes of air and water-vapor being in the ratio 5: 8 the volumes of $1 - m$ parts of air and m parts of water-vapor are in the ratio $5(1 - m) : 8m$ and consequently

$$(p - rf) : rf = 5(1 - m) : 8m$$

If the value of m found from this equation be substituted in the equation (d), section 21, we find the temperature correction

$$\varepsilon_r = \frac{3rf}{8p - 3rf}\vartheta \qquad (a)$$

It appears as a function of four variable quantities. But only three of these — pressure p, temperature ϑ, and humidity r—are independent, while the vapor pressure f is a known function of temperature.

To conveniently calculate ε_r we can first put $r = 1$, and calculate the temperature correction

$$\varepsilon_{100} = \frac{3f}{8p - 3f}\vartheta \qquad (b)$$

corresponding to 100 per cent humidity. This ε_{100} being calculated, the value of ε corresponding to any relative humidity is easily found. Division of (a) by (b) gives

$$\frac{\varepsilon_r}{\varepsilon_{100}} = \frac{8 - 3\dfrac{f}{p}}{8 - 3\dfrac{rf}{p}} \cdot r$$

where the temperature ϑ has dropped out. Numerical calculation easily shows that, even under unfavorable circumstances, the coefficient of r can be set equal to unity without producing any error in the tenths of the centigrade degree. Thus the equation is reduced to

$$\varepsilon_r = r\varepsilon_{100} \qquad (c)$$

As an immediate result of equations (b) and (c) we get the following rule for the calculation of the virtual-temperature correction ε_r for air of r per cent relative humidity: First calculate the correction ε_{100} for saturated air of the given temperature and pressure; then r per cent. of ε_{100} gives the required correction ε_r.

The virtual-temperature correction ε_{100} for saturated air is given in table 7 M of the Meteorological Tables as function of the pressure in m-bars and degrees of the centigrade thermometer.* The tabulated numbers are, as seen, all very small, and r per cent of any of them is easily found with sufficient accuracy.

When the pressure is given in millimeters of mercury, table 11 A of the Appendix gives the virtual-temperature correction for saturated air, expressed as above in degrees of the centigrade thermometer.

When the observations are made in inches of mercury and degrees of the Fahrenheit thermometer, it will be usually found most convenient, also, to calculate the virtual-temperature correction in Fahrenheit degrees and to perform at a later stage the transition to the other system of units. The virtual-temperature correction for saturated air, expressed in Fahrenheit degrees, and with the pressure in inches of mercury as argument, is given in table 12 A of the Appendix.

23. Virtual-Temperature Diagrams. — The calculation of the virtual-temperature correction for every special observation of a long series will be found to be a great waste of time. In such cases it is easy to find a curve, *the virtual-temperature diagram*, giving the relation between virtual temperature and pressure. This diagram is obtained by the following procedure: On coordinate paper the pressures are measured out along the axis of the ordinates and the temperatures along the axis of the abscissæ. In this plane the observed values of temperature and pressure give a number of points by use of which a curve representing the relation of pressure and temperature is drawn. This curve immediately gives the temperatures corresponding to the pressures 1100, 1050, 1000, 950, ... m-bars, serving as argument in table 7 M. The corresponding virtual-temperature corrections for saturated air are taken from this table with great ease, no interpolation respecting the pressure now being required. By means of these corrections a second curve is drawn, the curve of virtual temperature for saturated air. The curve for the virtual temperature corresponding to the observed relative humidities will run between these curves at a horizontal distance from the curve of real temperature, which is r per cent of the horizontal distance between the two curves. This final curve is then easily drawn by estimation, in accordance with the observed relative humidities.

The method is easily understood by inspection of the diagrams accompanying the examples worked out in Chapter VI. In each of them the curve to the left is that of real temperatures, as immediately given by the observed temperatures and pressures. The curve to the right is that of virtual temperature for saturated air, as obtained by the use of table 7 M as described above. The line between the two others is the required curve of virtual temperatures, as drawn by means of the observed relative humidities.

The observations being made in millimeters of mercury and centigrade degrees, or in inches of mercury and Fahrenheit degrees, the virtual-temperature curve will be obtained in exactly the same way, table 11 A or 12 A of the Appendix being used to obtain the curve, as shown in the examples of Chapter VI.

*The values of the vapor-tension f used for calculating table 7 M have been taken from Broch's well-known table (Travaux et Mémoires du Bureau International des Poids et des Mesures, T. 1, Paris, 1881) for temperatures above zero, and from Juhlin's table (Bihang till k. svenska Vetenskapsakademiens Handlingar, T. 17, Afdelning I, Stockholm, 1891), for temperatures below zero.

24. Virtual Temperature as a Function of the Height. — In some cases the height will appear instead of the pressure as one of the observed quantities. It will then be convenient to be able to calculate the virtual-temperature correction as a function not of pressure but of height. For this we must first know the average pressure for the heights to be used as argument in the table. Using the international kite and balloon ascents performed in Europe for the years 1901 to 1903, we have found the values of the average pressure in given dynamic heights *above the 1000 m-bar surface* (table A).

TABLE A.—*Average Pressures in Given Dynamic Heights above the 1000 m-bar Surface, Calculated from the International Kite and Balloon Ascents in Europe for the Period 1901–1903.*

Height (dynamic meters).	Pressure (m-bars).	Height (dynamic meters).	Pressure (m-bars).	Height (dynamic meters).	Pressure (m-bars).
3000	685	6500	428	10000	256
2500	731	6000	459	9500	276
2000	779	5500	492	9000	298
1500	830	5000	526	8500	321
1000	883	4500	563	8000	346
500	940	4000	600	7500	372
0	1000	3500	642	7000	399

Table A would give the average pressures in the corresponding heights above *sea-level*, if the pressure at sea-level was 1000 m-bars. This pressure being about 760 mm. of mercury, or 1013 m-bars, all pressures in table A would have to be increased by about 1.3 per cent in order to give the average pressures in the corresponding heights above sea-level. But this difference is quite insignificant for our present purpose.

By means of these values of the average pressure in the standard heights, table 8 M has been calculated from table 7 M. The virtual temperature varying very slowly with the pressure, even a great deviation of the actual pressure from the supposed average value will have no influence on the correctness of the results obtained from table 7 M.

The use of table 8 M is perfectly analogous to that of table 7 M. An example of a virtual-temperature diagram, drawn with the height as the independent variable is also given in Chapter VI. The heights are given originally in common meters. Before using table 8 M these are first to be transformed, by tables 3 M and 4 M, to dynamic meters.

The height being given originally in feet and the temperature in Fahrenheit degrees, tables 2 A and 3 A of the Appendix are first used to transform the heights from feet to dynamic meters, and afterwards table 13 A of the Appendix to draw the virtual-temperature diagram in Fahrenheit degrees.

25. Specific Volume and Density of the Air. — The value of the virtual temperature being found, it is easy to calculate the specific volume or the density by the equation of state, which gives

$$(a) \qquad \alpha = R\frac{\vartheta_r}{p}$$

$$(b) \qquad \rho = \frac{p}{R\vartheta_r}$$

Two variables ϑ_r and p appearing on the right side of each of these equations, and each of the variables having a wide range of variation, the complete tabulation of α and ρ would be very laborious and lead to very bulky tables. We shall therefore use the equations (*a*) or (*b*) for eliminating α or ρ of our equations. Afterwards we shall give an indirect way to obtain the geometric representation of the fields of specific volume or of density in the atmosphere. As an aid for this, table 14 M of Meteorological Tables, giving the value of the specific volume for the standard pressures, will be found useful.

26. Investigations of the Physical Properties of Sea-Water. — The physical properties of sea-water have been subject to elaborate investigations in connection with the international exploration of the northern European waters.*

The specific volume of sea-water and its reciprocal value, the density, depend upon three variables — pressure, temperature, and salinity. Generally the salinity is not determined directly, but deduced from the content of chlorine found by titration, s denoting the salinity and Cl the quantity of chlorine, both expressed in per milles ($^0/_{00}$) of weight. s and Cl are, according to Martin Knudsen, connected by the equation

$$(a) \qquad s = 0.030 + 1.8050\ \text{Cl}$$

By this equation, which is tabulated in Martin Knudsen's tables, we can pass from the independent variable Cl used by Martin Knudsen to the independent variable s, which we shall use consistently.

To express the results obtained for the specific volume or the density of sea-water, we shall introduce the following notations: $\alpha_{s\tau p}$ means the specific volume and $\rho_{s\tau p}$ the density of sea-water of salinity $s\ ^0/_{00}$, temperature $\tau°$ C., and sea-pressure of p decibars. By sea-pressure we then mean the total pressure diminished by the pressure exerted by the atmosphere against the surface of the sea. The decibar is employed as a practical unit instead of the unit centibar belonging to the m.t.s. system, because the pressure increases approximately by 1 decibar for every meter increase of depth.

Instead of writing the whole number representing a value of the density ρ, say the number 1.02674, practical hydrographers usually write the four last figures 26.74. This quantity being denoted by σ, we have thus

$$\sigma_{s\tau p} = (\rho_{s\tau p} - 1) \cdot 1000$$

* MARTIN KNUDSEN: Berichte über die Konstantenbestimmungen zur Aufstellung der hydrographischen Tabellen von Carl Forch, Martin Knudsen, und S. P. L. Sörensen. Mémoires de l'Academie Royale des Sciences de Danemark. Copenhague, 1902.

MARTIN KNUDSEN: Berechnung der hydrographischen Tabellen und Diskussion der Ergebnisse. Wissenschaftliche Meeresuntersuchungen herausgegeben von der Komission zur Untersuchung der deutschen Meere in Kiel, Band 2, 1903.

MARTIN KNUDSEN: Hydrographical Tables according to the measurings of Carl Forch, J. P. Jacobsen, Martin Knudsen, and S. P. L. Sörensen. Copenhagen and London, 1901.

V. WALFRID EKMAN: Die Zusammendrückbarkeit des Meerwassers, etc. Conseil permanent International pour L'Exploration de la Mer. Publication de Circonstance N° 43. Copenhague, 1908.

By measurements under atmospheric pressure on different samples of sea-water of different salinities at a series of different temperatures, Martin Knudsen has determined the quantity $\sigma_{s\tau 0}$. The result is contained in the following formulæ:

For the case of $\tau = 0$ the quantity σ_{s00} is determined as function of the quantity of chlorine by the equation

$$(b) \qquad \sigma_{s00} = -0.069 + 1.4708\,\mathrm{Cl} - 0.001570\,\mathrm{Cl}^2 + 0.0000398\,\mathrm{Cl}^3$$

Then the quantity $\sigma_{s\tau 0}$ is determined as a function of the temperature τ and the quality σ_{s00} found from (b) by the equation

$$(c) \qquad \sigma_{s\tau 0} = \Sigma_\tau + (\sigma_{s00} + 0.1324)[1 - A_\tau + B_\tau(\sigma_{s00} - 0.1324)]$$

the quantities Σ_τ, A_τ and B_τ being the following functions of temperature:

$$\Sigma_\tau = -\frac{(\tau - 3.98)^2}{503.570} \cdot \frac{\tau + 283}{\tau + 67.26}$$

$$(c') \qquad A_\tau = \tau(4.7867 - 0.098185\tau + 0.0010843\tau^2) \cdot 10^{-3}$$

$$B_\tau = \tau(18.030 - 0.8164\tau + 0.01667\tau^2) \cdot 10^{-6}$$

The quantities σ_{s00} and $\sigma_{s\tau 0}$ determined by these formulæ are tabulated in Martin Knudsen's tables. From the tabulated numbers we pass to the corresponding values $\rho_{s\tau 0}$ of the density by the formula

$$\rho_{s\tau 0} = 1 + \frac{\sigma_{s\tau 0}}{1000}$$

and from these we can pass by an inversion table (table 23 H) to the corresponding values of the specific volume. The equations (b) and (a) or the corresponding tables in Knudsen's collection allow us to bring in s as the independent variable.

V. Walfrid Ekman has determined the influence of the pressure upon the volume of sea-water of different salinities and at different temperatures. By these measurements the pressures were calculated from the compression of distilled water of 0° C., the measurements of Amagat* being, after an independent control of their reliability, used as a base for this calculation. Mr. Ekman has kindly furnished us with the following formula computed from his own measurements in connection with those of Amagat:

$$\begin{aligned}
\alpha_{s\tau p} = \alpha_{s\tau 0} - p\alpha_{s\tau 0} \cdot 10^{-9} \Bigg\{ & \frac{4886}{1 + 0.0000183p} - \Big[227 + 28.33\tau - 0.551\tau^2 + 0.004\tau^3\Big] \\
& + p \cdot 10^{-4}\Big[105.5 + 9.50\tau - 0.158\tau^2\Big] - 1.5p^2\tau \cdot 10^{-8} \\
(d) \qquad & - \frac{\sigma_{s00} - 28}{10}\Big[147.3 - 2.72\tau + 0.04\tau^2 - p \cdot 10^{-4}(32.4 - 0.87\tau + 0.002\tau^2)\Big] \\
& + \left(\frac{\sigma_{s00} - 28}{10}\right)^2 \Big[4.5 + 0.1\tau - p \cdot 10^{-4}(1.8 - 0.06\tau)\Big] \Bigg\}
\end{aligned}$$

* AMAGAT: Mémoires sur l'élasticité et le dilatation des fluides jusqu'aux très hautes pressions. Annales de Chimie et de Physique, t. 29, 1893, p. 544.

the quantities $\alpha_{s\tau 0}$ and σ_{s00} being calculated as explained above from the formulæ or the tables of Martin Knudsen. Mr. Ekman's discussion of his results shows that the specific volume calculated by the formula will not probably contain greater error than 0.00001 for the pressure of 1000 d-bars, and proportionally 0.0001 for 10,000 d-bars. Still more important for the oceanic dynamics is the following conclusion from Mr. Ekman's discussion: Differences in the specific volume of two samples of sea-water taken from the same depth, which have been calculated by this formula, will be perfectly reliable in the fifth decimal in all cases met with in the sea.

27. Tables for the Specific Volume of Sea-Water. — We shall never use the preceding formulæ directly in investigations in statics or dynamics of the sea. They will only serve for the construction of tables giving the specific volume or the density of sea-water for all necessary values of the independent variables. This tabulation, however, contains difficulties. The greatest depth hitherto sounded in the sea being 9636 meters, the sea-pressure can vary from 0 to about 10,000 d-bars. The temperature can vary from -2 to about 30° C. and the salinity from 0 to about 40 ‰. Modern observations being taken with a precision of about 0.01° C. and of 0.01 ‰ of salinity, the tables should not have greater intervals than 0.1° C. and 0.1 ‰ salinity. To be able to interpolate conveniently to any depth, the pressure should not be taken with greater intervals than 10 d-bars. The direct tabulation would thus involve the calculation of $320 \times 400 \times 1000 = 128{,}000{,}000$ different values of the specific volume. By printing 500 numbers on each page the tables would cover 256,000 pages. The direct tabulation must thus be given up, and we shall have to use in a more developed form the principles exemplified for the case of two variables in tables 3 M to 6 M and 3 H to 6 H, viz, to break up the quantity to be tabulated in a sum of terms, each of which is more easily subject to tabulation.

To carry this through in the present case, we can use a development analogous to that of Taylor. We first write

$$(a) \qquad \alpha_{s\tau p} = \alpha_{35,0,p} + \delta$$

Here $\alpha_{35,0,p}$ denotes the specific volume of sea-water of the constant salinity 35 ‰ and the constant temperature 0° C. under any pressure p. These special values of salinity and temperature are not very far from the average values in the deep oceans, and we shall therefore denote $\alpha_{35,0,p}$ as the *normal* specific volume of the sea-water under the pressure p. The value $\alpha_{s\tau p}$ representing the specific volume of any kind of sea-water under the same pressure p is then found from $\alpha_{35,0,p}$ by the addition of a correction δ, which we shall call the *anomaly* of the specific volume This correction will be a function of salinity, temperature, and pressure, and can be broken up in a series of terms

$$(b) \qquad \delta = \delta_s + \delta_\tau + \delta_{s\tau} + \delta_{sp} + \delta_{\tau p} + \delta_{s\tau p}$$

where the indices show the variables upon which the different terms depend.

Thus δ_s and δ_τ depend each on one variable, $\delta_{s\tau}$, δ_{sp}, $\delta_{\tau p}$ each on two, and only $\delta_{s\tau p}$ on all three variables. The main point is now to determine these terms so that the terms depending upon more than one variable become as small as possible. This is done if we give them the following values:

$$
(c) \quad
\begin{aligned}
\delta_s &= \alpha_{s,0,0} - \alpha_{35,0,0} \\
\delta_\tau &= \alpha_{35,\tau,0} - \alpha_{35,0,} \\
\delta_{s\tau} &= (\alpha_{s,\tau,0} - \alpha_{35,\tau,0}) - (\alpha_{s,0,0} - \alpha_{35,0,0}) \\
\delta_{sp} &= (\alpha_{s,0,p} - \alpha_{35,0,p}) - (\alpha_{s,0,0} - \alpha_{35,0,0}) \\
\delta_{\tau p} &= (\alpha_{35,\tau,p} - \alpha_{35,0,p}) - (\alpha_{35,\tau,0} - \alpha_{35,0,0}) \\
\delta_{s\tau p} &= [(\alpha_{s,\tau,p} - \alpha_{35,\tau,p}) - (\alpha_{s,\tau,0} - \alpha_{35,\tau,0})] - [(\alpha_{s,0,p} - \alpha_{35,0,p}) - (\alpha_{s,0,0} - \alpha_{35,0,0})]
\end{aligned}
$$

It is easily verified that the substitution of (c) and (b) in (a) makes this equation identical. Now, each of the quantities $\alpha_{35,0,p}$, δ_s, δ_τ, $\delta_{s\tau}$, δ_{sp}, $\delta_{\tau p}$, $\delta_{s\tau p}$ are easily tabulated separately. The values of $\alpha_{35,0,p}$ are found from Ekman's formula, putting $s = 35$, $\tau = 0$, and using the values $\alpha_{35,0,0} = 0.97364$ and $\sigma_{s00} = \sigma_{35,0,0} = 28.13$, calculated from Martin Knudsen's tables. The result for 1000 values of the pressure is given in table 8 H. The correction δ_s for salinity at temperature zero and the correction δ_τ for temperature at salinity 35 ‰ are both found from Martin Knudsen's formulæ or tables. The result is given in tables 9 H and 10 H for 400 values of the salinity and 320 values of the temperature, respectively.

Being differences of the second order, the quantities depending upon two variables $\delta_{s\tau}$, δ_{sp}, $\delta_{\tau p}$ are sufficiently small to be tabulated for ten times greater intervals of the independent variables, viz, for 40 values of the salinity, 32 values of the centigrade degrees, and 100 values of the pressure. $\delta_{s\tau}$ is found from Martin Knudsen's formulæ or tables. The expressions of δ_{sp} and $\delta_{\tau p}$, which are rather long in spite of the smallness of the numerical values, are formed from Ekman's formula. The results are given in tables 11 H, 12 H, and 13 H. The quantity $\delta_{s\tau}$ finally, depending upon three variables, is given by a very long expression deduced from Ekman's formula. But being a difference of the third order, it is sufficiently small to be tabulated for still greater intervals of the independent variables. The result is given in table 14 H as a system of 17 small tables, each corresponding to a certain salinity, while within each table temperature and pressure figure as the independent variables, the intervals of pressure being 1000 decibars.

The tabulation of the specific volume of sea-water has thus been accomplished by seven small tables covering 10 pages. This system of small tables is equivalent to the one table of 256,000 pages on account of the possible permutations in the sum (a) and (b) of the values taken from the different small tables.

The three last tables, involving the pressure as one of the independent variables, are not calculated completely, those combinations of the variables being left out which are not found in the sea. The general distribution of temperature and salinity in the sea is readily seen from the charts worked out by Dr. G. Schott.* The greatest variation of temperature is found on the surface of the sea, extending from the evident lower limit, the freezing-point of sea-water, between −1 and

* Wissenschaftliche Ergebnisse der deutschen Tiefsee-Expedition auf dem Dampfer "Valdivia," 1898–99. T. I. V. Oceanographie und maritime Meteorologie von Dr. G. Schott. Jena, 1902.

$-2°$ C., to the maximum values in the tropics, hardly anywhere exceeding 30° C. in the open sea. As we proceed downwards, the freezing-point of sea-water is retained as the lower limit, while the upper gradually decreases, but at a very different rate, in the open ocean and in the more or less closed seas. The temperature in the open sea will hardly anywhere exceed 10° C. at a depth of 1000 meters, and in still greater depths it will be found between the limits $+2°$ and $-2°$. In more or less closed seas the temperature may be much higher in corresponding depths. Thus, in the Red Sea there is a temperature of 21.5° C. at the depth of 2100 meters, in the Sulu Sea (between Borneo and Philippines) 10.2° C. at the depth of 4300 meters, and in the Mediterranean 13.9° C. at the depth of 4400 meters. The salinities also have their greatest range of variation at the surface, namely, from zero at the mouths of great rivers to 39 ‰ in the Mediterranean and 40.4 ‰ in the Red Sea. As we proceed downwards the low salinities rapidly disappear, but at a different rate and converging towards different limits in the open ocean and in the closed seas. In the open ocean the salinity rapidly converges towards the almost constant salinity of about 35 ‰. The Mediterranean has the higher salinity of about 39 ‰ and the Red Sea of about 40 ‰ at the bottom, while the Baltic has the low salinity of about 10 ‰ in its greatest depths, somewhat exceeding 400 meters. Lower salinity than that of the Baltic and higher than that of the Mediterranean or the Red Sea will hardly be found anywhere in corresponding depths. These general data have determined the limits of the three last tables.

The method of tabulation which we have used, while leading to very small and convenient tables, has a defect which must be mentioned. The result is found as the sum of seven terms. Each term may have an error of five units in the sixth decimal of the specific volume, or, on account of the double interpolations in the tables containing more than one variable, even somewhat more. Thus in exceptional cases errors may occur exceeding 3.5 in the fifth decimal, corresponding to an error of about 0.03 ‰ in the salinity and thus exceeding somewhat the errors of careful observations, which may be carried to about 0.01 ‰. This error may perhaps be of importance for the investigation of the conditions of equilibrium or motion in very homogeneous parts of the sea. It might have been avoided if we had calculated all our tables with one decimal more. But this would have made the use of the tables in the most common cases much less convenient. When more accurate tables are required it will probably be the best plan to construct three different sets of tables for three different types of sea-water: the oceanic type, the Mediterranean type, and the Baltic type. For the oceanic type the tables should be extended to all pressures, but for all greater pressures the temperature and salinity could be contained between very narrow limits, approaching 0° C. for temperature and 35 ‰ for salinity. For the Mediterranean type the tables should extend to pressures somewhat exceeding 4000 d-bars, while the temperature in greater depths has about 10 for its lower and 20 for its higher limit. The salinity in greater depths should have 35 for its lower and a little more than 40 for its higher limit. These tables might be used in closed tropic or subtropic seas, as the Mediterranean, the Red Sea, and the Sulu Sea. For the Baltic type the tables could be limited to the pressure of 500 d-bars, while the temperature must have all values from the freezing-point of water to that of the tropic sea, and the salinity all values from that of fresh water to that of oceanic water. These tables could be used for the Baltic, the St. Lawrence Bay, the mouths of great rivers, the shallow waters along the Arctic coasts, etc. The variations of the independent variables being limited in this way, the tables could be constructed for so small intervals of the independent variables that convenient differences would be obtained even if the specific volume was calculated with six decimals.

28. Control of the Accuracy of the Tables. — A question of the highest importance is that of the absolute reliability of the tables. The test is given in as direct form as possible by the annexed tables B and C. The first table contains the volumes of the samples of sea-water under atmospheric pressure examined by Martin Knudsen. These volumes are not given in absolute measure in Martin Knudsen's paper, but by the additional data they have been reduced from the relative measure employed in the course of the experiments to the values in absolute units given in table B. In the same manner table C contains the specific volumes of the samples of sea-water under different pressures examined by Ekman. These

TABLE B.—*Fundamental Values* $a_{s,\tau,0}$ *of Specific Volume of Sea-Water under Atmospheric Pressure.*

I. $s = 3.20$ ‰		II. $s = 8.35$ ‰		III. $s = 10.56$ ‰		IV. $s = 14.634$ ‰		V. $s = 18.818$ ‰	
τ	a	τ	a	τ	a	τ	a	τ	a
0.000	0.9974973	−0.296	0.9933732	−0.298	0.9916115	−0.164	0.9883820	−0.783	0.9851090
0.051	.9974948	0.000	.9933633	0.000	.9916041	0.000	.9883806	0.000	.9851091
5.085	.9974391	4.885	.9933790	5.000	.9916623	4.728	.9884897	4.986	.9852914
9.485	.9977004	9.936	.9937553	9.697	.9920317	9.981	.9889588	9.780	.9857721
14.895	.9983671	15.004	.9944515	14.907	.9927631	14.868	.9896916	14.793	.9865494
19.450	.9919899	20.212	.9954682	19.824	.9937177	20.118	.9907537	20.417	.9877318
25.319	1.0058375	24.754	.9965707	25.237	.9950581	24.661	.9918878	24.828	.9888584
30.088	1.0195311	29.899	.9980556	30.211	.9965190	30.607	.9936518	30.486	.9905582

VI. $s = 23.204$ ‰		VII. $s = 25.83$ ‰		VIII. $s = 28.956$ ‰		IX. $s = 35.37$ ‰		X. $s = 33.93$ ‰	
τ	a	τ	a	τ	a	τ	a	τ	a
−0.203	0.9816800	−0.262	0.9796612	−1.720	0.9772362	−2.668	0.9745978	−2.215	0.9734010
0.000	.9816837	0.000	.9796680	0.000	.9772745	−0.245	.9746578	0.000	.9734804
4.940	.9819294	5.310	.9799810	0.079	.9772773	0.000	.9746684	0.182	.9734897
9.737	.9824525	10.124	.9805596	5.016	.9776046	4.917	.9750309	4.777	.9738523
14.946	.9833107	14.569	.9813135	9.747	.9781858	10.036	.9757038	9.767	.9745102
19.879	.9843669	20.189	.9825299	14.518	.9790097	14.843	.9765761	14.784	.9754332
25.054	.9857070	25.068	.9838162	20.044	.9802288	19.941	.9777241	19.722	.9765432
30.834	.9874744	30.303	.9854087	24.988	.9815413	24.880	.9790510	24.635	.9778540
				30.683	.9832852	30.435	.9807592	29.810	.9794444

XI. $s = 34.93$ ‰		XII. $s = 35.05$ ‰		XIII. $s = 35.37$ ‰		XIV. $s = 39.35$ ‰	
τ	a	τ	a	τ	a	τ	a
−2.734	0.9725994	−2.488	0.9725204	−0.387	0.9723336	0.000	0.9693332
−0.024	.9726923	−0.028	.9726107	0.000	.9723540	0.266	.9693603
0.000	.9726936	0.000	.9726122	5.245	.9727937	5.066	.9699236
5.104	.9731106	5.096	.9730290	10.313	.9735072	9.670	.9706203
9.768	.9737433	10.225	.9737397	14.523	.9742959	14.868	.9716045
15.273	.9747742	15.287	.9746942	19.770	.9754929	20.366	.9728974
19.924	.9758557	19.917	.9757658	24.531	.9767758	24.798	.9741282
25.508	.9773670	25.645	.9773348	29.840	.9784143	30.327	.9758659
30.024	.9787782						

values do not appear explicitly in Mr. Ekman's paper either, but he has kindly calculated them for us as directly as possible from the compressibilities measured, using the necessary additional data from Amagat and Martin Knudsen.

TABLE C. — *Fundamental Values of $\alpha_{s,\tau,p}$ of the Specific Volume of Sea-Water under Different Pressures.*

Sea-pressure (d-bars).	$t=0$	$t=4.97°$	$t=9.97°$	$t=14.96°$	$t=19.96°$
I. Sea-water of salinity 31.130 ‰ (Cl = 17.230).					
0	0.975600	0.975953	0.976598	0.977493	0.978617
2000	.966722	.967312	.968147	.969189	.970425
4000	.958396	.959192	.960192	.961364	.962701
6000	.950561	.951535	.952679	.953966	.955394
II. Sea-water of salinity 38.525 (Cl. = 21.327).					
0	0.969960	0.970408	0.971130	0.972087	0.973257
2000	.961296	.961964	.962864	.963957	.965232
4000	.953166	.954028	.955079	.956293	.957659
6000	.945510	.946537	.947723	.949042	.950493

To control the accuracy of tables 8 H to 14 H, we can calculate from the tables the specific volume for those values of salinity, temperature, and pressure appearing in tables B and C. It will be seen, then, that all values are found with differences only in exceptional cases exceeding 1 or 2 units in the fifth decimal, these discrepancies being easily explained by the possible errors obtained as developed above by the addition of a sum of terms containing small errors. The test thus shows that no error of any importance can have been made, either by the calculation of the formulæ of interpolation from the observations or in the calculation of the tables from these formulæ.

29. Tables of the Density of Sea-Water. — The density being the reciprocal of the specific volume, tables for the density are easily deduced from those for the specific volume, provided that the same independent variables be retained. For reasons which will become evident later it will be convenient, however, to have the density registered as a function of the dynamical depth instead of as a function of the pressure. Of course there exists no intrinsic physical relation between depth and density. But some measures of precaution being taken, this method of tabulation can be used, thanks to the close relation between depth and pressure in the sea.

Using the index D to denote that the dynamic depth appears as an independent variable, we write, corresponding to section 27 (*a*) and section 27 (*b*):

(*a*) $$\rho_{s\tau D} = \rho_{35,0,D} + \varepsilon$$

(*b*) $$\varepsilon = \varepsilon_s + \varepsilon_\tau + \varepsilon_{s\tau} + \varepsilon_{sD} + \varepsilon_{\tau D} + \varepsilon_{s\tau D}$$

For the right-hand terms of the last equation we can write expressions corresponding to section 27 (*c*), ε being substituted for δ, ρ for α, and D for p.

The three terms corresponding to the case $p = D = 0$, viz, ε_s, ε_τ, $\varepsilon_{s\tau}$, are calculated directly from Martin Knudsen's formulæ or tables. Thus the only difficulty concerns the terms $\rho_{35,0,D}$, ε_{sD}, $\varepsilon_{\tau D}$, $\varepsilon_{s\tau D}$ containing the depth as an independent variable. To perform the transition from the variable p to the variable D, we must know with a sufficient approximation the relation between pressure and depth in the sea. For the case of normal sea-water of 35 $^0/_{00}$ and 0° C. this relation is easily determined by a method explained in the next chapter. The result is contained in table 7 H, which gives the dynamic depth of any given pressure for intervals of 10 d-bars. By interpolation in this table we can determine the pressure in any depth expressed by any integer number of dynamic meters. The result of these interpolations is given in table 15 H, which contains the pressures in depths expressed by any integer number of dynamic meters, registered for intervals of 10 dynamic meters. We can now by table 8 determine the specific volume of the sea-water corresponding to the pressures registered in table 15. These will be the specific volumes of normal sea-water for the depths figuring as arguments in table 15 H. Passing to the reciprocal values by use of the inversion-table 23 H, we get table 16 H, giving the normal density $\rho_{35,0,D}$ of sea-water in 1000 different dynamic depths.

For the calculation of the small quantities ε from the corresponding quantities δ, we can make use of simple approximation rules. Differentiating the equation connecting the density ρ with the corresponding specific volume α, we get $d\rho = -d\alpha/\alpha^2$. Applying this for the transition from the correction δ of any value of the specific volume to the corresponding correction ε of the density, we get

$$(a) \qquad \varepsilon = -\frac{\delta}{\alpha^2}$$

Using, as above, table 15 H to find the pressure corresponding to the given dynamic depth, and this pressure to find δ-values from tables 12 H or 13 H, the corresponding values of ε are calculated by equation (a). In this way the main tables 20 H and 21 H are calculated. These would give exactly the required corrections ε_{sD} and ε_{τ}, if the pressure in the depth considered had exactly the normal value given by table 15 H. But if the water above the level considered has other than the normal salinity 35 $^0/_{00}$ or other than the normal temperature 0° C., the pressure will be slightly different, and this will have a slight influence upon the density of the sea-water in the level considered. The anomaly of pressure in question can easily be estimated with sufficient approximation from the average salinity and the average temperature of the water above the level considered, and thus the corresponding correction of the density as the consequence of the compression found. These corrections are given in the small tables placed at the foot of the main tables 20 H and 21 H, having the average instead of the local values of salinity and temperature as argument.

It is seen that as α^2 never differs very much from unity, corresponding values of δ_{sp} and ε_{sD}, as well as of $\delta_{\tau p}$ and $\varepsilon_{\tau D}$, are very nearly like each other, but with opposite signs. Passing to the calculation of the term of the third order, $\varepsilon_{s\tau p}$, we can simply put $\alpha^2 = 1$, and identify the numbers expressing depths in dynamic meters with those expressing pressures in decibars. Table 22 H, giving the values of $\varepsilon_{s\tau D}$, is therefore identical with table 14 H, giving $\delta_{s\tau p}$, but with the difference that all terms appear with the sign reversed.*

30. Important Features of Specific Volume or Density of Sea-Water. — Tables 9 H and 17 H show the regular decrease of the specific volume, or the increase of density with increasing salinity. In the same way tables 8 H and 16 H show the regular decrease of the specific volume, or increase of density with increasing pressure. These tables do not show any marked peculiarities of the sea-water. But very marked peculiarities are shown by the volume-tables 10 H, 11 H, and 12 H and the corresponding density-tables 18 H, 19 H, and 20 H.

Table 10 H shows for sea-water of the salinity 35 ⁰/₀₀ a regular decrease of volume, and table 18 H the corresponding increase of density for decreasing temperature. No maximum of density is found. Table 10 H used together with 11 H, or 18 H used together with 19 H, shows that we have a minimum of specific volume or a maximum of density at 4° C. for fresh-water, at 2° for a salinity of about 9.5 ⁰/₀₀, at 0° for a salinity of about 19 ⁰/₀₀ and at —2° for a salinity of 28 to 29 ⁰/₀₀. But for the normal oceanic salinity of about 35 ⁰/₀₀, there exists no maximum of density, and we shall, in the case of equilibrium, always have warmer water above and colder below. This circumstance makes the equilibrium condition of the ocean quite different from that of fresh-water lakes.

*The first determinations of the compressibility of the sea-water were performed by P. G. Tait (*Challenger* Report, Physics and Chemistry, vol. II, 1889). The first hydrographical tables taking into account the compressibility were calculated by Sandström and Helland-Hansen (Report on Norwegian Fishery and Marine Investigations, vol. II, No. 4, Bergen, 1903). When our tables were first calculated, the only measurements performed upon sea-water were still those of Tait, which had not by far the exactitude of Ekman's. Especially, no care had been taken to determine the salinity of the samples of water experimented upon at the time when their compressibility was measured. We therefore combined Tait's measurements for sea-water with those of Amagat for distilled water, using Amagat's as absolute determinations and those of Tait only as relative comparisons of the compressibility of sea-water and fresh-water of the same temperature. In the developments, section 27 (*c*), we therefore had to subdivide every term where the pressure and the salinity entered into a main term depending upon the measurements of Amagat upon fresh-water and a correction term depending upon the corresponding measurements of Tait upon sea-water. Thanks to this method of calculation we had obtained tables which were in unexpectedly good accordance with the new tables, which we have now calculated after the manuscript of the first tables had been sent to press. The degree of accordance between the two sets of tables will be seen from the following data.

The specific volume of normal sea-water (Amagat-Tait) was smaller than that registered in our new table 8 H for the pressures from zero to about 2200 d-bars, the maximal error being 0.00007 at 800 to 900 d-bars. Then the volume was found greater from 2200 to 4400, the error varying between 0.00001 and 0.00002. For greater pressures the specific volume was always found smaller, the error increasing gradually to 0.0001 at 5700 d-bars, to 0.001 at 8500 d-bars, and to 0.002 at 10,000 d-bars. Thus the error here runs up to 1/500 of the total volume. It should be remarked, however, that these greater discrepancies only occur for the values which have been extrapolated, Tait's experiments being extended to the pressure of 4629 and Ekman's to the pressure of 6000 d-bars. For depths in the sea not exceeding 5000 meters the accordance is remarkably good, and hydrographic surroundings very seldom go to a greater depth. The degree of accordance is very well illustrated by the following fact: The normal depth of the isobaric surfaces calculated according to Tait never differs by so much as one decimeter from the corresponding depths according to Ekman (table 7 H) for the first 5000 meters. For greater depths there are gradually increasing discrepancies, the depth being found according to Tait 1 m. too small for the pressure of 8000 d-bars, 2 m. too small for the pressure of 9000 d-bars, and 3.6 m. too small for the pressure of 10,000 d-bars. But even these discrepancies are of a secondary importance, for, as will be seen later, an error in the estimation of the normal depth of the isobaric surfaces will have practically no influence upon the discussion of the state of equilibrium or motion. Of much greater importance are the much smaller corrections in tables 12 H and 13 H. The greater part of the numbers in these tables have remained unaltered. But still there is a marked difference, the numbers being found numerically too small in both tables, the discrepancies in the most extreme cases mounting to 0.00004 in the values of δ_{sp} and to 0.00007 in the values of $\delta_{\tau p}$. In spite of their smallness, these corrections are of real importance for the estimation of the conditions of equilibrium and motion in the sea.

The value of the quantity $\delta_{s\tau p}$ was underestimated, so that no tabulation was found necessary. This may have been an error due to the difficulties caused by the complicated method of calculation which had to be employed in order to eliminate so much as possible the errors due to the inaccuracies of Tait's measurements. Thus table 14 H has been calculated only by Ekman's formula.

The examples in Chapter VIII have been corrected according to the new tables, but the charts in Chapter IX were already printed. However, the changes in these charts would in most cases be almost microscopical.

Tables 12 H and 20 H show an increasing resistance of the sea-water against compression for increasing salinity, and tables 13 H and 21 H show, for the interval of the temperatures in question, a similar increased resistance against compression with increasing temperature. This dependence of the compressibility upon the temperature and the salinity is of great importance for the internal conditions of equilibrium or of motion in the sea. To consider a definite example: At a pressure of 5000 decibars, *i. e.*, at a depth of 5000 meters, water of 35 ‰ salinity and at the temperature of $-1°$ C. will have the same specific weight as water of 35.48 ‰ salinity at a temperature of $+1°$ C. But under the diminished pressure of 2000 decibars, *i. e.*, at the depth of 2000 meters, the specific volume of the first water will be 0.00015 greater than that of the second, and at a depth of 9000 meters the reverse will be the case. The extreme importance of these differences of compressibility will thus be perfectly clear.

31. Isosteric and Isopycnic Surfaces. — The value of the specific volume being known in a sufficient number of points in the atmosphere or the sea, we can represent the distribution of mass in each of these media by drawing a set of equiscalar surfaces, joining all points where the specific volume has certain constant values. We shall call these surfaces *isosteric surfaces.* If, on the other hand, the value of the density be known in a sufficient number of points, we may represent the distribution of mass by drawing surfaces of constant value of the density, or *isopycnic surfaces.*

The two fields representing the distribution of mass are closely related to each other, every isosteric surface being also an isopycnic surface, and *vice versa.* But one important difference should be emphasized: if the isosteric surfaces be drawn for unit-differences of the specific volume, the corresponding isopycnic surfaces will not have unit-differences of the density, and *vice versa.* This will be well illustrated if we consider the conditions in the atmosphere. Here the density decreases upwards, converging toward a very small limit, or perhaps toward zero. The specific volume, on the contrary, which is the reciprocal of the density, increases upwards, converging toward a very great limit, or perhaps toward infinity.

Drawing the isopycnic surfaces for unit-differences of the density (a unit of convenient magnitude being chosen), the thickness of the unit strata will increase upward, approximately in geometric series. If, on the other hand, the isosteric surfaces are drawn for unit-differences of the specific volume, the thickness of the corresponding unit-sheets will decrease, approximately in geometric series. Even in the sea there is a corresponding difference between the two systems of surfaces, only much less pronounced.

The equiscalar surfaces of the specific volume or the density being very nearly level, the gradient or the ascendant of these quantities will be directed very nearly along the plumb-line. For reasons which will appear later it will be more convenient to use the ascendants than the gradients in this case. The ascendant of the specific volume points upward, that of the density downward, forming a very small angle with the plumb-line.

CHAPTER IV.

PRINCIPLES OF HYDROSTATICS.

32. **Pressure, Isobaric Surfaces, and Gradient.**— The theory of pressure as met with under general conditions in strained elastic bodies or in moving viscous fluids is of great complexity. But in the special case of the equilibrium of any fluid, as well as in the case of the motion of a frictionless fluid, it is reduced simply to a scalar quantity. The field of a hydrostatic pressure can therefore be described according to the common principles for the description of scalar fields (section 16). Thus for the geometric representation of this field we draw surfaces of equal value of pressure, or *isobaric* surfaces. As a rule we shall draw them for unit-differences, so that they divide the space into a set of isobaric unit-sheets. To get unit-sheets of the proper thickness we are free to choose a unit-pressure of suitable magnitude.

The *pressure gradient*, or simply the *gradient* G, is given by the rate of decrease of the pressure p along the normal n to the isobaric surfaces

$$G = -\frac{dp}{dn} \tag{a}$$

and the component G_s of the gradient along any direction s is given by the rate of decrease of the pressure along this direction

$$G_s = -\frac{\partial p}{\partial s} \tag{b}$$

The isobaric surfaces and the unit-sheets, drawn for a unit of suitable magnitude, give the full representation of the field of the gradient G (section 17). The vector itself is directed along the normal to the surfaces, its numerical value being equal to the reciprocal thickness of the sheet. Its component G_s along any line is equal to the reciprocal length of that segment s of this line which is contained in the unit-sheet. In accordance with formula (f), section 17, we have finally

$$G_{s,m} = -\frac{p_1 - p_0}{s} \tag{c}$$

which gives the mean value along the curve s of the component of the gradient tangential to the curve. The mean tangential gradient is thus equal to the difference of pressure at the end-points of the curve, divided by the length of the curve.

33. **Dynamic Significance of the Pressure Gradient.** — Like every scalar quantity, pressure has a gradient. But the gradient of the pressure has at the same time a dynamical significance, making it the fundamental vector of hydrostatics and hydrodynamics.

Let us determine the elementary force component dF_s, which, as a consequence of the pressure, tends to move a volume element $d\tau$ of the fluid in the direction s. To consider the simplest case, let the volume element have the form of a straight cylinder with its axis in the direction s and with its bases normal to this direction. As the pressure in a perfect fluid acts normally to the surface, the pressure against the lateral surface can be disregarded, as giving no addition to the component of force along the axis s. We have thus only to consider the pressure against the two bases of the cylinder. Let the value of the pressure at the first base be p. At the other it will then be $p + \frac{\partial p}{\partial s} ds$, ds being the height of the cylinder. The area of each base being $d\sigma$, we find that the exterior fluid exerts the force $pd\sigma$ against the first and the oppositely directed force $-(p + \frac{\partial p}{\partial s} ds)\, d\sigma$ against the second base. From these two oppositely directed forces will therefore result the force $dF_s = -\frac{\partial p}{\partial s} ds d\sigma$. Now $dsd\sigma$ is the volume $d\tau$ of the element.. Further, $-\partial p/\partial s$ is the component G_s of the gradient in the direction s (section 33, *b*). We therefore get

$$dF_s = G_s d\tau \tag{a}$$

Thus the elementary force tending to move a volume element of the fluid in any direction is equal to the component of the gradient in this direction, multiplied by the volume of the element. Or, in other words: *The gradient represents the force per unit-volume due to the field of pressure in the fluid.*

By this we see that there is a close relation between potential gradient and pressure gradient. For both gradients represent moving forces. But there is this important difference, that the potential gradient represents the force of gravity *per unit-mass*, while the pressure gradient represents the force of pressure *per unit-volume* (section 3). To get the force of pressure per unit-mass we have to multiply the gradient by the specific volume, exactly as we get the force of gravity per unit-volume by multiplying the acceleration of gravity by the density of the body considered. Force of gravity and force of pressure, both referred to unit of mass, are therefore, respectively,

$$g \quad \text{and} \quad \alpha G \tag{b}$$

while the same two forces, both referred to unit-volume, are, respectively,

$$\rho g \quad \text{and} \quad G \tag{c}$$

The consistent use either of forces per unit-mass or of forces per unit-volume leads to mutually equivalent but formally different methods of formulating the principles and of treating the problems of hydrostatics. We shall develop both of them in parallel, as they are complements of each other from a practical point of view.

34. **Condition of Equilibrium in Terms of Forces per Unit-Mass.** — The condition of internal equilibrium of a fluid is fulfilled if the force of gravity and the

force of pressure are everywhere directed oppositely to each other, and if their amounts per unit-mass are equal,

(*a*) $$g = -\alpha G$$

Other forms for this condition are easily deduced. Remembering that the negative derivatives of ϕ and p along the direction s are equal to the components of g and G in this direction, we get

(*b*) $$\frac{\partial \phi}{\partial s} = -\alpha \frac{\partial p}{\partial s}$$

Writing equations of this form for each of the three rectangular axes x, y, z, we get the hydrostatic equations in their traditional form, referred to rectangular coördinates. For us, however, the introduction of artificial systems of coördinates, having no relation to the intrinsic geometry of our problems, will only cause complication. It will, on the contrary, be most convenient for us to have the condition of equilibrium referred as closely as possible to the natural coördinate surfaces, the level or equipotential surfaces. This is obtained if we multiply equation (*b*) by the line element ds, and use the differential formulæ

$$\frac{\partial \phi}{\partial s} ds = d\phi \qquad \frac{\partial p}{\partial s} ds = dp$$

Between the total increases $d\phi$ and dp of pressure and of potential along the line element ds, we thus get the relation

(*c*) $$d\phi = -\alpha dp$$

This equation gives in its simplest form the intrinsic relation which, in the case of equilibrium, exists between pressure, specific volume, and gravity potential.

35. Equilibrium Relation between the Fields of Potential, of Pressure, and of Specific Volume.— We have considered, independently of each other, the fields of potential, of pressure, and of mass, and the description of each field by means of its proper equiscalar surfaces and sheets. The condition of equilibrium which we have formulated gives a relation between these three fields which can be expressed as a relation between their surfaces and sheets. Expressed in this way the equilibrium relation will contain two distinct principles, the first of which is purely descriptive, dealing with the course of the surfaces, while the other is of metric nature, giving a numerical relation between the unit-sheets.

(I) *Principle of Coincidence of Surfaces.*— The gradients of potential and of pressure being oppositely directed, while the first of them is normal to the equipotential and the second to the isobaric surfaces, we at once conclude that isobaric and equipotential surfaces must coincide.

From this it follows that every isobaric sheet must coincide with an equipotential sheet. Let the two coinciding sheets be infinitely thin. The passage from the one limiting surface of the sheet to the other, then, gives a certain increase of potential $d\phi$, and a corresponding increase of pressure dp; all along the sheet $d\phi$ has the same value, and the same will be the case with dp. Their ratio $d\phi/dp$, therefore, is constant. But this ratio, taken with the negative sign, is, by equation (*c*), section 34, equal to the specific volume of the fluid in this sheet. We therefore conclude that the specific volume is constant all along the sheet. This condition being fulfilled for every infinitesimal sheet, it follows that the surfaces of equal specific volume must have the same course as those of equal pressure and of equal potential. Hence:

In the state of equilibrium there is coincidence between the isobaric, the isosteric, and the equipotential surfaces.

This remarkable coincidence of the equiscalar surfaces of three different fields is a necessary but not sufficient condition for equilibrium.

(II) *Principle of the Unit-Sheets.* — From equation (*c*), section 34, we further conclude that in every direction the variation of potential is α times more rapid than that of pressure. In reference to infinitesimal unit-sheets this means that every isobaric unit-sheet contains α equipotential unit-sheets. For practical reasons it will be important to have this principle formulated not only for infinitely thin sheets, but also for sheets of finite thickness. Integrating, therefore, equation (*c*), section 34, and denoting by α_m the mean value of the specific volume in the interval between the pressures p_1 and p_2, we get this relation between finite differences of potential and the corresponding finite differences of pressure:

$$\phi_2 - \phi_1 = -\alpha_m(p_2 - p_1) \tag{a}$$

Applying this to an isobaric unit-sheet, we get $p_2 - p_1 = 1$, and thus

$$\phi_2 - \phi_1 = -\alpha_m \tag{b}$$

Here $\phi_2 - \phi_1$ is the number of equipotential unit-sheets contained within the considered isobaric unit-sheet. Disregarding the sign, we thus get this numerical law:

In the state of equilibrium the number representing the mean specific volume of the fluid in an isobaric unit-sheet also represents the number of equipotential unit-sheets contained in the isobaric unit-sheet.

These two principles, taken in connection with the rule of signs that increasing potential gives decreasing pressure and *vice versa*, give the full equilibrium relation between the three fields — that of mass, that of pressure, and that of potential.

36. Determination of Heights or Depths of Given Pressures.— In the m.t.s. system of units the thickness of an equipotential unit-sheet is 1 dynamic decimeter. The number of equipotential unit-sheets contained in an isobaric unit-sheet is therefore the number of dynamic decimeters giving the thickness of the sheet. The principle of the unit-sheets, therefore, enables us to find the thickness of any isobaric sheet, adding the thicknesses of the successive unit-sheets and to determine thus the height or depth where the pressure has any given value. This is the dynamic principle of the barometric measurements of heights or of manometric measurements of depths.

Performing this operation practically, it will generally be convenient to pass from the m.t.s. units, dynamic decimeter and centibar, to the greater units, dynamic meter and decibar, or occasionally also to other decimal parts or multiples of the dynamic meter and the corresponding decimal parts or multiples of the bar.

As a first simple example we can consider pure imcompressible water of unit specific volume. Here there is full coincidence between isobaric and equipotential unit-sheets. The standard isobaric sheets of 1 decibar (section 5) have the thickness of 1 dynamic meter, exactly as the standard equipotential sheets (section 4). Disregarding the atmospheric pressure on the sea's surface, and considering only what we have called the sea-pressure (section 26), we get this simple rule for finding the depth where the pressure has a given value. The number expressing a given sea-pressure in decibars expresses at the same time the depth of this sea-pressure in dynamic meters. This rule, being exact for the case of pure incompressible water, remains a useful approximate rule also for the case of common sea-water.

As a second example we may consider sea-water of 35 ‰ salinity at 0° C. In table 8 H of Hydrographic Tables we have registered the specific volume of this water for the differences of pressure of 1 bar (10 decibars). The unit of gravity potential corresponding to this difference of pressure is the dynamic decameter. Forming the arithmetic mean of two and two successive numbers in table 8 H, we get the mean specific volume of the water in isobaric sheets of 1 bar, *i. e.*, the thickness of these sheets expressed in dynamic decameters. Adding these thicknesses from the surface downward, we get the depths of all isobaric surfaces for the interval of pressure of 1 bar. The dynamic depths found in this way are given in table 7 H of Hydrographic Tables, the units being again turned into dynamic meters and decibars. The equilibrium relation connecting dynamic depth and pressure according to this table is illustrated by the first vertical of fig. 1. The divisions to the right of this vertical give the pressures in decibars, and the divisions to the left the corresponding depths in dynamic meters. The second vertical of the figure gives in corresponding manner the equilibrium relation between pressure and specific volume, *i. e.*, the relation contained numerically in table 8 H.

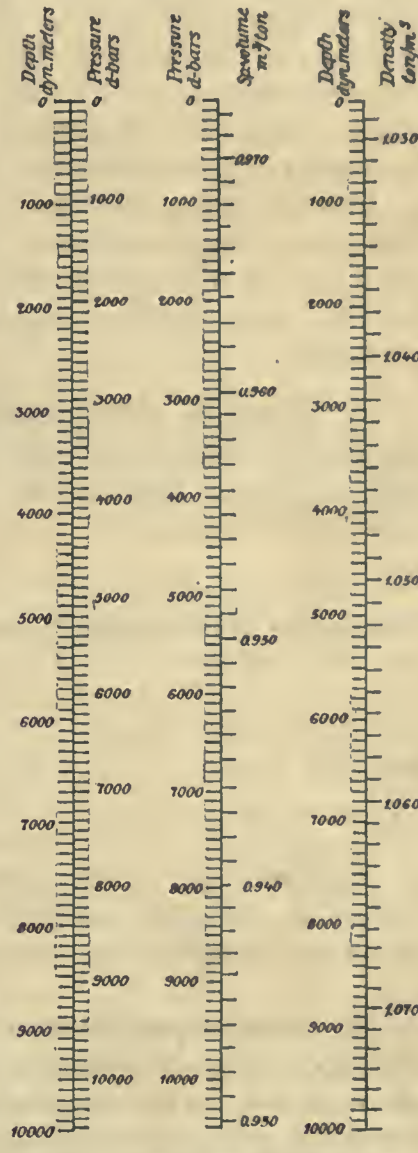

FIG. 1.—State of equilibrium of sea-water of 35 ‰ salinity and 0° C.

If we had constructed complete tables of the specific volume of atmospheric air, we should have been able to determine the heights of given pressures in the atmosphere in the same direct way. But as such tables would be very bulky, we have not calculated them, and we shall show later how to proceed without them. On this occasion, therefore, we only remark that the pressure at sea-level is very nearly 10 decibars. In the atmosphere we shall therefore have to count with 10 standard isobaric surfaces of the pressure from 10 to 1 decibars. These surfaces will divide the atmosphere into 10 standard isobaric sheets, the highest of which, however, has only a distinct lower limit, the standard surface of the pressure of 1 decibar, while the existence of the upper limit, the isobaric surface of pressure zero, may be open to discussion. As a consequence of the decrease of the pressure, the thickness of the standard sheets will increase upward. The thickness of each of them will vary with the virtual temperature as shown in table 9 M of Meteorologic Tables. The methods used for calculating this and other tables required for finding the height of given pressures in the atmosphere will be given in Chapter VI.

In the mercury column of a barometer we have the same number of standard sheets as in the atmosphere. The specific volume of the mercury being 0.073554, the thickness of the standard sheets is 0.073554 dynamic meter, or 0.075008 meter for the value of gravity at sea-level at 45° latitude. This is 75 millimeters, practically.

37. Condition of Equilibrium in Terms of Forces per Unit-Volume. — To express the condition of equilibrium we can also use the forces per unit-volume, section 33, (*c*). Equilibrium exists if the forces per unit-volume are equal and oppositely directed.

$$G = -\rho g \tag{a}$$

Proceeding as in section 35, we derive from this

$$\frac{\partial p}{\partial s} = -\rho \frac{\partial \phi}{\partial s} \tag{b}$$

and

$$dp = -\rho d\phi \tag{c}$$

Each of these equations may be formed from the corresponding equation of section 35 simply by multiplying by the density ρ. The difference between the equations is thus the slightest possible, but still important in its further consequences.

38. Equilibrium Relation between the Fields of Potential, Pressure, and Density. — On interpreting geometrically the condition of equilibrium in this form, characterized by the reference of the forces to the unit of volume, we find this difference only, that the field of mass is described by the distribution of density instead of, as previously, by the distribution of specific volume. We thus arrive at the following two slightly changed forms of the principles formulated in section 35 :

(I) *Principle of Coincidence of Surfaces.* — Every isosteric surface being at the same time an isopycnic surface, we immediately get from section 35 (I):

In the state of equilibrium there is coincidence between the isobaric, the isopycnic, and the equipotential surfaces.

(II) *Principle of the Unit-Sheets.* — As is immediately seen, equation (*a*), section 35, takes the changed form

$$(a) \qquad p_2 - p_1 = -\rho_m(\phi_2 - \phi_1)$$

and thus for an equipotential unit-sheet, $\phi_2 - \phi$ being equal to unity,

$$(b) \qquad p_2 - p_1 = -\rho_m$$

Therefore:

In the state of equilibrium the number representing the mean density of the fluid in an equipotential unit-sheet also represents the number of isobaric unit-sheets contained in the equipotential unit-sheet.

39. Determination of the Pressures at Given Heights or Depths. — The principle of the unit-sheets in its first form led to the method of barometric measurements of heights or of manometric measurements of depths (section 36). In its second form it leads to the solution of the inverse problem, namely, the determination of the pressure at given heights or depths. The m.t.s. isobaric unit-sheet represents the difference of pressure of 1 centibar. The number of such sheets contained in the equipotential unit-sheet therefore gives the difference of pressure in centibars between the surfaces limiting the equipotential unit-sheet. Adding these differences of pressure from level surface to level surface, we can determine the pressure at any level if it be known in an initial level. Performing it practically we may as above make use of other units of pressure and of gravity potential than those of the m.t.s. system.

Taking the same examples as above, there will be no difference so long as we consider pure incompressible water at maximum of density. The density being unity, the increase of pressure for each dynamic meter of depth will be 1 decibar, and the number representing the depth in dynamic meters will represent at the same time the sea-pressure expressed in decibars. As a second example we shall determine the pressure in given depths in sea-water of 35 ‰ salinity and 0°C. The hydrographical table 16 H gives the density of this water at all depths for intervals of 1 dynamic decameter. Forming the mean value of two and two successive numbers in this table, we get the average density of the sea-water in equipotential unit-sheets of 1 dynamic decameter, *i. e.*, the increase of pressure in bars from level surface to level surface. Adding these increases of pressure from sea-level downwards, we get the sea-pressure expressed in bars at all dynamic depths for intervals of 1 dynamic decameter. Then, on returning to the smaller units, the dynamic meter and decibar, these pressures are given in table 15 H in our Hydrographic Tables. The equilibrium relation between dynamic depth and pressure contained in this table is intrinsically the same as that contained in table 7 H. Graphically we arrive at the same representation from both tables, given by the first vertical of fig. 1. The third vertical represents the relations between dynamic depth and density contained in table 16 H.

40. Integral Forms of the Equation of Equilibrium. — In equations (*c*), section 34, and (*c*), section 37, the increase of potential $d\phi$ and the increase of pressure

dp are referred to the displacement along an element of line ds. Forming the sum for any succession of line elements, we get the equations referred to a curve of finite length, namely, from (c), section 34,

$$(a) \qquad \phi_2 - \phi_1 = -\int_{p_1}^{p_2} \alpha dp$$

and from (c), section 37,

$$(b) \qquad p_2 - p_1 = -\int_{\phi_1}^{\phi_2} \rho d\phi$$

The first of these equations gives the difference of potential, *i. e.*, the difference of dynamic height, between the isobaric surfaces of pressures p_2 and p_1. The second gives the difference of pressure between two equipotential surfaces of potentials ϕ_1 and ϕ_2.

The dynamic sense of the integrals forming the second member of equations (a) and (b) is easily found, as we have

$$-\alpha dp = \alpha G_s ds \qquad\qquad -\rho d\phi = \rho g_s ds$$

Thus the integral in (a) is the line-integral of the force of pressure per unit-mass. The integral in equation (b) is the line-integral of the force of gravity per unit-volume. On the other hand, the differences appearing on the left side of the equations $\phi_2 - \phi_1$ and $p_2 - p_1$ are the line-integrals of the force of gravity per unit-mass and of the force of pressure per unit-volume. Equation (a) thus shows that the force of gravity and the force of pressure, both referred to unit-mass, have oppositely equal line-integrals. In the same way, equation (b) shows that the force of gravity and the force of pressure, both referred to unit-volume, have oppositely equal line-integrals. One of the two oppositely equal line-integrals can always be expressed in finite form, namely, that of the force of gravity per unit-mass and that of the force of pressure per unit-volume.

The equations (a) and (b) enable us at once to derive a fundamental property of the integrals appearing on the right side. The values ϕ_1 and ϕ_2 of the potential in the end-points 1 and 2 of the curve depend only upon the situation of these points 1 and 2, and not upon the course of the curve s joining them. The same is the case with the values p_1 and p_2 of the pressure in the same two points. The integrals on the right side, therefore, must have the same property. Hence we conclude:

Under statical conditions the line-integral of the force of pressure per unit-mass

$$(c) \qquad -\int_{p_1}^{p_2} \alpha dp$$

as well as the line-integral of the force of gravity per unit-volume

$$(d) \qquad -\int_{\phi_1}^{\phi_2} \rho d\phi$$

are independent of the course of the curve and dependent only upon the positions of its end-points.

As a corollary we get this other theorem:

Under statical conditions the line-integrals of the force of pressure per unit-mass (c), *as well as the line-integral of the force of gravity per unit-volume* (d) *are zero for every closed curve.*

CHAPTER V.

IDEAL STATES OF EQUILIBRIUM IN THE ATMOSPHERE.

41. Analytical Integration of the Equation of Atmospheric Equilibrium. — The hydrostatic equation

$$(a) \qquad d\phi = -\alpha dp$$

contains three variable quantities, ϕ, p, α. Two of them, p and α, are connected with a third variable ϑ by the equation of state

$$(b) \qquad p\alpha = R\vartheta$$

ϑ being the true temperature of dry or the virtual temperature of moist air. By this equation we may introduce temperature ϑ as a variable in (a) instead of the specific volume α. This will generally be convenient, and the equation of atmospheric equilibrium then takes the form

$$(c) \qquad d\phi = -R\vartheta\frac{dp}{p}$$

Now, supposing a relation between temperature and pressure to be known,

$$(d) \qquad f_1(\vartheta, p) = 0$$

equation (c) is seen to be integrable immediately. To perform the integration we may choose either of two ways. We may use (d) to eliminate the pressure from the second member of (c). The integration then gives a relation between gravity potential and temperature

$$(e) \qquad f_2(\phi, \vartheta) = 0$$

Eliminating afterwards the temperature between (d) and (e), we get the relation of equilibrium connecting gravity potential and pressure

$$(f) \qquad F(\phi, p) = 0$$

Or we may, on the other hand, use equation (d) to eliminate the temperature from the second member of (c). The integration then immediately leads to the equilibrium relation (f) between gravity potential and pressure. The elimination of pressure between equations (f) and (d) will then lead to the relation (e) connecting gravity potential and temperature.

Again, we might have written equation (c) in the form

$$(g) \qquad \frac{d\phi}{\vartheta} = -R\frac{dp}{p}$$

The equation in this form is seen to be integrable at once if a relation between temperature and gravity potential be given, *i. e.*, a relation of the form (e). For the integration we again have the choice of either of two ways. We may use equation (e) to eliminate the gravity potential from the left member of (g). The integration then leads to the relation (d) between temperature and pressure. Then the elimination of the temperature between (d) and (e) leads to the equilibrium relation (f) connecting gravity potential and pressure. Or we might have used (e) to eliminate the temperature from equation (g). The integration would then have led directly to the equilibrium relation (f) between gravity potential and pressure, while elimination between (f) and (e) would have led to the corresponding relation (d) between temperature and pressure.

As will be inferred from the above discussion, we have to notice two cases of immediate integrability, the first characterized by a relation between temperature and pressure (d), the second by a relation between temperature and gravity potential (e). Between these two cases of integrability there is a full correspondence in this sense: that to a given relation between temperature and pressure (d) there will correspond a perfectly definite relation between temperature and gravity potential (e), and *vice versa*.

42. Atmosphere with Constant-Temperature Gradient. — Let us suppose temperature to be a linear function of gravity potential

$$\vartheta = \vartheta_0 - \gamma\phi \tag{a}$$

ϑ_0 being the temperature at sea-level and γ the temperature gradient

$$\gamma = -\frac{d\vartheta}{d\phi} \tag{a'}$$

which is in this case constant.

To find the relation between temperature and pressure, corresponding to the relation (a) between temperature and potential, we eliminate $d\phi$ between equations (a') and section 41 (g). This gives

$$\frac{d\vartheta}{\vartheta} = R\gamma\frac{dp}{p}$$

and hence after integration, p_0 being the pressure at sea-level,

$$\frac{\vartheta}{\vartheta_0} = \left(\frac{p}{p_0}\right)^{R\gamma} \tag{b}$$

We thus arrive at this important result:

If temperature be a linear function of gravity potential, with the temperature gradient γ, it will be proportional to the power $R\gamma$ of pressure, R being the gas constant. And vice versa: If temperature be proportional to any power $R\gamma$ of the pressure, it will be a linear function of gravity potential with the temperature gradient γ.

Eliminating the temperature between the equations (a) and (b), we arrive at the equilibrium relation between gravity potential and pressure, namely, (*c*) $$1 - \frac{\gamma}{\vartheta_0}\phi = \left(\frac{p}{p_0}\right)^{R\gamma}$$

Adding, finally, the equation of state

(*d*) $$p\alpha = R\vartheta$$

we can find the corresponding equilibrium values of the specific volume α or of its reciprocal, the density ρ.

The problem is thus fully solved. Summing up the results, we shall choose once the pressure and once the gravity potential as independent variable. In the first case we shall represent the distribution of mass by the specific volume α, in the second by the density ρ. Denoting by ϑ_0, p_0, α_0, ρ_0 the values of temperature, pressure, specific volume, and density at sea-level, we easily arrive at the following two schemes of formulæ:

(A) $$\vartheta = \vartheta_0\left(\frac{p}{p_0}\right)^{R\gamma} \qquad \alpha = \alpha_0\left(\frac{p}{p_0}\right)^{R\gamma-1} \qquad \phi = \frac{\vartheta_0}{\gamma}\left[1 - \left(\frac{p}{p_0}\right)^{R\gamma}\right]$$

(B) $$\vartheta = \vartheta_0\left(1 - \frac{\gamma}{\vartheta_0}\phi\right) \qquad \rho = \rho_0\left(1 - \frac{\gamma}{\vartheta_0}\phi\right)^{\frac{1}{R\gamma}-1} \qquad p = p_0\left(1 - \frac{\gamma}{\vartheta_0}\phi\right)^{\frac{1}{R\gamma}}$$

each of which represents the full solution of the problem.

43. Limit of the Atmosphere in Case of Constant-Temperature Gradient. — Temperature being a linear function of the gravity potential, and decreasing upwards, absolute zero will be reached at a certain finite height

(*a*) $$\phi_L = \frac{\vartheta_0}{\gamma}$$

Substituting this in the two last equations (B), section 43, and remembering that γ is positive when temperature decreases upwards, we get

$$\rho = 0 \qquad p = 0$$

Supposing, thus, the gas laws to be true even at absolute zero, we find the atmosphere to be limited by the level surface determined by (*a*).

For decreasing values of the temperature gradient γ the height of the atmosphere always increases and converges towards infinity when γ converges towards zero, *i. e.*, in the case of the isothermic atmosphere.

When γ is negative, and thus the temperature rises with the height, ϕ_L also is negative. The atmosphere remains unlimited upwards, while its analytical continuation below sea-level has the limit ϕ_L determined by equation (*a*).

44. States of Unstable Equilibrium. — In the extreme case $R\gamma = \infty$, *i. e.*, in the case of an infinite decrease of temperature with the height, we get $\phi_L = 0$. The atmosphere is, then, condensed to an infinitely thin sheet. For values of $R\gamma$ decreasing from ∞ to 1, we get values of the temperature gradient γ decreasing from ∞ to 0.00348, this last value representing a fall of temperature of 3.48° C. for every 100 dynamic meters of height. Extreme falls of temperature of this order of magnitude may exist locally under extraordinary conditions, as above a hot chimney or above a volcano in action. They may perhaps exist also for a short while over a heated area before the formation of a tornado. But the corresponding state of equilibrinm can not endure. For it is seen from the second equation (B) that as long as $R\gamma$ is comprised between ∞ and 1 there will be increase of density upward. The state of equilibrium is therefore completely unstable.

The limiting case

$$(a) \qquad R\gamma = 1$$

corresponding to a fall of temperature of 3.48° C. for every 100 meters, is interesting from a mathematical point of view. In this case the equations (A) and (B) reduce to the simple forms

$$(A') \qquad \vartheta = \vartheta_0 \frac{p}{p_0} \qquad \alpha = \alpha_0 \qquad \phi = R\vartheta_0 \left(1 - \frac{p}{p_0}\right)$$

$$(B') \qquad \vartheta = \vartheta_0 \left(1 - \frac{1}{R\vartheta_0}\phi\right) \qquad \rho = \rho_0 \qquad p = p_0 \left(1 - \frac{1}{R\vartheta_0}\phi\right)$$

These are all linear, those for the specific volume, $\alpha = \alpha_0$, and for the density, $\rho = \rho_0$, showing that specific volume and density are constant. As the pressure and the temperature thus both decrease with the height, they compensate each other in their influence upon the density of the air, the result being a perfectly *homogeneous atmosphere.*

Also, in the case of the homogeneous atmosphere the equilibrium is unstable. For if a mass of air be moved upwards, the adiabatic cooling will not suffice to bring it down to the temperature of the higher strata, to which it has been moved. Therefore, if once given the slightest displacement upwards, it will continue moving upwards, remaining always lighter than the adjacent air.

The height ϕ_L' of this homogeneous atmosphere has, according to (*a*) and section 43 (*a*), the value

$$(a') \qquad \phi_L' = R\vartheta_0$$

It merits attention that we may introduce the two limiting heights ϕ_L and ϕ_L' as fundamental parameters in our formulæ. To do this we have the expressions

$$(b) \qquad R\gamma = \frac{\phi_L'}{\phi_L} \qquad \frac{\gamma}{\vartheta_0}\phi = \frac{\phi}{\phi_L}$$

The ratios on the right side being independent of the units of gravity potential, we may also write

$$(b') \qquad R\gamma = \frac{H_L'}{H_L} \qquad \frac{\gamma}{\vartheta_0}\phi = \frac{H}{H_L}$$

measuring the height H and the limiting heights H_L and H_L' in dynamic meters. Equations (*b*) or (*b'*) give thus a perspicuous sense to expressions appearing in the equations (A) and (B).

Proceeding to values of $R\gamma$ smaller than 1, we come to states of less pronounced instability. The case $R\gamma = \frac{1}{2}$, corresponding to a decrease of temperature of 1.74° C. for every 100 dynamic meters of height, is interesting mathematically, temperature being in direct and specific volume in inverse proportion to the square root of the pressure, and the density being a linear function of the dynamic height. $R\gamma = \frac{1}{3}$ also gives simple formulæ, representing a state of equilibrium still unstable but greatly approaching the state of indifferent or adiabatic equilibrium.

45. Indifferent or Adiabatic Equilibrium. — The state of equilibrium will be indifferent if the adiabatic cooling of a mass of air, which is displaced upwards, will always bring its temperature to that of the air-masses in the new level. For in this case no force will arise tending to favor or to counteract the displacement. The distribution of temperature giving adiabatic equilibrium will be different according to the humidity of the air. Considering first the case of a perfectly dry atmosphere, let κ be the well-known ratio 1.4053 of the two specific heats of an ideal gas. Introducing

$$(a) \qquad R\gamma = \frac{\kappa - 1}{\kappa} = 0.2884$$

we see that the equations (A) and (B) take the forms

$$(\text{A}'') \qquad \vartheta = \vartheta_0 \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}} \qquad \alpha = \alpha_0 \left(\frac{p}{p_0}\right)^{-\frac{1}{\kappa}} \qquad \phi = \frac{\kappa}{\kappa - 1} R\vartheta_0 \left[1 - \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}}\right]$$

$$(\text{B}'') \qquad \vartheta = \vartheta_0 \left(1 - \frac{\kappa - 1}{\kappa R\vartheta_0}\phi\right) \qquad \rho = \rho_0 \left(1 - \frac{\kappa - 1}{\kappa R\vartheta_0}\phi\right)^{\frac{1}{\kappa-1}} \qquad p = p_0 \left(1 - \frac{\kappa - 1}{\kappa R\vartheta_0}\phi\right)^{\frac{\kappa}{\kappa-1}}$$

The two first equations (A″) are the well-known ones connecting temperature and pressure, and specific volume and pressure, respectively, in the case of an adiabatic change of state of an ideal gas. The state of equilibrium defined by equations (A″) or (B″) has therefore the following property: Proceeding upwards to decreasing pressure we find everywhere the temperature which a mass of dry air moved upwards would take on account of its adiabatic cooling. The temperature gradient in this atmosphere is

$$(a') \qquad \gamma = \frac{\kappa - 1}{\kappa R} = 0.0010048$$

representing a fall of temperature of 1.0048° C. for every 100 dynamic meters of height.

Moist air will have the same adiabatic temperature gradient (a') as dry air, as long as no condensation takes place. But as soon as condensation begins, the heat of condensation will partly compensate for the adiabatic cooling, and the adiabatic gradient will take one of the values given in table D.* While the adiabatic temperature gradient for dry air is constant, that for saturated air varies both with pressure and temperature, decreasing with decreasing pressure and increasing with decreasing temperature. The decrease upward both of pressure and temperature

* The table is taken from Hann's Meteorology (first edition), p. 241, with the difference that the pressure figuring as argument is reduced from millimeters of mercury to m-bars, while the fall of temperature is taken per 100 dynamic instead of per 100 common meters.

therefore counteract each other in their effect upon the fall of temperature, making its variation with the height very gradual. But still it will always increase upward, converging toward the limit 1.0048, which would be reached when all moisture had fallen out. To illustrate this increasing fall of temperature, the values corresponding to the case of a mass of air moved upwards from sea-level with the initial temperature of 15° C. are indicated by heavy-faced figures in the table.

TABLE D.—*Adiabatic Fall of Temperature per 100 Dynamic Meters for Saturated Air.*

Pressure (milli-bars).	Temperature (° C.).									
	−10	−5	−0	+0	+5	+10	+15	+20	+25	+30
300	0.52	0.46	0.40	0.42						
400	.58	.52	.45	.47	0.42					
500	**.68**	.57	.49	.51	.46	0.41	0.37			
600	.67	**.60**	.54	.56	.50	.45	.40			
700	.70	.64	**.57**	**.59**	.54	.48	.42	0.39		
800	.72	.66	.59	.61	**.56**	.50	.45	.41	0.38	
900	.75	.69	.62	.64	.59	**.58**	.48	.44	.40	0.37
1000	.77	.70	.64	.66	.61	.55	**.50**	.45	.41	.38

The case of adiabatic equilibrium for saturated air can not, therefore, be comprised in the case of equilibrium with constant-temperature gradients treated here. But as the increase of the gradient upward is gradual, we may with some approximation reckon with constant average values for sheets of moderate thickness. Thus the temperature gradient $\gamma = 0.0005$, corresponding to a fall of temperature of 0.5° C. for every 100 dynamic meters of height, is a value often used by practical meteorologists, and may be taken as an average value of the adiabatic temperature gradient for saturated air in the lower strata of the atmosphere.

46. States of Stable Equilibrium.— Passing to temperature gradients smaller than the adiabatic, we arrive at states of stable equilibrium. If in this case a mass of air be moved upward, the adiabatic cooling will bring it to a lower temperature than that of the surrounding masses and it will sink back again on account of its greater density.

Interesting mathematically is the case $R\gamma = 0$, that is, the case of a temperature-gradient zero,

$$(a) \qquad \gamma = 0$$

or of *isothermic atmosphere.* For greater gradients the atmosphere has been stated to be finite. But in this case it becomes infinite. At the same time the second member of the last equation (A), section 42, and of the two last equations (B), section 42, become indeterminate. But by the theory of indeterminate expressions, or by renewed integration of the equation of equilibrium (c), section 41, after the substitution $\vartheta = \vartheta_0$, we easily arrive at the following set of formulæ representing the state of isothermic equilibrium:

$$(\text{A}''') \qquad \vartheta = \vartheta_0 \qquad \alpha = \alpha_0 \frac{p_0}{p} \qquad \phi = R\vartheta_0 \text{ nat. log.} \frac{p_0}{p}$$

$$(\text{B}''') \qquad \vartheta = \vartheta_0 \qquad \rho = \rho_0 e^{-\frac{\phi}{R\vartheta_0}} \qquad p = p_0 e^{-\frac{\phi}{R\vartheta_0}}$$

Passing to the case of negative temperature gradients, *i. e.*, of increase of temperature upward, the height of the atmosphere remains infinite, the limit determined by formula (a), section 43, having only the analytical meaning of the limit of an imaginary continuation of the atmosphere below sea-level. This increase of temperature with the height is meaningless if it be extended to the whole atmosphere.

But "temperature inversion" is well known as a local phenomenon, limited to more or less narrow sheets, occurring specially often in the case of high pressure during winter. The main feature of this state from a dynamic point of view is a pronounced stability which can be overcome only by causes producing different distribution of temperature. Values of $R\gamma$ as $-\frac{1}{2}$ or -1 give very simple forms for equations (A) and (B) and represent increases of temperature with the height which may occur in the sheets of inversion, namely, 1.74 and 3.48° C. for every 100 meters of height.

47. Numerical Representation of the States of Equilibrium.— For the numerical representation of any definite state of equilibrium we have the choice between either of two methods.

TABLE E.—*Ideal States of Atmospheric Equilibrium. Argument, Pressure.*

Pressure (m-bars).	$1000\,\gamma = 3.48°$ C. (Homogeneous atmosphere.)			$1000\,\gamma = 1.0048°$ C. (Dry atmosphere in adiabatic equilibrium.)			$1000\,\gamma = 0.5°$ C.			$1000\,\gamma = 0$. (Isothermic atmosphere.)		
	Height (dynamic meters).	Temperature (° C.).	Specific volume (m³/ton).	Height (dynamic meers).	Temperature (° C.).	Specific volume (m³/ton).	Height (dynamic meters).	Temperature (° C.).	Specific volume (m³/ton).	Height (dynamic meters).	Temperature (° C.).	Specific volume (m³/ton).
0	7835	−273.0	784	27178	−273.0	∞	54600	−273.0	∞	∞	0	∞
100	7054	−245.7	784	13188	−132.5	4034	15368	− 76.8	5632	18047	0	7838
200	6270	−218.4	784	10092	−101.4	2464	11264	− 56.3	3111	12614	0	3919
300	5486	−191.1	784	7973	− 80.1	1846	8666	− 43.3	2198	9436	0	2612
400	4703	−163.8	784	6312	− 63.4	1504	6729	− 33.6	1718	7182	0	1959
500	3919	−136.5	784	4925	− 49.5	1284	5171	− 25.9	1419	5433	0	1568
600	3135	−109.2	784	3723	− 37.4	1127	3861	− 19.3	1214	4004	0	1306
700	2351	− 81.9	784	2656	− 26.7	1010	2726	− 13.6	1064	2796	0	1120
800	1568	− 54.6	784	1694	− 17.0	919	1721	− 8.6	949	1749	0	980
900	784	− 27.3	784	814	− 8.2	845	821	− 4.1	858	826	0	871
1000	0	0	784	0	0	784	0	0	784	0	0	784

We can use the pressure as argument and register temperature, specific volume, and height for suitable integer values of pressure. This method is used in table E, giving temperature, specific volume, and height for each of the standard isobaric surfaces. The four sections of the table correspond to four different temperature gradients: (1) that giving homogeneous atmosphere; (2) that giving adiabatic equilibrium in a perfectly dry atmosphere: (3) the gradient 0.0005 roughly representing in the lower strata adiabatic equilibrium of saturated air and in the higher strata stable equilibrium; (4) the gradient zero giving isothermic atmosphere. In all these cases the temperature is supposed to be zero centigrade at sea-level, and the height is measured in dynamic meters.

On the other hand, we can choose dynamic height as argument and register temperature, density, and pressure for certain standard heights. This is made in table F, the four sections of the table representing the same four cases as in table E.

TABLE F.—*Ideal States of Atmospheric Equilibrium. Argument, Dynamic Height.*

Height (dynamic meters).	$1000\,\gamma = 3.48°$ C. (Homogeneous atmosphere.)			$1000\,\gamma = 1.0048°$ C. (Dry atmosphere in adiabatic equilibrium.)			$1000\,\gamma = 0.5°$ C.			$1000\,\gamma = 0$. (Isothermic atmosphere.)		
	Pressure (m-bars).	Temperature (° C.).	Density (10^{-6} ton/m³).	Pressure (m-bars).	Temperature (° C.).	Density (10^{-6} ton/m³).	Pressure (m-bars).	Temperature (° C.).	Density (10^{-6} ton/m³).	Pressure (m-bars).	Temperature (° C.).	Density (10^{-6} ton/m³).
⋮							⋮	⋮	⋮	⋮	⋮	⋮
30,000	....	....	....	....	....	....	3.9	—150	11	21.8	0	28
29,000	....	....	....	....	....	....	5.1	—145	14	24.7	0	31
28,000	....	....	....	....	....	....	6.7	—140	17	28.1	0	36
27,000	....	....	....	0.00003	—271.2	0.005	8.6	—135	22	31.9	0	41
26,000	....	....	....	0.0188	—261.2	0.56	11.1	—130	27	36.3	0	46
25,000	....	....	....	0.158	—251.1	2.51	14.1	—125	33	41.2	0	53
24,000	....	....	....	0.586	—241.1	6.40	17.7	—120	40	46.8	0	60
23,000	....	....	....	1.514	231.0	12.6	22.2	—115	49	53.2	0	68
22,000	....	....	....	3.186	221.0	21.3	27.5	—110	59	60.4	0	77
21,000	....	....	....	5.877	210.9	33.0	34.0	—105	70	68.6	0	87
20,000	....	....	....	9.889	200.9	47.7	41.7	—100	84	77.9	0	99
19,000	....	....	....	15.54	—190.9	65.9	50.8	— 95	99	88.6	0	113
18,000	....	....	....	23.19	—180.8	87.6	61.6	— 90	117	100.6	0	128
17,000	....	....	....	33.19	—170.8	113	74.4	— 85	138	114.3	0	146
16,000	....	....	....	45.93	—160.7	143	89.3	— 80	161	129.8	0	166
15,000	....	....	....	61.82	—150.7	176	106.7	— 75	188	147.5	0	188
14,000	....	....	....	81.28	—140.6	214	127.0	— 70	216	167.6	0	214
13,000	....	....	....	104.7	—130.6	256	150.4	— 65	252	190.4	0	243
12,000	....	....	....	132.7	—120.5	303	177.5	— 60	290	216.3	0	276
11,000	....	....	....	165.7	—110.5	355	208.7	— 55	334	245.7	0	314
10,000	....	....	....	203.8	—100.4	411	244.4	— 50	382	279.2	0	356
9,000	....	....	....	248.0	— 00.4	473	285.3	— 45	435	317.2	0	405
8,000	....	....	....	298.6	— 80.4	540	331.7	— 40	496	360.3	0	460
7,000	106.9	—243.8	1276	356.1	— 70.3	612	384.5	— 35	563	409.4	0	522
6,000	234.5	—209.0	1276	421.1	— 60.3	689	444.5	— 30	637	465.1	0	593
5,000	362.0	—174.2	1276	494.2	— 50.2	773	512.2	— 25	719	528.4	0	674
4,000	489.6	—139.3	1276	575.8	— 40.2	861	588.6	— 20	810	600.3	0	766
3,000	617.2	—104.5	1276	666.7	— 30.1	956	674.6	— 15	911	682.0	0	870
2,000	744.8	— 69.7	1276	767.2	— 20.1	1057	771.2	— 10	1021	774.8	0	988
1,000	872.4	— 34.8	1276	878.1	— 10.0	1163	879.1	— 5	1143	880.2	0	1123
0	1000	0	1276	1000	0	1276	1000	0	1276	1000	0	1276

The two tables show essentially different features. The first has the important property of being finite, which gives a great practical advantage, while the second continues infinitely to infinite heights. It is important to remark also that the division of the atmosphere into isobaric sheets, as in table E, represents practically a division into sheets of equal mass, and thus, from certain points of view, of equal importance, while the division into equipotential sheets as in table F corresponds to a division into sheets of decreasing masses upward, and thus of decreasing importance.

The states of equilibrium represented by these tables are also illustrated by fig. 2, the method of representation being the same as that used in fig. 1 (p. 45) to illustrate the equilibrium in the sea.

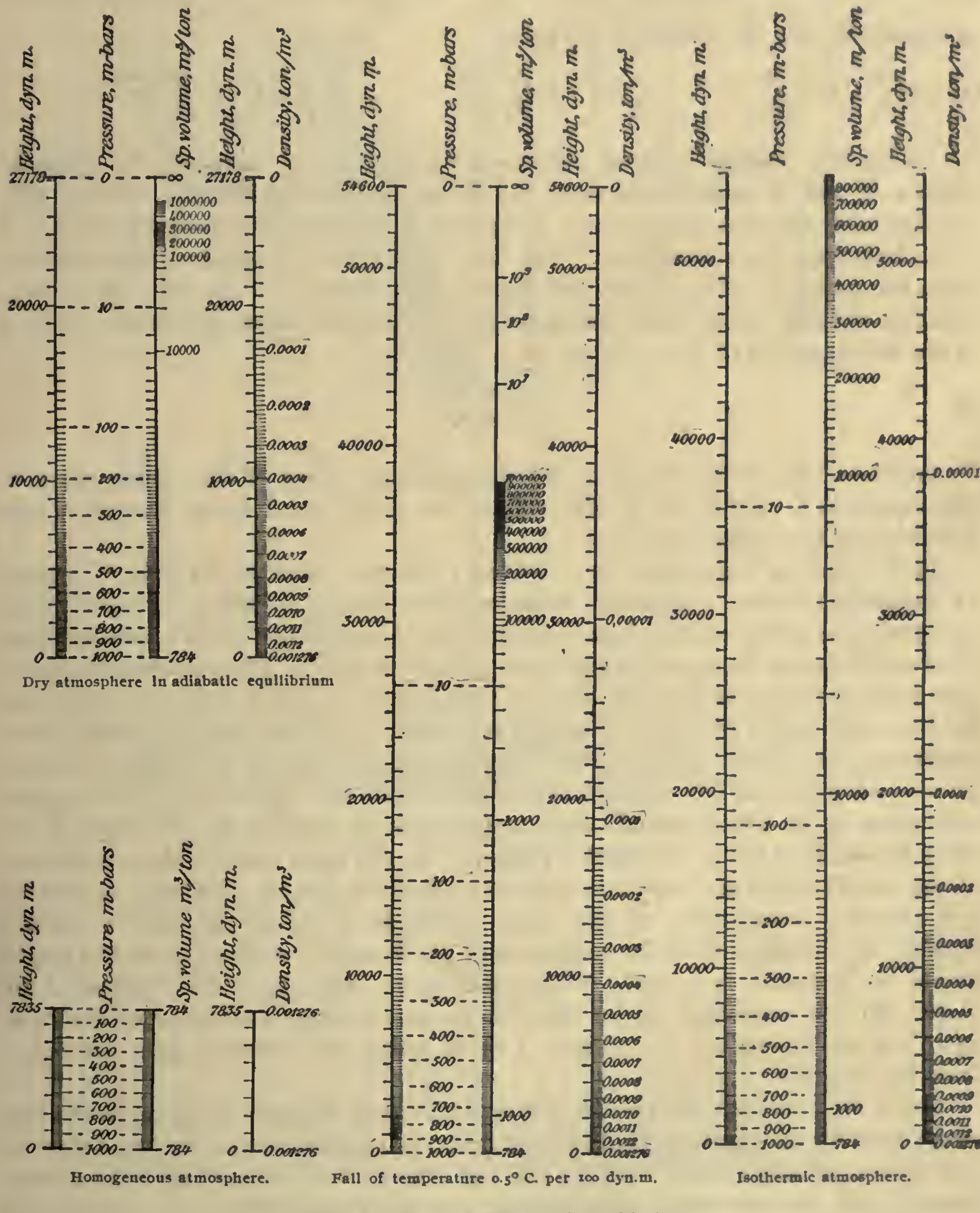

FIG. 2. — States of atmospheric equilibrium.

48. Graphical Representation of the States of Equilibrium. — To get a more comprehensive view of the states of equilibrium than that afforded by the numerical tables and the diagram, fig. 2, we may use a graphic method. Introducing according to (*b'*) section 44, the ratios $\frac{H}{H_L}$ and $\frac{H_L}{H_L'}$ in (B), section 42, this system of equations may be written in the form

$$(a) \qquad \frac{\vartheta}{\vartheta_0} = 1 - \frac{H}{H_L} \qquad \frac{\rho}{\rho_0} = \left(1 - \frac{H}{H_L}\right)^{\frac{H_L}{H_L'} - 1} \qquad \frac{p}{p_0} = \left(1 - \frac{H}{H_L}\right)^{\frac{H_L}{H_L'}}$$

To see the content of these equations we may use as ordinates the dynamic heights H, and as abscissæ the ratio H_L/H_L' of the limiting heights. Doing this, we range the different atmospheres according to their heights compared with that of the homogeneous atmosphere. The ratio itself has a real physical meaning only when it is positive. But to every value, positive or negative, of the ratio there corresponds a definite value, positive or negative, of the temperature gradient according to the first equation (*b'*), section 44, or

$$(b) \qquad \gamma = \frac{1}{R} \frac{H_L'}{H_L}$$

To facilitate the interpretation of the diagram the values of 1000 γ according to this equation, *i. e.*, the fall of temperature for every 100 dynamic meters, is also shown along the axis of abscissæ.

In the plane of coördinates thus defined a constant value of the ratio ϑ/ϑ_0 gives an isothermic curve, a constant value of the ratio ρ/ρ_0 an isopycnic curve, and a constant value of the ratio p/p_0 an isobaric curve. Choosing a set of values for these three ratios we get three systems of curves, drawn in fig. 3. The three sets of curves give a full representation of the state of equilibrium for every value of the ratio of the limiting heights H_L/H_L'. Fixing a certain value for this ratio, or for the temperature gradient, we get a definite vertical line in each of the three diagrams. The intersections of this vertical, for instance with the isothermic curve 0.1, give the height H at which the absolute temperature is reduced to one-tenth of the value ϑ_0 which it has at the earth's surface. In the same manner the intersection of the vertical with the isopycnic curve 0.1 gives the height H where the density ρ is reduced to one-tenth of the value ρ_0 which it has at the earth's surface. Finally, the intersection of the vertical with the isobaric curve 0.1 gives the height where the pressure is reduced to one-tenth of its value p_0 at the surface of the earth. Fixing according to the equation of state a consistent set of values ϑ_0, ρ_0, p_0 at the earth's surface, the values of these quantities at any heights are found from the diagram.

As to the course of the curves, it is seen that each diagram contains, on the side of the positive temperature gradients (decrease of temperature upwards), a straight line forming an angle of 45° with the axis and representing respectively temperature, density, and pressure zero. The ordinates of this straight line give the limiting height of the atmosphere for all positive finite values of the temperature gradient, the value being ∞ for $H_L/H_L' = \infty$, *i. e.*, for gradient zero or isothermic atmosphere. On the negative side of the axis of ordinates no such limiting curve exists, the atmosphere being always unlimited in the case of increase of temperature upward. Both isobaric and isopycnic curves converge at infinity towards horizontal

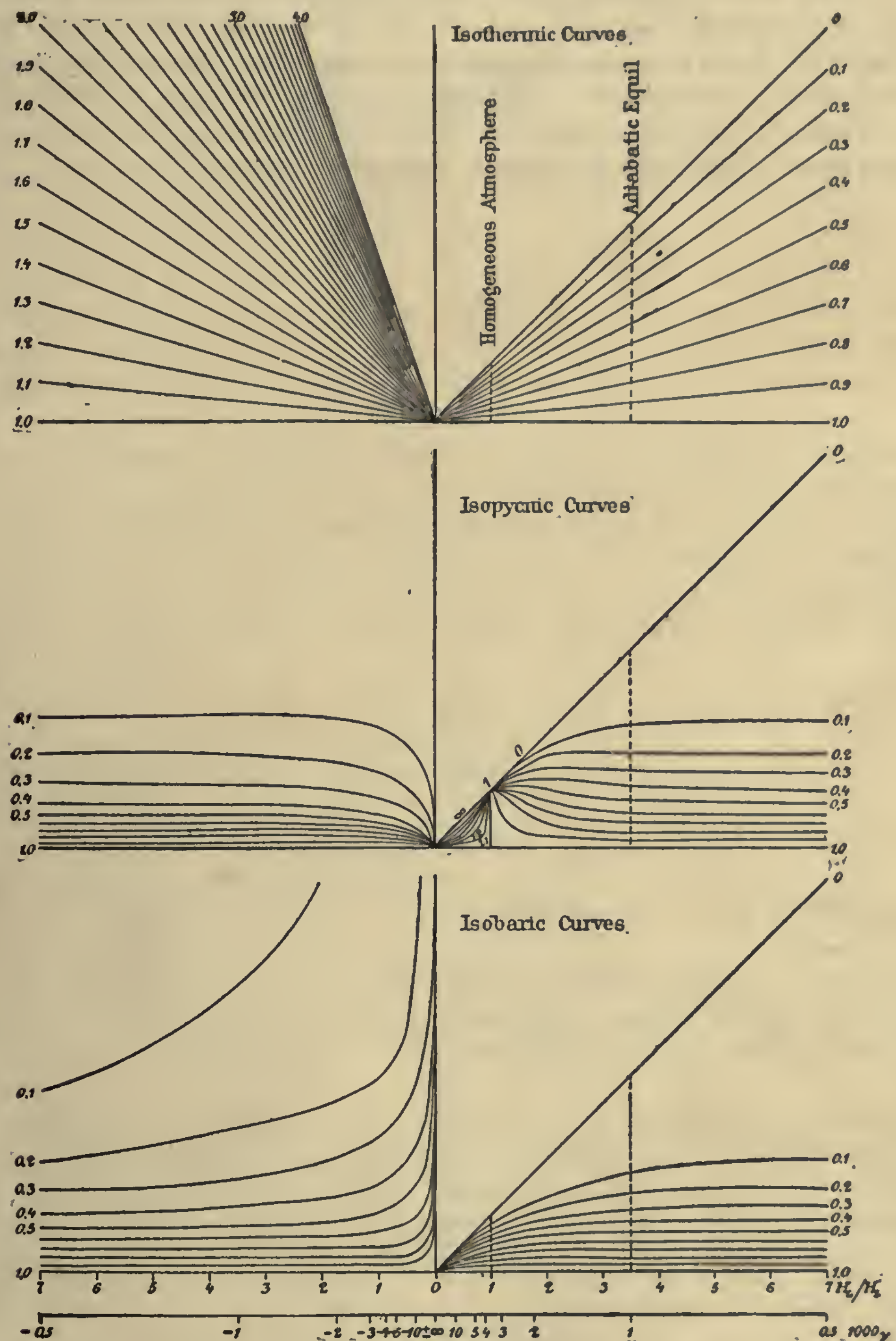

FIG. 3. — States of atmospheric equilibrium for different values of the ratio H_L/H_L' of the limiting heights.

asymptotes, the asymptotic values representing the case of the isothermic atmosphere. The course of the isobaric curves is relatively simple. For positive infinite values of the temperature gradient they all begin at zero. The ordinates increase with decreasing values of the gradient, converging towards infinity for an infinite negative value of this gradient. The course of the isopycnic curves is more complicated. They also all begin at zero for infinite positive values of the temperature gradient. Then they pass through one point — that representing the limit of the homogeneous atmosphere. Afterwards, passing through a maximum, they return to zero for infinite negative values of the gradient, giving as we approach this point an infinitely rapid decrease of the density upward.

CHAPTER VI.

PRACTICAL SOLUTION OF THE HYDROSTATIC PROBLEM FOR THE ATMOSPHERE.

49. **Four Forms of the Problem.** — In the preceding simple cases we have used two different methods of registering numerically the equilibrium relation between pressure and dynamic height. We have registered either *the height of given pressures* or *the pressure at given heights.* In cases of practical occurrence, when any analytical form to the equilibrium relation can not be given, we shall always try to find the result in one of the same two forms, as a table containing the heights of given pressures or as a table containing the pressures in given heights.

On the other hand, the observed data from which the equilibrium relation may be deduced will generally be given in one of two forms. The observed quantities may be the correlated values of *pressure*, temperature, and humidity, or of *height*, temperature, and humidity. From the first set we can calculate the virtual temperature for given values of pressure; from the second the virtual temperature at given heights. The practical problem, therefore, will present itself in one of the following four forms:

(1) To calculate the heights corresponding to given pressures, the virtual temperatures for these values of the pressures being known.

(2) To calculate the heights corresponding to given pressures, the virtual temperatures at given heights being known.

(3) To calculate the pressures at given heights, the virtual temperatures for given values of the pressure being known.

(4) To calculate the pressures at given heights, the virtual temperatures at given heights being known.

Of these four problems the first is by far the simplest, and at the same time practically the most important. We shall therefore first direct our attention to the practical solution of this problem. The others will afterwards easily be solved.

50. **Fundamental Formula.** — As already remarked, the hydrostatic equation in its first integral form, (a), section 40, gives the difference of potential corresponding to a given difference of pressure. Passing from the potential ϕ expressed in dynamic decimeters to the dynamic height H expressed in dynamic meters, and passing simultaneously from centibar to decibar as unit-pressure, the equation takes the form

$$(a) \qquad H_b - H_a = -\int_{p_a}^{p_b} \alpha dp$$

Eliminating the specific volume by the equation of state

(*b*)
$$\alpha = \frac{R\vartheta_r}{p}$$

we get

(*c*)
$$H_b - H_a = -R\int_{p_a}^{p_b} \vartheta_r \frac{dp}{p}$$

or

(*d*)
$$H_b - H_a = -R\int_{p_a}^{p_b} \vartheta_r d \text{ nat. log. } p$$

Considering henceforth nat. log. p as the independent variable, and taking outside the integral sign the average value ϑ_{ab} of the virtual temperature, we get the simple formula

(*e*)
$$H_b - H_a = R\vartheta_{ab} \text{ nat. log. } \frac{p_a}{p_b}$$

giving the dynamic height from the isobaric surface p_a to the isobaric surface p_b.

The defined average value ϑ_{ab} of the variable virtual temperature ϑ_r has a simple meaning: It is that constant temperature which, substituted for the variable temperature ϑ_r gives the sheet between the two isobaric surfaces p_a and p_b its true thickness. This average value is easily found by the virtual-temperature diagram, this diagram being drawn with logarithmic scale for the pressure, as in fig. 5, example 1 below. Here the horizontal lines correspond to constant pressures and the vertical lines to constant temperatures. Three curves running close together are seen in the diagram. The middlemost is that representing the virtual temperatures derived from the observations. The vertical segments of line give the required average values of the virtual temperature of each of the standard isobaric sheets. Each segment is drawn so that the two triangular areas limited by the urve, the segment, and the two standard isobaric lines are equal. These vertical segments may generally be drawn free-hand with a precision exceeding that of the observations from which the curve of virtual temperature has been derived. Of course greater precision, if required, may be obtained by use of a planimeter.

51. **Fundamental Tables.** — The sheet between the isobaric surfaces p_a and p_b will generally contain a set of isobaric standard sheets. The height $H_b - H_a$ can therefore be calculated as the sum of three terms: (1) the height from the isobaric surface p_a to the nearest standard surface; (2) the height from this standard surface to a certain higher standard surface; (3) the height from the last standard surface to the isobaric surface p_b.

To find height (2), we determine the thickness of any standard sheet. Let n be the pressure in any standard surface. The pressure being measured in decibars, n will have one of the values 1, 2, 3, . . . 10. The thickness $H_{n,\,n-1}$ of the standard sheet between the surfaces n and $n-1$ is obtained if in the fundamental formula section 50 (*e*) we introduce $p_a = n$, $p_b = n - 1$. Substituting further for R its numerical value when the pressure is expressed in decibars, $R = 28.7$, and writing

$273+\tau$ instead of ϑ, τ being the virtual temperature counted from the freezing-point of water, we get this expression for the thickness of the standard sheet:

$$(a) \qquad H_{n,n-1} = 28.7\,(273 + \tau_{n,n-1}) \text{ nat. log. } \frac{n}{n-1}$$

Height (2) can evidently be found as the sum of a certain number of heights given by formula (*a*).

The determinations of the heights (1) and (3) are different forms of one problem, namely, the determination of the distance $H_{n,p}$ from a certain standard surface of pressure n to any isobaric surface of pressure p. By the fundamental formula this distance is

$$H_{n,p} = 28.7\,(273 + \tau_{n,p}) \text{ nat. log. } \frac{n}{p}$$

$\tau_{n,p}$ being the average virtual temperature for the sheet of air between the isobaric surfaces n and p. This formula containing two continuously variable quantities, $\tau_{n,p}$ and p, is not immediately suited for tabulation. But it may be written as a sum of two terms, a principal term H_0 containing only one variable p, and a correction term ΔH containing two variables, namely, H_0 and τ. We thus write

$$(b) \qquad H_{n,p} = H_0 + \Delta H$$

giving H_0 and ΔH respectively the values

$$(b') \qquad H_0 = 7835 \text{ nat. log. } \frac{n}{p}$$

$$(b'') \qquad \Delta H = H_0 \frac{\tau_{n,p}}{273}$$

Thus, tabulating the three formulæ (*a*), (*b′*), and (*b″*), we shall easily be able to calculate the height from any isobaric surface p_a to any isobaric surface p_b, the required values of the virtual temperature being given. Only three tables would therefore be necessary. But for practical reasons, however, we shall give two different tabulations of formula (*b′*), arranging the table in a special form for the important case of p_a being the pressure at the earth's surface. Thus in the second tabulation of formula (*b′*) the height (1) is the height from the ground to the nearest standard surfaces. We then get the following four tables.

(A) *Table 9 M.—Mutual distances measured in dynamic meters between standard isobaric surfaces.*—This table contains nine small tables in succession, each giving, according to formula (*a*), the thickness of one of the standard sheets of the atmosphere for practically occurring values of the average virtual temperature. These successive tables are separated the one from the other by horizontal lines representing standard surfaces, the pressures of which are added in millibars.

(B) *Table 10 M.—Distances in dynamic meters, measured from the standard isobaric surfaces to points of given pressure, the average virtual temperature of the sheet being 0° C.*—This table contains ten small tables in succession, calculated according to formula (*b′*). Each gives the distance from one standard isobaric surface to all other isobaric surfaces situated below the next higher and above the next lower standard surface. The distance is counted positive upward and negative downward. It gives the first approximation value H_0 of the height $H_{n,p}$ from the standard isobaric surface n to a point of the pressure p.

(C) *Table 11 M.—Distances in dynamic meters from the earth's surface to the nearest standard isobaric surfaces, the average virtual temperature of the sheet being 0° C.*—This table differs from the preceding one only in the arrangement. As argument appears the pressure observed at the earth's surface. The tabulated quantities are the distances to the two nearest standard surfaces above and to the nearest standard surface below the point where the pressure is observed. The distance upward from the earth is counted positive and the distance downward negative. Of course the standard surface below the earth has no real existence. It may, however, for obvious reasons, be useful to bring it under consideration. The table gives the first approximation H_0 to the height $H_{p,n}$ from the earth's surface, where the pressure is p, to the standard surface of pressure n.

(D) *Table 12 M.—Corrections to tables 10 M and 11 M for temperature.*—This table is calculated according to formula (b'') with the two arguments, the average virtual temperature $\tau_{n,p}$ between the two surfaces n and p and the height H_0 found from table 10 M or 11 M. This correction for temperature may be either positive or negative according to the sign of the temperature $\tau_{n,p}$. But as it has equal numerical values for equal numerical values of τ, it will not be necessary to introduce signs in the table. The first part of the table, extending to the value 1100 dynamic meters for height H_0 and to values ± 34° C. for temperature, will be sufficient for most cases practically occurring. The continuation gives the extension to the limiting height of 10,000 dynamic meters and to the values ± 100° C. of temperature.

52. Calculation of the Height Corresponding to a Given Pressure.—If the virtual-temperature diagram be given as a curve connecting virtual temperature and pressure, tables 9 M to 12 M will at once enable us to calculate the height corresponding to any pressure. From the diagram (fig. 5), we take the average virtual temperature, first between the earth's surface, where the pressure is p_a, and the lowest standard surface, then between the successive standard surfaces, and finally between the highest of these and the isobaric surface of the given pressure p_b. By tables 11 M and 12 M we then find the height of the lowest standard surface above the earth, by table 9 M the thickness of the successive standard sheets, and by tables 10 M and 12 M the height of the given isobaric surface above the highest standard surface. Adding these heights we get the height of the isobaric surface p_b above the ground, and adding the height of the ground above sea-level we get the height above sea-level of the given isobaric surface. As all the other heights, that of the ground is to be expressed in dynamic meters. The first of the problems defined in section 49 will then be solved.

Suppose now the virtual-temperature diagram to be given with the heights in dynamic meters as ordinates (fig. 6). The pressure observed at the station at the earth's surface gives by means of table 11 M an approximation value H_0 of the height above the station of the lowest standard isobaric surface. By means of this approximate value we may, with sufficient precision, take from the diagram the average virtual temperature of the sheet. This temperature used in table 12 M gives the correction ΔH, which added to the approximation value H_0 gives with sufficient correctness the height of the surface above the station. Adding the height of the station we get the height of the surface above sea-level. This height being known, we estimate a value for the height to the next standard surface. This is easily done with fair approximation by the inspection of the virtual-temperature diagram and by comparison with the corresponding heights in table 9 M. For this estimate of height the value of the average virtual temperature is taken from the diagram. Using this value in table 9 M, we generally find the height to the next standard surface with sufficient precision. Otherwise the operation may be repeated, giving for every repetition a more accurate value. The final value of the height found in this manner added to the height of the first standard surface gives the height of the second standard surface. Then the distance to the next standard surface is estimated, the corresponding average virtual temperature determined from the diagram, and this temperature used to find a better value for the distance by means of table 9 M, and so on.

To complete the solution of the problem we finally determine the distance $H_{n,p}$ from one of the standard surfaces, the height of which is found, to a neighboring isobaric surface of the given pressure p_b. An approximation value H_0 of the height is found at once from table 10 M. Using this approximate value we find the average virtual temperature of the sheet from the diagram, and by means of this temperature we find from table 12 M the correction ΔH, which, added to the first approximation value H_0, gives the required height $H_{n,p}$.

The second of the problems defined in section 49 is thus solved.

53. Calculation of the Pressure at a Given Height. — Let H be the given height at which the pressure is to be found. We then determine first, as described in the preceding section, the height of the standard isobaric surfaces. Now, let p_n be the standard surface whose height H_n is nearest the given height H. The problem is then reduced to finding the pressure p at the height $H - H_n$ above the standard surface of pressure n.

Now, the height $H - H_n$ is the quantity tabulated in table 10 M, and the argument is the corresponding pressure p. Then if the average virtual temperature of the sheet of air between the heights H_n and H happens to be 0° C., table 10 M used inversely immediately gives the required pressure.

As a rule, however, this average temperature will have another value, τ. This temperature being known, we can avail ourselves of a simple artifice, determining a difference of height $H' - H_n$ defined by the property of being the height, which, used in table 10 M, gives the required pressure p.

The difference of pressure corresponding to a given difference of height is in inverse proportion to the specific volume of the sheet of air between the two heights, and therefore also in inverse proportion to the average virtual temperature of this sheet, reckoned from absolute zero. Thus the two heights $H' - H_n$ and $H - H_n$ must be in the proportion

$$H' - H_n = (H - H_n)\frac{273}{273 + \tau}$$

Subtracting $H - H_n$, we find the following value for the required correction:

$$\Delta H = H' - H = -(H - H_n)\frac{\tau}{273 + \tau}$$

This may finally be written in the form

$$(a) \qquad \Delta H = (H - H_n)\frac{\tau'}{273}$$

the auxiliary quantity τ' having the value

$$(b) \qquad \tau' = -\frac{273\tau}{273 + \tau}$$

Formula (a) has the same form as formula (b''), section 51, tabulated in table 12 M. But, to use table 12 M for the determination of the height correction ΔH in the case now treated, we have to use the artificial temperature τ' instead of the real temperature τ. This artificial temperature is tabulated according to formula (b) in table 13 M of our Meteorological Tables. Using this table in connection with tables 9 M to 12 M, we can calculate the pressure at any given height.

The practical procedure will turn out somewhat differently according as the virtual temperature is known at a given height or for a given pressure. In both cases we first determine the height of the standard isobaric surface as described in the preceding article. Then, if the virtual temperature be known for a given height, we immediately find from the diagram the average virtual temperature τ for the sheet between the heights H and H_n. On the other hand, if the virtual temperature be known for a given pressure, we use table 10 M to find an approximate value p' of the pressure at the height H. Taking from the diagram the average virtual temperature between the pressures n and p' we get a temperature τ, which with sufficient approximation can be identified with the average virtual temperature of the sheet between the heights H_n and H.

This temperature τ being found, we take the corresponding artificial temperature τ' from table 13 M. Using this and the height $H - H_n$ in table 12 M, we find the required correction ΔH. This correction added to the height $H - H_n$ gives the height $H' - H_n$, which used in table 10 M gives the required pressure p at the height H.

The third and fourth of the problems defined in section 49 are thus solved.

54. Examples of a Complete Interpretation of the Results of a Meteorological Ascent.— On pages 68–75 are given the schemes for the complete hydrostatic application of the observations obtained, under different suppositions, from an ascent in the air with meteorological instruments. It will be evident that, the hydrostatic results contained in these schemes being once worked out, a set of supplementary results of general meteorological interest might easily have been obtained. We may for instance mention temperatures and humidities at given heights for given pressures, or average values of these quantities for given height-sheets or pressure-sheets. But in order not to complicate the schemes we have taken up only what is of interest for as full an illustration as possible of the developed hydrostatic methods.

The examples are derived from the observations obtained by the celebrated balloon ascent by Berson and Süring from Berlin July 31, 1901, to the greatest height yet attained by man.* The height of the station, Tegel at Berlin, was 40 meters or 39 dynamic meters above sea-level. The observed quantities during the ascent were time, pressure in millimeters of mercury, temperature centigrade, and relative humidity. From the general remarks in the preceding articles and by the small examples added to each table in the Meteorological Tables, the schemes will easily be understood. We shall therefore content ourselves with a few general remarks.

In the first example (page 68) we have made a direct use of the observed data only supposing the pressure to have been observed in millibars instead of in millimeters of mercury.

This first example being worked out, we have constructed the second, considering the calculated heights (column 24 of table J) as observed quantities, column 2 of table K. We have preferred thus to derive example 2 artificially from example 1, instead of taking an independent example, where the heights have been really observed; for the analogy and the contrast of the methods are better illustrated when both are used to work out the same case of atmospheric equilibrium. Comparing the two schemes, we see that the difference amounts mainly to an interchange of the order of the columns, followed by a passage from direct methods to methods of estimation, or *vice versa*.

In connection with this second example it is important to emphasize that a observed heights should be considered only those found according to rational geometrical methods, as for instance when the height of a kite is determined by the angle and the length of the kite-line. The use, on the contrary, of a barometer with height-scale instead of pressure-scale is unscientific. It gives less trustworthy results, and at the same time additional labor; for the working out of the results according to example 2 is more laborious than the corresponding work according to example 1. In some cases both pressure and height may be observed. The observations then give directly the equilibrium relation between pressure and height. But on account of the imperfections of the aneroid barometer, the relation found in this direct way will be much less accurate than that found by one of the above methods, the observations either of pressure or of height being provisionally set aside. The derivation of the results according to both methods, once omitting

* Veröffentlichungen des K. Preussischen Meteorologischen Instituts. R. Assmann und A. Berson: Ergebnisse der Arbeiten am Aeronautischen Observatorium 1900–1901. Berlin, 1902. p. 227.

the observed pressures and once the observed heights, will give a valuable control, especially useful in correcting the records of the barometer.

The common result of examples 1 and 2 is illustrated graphically by the three verticals of fig. 4, the principle of the representation being the same as that used previously in figs. 1 and 2. The first vertical, representing the equilibrium relation between dynamic height and pressure, may be constructed from columns 5 and 8 or 11 and 18 of table J, or from the corresponding columns 6 and 9 or 12 and 18 of table K. The divisions on the second vertical, representing the equilibrium relation between pressure and specific volume, are drawn according to the figures in columns 5, 7, and 10 of table J, or the corresponding columns 6, 8, and 11 of table K. The divisions on the third vertical, giving the equilibrium relation between dynamic height and density, are drawn in accordance with the figures contained in columns 11 and 19 of table J, or 12 and 19 of table K: To obtain greater accuracy the specific volumes in column 10, table J, or column 11, table K, have been changed into densities and used to correct the divisions.

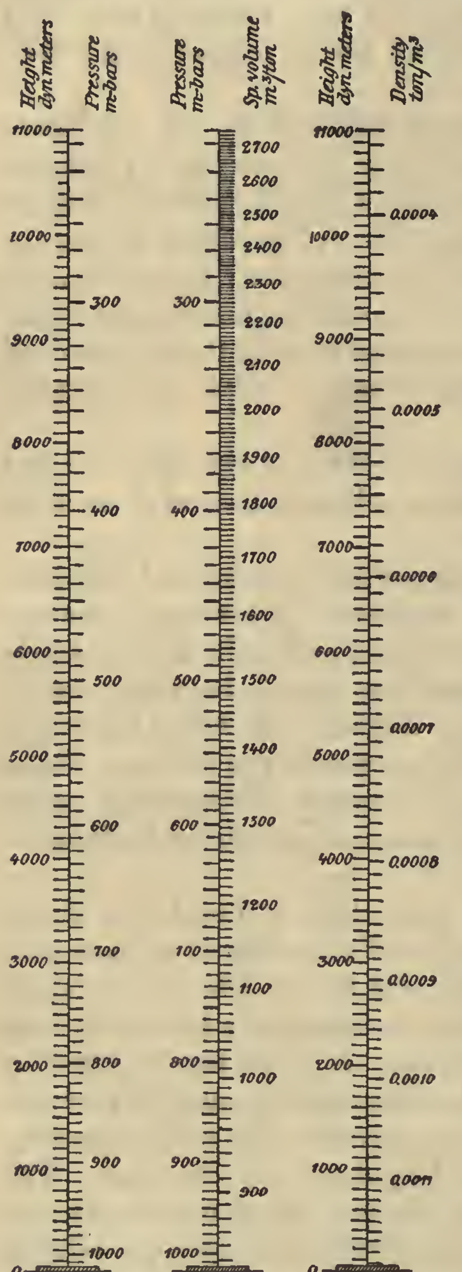

Fig. 4.—State of atmospheric equilibrium above Berlin, July 31, 1901.

We shall later make important practical applications of verticals as those of fig. 4, drawing vertical sections as in figs. 13, 14, 21, 24 below. The most important use will be made, however, of the numbers contained in columns 8 and 7 of table J, respectively, 9 and 8 of table K, *i. e.*, the numbers giving the height of the standard isobaric surfaces and the average specific volume of the air between them, and the numbers contained in columns 18 and 19 of both tables, *i. e.*, the numbers giving the pressure in standard level surfaces and the average density of the air between them. From such numbers as these we shall draw synoptical charts such as those found in Chapter VII, representing in two different ways the distribution of pressure and of mass in the atmosphere.

Example 1.—*Observed time, pressure (m-bars), temperature (° C.), humidity (per cent).* (Table J.)—From the observed pressures and temperatures (columns 2 and 3) the curve of true temperatures is drawn (curve to the left in fig. 5). By means of table 7 M the curve of virtual temperature for saturated air is drawn (curve to the right). Using the percentages of humidity (column 4) the curve of virtual temperature is drawn between the other two curves. (See section 23.)

The vertical segments of line determining the average virtual temperature of the standard sheets are drawn, and the corresponding temperatures read off (column 6). The mutual distances between the standard surfaces (column 7) and the heights of these surfaces (column 8) are determined. In addition the virtual temperatures at the standard surfaces (column 9) may be read off from the diagram, and the corresponding specific volume of the air determined (column 10).

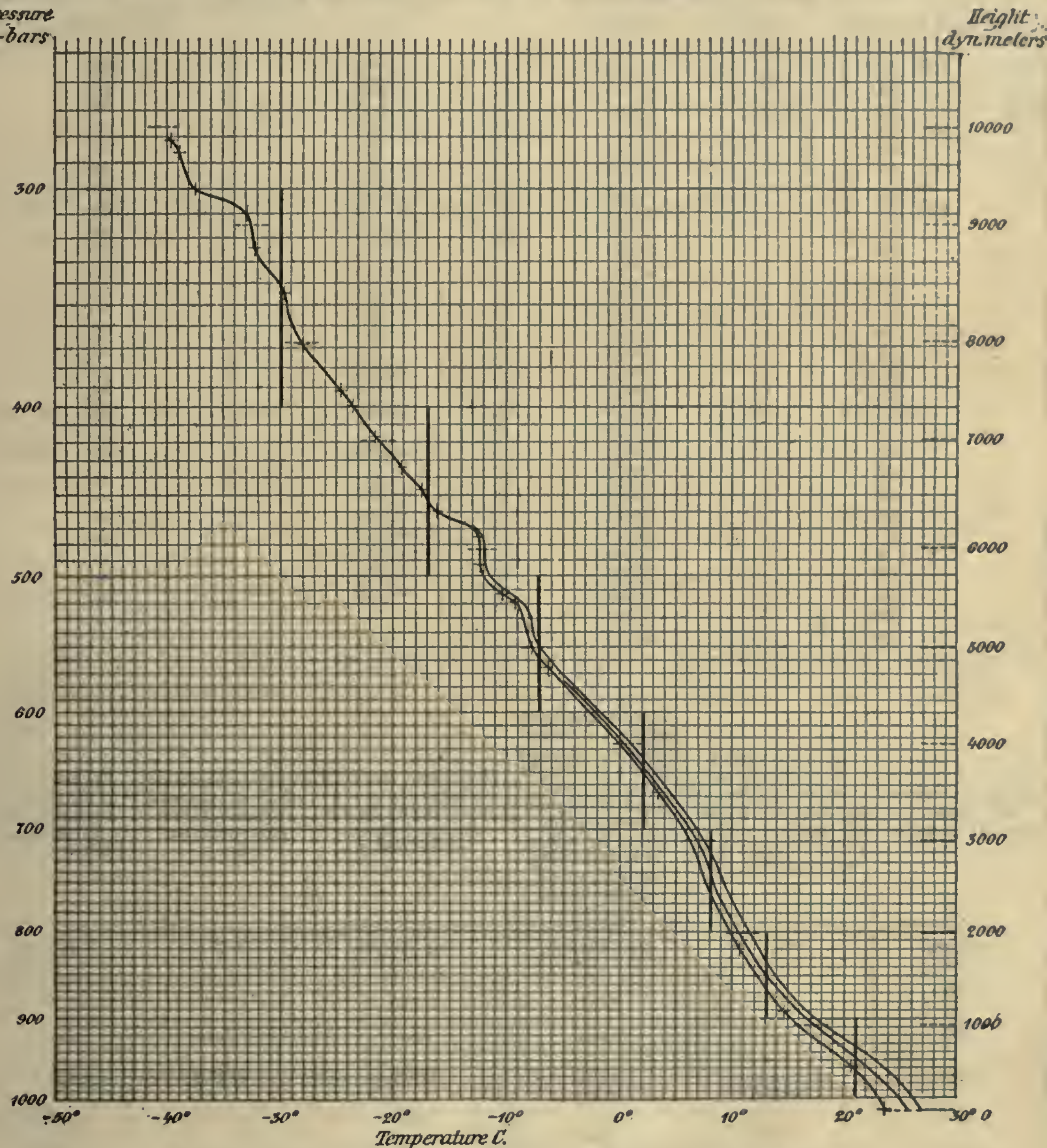

FIG. 5. — Virtual-temperature diagram, with logarithmic pressure-scale.

Columns 11 to 19 give the solution of the inverse problem, the determination of the pressure at a standard height. The dotted horizontal lines in the diagram represent the approximate situation of the level surfaces, these lines being drawn according to the approximate pressures in the heights given in column 14. Not to complicate the figure, the vertical segments giving the virtual temperatures in column 15 are not drawn.

Columns 20 to 24 give the determination of the heights from which the observations in columns 1 to 4 are taken. The horizontal and vertical lines in the diagram required for this determination are not given in fig. 5. It should be emphasized that the determination of these heights is independent of the solution of the inverse problem (columns 11 to 19) and dependent only upon the knowledge of the heights of the standard isobaric surfaces (column 8).

TABLE J (EXAMPLE 1).

1	2	3	4
Observed time.	Observed pressure.	Observed temperature.	Observed relative humidity.
15th meridian.	m-bars.	° C.	Per cent.
10h 50m	1015.9	23.4	72
54	957.9	20.4	58
57	889.9	14.6	62
11 00	819.3	10.5	46
16	665.3	3.5	23
30	563.9	− 6.3	42
44	549.3	− 7.8	17
12 16	517.3	− 9.2	84
28	511.9	−10.5	31
40	492.0	−12.5	46
57	473.9	−12.5	38
1 00	460.0	−16.1	51
11	449.0	−17.3	70
20	433.7	−19.2	67
34	421.0	---	--
39	416.4	−21.6	74
46	400.4	−23.3	83
51	392.0	−24.7	85
2 1	383.7	---	--
13	369.3	−27.8	95
25	357.9	---	--
28	345.3	−29.6	--
52	324.6	−32.2	--
55	309.7	−33.0	--
3 3	300.0	−37.3	--
10	285.9	−38.9	--
28	280.6	−39.7	--
32	270.0	---	--
40	257.3	---	--

5	6	7	8	9	10
Standard pressures and pressure (1015.9) at station.	Average virtual temperature in standard sheets and in sheet below lowest standard surface (+25.1) found from diagram (fig. 5).	Mutual distances between standard surfaces, found by table 9 M; height 135, of lowest standard surface above station, found by tables 11 M and 12 M; height, 39, of station above sea-level.	Heights of standard surfaces above sea-level, found by addition of figures of column 7.	Virtual temperature of air at standard surfaces, found from diagram (fig. 5).	Specific volume of air at standard surfaces, found by table 14 M (m^3/ton = cm^3/gr.).
m-bars.	° C.	Dyn. meters.	Dyn. meters.	° C.	m^3/ton.
100	----	----	----	----	----
200	----	----	----	----	----
300	----	----	9365	−37.3	2255
	−29.8	2009			
400	----	----	7356	−23.7	1789
	−17.0	1640			
500	----	----	5716	−11.9	1499
	− 7.2	1391			
600	----	----	4325	− 2.1	1296
	+ 2.2	1218			
700	----	----	3107	+ 5.8	1143
	+ 8.0	1077			
800	----	----	2030	+10.1	1015
	+13.0	967			
900	----	----	1063	+16.5	923
	+21.0	889			
1000	----	----	174	+24.7	855
	+25.1	135			
1015.9	----	----	----	----	----
		39			

TABLE J (EXAMPLE 1)—Continued.

11	12	13	14	15	16	17	18	19
Standard heights.	Nearest standard isobaric surfaces.	Distances from standard isobaric surfaces of column 12, heights of which are given in column 8, to standard heights of column 11.	Approximate pressures in standard heights, obtained by table 10 M as pressures corresponding to distances of column 13.	Average virtual temperatures for sheets between standard isobaric surfaces and isobaric surfaces of column 14, found from diagram (fig. 5).	Corresponding artificial temperature according to table 13 M.	Distances in column 13 artificially changed by addition of height corrections obtained from table 12 M as corresponding to distances in column 13 and temperatures in column 16.	Pressures in standard heights, found from table 10 M as pressures corresponding to artificial distances in column 17.	Average density of air in level sheets between standard heights of column 11. The figures are ten times the differences between pressures of column 18 (ton/m³ = gr/cm³).
Dyn. meters.	m-bars.	Dyn. meters.	m-bars.	° C.	° C.	Dyn. meters.	m-bars.	σ^{-4} ton/m
10000	300	+635	277	−39	45.5	+741	272.9	----
								435
9000	300	−365	314	−34	39	−417	316.4	----
								488
8000	400	+644	368	−26	29	+713	365.2	----
								551
7000	400	−356	419	−22.5	24.5	−388	420.3	----
								611
6000	500	+284	482	−12	12.5	+297	481.4	----
								682
5000	600	+675	550	− 5	5	+687	549.6	----
								759
4000	600	−325	625	− 1	1	−326	625.5	----
								840
3000	700	−107	709	+ 6	− 6	−105	709.5	----
								935
2000	800	− 30	803	+10.5	−10	− 29	803.0	----
								1038
1000	900	− 63	907	+17	−16	− 59	906.8	----
								1138
0	1000	−174	1022	+25	−23	−160	1020.6	----

20	21	22	23	24
Approximate values, found from table 10 M, of distances from nearest standard isobaric surfaces to isobaric surfaces of column 2.	Average virtual temperature of sheets between standard isobaric surfaces and surfaces of column 2, found from diagram (fig. 5).	Distances in column 20, corrected for temperature by table 12 M.	Heights of isobaric surfaces in column 2 = heights where observations of columns 1 to 4 are taken, found from columns 8 and 22.	Heights where observations are taken, reduced from dynamic meters to meters by tables 5 M and 6 M, using $g_0 = 9.8128$.
Dyn. meters.	° C.	Dyn. meters.	Dyn. meters.	Meters.
− 123	25	− 135	39	40
+ 337	23	+ 365	539	549
+ 89	16	+ 94	1157	1180
− 187	10.5	− 194	1836	1872
+ 399	5	+ 406	3513	3583
+ 486	− 4.5	+ 478	4803	4900
+ 692	− 5	+ 679	5004	5104
− 267	−10.5	− 257	5459	5568
− 184	−11	− 177	5539	5650
+ 126	−12	+ 120	5836	5954
+ 420	−12.5	+ 401	6117	6241
+ 653	−13	+ 622	6338	6467
+ 843	−14	+ 800	6516	6648
− 634	−21	− 585	6771	6908
− 401	−22	− 369	6987	7130
− 315	−22.5	− 289	7067	7211
− 8	−23.5	− 7	7349	7499
+ 158	−24	+ 144	7500	7653
+ 326	−24.8	+ 297	7653	7810
+ 626	−26	+ 566	7922	8083
+ 871	−26.5	+ 786	8142	8309
−1102	−32.5	− 971	8394	8567
− 618	−33.5	− 542	8823	9004
− 250	−35	− 218	9147	9336
0	−37	0	9365	9559
+ 377	−38	+ 324	9689	9891
+ 524	−38.5	+ 450	9815	10019
+ 826	−39	+ 708	10073	10283
+1203	−40	+1027	10392	10610

Example 2. — *Observed height (meters), temperature (°C.), and humidity (per cent).* (Table K). — The observed geometric height (column 2) is changed into dynamic height (column 5). From these heights and the observed temperatures the curve of true temperature is drawn (curve to the left in fig. 6). By means of table 8 M the curve of virtual temperature for saturated air is drawn (curve to the right). Using the percentages of humidity (column 4) the curve of virtual temperature is drawn between the two others.

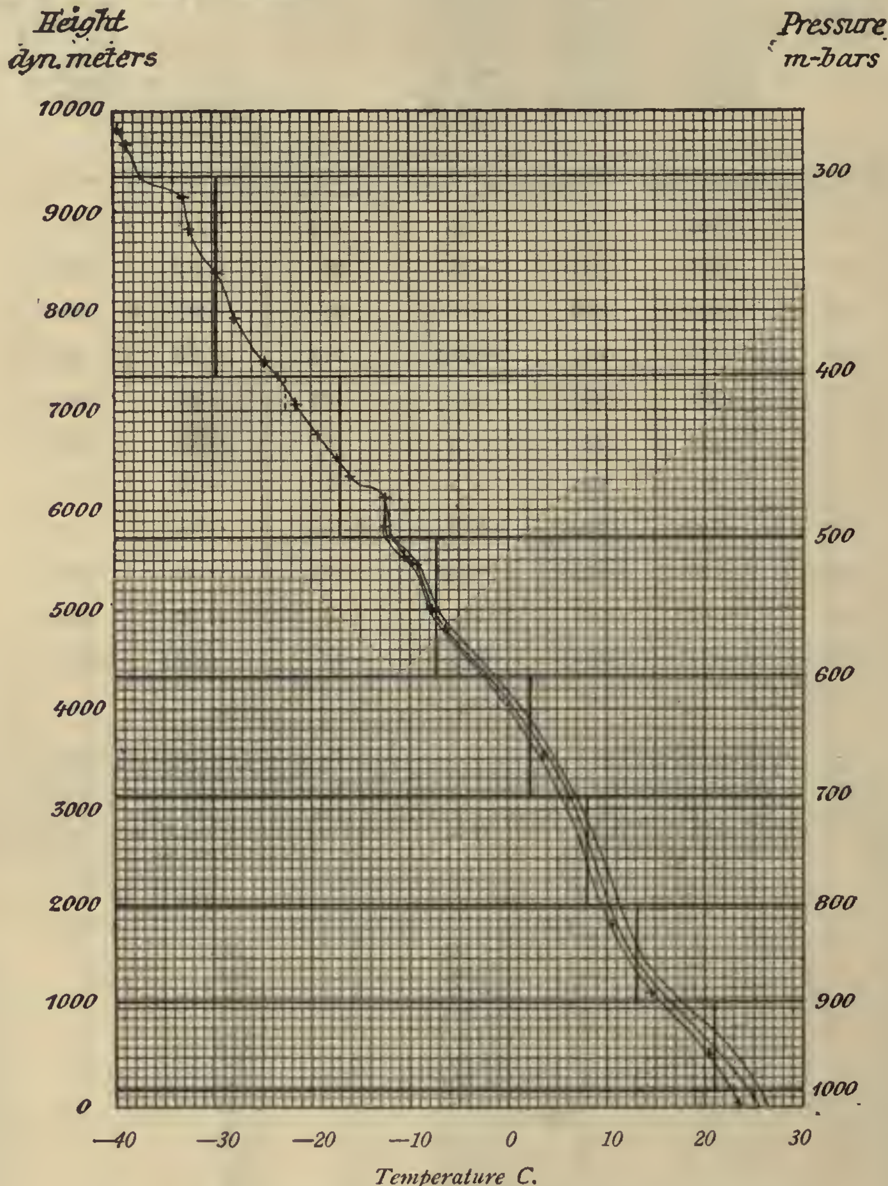

Fig. 6. — Virtual-temperature diagram, with dynamic height as ordinate.

The heavy horizontal lines represent the standard isobaric surfaces, successively drawn according to the estimated thickness of the standard sheets (section 52). The vertical segments of line give the average virtual temperatures of the sheets (column 7) by which the more accurate thickness of the sheets (column 8) and the heights of the surfaces (column 9) are determined.

The solution of the inverse problem, the determination of the pressure at a given height (columns 12 to 19) is found mainly by the same method as in the preceding example, except that one operation less is necessary to find the required virtual temperatures (column 15). Not to complicate the figure, the vertical segments giving these temperatures are not drawn.

Columns 20 to 24 give the determination of the pressures at the heights where the observations were taken. The lines in the diagram required for this determination are not given in fig. 6. The determination of these pressures is independent of the solution of the inverse problem, and dependent only upon the knowledge of the heights of the standard isobaric surfaces (column 9).

The discussion and the comparison of the examples 1 and 2 is important in connection with the practical question as to the choice of method of observation, as well as in connection with the theoretical question as to the choice of method for representing the result of the observations.

It is seen that from the point of view of the computer it is no advantage at all to have the height instead of the pressure as one of the observed quantities. Observed heights being always geometric heights, they have first to be changed into dynamic heights, and then the average virtual temperature of the standard isobaric sheets must be found by the method of conjectures instead of by the direct way which can be followed when the pressure is one of the observed quantities. When some practice is acquired, these conjectures can easily be made with sufficient precision to make repetitions of the operation superfluous. But still the convenience of the direct method can not be attained. Thus, as far as the observations of pressure can be obtained with the same precision as those of height, the observations of pressure should be preferred as those giving least trouble to the computer. In no case the values of pressure should be left out in the publications of the result of the meteorological ascents, as it is unfortunately sometimes done, height being substituted for pressure as the result of the calculations. But especially we must warn, as we have already done, against the use of barographs with height scale instead of pressure scale. For in addition to the increased trouble to the computer, this method will give much less trustworthy results.

On the other hand, it is seen that the calculation of the height of standard pressures is in all cases easier than the calculation of pressure in standard heights. This is equally true whether it is pressure or geometric height which is observed. Even if it may be possible to further simplify the methods developed for calculating pressure in standard dynamic heights, it is not probable that it should be possible to attain the simplicity of the method given for calculating the dynamic height of standard pressures.

In the choice between the two theoretically equivalent methods of representing the distribution of pressure, viz, that of registering the height of standard pressures or that of registering pressure in standard heights, we have thus found an important practical reason for preferring the first method, that of registering the height of standard pressures.

Whichever method of observation be used, and whichever method of representing the results be preferred, it is seen that the fundamental operation remains that of drawing and interpreting the virtual-temperature diagram. What can be done to facilitate this work will therefore be of the highest practical importance. In this respect the hints given in the next section (55) will be useful.

TABLE K (EXAMPLE 2).

1	2	3	4	5
Observed time.	Observed height.	Observed temperature.	Observed relative humidity.	Heights of column 2 reduced to dynamic heights by tables 3 M and 4 M, using $g_0 = 9.8128$.
15th meridian.	Meters.	° C.	Per cent.	Dyn. meters.
10h 50m	40	23.4	72	39
54	549	20.4	58	539
57	1180	14.6	62	1157
11 00	1872	10.5	46	1836
16	3583	3.5	23	3513
30	4900	− 6.3	42	4803
44	5104	− 7.8	17	5004
12 16	5568	− 9.2	84	5459
28	5650	−10.5	31	5539
40	5954	−12.5	46	5836
57	6241	−12.5	38	6117
1 00	6467	−16.1	51	6338
11	6648	−17.3	70	6516
20	6908	−19.2	67	6771
34	7130	----	--	6987
39	7211	−21.6	74	7067
46	7499	−23.3	83	7349
51	7653	−24.7	85	7500
2 1	7810	----	--	7653
13	8083	−27.8	95	7922
25	8309	----	--	8142
28	8567	−29.6	--	8394
52	9004	−32.2	--	8823
55	9336	−33.0	--	9147
3 3	9559	−37.3	--	9365
10	9891	−38.9	--	9689
28	10019	−39.7	--	9815
32	10283	----	--	10073
40	10610	----	--	10392

6	7	8	9	10	11
Standard pressures and pressure 1015.9 at station.	Average virtual temperature of standard sheets, and of sheet below lowest standard surface, found from diagram (fig. 6) by use of conjectured thicknesses of sheets.	Mutual distances between standard surfaces, found by table 9 M; height 135 of lowest standard surface above station, found by tables 11 M and 12 M; height 39 of station above sea-level.	Heights of standard isobaric surfaces above sea-level, found by addition of figures in column 8.	Virtual temperature of air at standard surfaces found from diagram (fig. 6).	Specific volume of air at standard surfaces found by table 14 M (m³/ton = cm³/gr.).
m-bars.	° C.	Dyn. meters.	Dyn. meters.	° C.	m³/ton.
100	----	----	----	----	----
200	----	----	----	----	----
300	----	----	9365	−37.3	2255
	−29.8	2009			
400	----	----	7356	−23.7	1789
	−17.0	1640			
500	----	----	5716	−11.9	1499
	− 7.2	1391			
600	----	----	4325	− 2.1	1296
	+ 2.2	1218			
700	----	----	3107	+ 5.8	1143
	+ 8.0	1077			
800	----	----	2030	+10.1	1015
	+13.0	967			
900	----	----	1063	+16.5	923
	+21.0	889			
1000	----	----	174	+24.7	855
	+25.1	135			
1015.9	----	----	----	----	----
		39			

TABLE K (EXAMPLE 2)—Continued.

12	13	14	15	16	17	18	19
Standard heights.	Nearest standard isobaric surfaces.	Distances from standard isobaric surfaces of column 13, heights of which are given in column 9, to standard heights of column 12.	Average virtual temperatures for sheets between standard heights in column 12 and heights of standard isobaric surfaces in column 9, found from diagram (fig. 6).	Corresponding artificial temperatures according to table 13 M.	Distances in column 14, artificially changed by addition of height corrections obtained from table 12 M as corresponding to heights in column 14 and temperatures in column 16.	Pressure in standard heights, found from table 10 M as pressures corresponding to distances in column 17.	Average density of air in level sheets between standard heights of column 12. The figures are ten times the differences between pressures of column 18.
Dyn. meters.	m-bars.	Dyn. meters.	° C.	° C.	Dyn. meters.	m-bars.	10^{-4} ton/m³.
10000	300	+635	-39	+45.5	+741	272.9	----
							435
9000	300	-365	-34	+39	-417	316.4	----
							488
8000	400	+644	-26	+29	+713	365.2	----
							551
7000	400	-356	-22.5	+24.5	-388	420.3	----
							611
6000	500	+284	-12	+12.5	+297	481.4	----
							682
5000	600	+675	- 5	+ 5	+687	549.6	----
							759
4000	600	-325	- 1	+ 1	-326	625.5	----
							840
3000	700	-107	+ 6	- 6	-105	709.5	----
							935
2000	800	- 30	+10.5	-10	- 29	803.0	----
							1038
1000	900	- 63	+17	-16	- 59	906.8	----
							1138
0	1000	-174	+25	-23	-160	1020.6	----

20	21	22	23	24
Distances from standard isobaric surfaces (column 9) to heights in column 5.	Average virtual temperature for sheets in column 20.	Corresponding artificial temperatures according to table 13 M.	Distances in column 20 artificially changed by addition of height corrections obtained from table 12 M, using distances in column 20 and temperatures in column 22.	Pressures in observed heights of column 2, found from table 10 M as pressures corresponding to artificial distances in column 23.
Dyn. meters.	° C.	° C.	Dyn. meters.	m-bars.
- 135	25	-22.9	- 123	1015.9
+ 365	23	-21.2	+ 337	957.9
+ 94	16	-15.1	+ 89	889.9
- 194	10.5	-10.1	- 187	819.3
+ 406	5	- 4.9	+ 399	665.3
+ 478	- 4.5	+ 4.6	+ 486	563.9
+ 679	- 5	+ 5.1	+ 692	549.3
- 257	-10.5	+10.9	- 267	517.3
- 177	-11	+11.5	- 184	511.9
+ 120	-12	+12.6	+ 126	492.0
+ 401	-12.5	+13.2	+ 420	473.9
+ 622	-13	+13.7	+ 653	460.0
+ 800	-14	+14.8	+ 843	449.0
- 585	-21	+22.8	- 634	433.7
- 369	-22	+23.9	- 401	421.0
- 289	-22.5	+24.5	- 315	416.4
- 7	-23.5	+25.7	- 8	400.4
+ 144	-24	+26.3	+ 158	392.0
+ 297	-24.8	+27.3	+ 326	383.7
+ 566	-26	+28.7	+ 626	369.3
+ 786	-26.5	+29.3	+ 871	357.9
- 971	-32.5	+36.8	-1102	345.3
- 542	-33.5	+38.1	- 618	324.6
- 218	-35	+40.1	- 250	309.7
0	-37	+42.8	0	300.0
+ 324	-38	+44.2	+ 377	285.9
+ 450	-38.5	+44.8	+ 524	280.6
+ 708	-39	+45.5	+ 826	270.0
+1027	-40	+46.9	+1203	257.3

55. Remarks on Virtual-Temperature Diagrams. — When many calculations are to be performed, much time may be saved if convenient blanks be prepared for the drawing of the virtual-temperature diagrams. The use of tables 7 M or 8 M may be completely dispensed with if the horizontal lines on these blanks be provided with *divisions showing the distance from the curve of true to that of*

TABLE L.— *Virtual-temperature divisions on lines representing standard isobaric surfaces. (Vertical columns in the table correspond to horizontal lines on the virtual-temperature diagram.)*

Pressure (m-bars).							
1000	900	800	700	600	500	400	300
−9.8	−10.7	−11.8	−13.1	−14.5	−16.0	−17.5	−19.7
−9.6	−10.5	−11.6	−12.9	−14.2	−15.7	−17.2	−19.4
−9.4	−10.3	−11.4	−12.6	−13.9	−15.4	−16.9	−19.1
−9.2	−10.1	−11.3	−12.4	−13.6	−15.1	−16.5	−18.8
−9.0	−9.9	−11.0	−12.1	−13.3	−14.8	−16.2	−18.5
−8.8	−9.7	−10.7	−11.8	−13.1	−14.5	−15.9	−18.2
−8.6	−9.5	−10.5	−11.6	−12.8	−14.2	−15.6	−17.9
−8.4	−9.3	−10.2	−11.3	−12.5	−13.8	−15.3	−17.5
−8.2	−9.0	−9.9	−11.0	−12.2	−13.4	−14.9	−17.1
−8.0	−8.8	−9.6	−10.7	−11.8	−13.0	−14.5	−16.7
−7.8	−8.5	−9.3	−10.3	−11.4	−12.6	−14.2	−16.3
−7.6	−8.3	−9.0	−10.0	−11.0	−12.3	−13.8	−15.9
−7.3	−8.0	−8.7	−9.7	−10.5	−11.9	−13.4	−15.5
−7.0	−7.7	−8.4	−9.3	−10.1	−11.5	−13.0	−15.1
−6.7	−7.4	−8.1	−8.9	−9.7	−11.1	−12.6	−14.6
−6.4	−7.0	−7.7	−8.5	−9.3	−10.6	−12.1	−14.1
−6.0	−6.6	−7.3	−8.1	−8.9	−10.1	−11.6	−13.6
−5.6	−6.2	−6.9	−7.7	−8.5	−9.6	−11.0	−13.0
−5.2	−5.8	−6.5	−7.3	−8.0	−9.1	−10.4	−12.3
−4.8	−5.4	−6.0	−6.8	−7.6	−8.5	−9.8	−11.6
−4.4	−4.9	−5.5	−6.3	−7.0	−7.8	−9.2	−10.8
−4.0	−4.5	−5.1	−5.7	−6.4	−7.2	−8.5	−10.0
−3.6	−4.1	−4.6	−5.1	−5.7	−6.5	−7.7	−9.2
−3.2	−3.6	−4.1	−4.5	−5.1	−5.8	−6.9	−8.3
−2.7	−3.1	−3.6	−3.9	−4.4	−5.0	−6.0	−7.3
−2.2	−2.6	−3.0	−3.2	−3.6	−4.2	−5.0	−6.2
−1.7	−2.0	−2.3	−2.5	−2.8	−3.3	−3.9	−5.0
−1.2	−1.4	−1.6	−1.7	−2.0	−2.3	−2.7	−3.7
−0.6	−0.7	−0.8	−0.9	−1.0	−1.2	−1.4	−1.9
0	0	0	0	0	0	0	0
0.6	0.7	0.8	0.9	1.0	1.3		
1.3	1.5	1.7	1.9	2.1	2.7		
2.0	2.3	2.6	2.9	3.3	4.3		
2.7	3.2	3.6	4.0	4.6	6.0		
3.5	4.0	4.6	5.2	6.1	8.0		
4.3	5.0	5.7	6.5	7.7	10.3		
5.1	6.0	6.9	7.9	9.6			
6.0	7.0	8.2	9.5	11.8			
7.0	8.2	9.6	11.2	14.3			
8.0	9.4	11.2	13.2				
9.1	10.8	13.0	15.5				
10.3	12.4	15.0					
11.6	14.1	17.3					
13.1	16.0	19.9					
14.7	18.2						
16.5	20.8						
18.5	23.7						
20.7	27.5						
23.4	31.9						
26.5							
30.3							
35.4							
42.2							

TABLE M.— *Virtual-temperature divisions on lines representing standard levels. (Vertical columns in the table correspond to horizontal lines on the virtual-temperature diagram.)*

Height (dynamic meters).							
0	1000	2000	3000	4000	5000	6000	7000
−9.8	−10.9	−12.0	−13.3	−14.5	−15.6	−16.6	−17.5
−9.6	−10.7	−11.8	−13.1	−14.2	−15.3	−16.3	−17.2
−9.4	−10.5	−11.6	−12.8	−13.9	−15.0	−16.0	−16.9
−9.2	−10.3	−11.5	−12.6	−13.6	−14.7	−15.7	−16.5
−9.0	−10.1	−11.2	−12.3	−13.3	−14.4	−15.4	−16.2
−8.8	−9.9	−10.9	−12.0	−13.1	−14.1	−15.1	−15.9
−8.6	−9.7	−10.7	−11.8	−12.8	−13.8	−14.8	−15.6
−8.4	−9.5	−10.4	−11.5	−12.5	−13.5	−14.4	−15.3
−8.2	−9.1	−10.1	−11.2	−12.2	−13.1	−14.0	−14.9
−8.0	−8.9	−9.7	−10.9	−11.8	−12.7	−13.6	−14.5
−7.8	−8.6	−9.4	−10.6	−11.4	−12.3	−13.2	−14.2
−7.6	−8.4	−9.1	−10.2	−11.0	−12.0	−12.9	−13.8
−7.3	−8.1	−8.8	−9.8	−10.5	−11.6	−12.5	
−7.0	−7.8	−8.5	−9.4	−10.1	−11.2	−12.1	
−6.7	−7.5	−8.2	−9.0	−9.7	−10.8	−11.7	
−6.4	−7.1	−7.8	−8.6	−9.3	−10.3	−11.2	
−6.0	−6.7	−7.4	−8.2	−8.9	−9.8	−10.7	
−5.6	−6.3	−7.0	−7.8	−8.5	−9.3	−10.2	
−5.2	−5.9	−6.6	−7.4	−8.0	−8.8	−9.6	
−4.8	−5.5	−6.1	−6.9	−7.6	−8.3	−9.0	
−4.4	−5.0	−5.6	−6.4	−7.0	−7.6	−8.3	
−4.0	−4.6	−5.0	−5.8	−6.4	−7.0	−7.7	
−3.6	−4.2	−4.5	−5.2	−5.7	−6.3	−7.0	
−3.2	−3.7	−4.0	−4.6	−5.0	−5.6	−6.2	
−2.7	−3.2	−3.5	−3.9	−4.4	−4.8	−5.4	
−2.2	−2.7	−2.9	−3.2	−3.6	−4.1	−4.7	
−1.7	−2.1	−2.2	−2.5	−2.8	−3.2	−3.5	
−1.2	−1.4	−1.6	−1.7	−2.0	−2.2	−2.5	
−0.6	−0.7	−0.8	−0.9	−1.0	−1.1	−1.3	
0	0	0	0	0	0	0	
0.6	0.7	0.8	0.9	1.0	1.2		
1.2	1.5	1.7	1.9	2.1	2.6		
1.8	2.3	2.6	2.9	3.3	4.0		
2.5	3.3	3.7	4.1	4.6	5.6		
3.2	4.1	4.7	5.3	6.1	7.5		
3.9	5.1	5.8	6.6	7.7	9.7		
4.7	6.1	7.1	8.0	9.6			
5.5	7.2	8.4	9.7	11.8			
6.4	8.4	9.9	11.4	14.3			
7.3	9.6	11.6	13.5				
8.2	11.1	13.4	15.9				
9.2	12.7	15.5					
10.3	14.5	17.9					
11.6	16.5	20.7					
13.0	18.8						
14.5	21.5						
16.2	24.6						
18.0	28.6						
20.0	33.5						
22.3							
25.0							
28.2							
32.1							

virtual temperature for saturated air. The annexed table L shows how these divisions should be drawn in uninterrupted succession on those horizontal lines which in the diagram represent standard pressures. Table M shows how the corresponding divisions should be drawn on those horizontal lines which in the diagram represent standard heights.

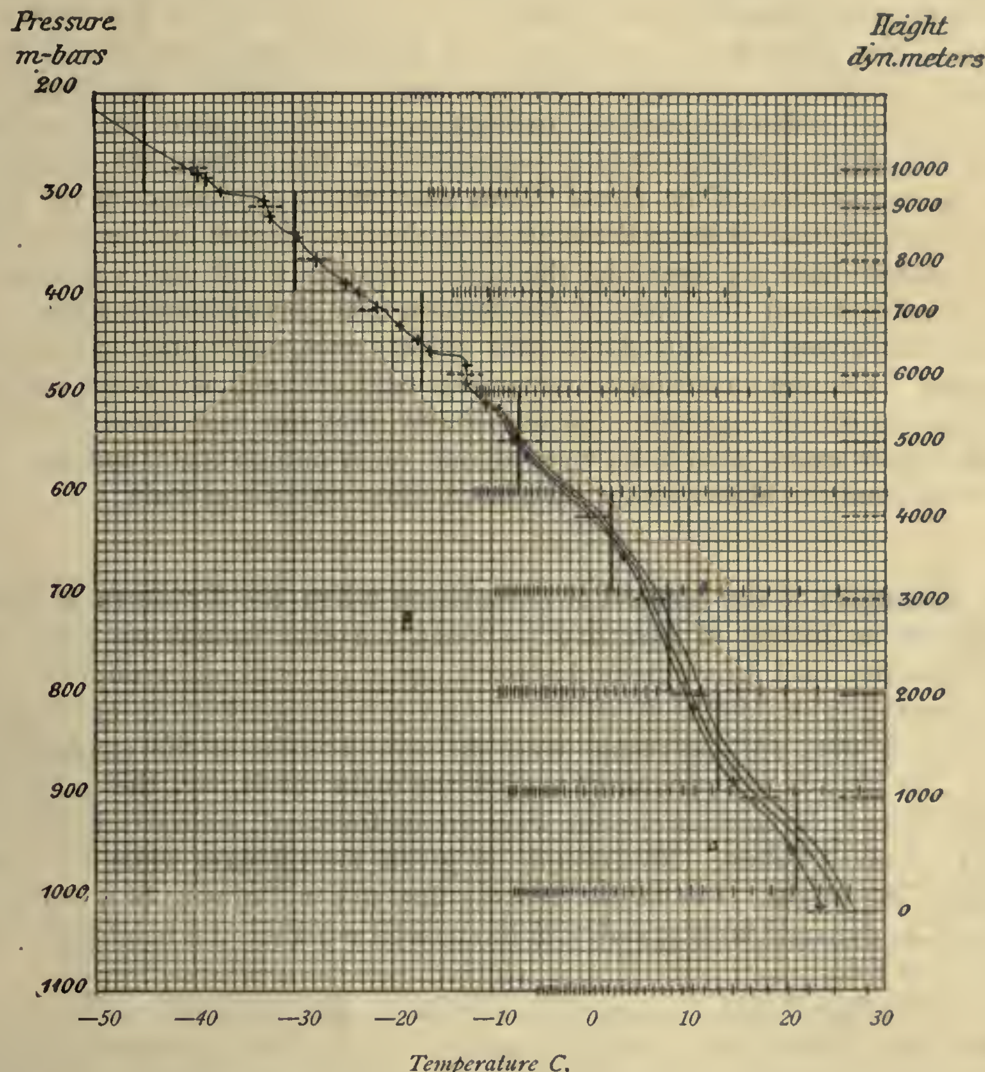

Fig. 7. — Virtual-temperature diagram with common pressure-scale and with virtual-temperature divisions.

In both tables the figures give the distances of the points of division from that ordinate, which in the diagram represents 0° C. They are found by a simple process of summation of the numbers contained in the tables of virtual temperature, respectively 7 M and 8 M. Examples of these divisions taken from table L and introduced on the lines representing standard pressures are shown in fig. 7 as well as in several of the following diagrams.

Theoretically it is correct to use a logarithmic scale of pressure. Practically, however, *the common pressure-scale* may be used without introducing error of any importance. Fig. 7 gives the same virtual-temperature diagram as fig. 5, but with a difference of scale. It is seen that the average virtual temperatures deduced from this diagram are practically the same as those deduced from fig. 5.

In reality the temperatures will be found a little too high from the diagrams with the common pressure-scale. The amount of error can be determined theoretically if we suppose the curve to be a straight line in the one of the two diagrams. It will then run up to 0.008 $(\tau_{10} - \tau_9)$ for the isobaric sheet between the 1000 and the 900 m-bar surfaces, and to 0.057 $(\tau_2 - \tau_1)$ for the sheet between the 200 and the 100 m-bar surface, τ_{10} and τ_9 respectively, τ_2 and τ_1 being the temperatures at the limiting surfaces of the sheet. Thus a temperature difference of 10° between the limiting surfaces will bring the error up to about 0.1 degree for the lower sheets, and somewhat above 0.5 degree for the highest sheet. These errors will generally be much smaller than the errors of observation from these different sheets.

While the errors introduced are thus unimportant, many practical advantages are gained. Common coordinate paper can be used, and we avoid the special inconvenience of the logarithmic scale, namely, that the best observations, those from the lower strata, have to be worked out on a minute scale, and the inferior ones, those from the higher strata, by constructions on a large scale.

56. Examples of the Method of Calculation when the Pressure is Given in Millimeters or Inches of Mercury. — When the observations are given in irrational units, the most direct method is to change at once all observations to rational units using the tables of the Appendix. And this change will be necessary, if it be desired to work out the results with the completeness of the examples given above. But if it be only required to find the main result, viz, the height of the standard isobaric surfaces, a shorter way can be followed, which is illustrated by the two examples below. No example is given for the case when the observed quantity is the height. For in this case the first step will always be a change from geometric to dynamic height, and it is immaterial whether the height is observed in meters or feet.

It is to be hoped that the time will soon come when all observations obtained from the higher strata are recorded in rational units. But as a vast amount of such observations has already been produced and recorded according to the different systems of irrational units, it will for some time to come be a question of great importance to be able to work out with as little waste of labor as possible the most important results in rational units from the data given in irrational units.

The simplest method of doing this, under the supposition that no other auxiliaries are at hand than our tables and common coordinate paper, is illustrated by the examples 3 and 4 below. As will be seen immediately from these examples, several operations will drop out, and no little amount of time will be saved, if special blanks be prepared, containing, besides the common coordinate lines, also some special auxiliary lines and auxiliary divisions. These are also shown on figs. 8 and 9 belonging to the examples. When extended work of this kind is to be performed, the best method of saving time will therefore be to print such special blanks.

When the observations of pressure are recorded in millimeters of mercury and those of temperature in centigrade degrees, the blanks should contain (see fig. 8): (1) horizontal lines representing the standard isobaric surfaces; (2) virtual-temperature divisions on each of these lines. These divisions are obtained by using table L, page 76, as explained in the preceding section. If these blanks be used the somewhat time-wasting work of drawing by hand the lines representing the standard isobaric surfaces drops out. Further, the virtual-temperature divisions allowed us to draw the virtual-temperature diagram without using table 11 A of the Appendix. It is thus seen that in using these special blanks, the height of the standard isobaric surfaces can be determined with practically the same ease as if the observations of pressure had been taken in rational units. Some supplementary results, as for instance the specific volume of the air at the standard isobaric surfaces, are also obtained with the same ease. But if the working out of the example should be carried still further, if it be required, for instance, to determine pressure in given heights, or to find the heights at which the observations were taken, it will be the best plan to change from the beginning the observed pressures from millimeters of mercury to millibars, and to proceed as in example 1.

When the observations of pressure are given in inches of mercury, and those of temperature simultaneously in Fahrenheit degrees, the blanks should contain (see fig. 9): (1) special divisions along the axis of abscissæ representing the centigrade degrees, while the main divisions are used to represent the Fahrenheit degrees; (2) horizontal lines representing the standard isobaric surfaces; (3) virtual-temperature divisions on each of these lines. These divisions are found by using table L, p. 76, in connection with the centigrade divisions along the axis of abscissæ. If these blanks be used, the following facilitations are obtained: The special drawing by hand of each line representing a standard isobaric surface is no more required. The virtual-temperature divisions allow us to draw the virtual-temperature diagram without being obliged to refer to table 12 A of the Appendix. The use of table 9 A of the Appendix for the transition from Fahrenheit to centigrade degrees is no more required. In this way column *6a*, table O, drops out, the centigrade temperature recorded in column *6b* being found directly from the diagram. It is seen that in this way, by the use of these special blanks, the height of the standard isobaric surfaces are found with practically the same ease as if the observations of pressure had been recorded in m-bars and those of temperature in centigrade degrees. As in the preceding case, some supplementary results are also easily obtained, such as the specific volume of the air at the standard isobaric surfaces. Even these supplementary calculations are simplified by the centigrade divisions along the axis of abscissæ, column *9a*, of table O, dropping out when these divisions are at hand. But if the example should be worked out still more in detail, if it be required to determine pressure in given heights, or to find the heights at which the different observations were taken, it will be the best plan, exactly as in the preceding case, to change at once the given observations to rational units, and proceed as in example 1.

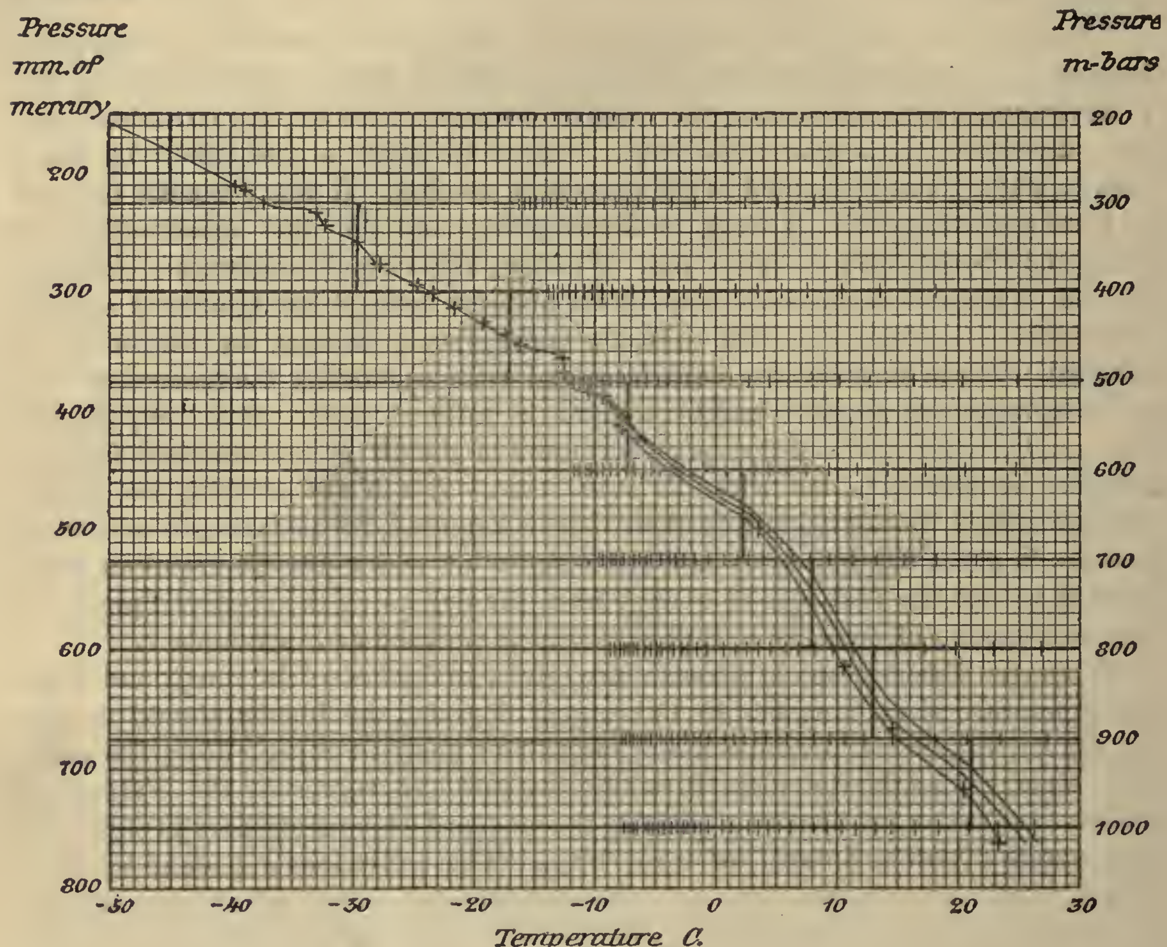

FIG. 8. — Virtual-temperature diagram, pressure in millimeters of mercury.

EXAMPLE 3. — *Observed time, pressure (millimeters of mercury), temperature (°C.), and humidity (per cent).* (Table N.) — From the observed pressures and temperatures (columns 2 and 3) the curve of true temperature is drawn (the curve to the left in fig. 8). The curve of virtual temperature for saturated air (curve to the right) is drawn by means of table 11 A of the Appendix.

TABLE N (EXAMPLE 3).

1	2	3	4	5	6	7	8	9	10
Observed time.	Observed pressure.	Observed temperature.	Observed humidity.	Standard pressures and pressure 1015.9 at the station, the latter found by table 7 A of Appendix.	Average virtual temperature in standard sheets and in sheet below lowest standard surface (+25.1) found from diagram (fig. 8).	Mutual distances between standard surfaces, found by table 9 M; height 135 of lowest standard surface above station, found by table 11 M and 12 M; height 39 of station above sea-level.	Heights of standard surfaces above sea-level, found by addition of figures of column 7.	Virtual temperature of air at the standard surfaces, found from diagram (fig. 8).	Specific volume of air at standard surfaces, found by table 14 M.
15th meridian.	mm. Hg.	°C.	Per cent.	m-bars.	°C.	Dyn. meters.	Dyn. meters.	°C.	m³/ton.
10h 50m	762.0	23.4	72	100	....	....	....	....	....
54	718.5	20.4	58						
57	667.5	14.6	62	200	....	....	....	....	....
11 00	614.5	10.5	40						
16	499.0	3.5	23	300	....	....	9365	−37.3	2255
30	423.0	− 6.3	42		−29.8	2009			
44	412.0	− 7.8	17	400	....	....	7356	−23.6	1790
12 16	388.0	− 9.2	84		−17.0	1640			
28	384.0	−10.5	31	500	....	....	5716	−11.9	1499
40	369.0	−12.5	46		− 7.2	1391			
57	355.5	−12.5	38	600	....	....	4325	− 2.0	1296
1 00	345.0	−16.1	51		2.2	1218			
11	336.8	−17.3	70	700	....	....	3107	+ 6.0	1144
20	325.3	−19.2	67		8.0	1077			
34	315.8	....	..	800	....	....	2030	+10.2	1016
39	312.3	−21.6	74		13.0	967			
46	300.3	−23.3	83	900	....	....	1063	+16.7	924
51	294.0	−24.7	85		21.0	889			
2 1	287.8	....	..	1000	....	....	174	+24.7	854
13	277.0	−27.8	95		25.1	135			
25	268.5	....	..	1015.9	....	....	....	....	....
28	259.0	−29.6	..			39			
52	243.5	−32.2	..						
55	232.3	−33.0	..						
3 3	225.0	−37.3	..						
10	214.5	−38.9	..						
28	210.5	−39.7	..						
32	202.5	....	..						
40	193.0	....	..						

Observing the percentages of humidity in column 4, the curve of virtual temperature is drawn between the two other curves. Then the horizontal lines representing the standard pressures are drawn according to the following table of the values of the standard pressures in millimeters of mercury:

m-bars	1000	900	800	700	600	500	400	300	200	100
Millimeters of mercury	750	675	600	525	450	375	300	225	150	75

The standard isobaric sheets being thus marked in the diagram, their average virtual temperature is determined by drawing the vertical segments of line in the usual way. Then the determination of the thickness of the standard sheets and the height of the standard surfaces follow as before (columns 5 to 8), as well as the determination of the virtual temperature and the specific volume of the air at the standard surfaces (columns 9 and 10).

EXAMPLE 4. — *Observed time, pressure (inches of mercury), temperature (° F.), and humidity (per cent).* (Table O). — From the observed pressures and temperatures (columns 2 and 3) the curve of true temperature Fahrenheit is drawn (the curve to the left in fig. 9). Then the curve of virtual temperature for saturated air is drawn (curve to the right) by means of table 12 A of the Appendix. Finally, the curve of virtual temperature is drawn between the two others in accordance with the percentages of humidity (column 4). Then the horizontal lines representing the standard surfaces are drawn in accordance with the following table giving the value of the standard pressures in inches of mercury:

m-bars	1000	900	800	700	600	500	400	300	200	100
Inches mercury	29.53	26.58	23.62	20.67	17.72	14.77	11.81	8.86	5.91	2.95

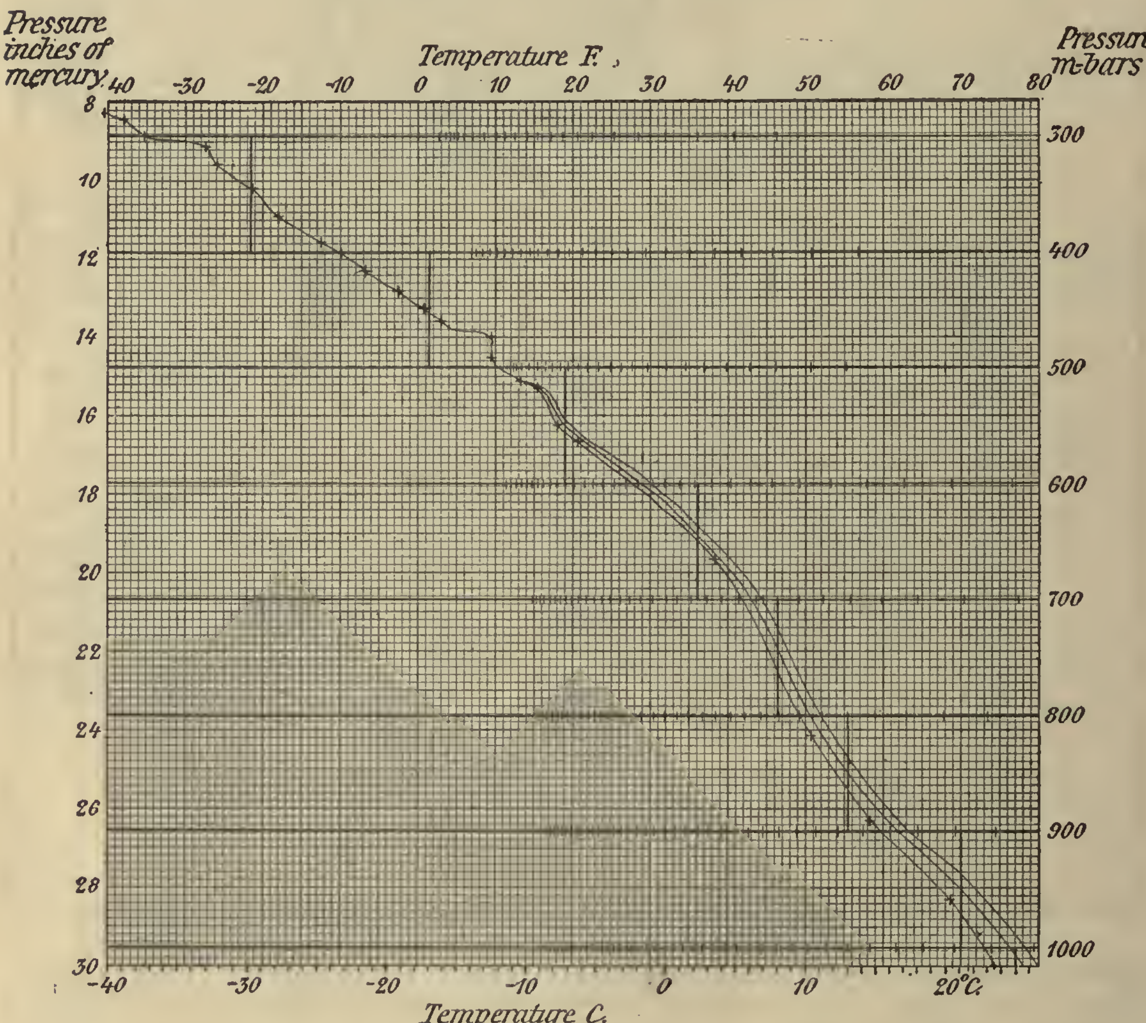

FIG. 9. — Virtual-temperature diagram; pressure in inches of mercury, temperature in degrees Fahrenheit.

The standard isobaric sheets being thus marked in the diagram, their average virtual temperatures are determined in the usual way by drawing the vertical segments of line. The diagram gives these temperatures in degrees Fahrenheit (column 6*a*), but by table 9 A of the Appendix they are changed into degrees centigrade. Afterwards the thickness of the standard sheets (column 7), and the height of the standard surfaces (column 8), are determined as in the preceding examples. From the diagram also the temperature Fahrenheit at the standard surfaces can be read off, and from this temperature, changed into centigrade (column 9*b*), we find the specific volume of the air at the standard surfaces (column 10).

TABLE O (EXAMPLE 4).

1	2	3	4	5	6a	6b	7	8	9a	9b	10
Observed time.	Observed pressure.	Observed temperature.	Observed humidity.	Standard pressures and pressure 1015.9 at station, the latter found by table 8 A of Appendix.	Average virtual temperatures in standard sheets and in sheet below lowest standard surface (+77.0), found by diagram (fig. 9).	Corresponding temperatures centigrade, found by table 9 A of Appendix.	Mutual distances between standard surfaces, found by table 9 M; height 135 of lowest standard surface above earth, found by tables 11 M and 12 M; height 39 of station above sea-level.	Heights of standard surfaces above sea-level, found by addition of figures of column 7.	Virtual temperature of air at standard surfaces found from diagram (fig. 9).	Corresponding temperatures centigrade, found by table 9 A of Appendix.	Specific volume of air at standard surfaces found by table 14 M.
15th meridian.	Ins. Hg.	° F.	Per cent.	m-bars.	° F.	° C.	Dyn. met.	Dyn. met.	° F.	° C.	m³/ton.
10ʰ 50ᵐ	30.00	74.2	72	100	....	....	....	....	....	....	....
54	28.29	68.7	58								
57	26.28	58.3	62	200	....	....	....	....	....	....	....
11 00	24.19	50.8	46								
16	19.65	38.3	23	300	....	....	....	9365	−35.2	−37.3	2255
30	16.65	20.7	42		−21.5	−29.7	2009				
44	16.22	18.0	17	400	....	....	....	7356	−10.5	−23.6	1790
12 16	15.28	15.4	84		+ 1.5	−17.0	1640				
28	15.12	13.2	31	500	....	....	....	5716	+10.5	−11.9	1499
40	14.53	9.5	46		+19.0	− 7.2	1391				
57	14.00	9.5	38	600	....	....	....	4325	+28.5	− 2.0	1296
1 00	13.59	3.0	51		+36.0	+ 2.2	1218				
11	13.26	0.8	70	700	....	....	....	3107	+42.8	+ 6.0	1144
20	12.81	− 2.6	67		+46.5	+ 8.0	1077				
34	12.43	....	..	800	....	....	....	2030	+50.3	+10.2	1016
39	12.30	− 6.8	74		+55.5	+13.0	967				
46	11.82	−10.0	83	900	....	....	....	1063	+62.0	+16.7	924
51	11.58	−12.5	85		+70.0	+21.1	889				
2 1	11.33	....	..	1000	....	....	....	174	+76.5	+24.7	854
13	10.90	−18.0	95		+77.0	+25.0	135				
25	10.57	....	..	1015.9	....	....	....	....	....	....	....
28	10.20	−21.3	..				39				
52	9.59	−26.0	..								
55	9.14	−27.3	..								
3 3	8.86	−35.2	..								
10	8.44	−38.0	..								
28	8.29	−39.5	..								
32	7.97	....	..								
40	7.60	....	..								

57. Example of Rapid Derivation of the Main Hydrostatic Results of a Meteorological Ascent. — We have given examples above of the derivation hydrostatically of the results of a meteorological ascent in as complete a form as possible. We have shown how to calculate all quantities with pressure and with height as independent variable, and have also taken up problems of a more secondary interest from a meteorological point of view, such as the calculation of the heights at which the different readings of the instruments were taken.

We shall now show how with the smallest loss of time we may deduce the most important hydrostatic results. This rapid reduction of the data of a meteorological ascent may soon be of practical importance. It is already proved possible to carry out meteorological ascents every day, in all kinds of weather.*

* Compare, for instance, R. Assmann: Ergebnisse der Arbeiten des K. Preussischen Aeronautischen Observatoriums bei Lindenberg im Jahre 1905. Braunschweig, 1906. List of ascents during the year 1905, pp. xxvi–xxix.

Simultaneous ascents from a system of stations for aeronautical meteorology may therefore be organized, and the problem will present itself, how to use the results of these ascents for the daily forecasts of the weather. The meteorologist at each station must therefore be able, as soon as possible, to send off a telegram giving the main result of his ascent. We shall here take under consideration the preparatory work for sending this telegram as far as the hydrostatic state of the atmosphere above the station is concerned. The further work at the central bureau after the reception of the telegrams will be discussed in the next chapter.

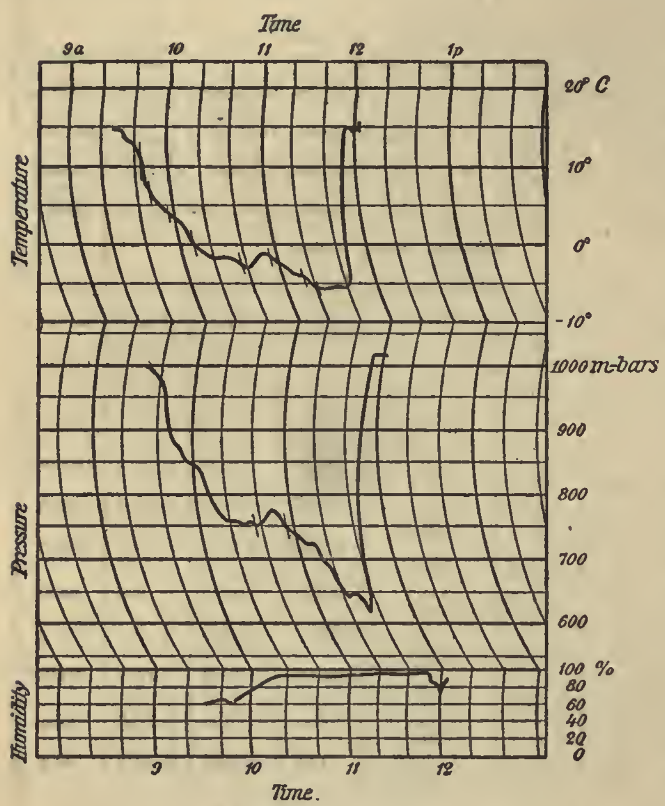

Fig. 10.— Meteorogram, Berlin, August 28, 1901.

The hydrostatic state of the atmosphere above a station is given if we know the height above sea-level of the standard isobaric surfaces. To enable the central bureau to find these heights it will be sufficient if the telegram contain (1) the height of the lowest standard isobaric surface; (2) the average virtual temperature of the standard isobaric sheets.

We have thus to show what the meteorologist at the station has to do from the moment he receives the meteorogram representing the results of the ascent on his table, in order to find the results (1) and (2) to be telegraphed.

Discussions of instrumental technics will not be taken up in this treatise. But it is important to remind the reader of the existence of two kinds of registering meteorographs. The first, which is most commonly used, contains a clock, and all the meteorological elements are registered as functions of the time. The second contains no clock. The barometer produces the motion of the paper on which the curves for the other meteorological elements are thus registered as functions of the atmospheric pressure. We shall show how to interpret a meteorogram obtained by each of these two kinds of instruments.

(A) *Meteorological elements registered as a function of time.* — Fig. 10 represents a meteorogram obtained by a kite flight from Tegel at Berlin, August 28, 1901.* The circle-arcs are coordinate curves of equal time. The curve in the

* R. Assmann und A. Berson: Ergebnisse der Arbeiten am Aeronautischen Observatorium 1900–1901, p. 259. The figure is changed, in as much as the coordinate curves on the barogram are drawn for m-bars instead of for millimeters of mercury.

first section of the diagram represents the variation of temperature, that in the second the variation of pressure, and that in the third the variation of humidity, all as functions of time. If there are instrumental errors, the corrections are supposed to be introduced graphically upon the diagram, the curves of fig. 10 being such corrected curves.

In order to derive from this meteorogram the curve of virtual temperature, we have to determine sets of corresponding values of pressure, temperature, and humidity. To do this most conveniently, we start with the points where the barometer-curve cuts the lines for 1000, 950, 900, . . . m-bars. Using a pair of compasses, we mark the corresponding points on the thermometer and hygrometer curves. Reading the temperature corresponding to the marked points on the temperature curve, we draw the curve of true temperature in the diagram (fig. 11). Then, using the virtual-temperature divisions, we draw the curve of virtual temperature for saturated air. Finally, using the humidities corresponding to the marked points on the hygrometer curve of fig. 10 we draw in the diagram fig. 11 the curve of virtual temperature between the two other curves. The vertical segments of the line giving the average virtual temperatures of the standard sheets are drawn, as well as the segment (invisible on account of its shortness), giving the average virtual temperature (+ 16) of the air between the lowest standard surface and the earth. By means of this temperature and the pressure 1001.2 at the station, we find the height 10 dynamic meters of the 1000 m-bars surface above the earth, using tables 11 M and 12 M as described previously. Adding the height (39) of the station, we get the height (49) of this standard surface above sea-level. The figures to be telegraphed are then

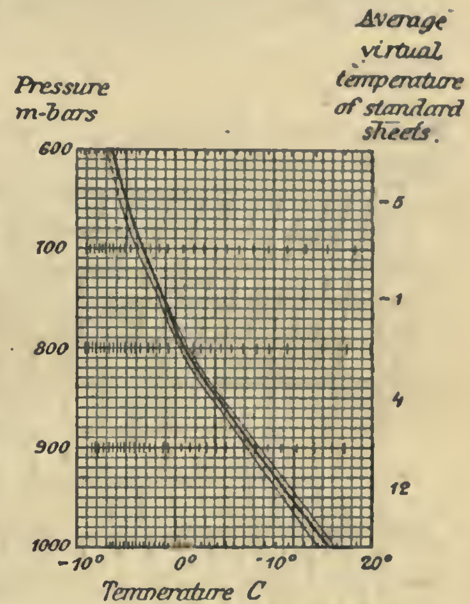

FIG. 11.—Virtual-temperature diagram, Berlin, August 28, 1901.

(*a*) 49, 12, 4, −1, −5

the first, 49, being the height of the 1000 m-bars surface, and the four other numbers the virtual temperature of the standard sheets.

The interval of time from the moment the meteorologist has obtained the meteorogram (fig. 10) on his desk until he has found the figures (*a*) to be telegraphed ought not to exceed ten to fifteen minutes.

(B) *Meteorological elements registered as function of pressure.*—The curve to the left in fig. 12 is recorded by Professor Assmann's baro-thermograph* at an ascent with a registering balloon on July 4, 1901.†

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* R. Assmann und A. Berson, *l. c.*, p. 42.

† R. Assmann und A. Berson, *l. c.*, p. 209. The original figure is changed by introduction of coordinate curves representing the pressure in m-bars, and by change of positive direction on the axis of temperatures.

This recorded curve is the curve of true temperature. Using the virtual-temperature divisions, we draw the curve to the right, the curve of virtual temperature for saturated air. No humidity having been registered, we suppose the relative humidity to have had the average value of 70 per cent and draw the curve of virtual temperature at a distance from the curve of true temperature equal to 70 per cent of the distance between the two curves. Then the circle arcs are drawn, which, in this system of coordinates, represent the average virtual temperatures of the standard sheets, as well as the arc (invisible in the figure on account of its shortness) representing the average virtual temperature (+12) of the air between the lowest standard surface and the earth. Using this temperature and the pressure (1005.2) at the station, we find by tables 11 M and 12 M the height (42 dynamic meters) of the lowest standard surface above the station, and adding the height (39) of the station, we find the height (81 dynamic meters) of this surface above sea-level. The figures to be telegraphed

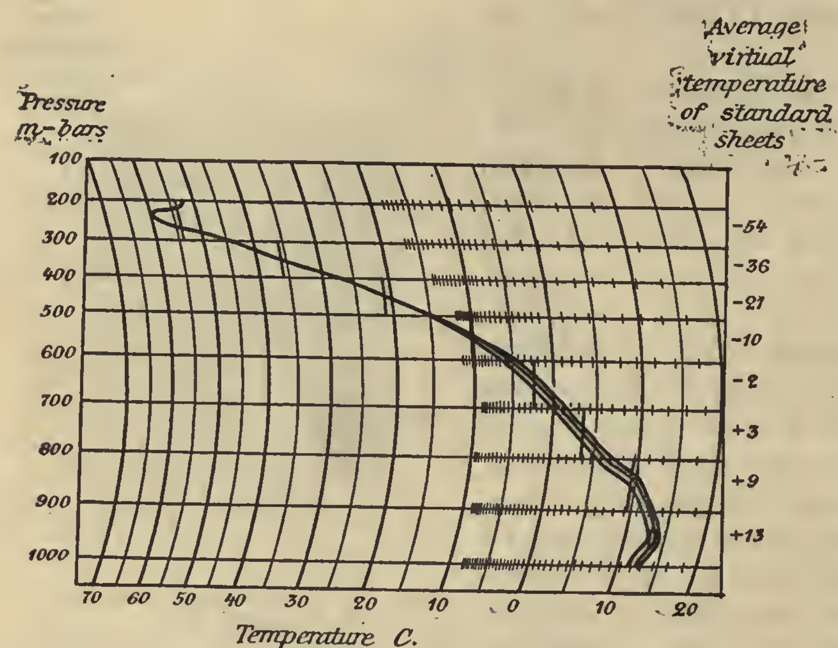

FIG. 12.—Meteorogram, Berlin, July 4, 1901.

(*b*) 81, 13, 9, 3, −2, −10, −21, −36, −54

are thus found. The time from the moment the meteorologist has obtained the meteorogram until he has found the figures (*b*) ought not to exceed five minutes.

58. Extrapolation of the Virtual-Temperature Diagrams. — The virtual-temperature diagram obtained from the observations of a meteorological ascent may be prolonged some distance upward, so as, for instance, to attain the next standard isobaric surface. If the prolongation be not too long, a slight deviation from the course which real observations would have given will have no great influence upon the calculated heights or pressures. Short extrapolations of this kind have been used occasionally in the examples given above.

A special kind of extrapolation will be of great importance, namely, those from the earth's surface. Complete observations along a vertical in the atmosphere will always remain rare, while we may get abundant observations from stations at the earth's surface. The observation of pressure, temperature, and humidity gives one point of the virtual-temperature diagram for a vertical in the atmosphere passing through the station. If the curve could be continued upwards somewhat from this point, we should be able to solve the hydrostatic problem for a vertical of moderate height. The solution would give a perfectly satisfactory accuracy in sufficiently small heights above the station, but of course decreasing accuracy with increasing height. Experience would gradually show to what height the extrapolation might be ventured.

As a guide for extrapolations of the virtual-temperature diagram, table 16M of Meteorological Tables has been constructed. It has been obtained by a statistical study of the results of the international balloon and kite ascents for the three years 1901, 1902, and 1903. The table gives the correction, which should be added to the virtual temperature at the station in order to give the virtual temperature at the heights above the station figuring as argument. The little table headed "Under the earth's surface" gives the correction for extrapolations downwards, based upon the common supposition of a decrease of the temperature of 0.5 degree per each 100 meters, used generally at present for "reductions to sea-level" of barometric records.

An observation being given, taken at a station at the earth's surface, table 16M thus enables us to draw the virtual-temperature diagram for a vertical through the station. The ordinates being the heights, we have to use the method shown in example 2, page 72, for calculating from this diagram the heights corresponding to given pressures or the pressures at given heights, negative heights below the earth's surface being also for theoretical reasons included.

We emphasize that table 16M should serve only as a guide in extrapolating virtual temperatures, and that it must be used with caution. Preferably a table of this kind should be made for each meteorological station, based solely upon data from ascents in air from this station. Great differences, dependent on the situation of the station, would probably be found. Thus for stations situated on high isolated mountains the temperature inversions (positive temperature corrections) given in table 16M in case of high pressure during winter, would probably not be found, and the gradients under ordinary conditions would probably be found smaller than above low land. When these gradients are determined by ascents undertaken from mountains, the value of meteorological observations at stations on mountains will be very much increased.

59. Extrapolation of Average Virtual Temperatures.—The method developed in the preceding article is important, because it enables us to find the hydrostatic state of the atmosphere near the earth's surface every day by common meteorological observations, quite independently of ascents in the air. Since it therefore furnishes methods which at once might be introduced into the daily meteorological work for the forecast of the weather, it will be important to simplify the operations to be performed as much as possible.

If the problem be to find completely the hydrostatic state, both as to the height corresponding to any given pressure, and as to the pressure at any given height, no simplification is possible. In such a case the virtual-temperature diagram must be drawn. But limiting the problem to the determination of the heights of the lowest standard isobaric surfaces, a convenient short cut is easily found. Instead of using table 16 M, giving the local values of the virtual temperatures, we use table 15 M, which gives in a corresponding manner the average virtual temperatures for the sheets of air between the station and the heights figuring as argument. This table is deduced from the preceding one by a process of integration, the principle of which will be clear in itself.

This table being given, we may proceed as follows in order to find the heights of the nearest standard isobaric surfaces: Using the observed pressure we find from table 11 M approximate values of the heights of these surfaces above the station. Using these approximate height values and table 15 M, we find the average virtual temperatures of the corresponding sheets. These virtual temperatures enable us to correct the approximate heights already found by means of table 12 M. The complete procedure is seen by the example annexed to the table.

This method of calculating the height above or the depth below the earth of standard isobaric surfaces is analogous to the method of "reduction to sea-level" of the barometric observations taken at stations situated at heights above sea-level. We emphasize some important differences however — our aim is always to find the heights of isobaric surfaces really existing in the air. The main reductions are therefore made upward and not downward. Consequently the result of the reduction is capable of being controlled by actual observations made in the open air, while reductions to the interior of the earth involved in the reduction to sea-level can not be made the subject of any kind of test by actual observations made at the place for which the pressure is calculated. Further, the reductions to sea-level are made generally according to a schematic method, using under all conditions the same temperature gradient. We have retained this gradient for small reductions downwards, while in working out the part of table 15 M to be used for reductions upwards, we have tried to introduce individual temperature gradients according to the different types of weather. In this direction probably much progress could be made by a statistical study of the results of ascents in the air, as remarked above.

CHAPTER VII.

SYNOPTIC REPRESENTATION OF THE FIELDS OF PRESSURE AND OF MASS IN THE ATMOSPHERE.

60. Quasi Static State. — Setting aside extraordinary phenomena, as, for instance, waterspouts, we may characterize atmospheric motions as slow motions going on near a state of equilibrium. Comparing simultaneous barometric records taken from two different places, we find the conditions of equilibrium apparently fulfilled, if the two places are at small or moderate distances from each other. Only as the distance increases do we find a gradual departure from the fulfillment of these conditions. As long, therefore, as the distance is small, the correct result will be produced if we use the barometric records for calculating the difference of height between the two barometers. The instruments will especially show the same pressure if they are placed on the same level. But if we sufficiently increase the distance between them, they must finally be placed on distinctly different levels in order to show the same pressure.

Thus, in reality, there is a deviation from the principle of the coincidence of surfaces (sections 35, 38). But the angle of intersection is so small that we must follow the surfaces over great distances in order to find an appreciable separation. On the other hand, proceeding along the plumb-line, we can not attain distances sufficiently great in order to prove an unquestionable deviation from the principle of the unit-sheets (sections 35, 38). Owing to the great lateral and small vertical extent of the atmosphere, we have therefore this peculiar relation, characterizing what we may call the quasi static state of the atmosphere:

The condition of equilibrium is apparently fulfilled along every vertical line. But as we proceed in a horizontal direction, there is a gradual change from vertical to vertical in this apparent state of equilibrium.

This important principle forms the basis of all practical investigations in atmospheric dynamics. In making use of it, it is important to remark that we need not take the expression "vertical" in the narrow sense of the word. We can consider the greatest angle of inclination of the isobaric surfaces as a kind of critical angle. Every curve whose angle of inclination is everywhere great in comparison with this critical angle will be called a *quasi vertical* curve, while curves whose angle of inclination is of the same order of magnitude or smaller than this critical angle will be called a *quasi horizontal* curve. The latter curves may attain lengths comparable to the lateral extent of the atmosphere, while the first remain short in the same sense as the true vertical curves are short. Following a quasi vertical curve, we can not therefore attain sufficient distances to be able to observe any appreciable departure from the hydrostatic conditions, and the principle stated above can therefore at once be extended from true vertical to quasi vertical curves.

61. Consequences of the Principle of the Quasi Static State.— In the preceding chapter we have shown how the results of meteorological ascents could be worked out according to the principles of hydrostatics.

In the case of true equilibrium, one ascent would be sufficient to give the state of the whole atmosphere. For according to the principle of coincidence of surfaces, the state will be the same at all points contained in the same level surface. Therefore, if we know the state at the points of a curve cutting a set of level surfaces, we also know the state at all points of these level surfaces.

Now, the actual state of the atmosphere is not one of true equilibrium. But owing to the principle of the quasi static state, the hydrostatic methods may still be used to a certain extent. The curve along which the ascent of a kite or balloon has taken place is always a quasi vertical curve. Along every curve of this kind the conditions ot equilibrium are fulfilled with sufficient approximation to entitle us to use the principles of hydrostatics. The states recorded by the instruments at the different points of this curve may be interpreted as if recorded at points of corresponding heights in a true vertical. By means of the developed hydrostatic methods we therefore find the distribution of pressure and of mass along this vertical. Although calculated upon a supposition not strictly fulfilled, the distributions of pressure and of mass found in this way will be very nearly the true ones.

This does not, however, entitle us to draw any conclusion as to the distribution of pressure and of mass along other verticals. For verticals of sufficient mutual separation, the distributions will generally be distinctly different, and must be found by independent observations. But this being done, we can easily calculate by interpolation the distribution of pressure and of mass also for all interjacent verticals, and thus find this distribution in the whole atmosphere.

Before concluding the consideration of atmospheric statics, we shall develop the geometrical methods of representing synoptically the results obtained by this method.

62. Method of Drawing Charts Representing Scalar Fields. — It will be useful first to exemplify some practical methods of drawing charts representing scalar fields in a plane.

If the values of a scalar quantity be known at a number of points in a plane, we always know how many equiscalar curves will pass between any two of these points, the curves being drawn for fixed intervals, say for unit-differences of the scalar quantity. By this condition the course of the equiscalar curves is determined to some extent, the more accurately so the greater the number of points in which the value of the scalar quantity is known. Therefore, knowing the value of such a quantity at a sufficient number of points, we can draw the equiscalar curves with sufficient accuracy, and thus arrive at the graphic representation of the scalar field in the plane.

This is the well-known method of drawing isothermic charts from the observations of temperature, isobaric charts from the observations of pressure, topographic charts from measurements of heights, and so on.

63. Arithmetical and Graphic Methods of Adding and Subtracting Scalar Fields. — Let α_1 and α_2 be scalar quantities of the same kind, say both temperatures, both pressures, or both heights above sea-level. From the fields α_1 and α_2 we shall often have to deduce the field of their sum

(*a*) $$\alpha_1 + \alpha_2$$

or the field of their difference

(*b*) $$\alpha_1 - \alpha_2$$

For the solution of this problem two methods offer themselves.

(A) *Arithmetical method.* — We may form arithmetically the sum (*a*), likewise the difference (*b*), at a certain number of points. The numbers representing these sums or differences are noted in the plane, and the equiscalar curves drawn according to the method mentioned in the preceding article.

This method will be especially convenient when the values of both quantities α_1 and α_2 are observed at the same points. The sums or differences used in drawing the chart can then be derived directly from the observed quantities.

(B) *Graphic method.* — The curves are supposed to be drawn *for the same interval* in the field of α_1 as in that of α_2. Superimposed upon each other, the curves divide the plane into a set of curvilinear parallelograms. We can then draw two sets of diagonal curves, and it will easily be verified that the one set represents the field of the sum $\alpha_1 + \alpha_2$, and the other that of the difference $\alpha_1 - \alpha_2$.

This graphic method of forming the sum or difference of two scalar fields is most convenient, and will be much used below. It will generally prove practicable to draw the curves of the different systems on different papers laid upon each other. Transparent paper may be used, or, more conveniently, common paper placed on a sheet of glass and strongly illuminated from below.

The arithmetical and the graphic methods supplement each other in a valuable manner. The latter gives both systems of diagonal curves sharply, if the two sets of originally given curves cut each other at nearly right angles. But if the angles approach 0° or 180°, only the one system, that containing the long diagonals of the parallelograms, will be sharply defined. The other, that containing the short diagonals, will be very indeterminate, greatly varying with small errors in the course of the originally given curves. This second set is therefore obtained better by the arithmetical method, especially if the arithmetical sums or differences can be formed from originally measured and not from interpolated values of the two scalar quantities α_1 and α_2.

64. Charts of Absolute and of Mutual Topography of Isobaric Surfaces. — The synoptic representations of the fields of pressure and of mass are worked out on charts containing the situation of the stations from which the observations have been made.

On a chart of this description we can mark the height of a certain isobaric surface above each station. Then, by aid of these heights, we draw, as explained in section 62, a topographic chart showing the configuration of the isobaric surface. In this way we can draw a topographic chart for every standard isobaric surface reached by the ascents. A set of such charts gives a perspicuous representation of the distribution of pressure in the investigated part of the atmosphere. Examples are given in figs. 13 and 19.

To find the correlative representation of the distribution of mass, we have to remember that the figures representing the mutual distances from one standard surface to the next at the same time represent the average specific volume of the air in the sheets between the surfaces. A chart of *mutual topography* of two successive standard isobaric surfaces will therefore also represent the field of average specific volume in the sheet between the two surfaces. We may draw these charts directly from the figures representing the thickness of the sheets (example 1, column 7, table J, p. 70 ; example 2, column 8, table K, p. 74) or indirectly from the charts of absolute topography, using the method of graphic subtraction, the latter method being, however, less accurate. Charts of this description are given in figs. 14 and 20.

Of course we might also have represented the distribution of mass by topographic charts of the isosteric surfaces. But, however interesting these might be, the above representation, obtained in immediate connection with the distribution of pressure, will generally be found the more useful, apart from the greater facility with which it is found.

In order better to conceive the topography represented by the charts it will be useful to draw profile curves of the isobaric surfaces. A set of such verticals as the second of fig. 4 are drawn at horizontal distances corresponding to the distances between the stations. Joining the points belonging to the same isobaric surface, we get the profile curves. Taking the other set of divisions on the same verticals, we may also get the profile curves of the isosteric surfaces. If both sets of profile curves be drawn on the same diagram, as in figs. 17 and 21, they intersect each other, showing the deviation from the hydrostatic principle of the coincidence of the surfaces. These vertical sections are not of the same practical interest as the charts of absolute and relative topography, but have still interesting theoretical properties, and enable us to get a more complete conception of the content of the charts. An important geometrical relation between the section containing the profile curves and the charts of relative topography will be given presently (section 73).

65. Charts of Absolute Pressure and of Mutual Pressure Differences in Level Surfaces. — On the chart containing the stations we can then note the numbers representing the pressure found at a certain level, and guided by these numbers draw an isobaric chart for this level, in the same manner as such charts are drawn for sea-level in the daily weather service. A set of such charts, drawn for a set of standard levels, will give as complete a representation of the distribution of pressure as the preceding one by topographic charts of isobaric surfaces. Examples of such isobaric charts at different levels are given in figs. 15 and 22. As too many charts would be acquired if drawn for every standard level, *i. e.*, for every dynamic meter of height, we have only drawn them for intervals a thousand times greater, *i. e.*, for level differences of 1,000 dynamic meters. The pressures represented by the isobaric curves are added in m-bars.

To find the correlative representation of the distribution of mass, we have to remember that the difference of pressure from one standard equipotential surface to another is equal to the average density of the air in the sheet between them. By arithmetical or graphic subtraction of the fields of pressure in the level surfaces limiting an equipotential sheet we therefore get a chart representing the average distribution of density in this sheet. Such charts are given in figs. 16 and 23. The figures added to the curves give the mutual pressure differences in m-bars. As they refer to level sheets of 1000 dynamic meters interval they will, after division by 10^5, give the average densities of the sheets.

A valuable complement to these charts of absolute pressure and of pressure difference are vertical sections like those of figs. 18 or 24. These are obtained by means of verticals like the third of fig. 4. A set of such verticals being drawn at proper mutual distances, points representing the same dynamic height are united by curves, and in like manner points representing the same value of density. In this way we obtain the profile curves of the equipotential and isopycnic surfaces, those of the equipotential surfaces being drawn simply as horizontal equidistant lines.

An important relation between these vertical sections and the corresponding charts will be developed below (section 73).

66. Construction for Lower Levels of Charts of Absolute and of Mutual Topography from Observations Made at the Earth's Surface.— In drawing the charts described in principle in the preceding articles, it is important to make as complete a use as possible of the observations from the stations at the earth's surface. For these observations are abundantly at hand, while those from the open air will always remain relatively scarce. By means of the method of extrapolation developed in sections 58 and 59, it will be possible from the observations at the earth's surface to draw charts for the lower sheets of the atmosphere.

From stations near sea-level the heights of the three lowest standard surfaces may be found, and from many mountain stations the heights also of the fourth and fifth and even higher surfaces may be determined with satisfactory accuracy. The common meteorological observations will therefore enable us to draw topographic charts of the three, four, or even five lowest standard isobaric surfaces. The three first charts of fig. 19 are obtained in this way, only slightly corrected and extended afterwards by the results obtained by ascents in the air. It is important to remark that charts of this kind can be obtained every day from the regular meteorological observations, and with the same ease as the charts for sea-level now in use.

It is of course always desirable to derive the charts directly from the original observations, and not from these observations after they have been "reduced to sea-level." But often, when past atmospheric states must be worked out from published observations, these are accessible only in the distorted form of isobaric charts for sea-level. Re-reductions to higher levels are thereby made more troublesome and less trustworthy. But it is important to notice that it is very easy to change an isobaric chart for sea-level into a topographic one for the 1000 m-bar surface, provided the isothermic chart be known besides the isobaric.

To perform this change when the isobaric chart is drawn for millimeters of mercury and the isothermic for degrees of the centigrade thermometer, table 18 A of the Appendix is used. The table shows that the level curve of the height zero coincides with the isobaric curve of a pressure of 750 mm. mercury at sea-level, independently of the temperature. The level curve of a height of 50 dynamic meters coincides almost completely with the isobaric curve for a pressure of 755 mm. mercury, deviating for high temperatures towards the isobaric curve of 754 mm. mercury, and for low towards the isobaric curve of 756 mm. mercury. In the same way the curve of 100 dynamic meters of height closely follows the isobaric curve of 760 mm. mercury, with small deviations towards higher pressure for low temperature and towards lower pressure for higher temperature, and so on. Using this table and the isothermic chart, slight changes are easily made in the isobaric curves, giving thus the level curves representing the topography of the 1000 m-bars surface.

Table 19 A of the Appendix serves the same purpose, in the case of the isobaric chart being drawn for inches of mercury and the isothermic for Fahrenheit degrees. This table has been used to draw the topographic chart for the 1000 m-bars surface in fig. 13 from the corresponding isobaric chart for sea-level published by the U. S. Weather Bureau.

The principle for the calculation of tables of this kind is explained in the next article, where tables serving an analogous purpose are described.

From the charts of absolute topography, obtained by extrapolation from below, those of relative topography, representing the distribution of mass in the sheets between the standard surfaces, may be deduced at once by the method of arithmetical or graphic subtraction. The arithmetical method will generally be found preferable on account of the acuteness of the angles of intersection of the curves of absolute topography (section 63). Two charts obtained in this way are given in fig. 20.

67. Construction for Lower Levels of Charts of Absolute Pressure and of Pressure Differences from Observations at the Earth's Surface. —Drawing the extrapolated virtual-temperature diagram as explained in section 58, and calculating the pressure in standard levels, we can draw the isobaric charts of absolute pressure in these levels. Afterwards, by the method of arithmetical or graphic subtraction (the first being generally preferable), the charts of relative pressure, representing the distribution of density in the level sheets, can be drawn.

On the other hand, if the charts of absolute topography of standard isobaric surfaces be drawn, it is easy to change them into isobaric charts for corresponding standard levels. To see this we remark that the level curves on isobaric surfaces and the isobaric curves on level surfaces belong to one family, the curves of intersection between isobaric and level surfaces. The level curves on an isobaric surface and the isobaric curves on a level surface from about the same height in the atmosphere will therefore resemble each other. Further, the standard isobaric surfaces of pressures 1000, 900, 800, 700, 600, 500, 400, and 300 m-bars are nearly in the levels of 0, 1000, 2000, 3000, 4000, 5000, 7000, and 9000 dynamic meters, and therefore only a small correction is required to change the level curves of these isobaric surfaces into isobaric curves at the corresponding levels.

The principles for finding these corrections are easily seen. The isobaric curve 700 m-bars in the level surface 3000 dynamic meters is identical with the given level curve 3000 dynamic meters on the isobaric surface 700 m-bars. The isobaric curve 705 m-bars will run where the sheet of air between the isobaric and the level surface exerts the pressure of 5 m-bars. In order to exert this pressure the sheet must have the thickness of 56 dynamic meters if it has the temperature of 0° C., the thickness of 58 dynamic meters if it has the temperature of 10° C., and so on. This is seen at once from tables 10 M and 12 M. The required isobaric curve of 705 m-bars will thus coincide with the given level curve of 3056 dynamic meters where the sheet has the temperature of 0° C., with the given level curve 3058 dynamic meters where the sheet has the temperature of 10° C., and so on. These temperatures and the level curves being given, the isobaric curves can thus be drawn.

To avoid the laborious use of tables 10 M and 12 M, table 17 M has been derived from them; as one argument appears the pressures along any isobaric curve to be drawn, and as the other the virtual temperatures in the given isobaric surface. These are always known (see example 1, column 9, table J, p. 70, example 2, column 10, table K, p. 74). To these temperatures at the surface will correspond a definite average temperature of the sheet if we make the common supposition of a fall of temperature of 0.5° C. for every 100 dynamic meters of height. On account of the smallness of the reductions a greater accuracy than that obtained under this simple supposition will never be required. Using this supposition, the tabulated numbers are derived from tables 10 M and 12 M. They indicate with which level curves the required isobaric curves should coincide. Using these tables and the given topographic chart and temperature chart for the isobaric surfaces, the required isobaric charts can be drawn with great ease.

68. Correction of Charts for Lower Levels and Construction of Charts for Higher Levels by Means of Observations Obtained from Ascents.— If results from simultaneous ascents in the air were available in sufficient number, charts of absolute and of mutual topography, or of absolute pressure and pressure differences, could be drawn directly and independently of each other for every level. But as long as these ascents remain comparatively rare, it will be advisable first to draw all charts which can be obtained by extrapolation from the stations at the earth's surface as completely as possible.

This being done, our first task will be to correct the charts according to the absolute values obtained from the ascents. This is easily done for charts of absolute topography or of absolute pressure. The values obtained from the ascents are noted on the charts, and the whole set of curves displaced or changed so as to suit these values. As a rule this is easily done without any noticeable change in the qualitative course of the curves. These corrections have been made on the charts of figs. 19 and 22.

Greater difficulty will be found in correcting charts of mutual topography or of pressure differences, because their curves have a very complicated course, evidently in great measure depending upon the topography of the land and the distribution of land and sea. It is not easy to see how to change the course of such curves so as to suit the small number of correct values obtained by the ascents. In the examples worked out below we have therefore desisted from making this correction.

In fig. 20 are given side by side two charts of mutual topography obtained by extrapolation from 219 stations at the earth's surface, and two as obtained from the results of ascents in the air from 5 stations. If ascents had been made at a sufficient number of places the curves of the latter charts would probably have had mainly the same course as those of the extrapolated charts, but with slightly changed situations of the different curves, and it would have involved no difficulty to correct the extrapolated charts by the fundamental values obtained by the ascents.

As to the charts for higher levels, those of mutual topography or of pressure differences are drawn directly from the results of the ascents. Afterwards we use the following method for drawing the charts of absolute topography not obtained by extrapolations from below: The chart of mutual topography of two surfaces is placed upon that of the absolute topography of the lower one. Then the absolute topography of the upper one is obtained by graphic addition. The chart thus obtained is then corrected in accordance with the absolute heights found from the ascents and from the observations on mountains of a sufficient height. For the present, however, the latter observations must be used with caution because of our ignorance of temperature gradients above mountains (section 58). This chart being drawn, we place upon it the next chart of mutual topography, proceed in the same manner, and so on.

The charts of absolute pressure in the higher standard levels are found by a completely analogous procedure.

In drawing charts in this way, one after the other by graphic addition, there is this advantage — that the characteristic feature of the distribution of pressure as known from the numerous observations from the earth's surface does not disappear as we proceed upward, as would have been the case if each chart had been drawn independently of the others by means of the small number of calculated values.

69. Remarks on the Rapid Work Essential for Daily Weather Service. — In the preceding articles we have shown in detail how to find and represent as completely as possible the distribution of pressure and mass in the atmosphere. Nothing would prevent the use of these methods in the daily meteorological service for the forecasts of the weather. But then it becomes a question of vital importance how to be able to draw the whole system of charts with as short a delay in time as possible.

We have, then, first to make a choice between the two methods, developed side by side — that of representing the absolute and the relative topography of isobaric surfaces, or that of representing the absolute and relative pressure in level surfaces. There is no doubt as to what choice to make; the charts of absolute and relative topography can be found by a smaller number of operations, and therefore be ready within a shorter time. It may be possible that the method of constructing the isobaric charts in level surfaces might be developed to a greater degree of simplicity than is done here. But it is not probable that the simplicity of the other method could be reached. The preference in favor of the first method is due to the greater theoretical simplicity of the problem of determining the height corresponding to a given pressure compared with that of determining the pressure at a given height.

The superiority of the charts of absolute and relative topography being admitted, the meteorologists at the central bureau have to work out such charts from two sets of telegrams, giving (1) the observations from the common meteorological stations, (2) the height of the lowest standard surface and the virtual temperature of the standard sheets above the aeronautical stations from which ascents have been made (section 57).

From the first set of observations the charts of absolute and relative topography are drawn as described above for the lower levels. They can be drawn independently of each other, and accordingly simultaneously by different workers. As the drawing of each chart is of precisely the same nature as the drawing of an isobaric chart for sea-level, nothing prevents the whole set from being ready within an interval of time not exceeding that required for drawing the single isobaric chart for sea-level.

From the telegraphed values of the virtual temperatures of the standard isobaric sheets the higher-level charts of relative topography are drawn. In doing this it is not necessary first to change by table 9 M the telegraphed virtual temperatures into heights. The curves for constant thickness of a sheet are curves for certain constant values of the virtual temperature. We may therefore note these temperatures on the chart and draw the curves for constant thickness of the sheet directly from them, table 9 M showing which virtual temperature corresponds to a required value of the vertical distance.

The charts of relative topography being drawn, the corresponding charts of absolute topography are found by the method of graphic addition. Finally, if required, the charts are corrected according to the absolute heights, which to save time may have been calculated by another computer. But as long as observations from the open air are rare, one man will probably be able to perform all the work for the higher-level charts during the time required by the other workers to draw the lower-level charts. Thus, by good organization there is nothing to prevent the whole system of charts giving the distribution of pressure and mass in the atmosphere for all heights reached by extrapolations from below and by direct ascents in the air, being ready within an interval of time not greatly exceeding that required for drawing such charts as are now used for sea-level.

70. Example 1. — Atmospheric Conditions over North America, September 23, 1898. — The first simultaneous meteorological kite ascents were organized by the U. S. Weather Bureau during the summer of 1898.* September 23 seven ascents succeeded, five of which were fairly simultaneous, between 7 and 11 o'clock in the morning, and thus simultaneous also with the common meteorological observations at 8 o'clock, time of the seventy-fifth meridian. Two of the ascents, from North Platte and from Dodge City, came between 2 and 5 in the afternoon. During the days September 21 to 24, kite ascents were made also from the Blue Hill Meteorological Observatory near Boston.† None of them were simultaneous with

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*See H. C. FRANKENFIELD: Vertical Gradients of Temperature, Humidity, and Wind Direction. A preliminary report on the kite observations of 1898. Weather Bureau Bulletin F. Washington, 1899. The original results of the kite ascents have not been published. Those used below have been kindly communicated by the Weather Bureau.

†H. HELM CLAYTON: Studies of Cyclonic and Anticyclonic Phenomena with Kites. Bulletin No. 1, 1899, of Blue Hill Meteorological Observatory.

those of the Weather Bureau. But by a method of interpolation to be explained below they have been reduced to simultaneousness with the others. Besides the results of these kite ascents we have had at our disposal the synoptic charts of the Weather Bureau for this day, but not the original observations from the stations at the earth's surface.

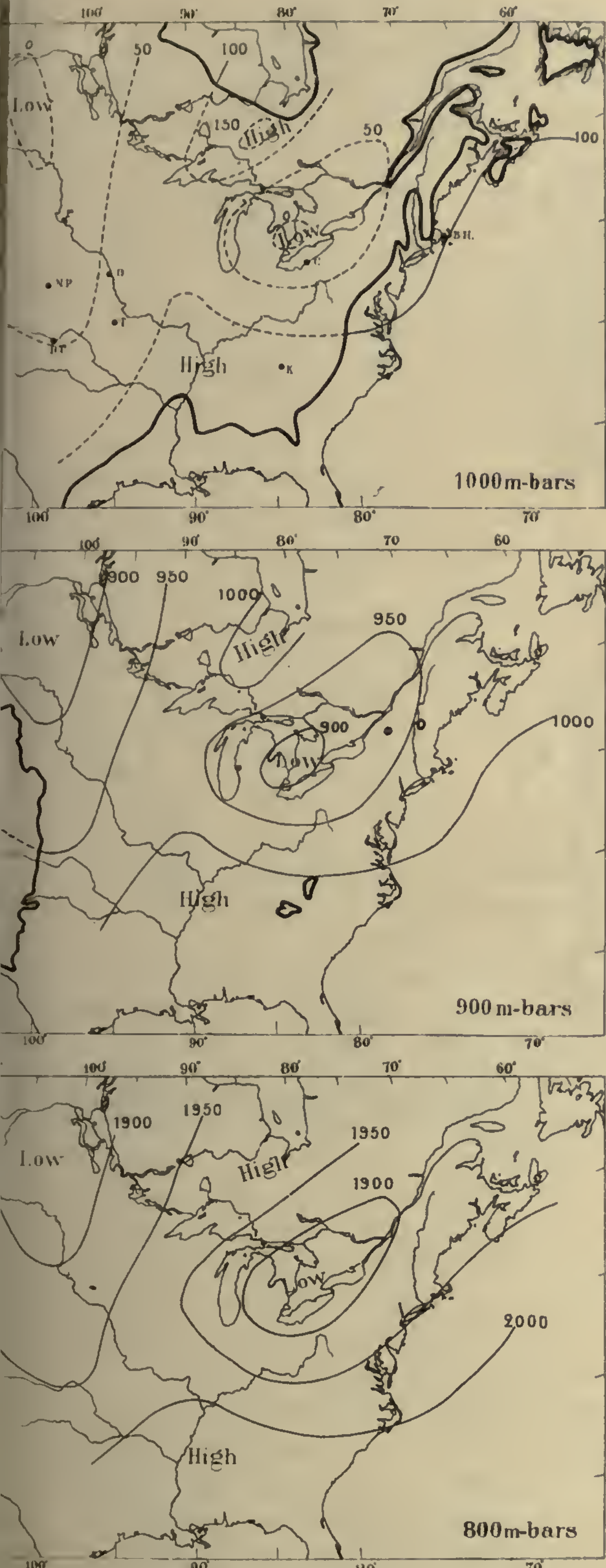
Table P contains for each of the kite-flights the calculated dynamic heights of the three lowest standard isobaric surfaces (first column under each station), and the mutual distances between these surfaces (second column under each station).

TABLE P.—*Dynamic heights of standard isobaric surfaces and mutual distances between them, United States, September 23, 1898.*

Station:	Blue Hill.		Cleveland.		Dodge City.		Knoxville.		North Platte.		Omaha.		Pierre.		Topeka.	
Dynamic height:	188		210		739		296		840		370		477		291	
Pressure (m-bars).																
800	1956		1882		1953		1997		1916		1958		1905		1958	
		975		970		1011		974		991		996		996		996
900	981		912		942		1023		925		962		909		962	
		886		888				894				888		879		904
1000	95		24				129				74		30		58	

From the figures contained in table P and from the charts of the Weather Bureau, the charts in figs. 13 and 14 have been drawn. The level curves used to represent the absolute topography of the standard isobaric surfaces (fig. 13) are drawn continuously where these surfaces run in the open air, while they are dotted where they represent only the ideal continuation of these surfaces below the earth. From a topographic chart the curves of intersection of the isobaric surfaces with the earth (heavy curves in fig. 13) have been obtained. The curves representing the mutual topography of successive standard isobaric surfaces are drawn continuously only as long as both surfaces run in the open air, while they are dotted as soon as the lower surface cuts the earth. The curves of intersection both of the upper and the lower surface are drawn as heavy curves, and the portion of land rising above the upper surface is shaded.

The topography of the 1000 m-bar surface (first chart of fig. 13) is derived from the isobaric chart of the Weather Bureau, table 19 A of the Appendix being used as explained in section 66. The charts of mutual topography (fig. 14) have been drawn directly from the figures of table P. The situation of the kite stations is marked on the first chart of fig. 13. In drawing the curves the observations from the two stations North Platte and Dodge City, where the ascents came 6 to 8 hours too late, have also been used, only with less attention given to them than to the others. The chart of absolute topography of the 900 m-bar surface (second chart of fig. 13) has been obtained by graphic addition of the first chart of fig. 13 and the first of fig. 14, and the chart of the 800 m-bar surface in the same manner by graphic addition of the second of fig. 13 and the second of fig. 14. Afterwards they have been corrected according to the absolute heights given in table P. As we have not had, as already mentioned, at our disposal the original observations from the stations at the earth's surface, we have refrained from every extrapolation from below.



13. Field of pressure, Sept. 23, 1898, represented by topographic charts giving the height of isobaric surfaces in dyn. meters.

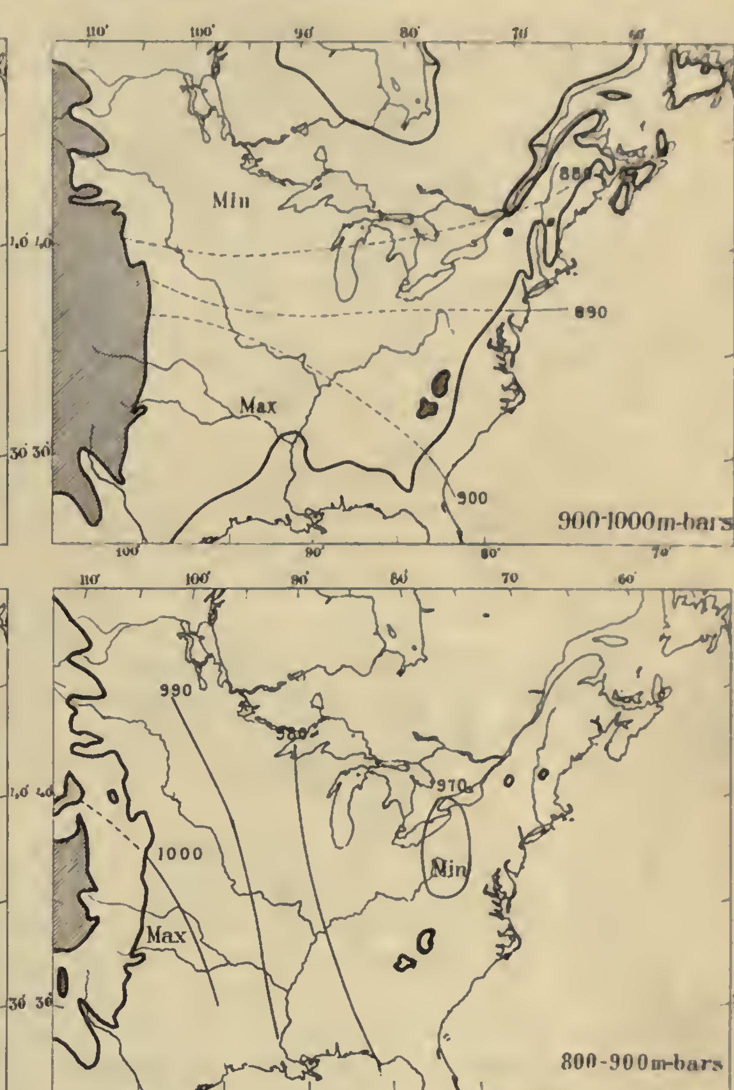


Fig. 14. Field of mass, Sept. 23, 1898, represented by charts giving the average specific volume (m^3/ton) of the air in isobaric sheets, or the thickness of these sheets in dyn. meters.

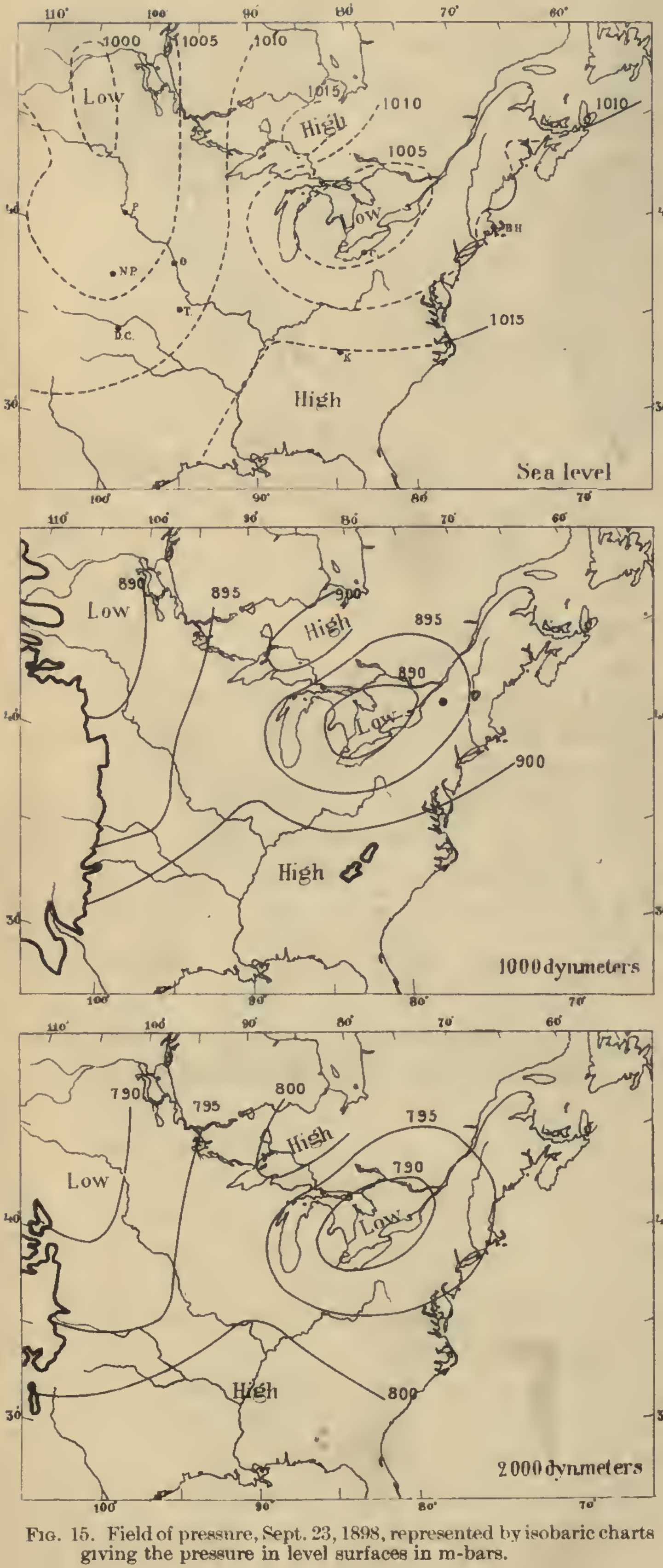


Fig. 15. Field of pressure, Sept. 23, 1898, represented by isobaric charts giving the pressure in level surfaces in m-bars.

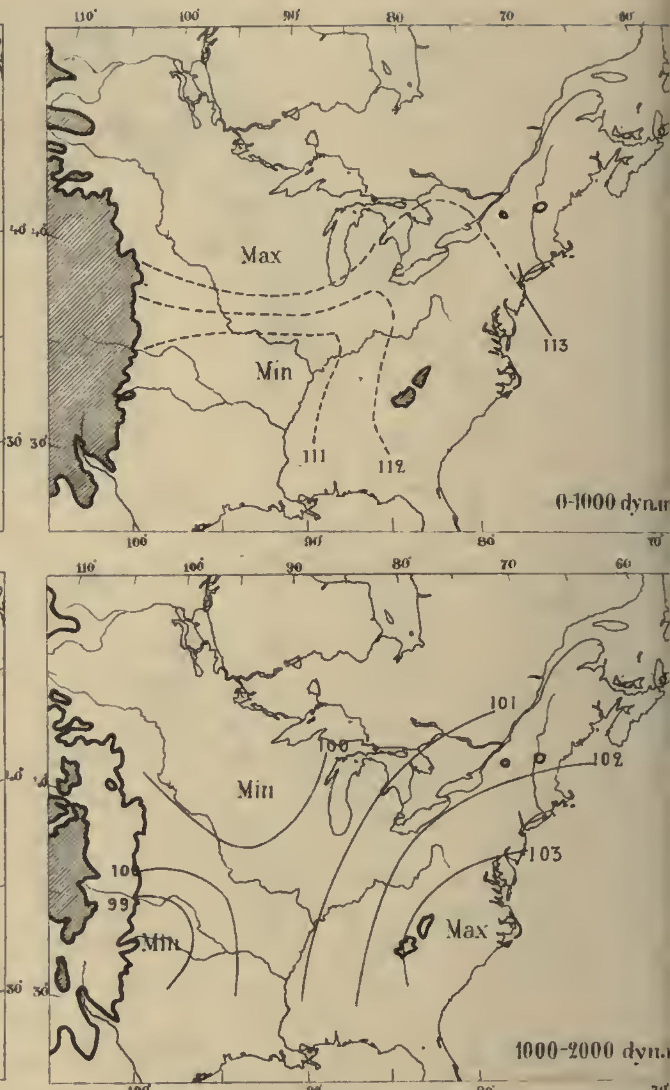


Fig. 16. Field of mass, Sept. 23, 1898, represented by charts giv average density (10^{-5}ton/m³) of the air between level su faces, or the difference of pressure between these surfac in m-bars.

The 1000 m-bar surface rises to a height of 100 dynamic meters or more above the Atlantic Ocean, Hudson Bay, and the nearest parts of the coast; it cuts the earth's surface along a line running approximately parallel to the coast, and shows a marked depression in the region about the Great Lakes, where it goes down to or below sea-level. The 900 m-bar surface runs at an average height of 950 dynamic meters, and cuts the earth along the lower slope of the Rocky Mountains as well as about a few of the higher peaks of the Appalachian Mountains, which rise as islands above the surface. The 800 m-bar surface runs at a height of 1950 dynamic meters and cuts the earth only along the upper slope of the Rocky Mountains. Both show the same depression as the 1000 m-bar surface in the region about the Great Lakes.

The curves of the first chart, fig. 14, giving the mutual topography of the 900 m-bar and the 1000 m-bar surface, run mainly east and west, indicating a decreasing distance between the surfaces as we proceed from south to north. The curves of the next chart, giving the mutual topography of the 800 m-bar and the 900 m-bar surfaces, have a very different course, running mainly north and south, and indicating a decreasing distance between the surfaces as we proceed from west to east. Interpreted as charts of the distribution of mass in the standard isobaric sheets, the first shows decreasing specific volume, *i. e.*, increasing concentration of mass, as we proceed from south to north, while the second indicates a corresponding concentration of mass as we proceed from west to east, the greatest concentration apparently being found a little south of the greatest depression of the isobaric surfaces.

Fig. 17 is a vertical section showing the profile curves of the isobaric and the isosteric surfaces. This section is not, however, derived from the observations used in drawing the charts, but from the successive ascents performed at Blue Hill on each of the four days September 21 to 24. Supposing the cyclone to have moved during these days without undergoing any change in its interior constitution, the section obtained in this way would have given the same result as a set of simultaneous ascents from four properly chosen stations on any of these days. We waive the question as to the invariability of the cyclone during this time, and therefore also as to what approximation the four successive ascents from Blue Hill may be interpreted as four simultaneous ascents from different stations. The sections obtained by successive ascents from the same stations will always be of value in themselves, and in this case in enabling us to find by interpolation the state of the atmosphere above Blue Hill at the time of the Weather Bureau ascents September 23. The point marked *B. H.* indicates the vertical of the moving cyclone which was above Blue Hill at the time, and from its sections with the isobaric and the isosteric curves the numbers under the column Blue Hill in table P have been derived.

Table Q contains the result of the same kite ascents as table P, but worked out for the absolute pressures at given levels and the pressure differences from level to level. The corresponding synoptical representation of the state of the atmosphere is given in figs. 15, 16, and 18. The isobaric curves are drawn continuously or dotted according as they represent real pressure in the open air or ideal pressure below the earth's surface. The curves of intersection of the different levels with the earth's surface are drawn heavy.

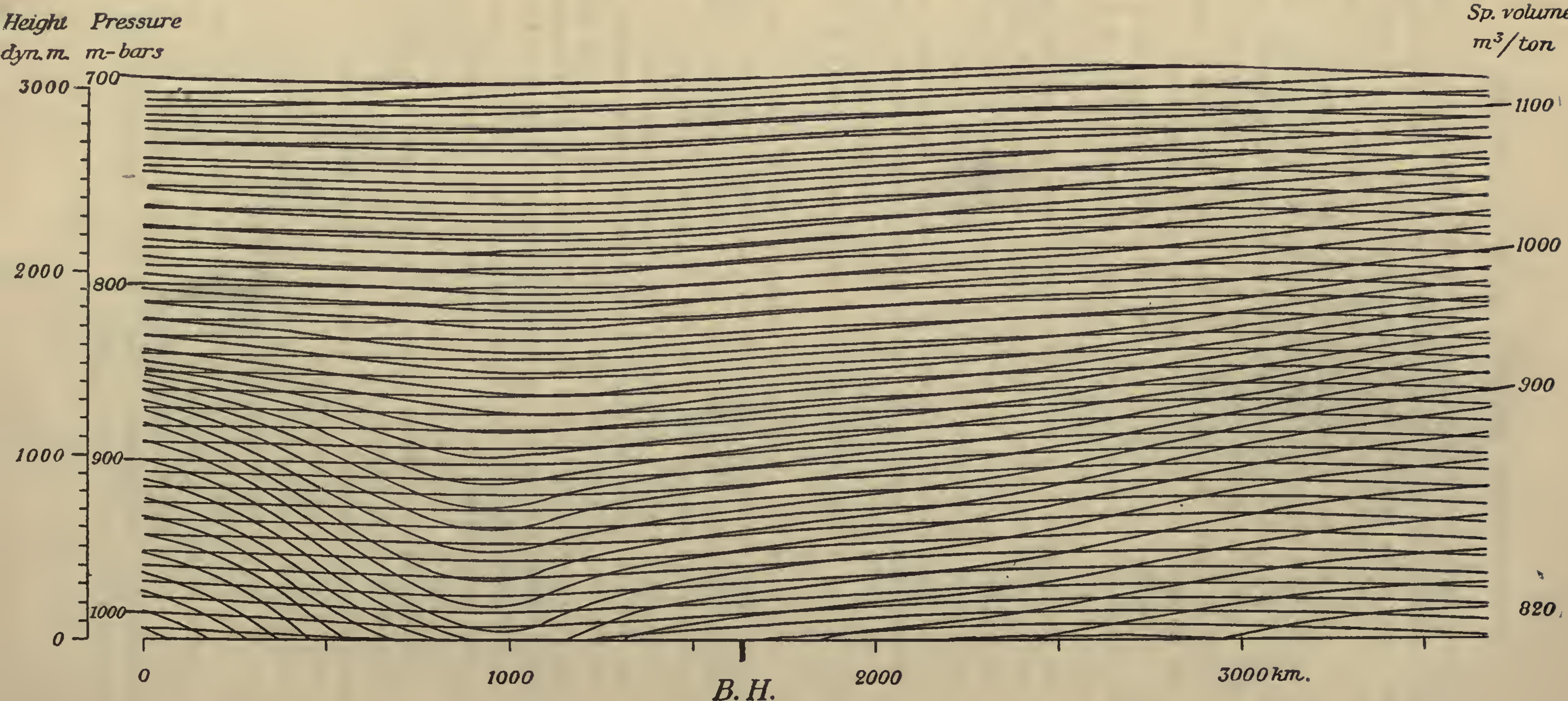


FIG. 17.—United States, September 23, 1898. Profile curves of isobaric and of isosteric surfaces. Every parallelogram represents 10 m. t. s. isobaric-isosteric unit-tubes.

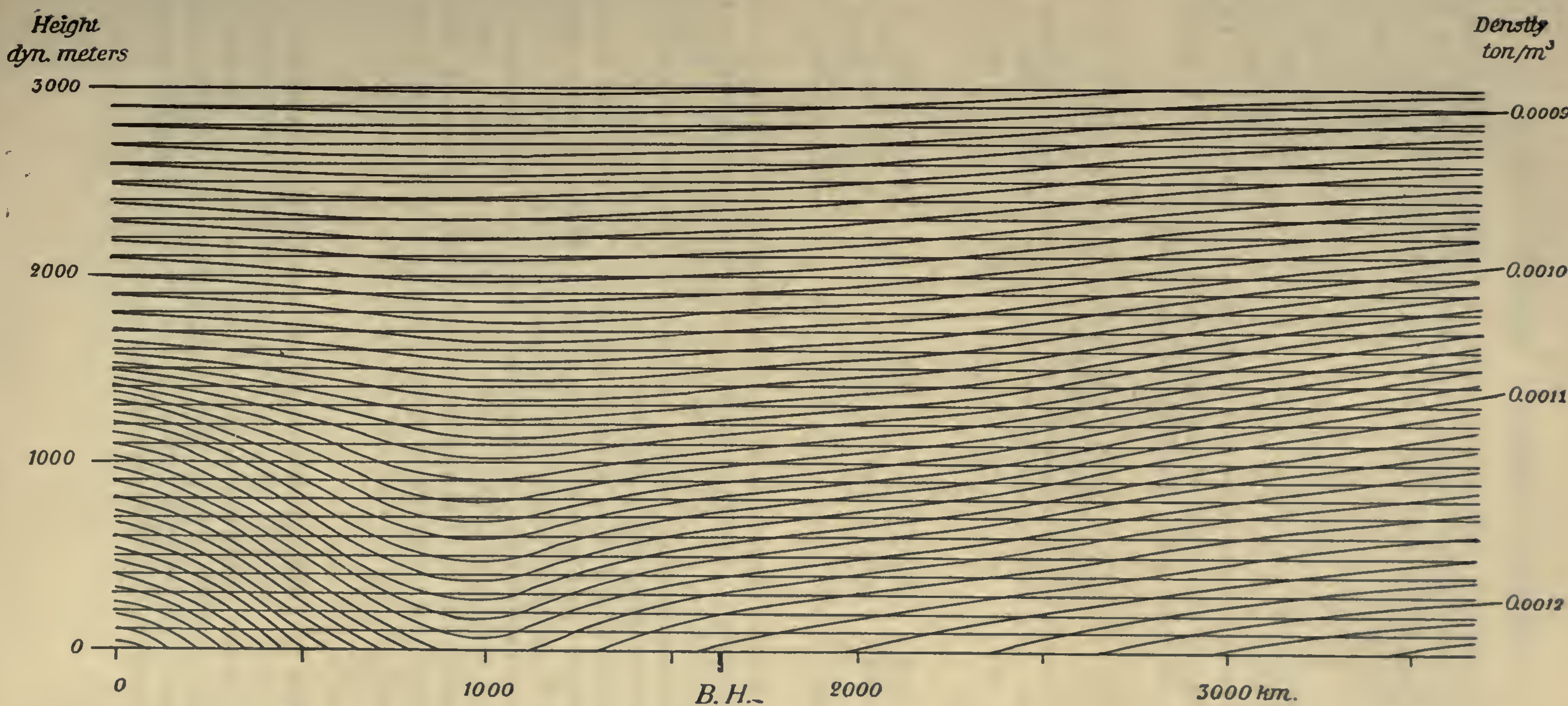


FIG. 18. — United States, September 23, 1898. Profile curves of equipotential and of isopycnic surfaces. Every parallelogram represents 10^{-8} m. t. s. equipotential-isopycnic unit-tubes.

The first chart of fig. 15 gives the pressure at sea-level. It is derived from the isobaric chart of the Weather Bureau, only changed by table 8 A of the Appendix from inches of mercury to millibars. The two charts of pressure-differences (fig. 16), from level 0 to that of 1000, and from this level to that of 2000 dynamic meters, are drawn from the figures in table Q. The curves of these charts are dotted from the point where the lower limiting surface of the sheet cuts the ground and stop where the upper surface cuts the ground, the part of the earth rising above this upper surface being shaded. The second chart of absolute pressure (fig. 15) is obtained by graphic addition of the first charts of fig. 15 and the first of fig. 16, and in the same way the third chart of absolute pressure is obtained by graphic addition of the second chart of fig. 15 and the second of fig. 16.

TABLE Q.—*Pressure (m-bars), in standard level surfaces, and differences of pressure between them. United States, September 23, 1898.*

Station:	Blue Hill.	Cleveland.	Dodge City.	Knoxville.	North Platte.	Omaha.	Pierre.	Topeka.
Dynamic height:	188	210	739	296	840	370	477	291
Height (dynamic meters).								
2000	795.8	788.6	795.6	799.7	791.9	796.0	790.9	796.0
	102.2	102.0	98.4	102.9	100.2	100.0	99.5	100.0
1000	898.0	890.6	894.0	902.6	892.1	896.0	890.4	896.0
	113.4	112.3		112.4		113.0	113.3	110.7
0	1011.4	1002.9		1015.0		1009.0	1003.7	1006.7

The three charts of absolute pressure show barometric depressions in the region about the Great Lakes. Interpreting the charts of pressure differences as charts of mass distribution, we get mainly the same results as from the corresponding charts of relative topography. Where these indicate a minimum of specific volume, those of relative pressure give a maximum of density, and *vice versa*.

Fig. 18 is a vertical section showing the profile curves of the level and the isopycnic surfaces. As in the corresponding section of fig. 17, these profile curves are drawn not from the simultaneous kite ascents at different stations, but from the successive ascents at Blue Hill. Thus if the cyclone has passed without undergoing any interior change (which can not be asserted), these profile-curves will correspond exactly to the same atmospheric state as that represented by the chart. But the construction of the section in this way was of special importance in enabling us to find graphically the absolute and relative pressures in the column Blue Hill in table O.

71. Example 2. — Atmospheric Conditions over Europe, November 7, 1901. — In further illustration we shall consider a second example. It will differ from the preceding one by the greater completeness of the observations. On the one hand, the ascents have reached much greater heights, and on the other the observations from numerous stations at the earth's surface have been available to us in their original form, and not only after reduction to sea-level.

On November 7, 1901, in the morning and forenoon there ascended* from Paris one registering balloon with two instruments; from Strassburg two registering and one manned balloon; from Berlin two registering and one manned balloon; from Vienna one registering and two manned balloons. From St. Petersburg one registering balloon ascended on the following morning, November 8. This ascent has been treated as simultaneous with the others, our aim being only to exemplify the technics of our methods, not to discuss the true state of the atmosphere on this particular occasion.

TABLE R.—*Dynamic heights of standard isobaric surfaces and mutual distances between them, computed from ascents, Europe, November 7, 1901.*

Station:	Paris.	Strassburg.	Berliu.	Wien.	Petersburg.
Latitude:	48° 48′	48° 35′	52° 30′	48° 15′	59° 56′
Longitude:	2° 29′	7° 46′	13° 23′	16° 21′	30° 16′
Dynamic height:	48	141	48	198	5
Pressure (m-bars).					
200	11427		11303		
	2491		2502		
300	8936	9057	8801	8947	8136
	1891	1941	1850	1883	1759
400	7045	7116	6951	7064	6377
	1563	1576	1563	1563	1428
500	5482	5540	5388	5501	4949
	1335	1340	1335	1345	1251
600	4147	4200	4053	4156	3698
	1159	1173	1155	1177	1106
700	2988	3027	2898	2979	2592
	1027	1047	1023	1035	993
800	1961	1980	1875	1944	1599
	926	937	923	933	899
900	1035	1043	952	1011	700
	838	841	841	850	820
1000	197	202	111	161	—120

The results of these ascents have been worked out according to the methods developed in the preceding chapter. In cases where two or more balloons have ascended from the same place, the observations from all the ascents have been introduced in the same diagram and the curve of virtual temperatures drawn so as to suit all observations as closely as possible, attention being paid to the different observations in proportion to their probable value. Thus as long as observations from manned balloons are available, the curve is drawn through points representing these observations. The distance of the observations of the registering balloons from this curve gives valuable corrections to the records of the registering balloon, which may be applied for the greater heights not reached by the manned balloons.

Table R contains the results of the ascents worked out as absolute heights of the standard isobaric surfaces and as distances from surface to surface, while table T contains the same results in the form of absolute pressures at standard levels and differences of pressure from level to level.

* Publications de la Commission Internationale pour l'Aérostation scientifique. Observations des ascensions internationales simultanées et des stations de montagne et de nuages 1901, pp. 390–410. Strassburg, 1903. The use of this important publication is very much impeded by the fact that the observations taken at the common meteorological stations on the days of the ascents are not accessible till many years afterwards, according as the meteorological year books of the different countries appear. This circumstance has forced us to choose our example from the first year of the international ascents, when the aeronautical stations were less numerous and the self-registering instruments less trustworthy than they are at present.

TABLE S.—*Dynamic heights of standard isobaric surfaces and mutual distances between them, computed by extrapolation from observations taken at the earth's surface, Europe, November 7, 1901.*

Station:	Angmagsalik.	Upernivik.	Jakobshavn.	Godthaab.	Ivigtut.	Vestmannö.	Styckisholm.	Grimsey.	Berufjord.	Thorshavn.
Latitude:	65° 36½′	72° 47′	69° 13′	64° 10½′	61° 12′	63° 26′	65° 5′	66° 33′	64° 40′	62° 2½′
Longitude:	37° 33½′ W.	56° 7′ W.	51° 2′ W.	51° 43½′ W.	48° 10′ W.	20° 15′ W.	22° 46′ W.	18° W.	14° 19′ W.	6° 45′ W.
Dynamic height:	31	13	12	9	5	8	11	7	18	9
Pressure (m-bars).										
800	1791	1851	1812	1721	1700	1896	1857	1838	1855	1896
	903	883	903	909	919	928	917	919	936	934
900	888	968	909	812	781	968	940	919	919	962
	816	855	817	823	830	838	830	830	847	845
1000	72	113	92	—11	—49	130	110	89	72	117

Station:	Vardö.	Alten.	Bodö.	Brönnö.	Christiansund.	Florö.	Bergen.	Skudesnæs.	Mandal.	Färder.
Latitude:	70° 22′	69° 58′	67° 17′	65° 28′	63° 7′	61° 36′	60° 23′	59° 9′	58° 2′	59° 2′
Longitude:	31° 8′	23° 15′	14° 24′	12° 13′	7° 45′	5° 2′	5° 21′	5° 16′	7° 27′	10° 32′
Dynamic height:	10	10	7	10	22	8	21	4	5	9
Pressure (m-bars).										
800	1534	1573	1664	1718	1805	1853	1856	1874	1847	1810
	915	909	909	919	920	925	925	931	925	928
900	619	664	755	799	885	928	931	943	922	882
	830	826	826	831	832	837	837	843	837	842
1000	—211	—162	—71	—32	53	91	94	100	85	40

Station:	Christiania.	Dovre.	Karesuando.	Gellivara.	Iockmock.	Haparanda.	Stensele.	Östersund.	Hernösand.	Falun.
Latitude:	59° 55′	62° 5′	68° 26′	67° 8′	66° 36′	65° 50′	65° 4′	63° 11′	62° 38′	60° 37′
Longitude:	10° 43′	9° 7′	22° 30′	20° 40′	19° 51′	24° 9′	17° 11′	14° 38′	17° 57′	15° 38′
Dynamic height:	24	631	325	358	253	9	321	308	15	114
Pressure (m-bars).										
700		2784	2565	2621	2630		2667	2718		
		1014	992	998	1006		1020	1024		
800	1771	1770	1573	1623	1624	1597	1647	1694	1646	1700
	916	908	895	897	903	919	914	918	918	916
900	855	862	678	726	721	678	733	776	728	784
	827	820	812	816	822	817	826	823	830	830
1000	28	42	—134	—90	—101	—139	—93	—47	—102	—46

Station:	Upsala.	Askersund.	Visby.	Vexiö.	Skagen.	Vestervig.	Fanö.	Kjöbenhavn.	Hammershus.	Nikolaistad.
Latitude:	59° 52′	58° 53′	57° 39′	56° 33′	57° 44′	56° 47′	55° 27′	55° 41′	55° 17′	63° 4′
Longitude:	17° 38′	14° 55′	18° 18′	14° 49′	10° 38′	8° 20′	8° 24′	12° 36′	14° 38′	21° 40′
Dynamic height:	23	94	16	164	3	25	5	13	15	
Pressure (m-bars).										
800	1704	1738	1723	1775	1827	1860	2008	1825	1815	1605
	925	912	929	920	935	935	968	933	934	909
900	779	826	794	855	892	925	1040	892	881	696
	826	828	841	833	846	845	873	845	845	826
1000	—47	—2	—47	22	46	80	167	47	36	—130

Station:	Kajana.	Tammerfors.	Hangö.	Valencia.	Aberdeen.	Falmouth.	Kew.	Deerness.	Landale.	Scarborough.
Latitude:	64° 14′	61° 30′	59° 46′	51° 56′	57° 10′	50° 9′	51° 28′	58° 56′	56° 41′	54° 18′
Longitude:	27° 44′	23° 45′	22° 57′	10° 15′ W.	2° 6′ W.	5° 4′ W.	0° 19 W.	2° 45′ W.	5° 41′ W.	0° 24′ W.
Dynamic height:	131	89	5	13	26	55	10	52	7	38
Pressure (m-bars).										
800	1596	1618	1650	1966	1958	1988	1964	1963	2012	1999
	908	910	913	925	930	937	927	947	953	948
900	688	708	737	1041	1028	1051	1037	1016	1059	1051
	827	828	836	831	855	844	835	854	858	855
1000	—139	—120	—99	210	173	207	202	162	201	196

TABLE S.—*Dynamic heights of standard isobaric surfaces and mutual distances between them, computed by extrapolation from observations taken at the earth's surface, Europe, November 7, 1901*—Continued.

Station:	Croukbourne.	Hillington.	Churchstoke.	St. David's Peirbroke.	Birr Castle.	Ben Nevis.	Parc du St. Maur.	Tour Eiffel.	Nantes.	Besançon.
Latitude:	54° 10′	52° 48′	52° 31′	51° 53′	53° 6′	56° 48′	48° 48′	48° 52′	47° 15′	47° 15′
Longitude:	4° 29′ W.	0° 33′	3° 5′ W.	5° 16′ W.	7° 55′ W.	5° W.	2° 29′	2° 17′	1° 34′ W.	5° 59′
Dynamic height:	42	28	164	66	53	1318	48	329	41	305
Pressure (m-bars).										
700						2995				
						1044				
800	2030	1971	2000	2036	2012	1951	1934	2022	1957	1937
	956	934	937	954	938	928	914	947	920	908
900	1074	1037	1063	1082	1074	1023	1020	1075	1037	1029
	862	841	843	859	846		818	849	829	815
1000	212	196	220	223	228		202	226	208	214

Station:	Puy de Dôme.	Puy de Dôme.	Lyon.	Pic du Midi.	Perpignan.	Saint Honorine du Fay.	Toulouse.	Marseille.	Brest.	Langres.
Latitude:	45° 46′	45° 46′	45° 42′	42° 56′	42° 42′	49° 5′	43° 37′	43° 18′	48° 23′	47° 52′
Longitude:	3° 5′	2° 58′	4° 47′	0° 8′	2° 53′	0° 30′ W.	1° 27′	5° 23′	4° 30′ W.	5° 20′
Dynamic height:	380	1438	293	2793	31	116	190	73	64	457
Pressure (m-bars).										
600				4182						
				1183						
700		3038		2999						
		1059		1030						
800	1938	1979	1974	1969	2000	1947	1966	1976	1969	1982
	911	931	932		957	916	941	930	927	937
900	1027	1048	1042		1043	1031	1025	1046	1042	1045
	817		827		862	824	844	867	835	838
1000	210		215		181	207	181	179	207	207

Station:	Mont Ventoux.	Bagnères de Bigorre.	Mont Aigoual.	Helder.	Vlissingen.	Bern.	Genf.	Zürich.	Rigi Kulm.	Sils-Maria.
Latitude:	44° 10′	43° 4′	44° 7′	52° 58′	51° 26′	46° 57′	46° 12′	47° 23′	47° 3′	46° 26′
Longitude:	4° 57′	0° 9′	3° 35′	4° 44′	3° 36′	7° 26′	6° 9′	8° 33′	8° 30′	9° 46′
Dynamic height:	1862	536	1524	5	8	560	397	483	1751	1773
Pressure (m-bars).										
600	4148								4224	4169
	1172								1198	1165
700	2976	2953	3028			2992			3026	3004
	1025	1011	1055			1026			1051	1023
800	1951	1942	1973	1999	1980	1966	1989	1976	1975	1981
	922	915	931	961	932	918	937	926	931	906
900	1029	1027	1042	1038	1048	1048	1052	1050	1044	1075
		830		865	857	822	832	831		
1000		197		173	191	226	220	219		

Station:	Castasegna.	Lugano.	Basel.	Säntis.	Memel.	Neufahrwasser.	Swinemünde.	Hamburg.	Margrabowa.	Breslau.
Latitude:	46° 20′	46° 0′	47° 33′	47° 15′	55° 43′	54° 24′	53° 56′	53° 33′	54° 2′	51° 7′
Longitude:	9° 31′	8° 57′	7° 35′	9° 20′	21° 7′	18° 40′	14° 16′	9° 59′	22° 30′	17° 2′
Dynamic height:	686	269	273	2451	12	4	10	25	159	144
Pressure (m-bars).										
600				4218						
				1193						
700	3031			3025						
	1065			1038						
800	1966	1973	1958	1987	1725	1772	1839	1893	1773	1876
	928	929	919		936	930	939	937	929	923
900	1038	1044	1039		789	842	900	956	844	953
	830	834	826		847	835	843	847	840	838
1000	208	210	213		−58	7	57	109	4	115

TABLE S.—*Dynamic heights of standard isobaric surfaces and mutual distances between them, computed by extrapolation from observations taken at the earth's surface, Europe, November 7, 1901*—Continued.

Station:	Ratibor.	Nordhausen.	Helgoland.	Aachen.	Eichberg.	Schneekoppe.	Wasserleben.	Brocken.	Potsdam.	Strassburg.
Latitude:	50° 6′	51° 30′	54° 10′	50° 47′	50° 55′	50° 44′	51° 56′	51° 48′	52° 23′	48° 35′
Longitude:	18° 13′	10° 48′	7° 51′	6° 6′	15° 48′	15° 44′	10° 45′	10° 37′	13° 4′	7° 46′
Dynamic height:	197	214	41	201	332	1579	153	1126	83	141
Pressure (m-bars).										
600						4075				
						1166				
700						2909		2926		
						1037		1031		
800	1886	1930	1928	1965	1882	1872	1916	1895	1886	1946
	918	938	950	937	926	924	935	921	933	919
900	968	992	978	1028	956	948	981	974	953	1027
	833	841	855	841	839		847		845	822
1000	135	151	123	187	117		134		108	205

Station:	Mülhausen.	Gr. Belchen.	Karlsruhe.	Villingen.	Höckenswand.	Hohenheim.	Pilatus.	Untersberg.	Zugspitze.	Schmittenhöhe.
Latitude:	47° 45′	47° 53′	49° 1′	48° 4′	47° 44′	48° 43′	46° 59′	47° 43′	47° 25′	47° 20′
Longitude:	7° 10′	7° 6′	8° 27′	8° 27′	8° 10′	9° 13′	8° 16′	12° 2′	11° 59′	12° 44′
Dynamic height:	237	1366	124	701	986	304	2027	1632	2907	1928
Pressure (m-bars).										
500									5572	
									1373	
600							4227	4195	4199	4214
							1197	1197	1180	1191
700		3041		2916	2998		3030	2998	3019	3023
		1056		992	1028		1046	1052	1028	1043
800	1952	1985	1949	1924	1970	1841	1984	1946	1991	1980
	919	939	921	889	921	914		929		926
900	1033	1046	1028	1035	1049	927		1017		1054
	825		829	805	836	820				
1000	208		199	230	213	107				

Station:	Schneeberg.	Kuttenplan.	Budweis.	Lemberg.	Czernowitz.	Bregenz.	Innsbruck.	Salzburg.	Sonnblick.	Obir.
Latitude:	47° 45′	49° 54′	48° 58′	49° 50′	48° 17′	47° 30′	47° 16′	47° 48′	47° 3′	46° 30′
Longitude:	15° 50′	12° 43′	14° 28′	24° 1′	25° 56′	9° 45′	11° 24′	13° 2′	12° 57′	14° 29′
Dynamic height:	1414	513	381	301	237	431	560	420	3044	2001
Pressure (m-bars).										
500									5554	
									1380	
600									4174	4223
									1180	1198
700	3047	2924					2975		2994	3025
	1067	1015					1020			1046
800	1980	1909	1930	1876	1875	1984	1955	1944		1979
	927	909	930	919	911	927	910	927		
900	1053	1000	1000	957	964	1057	1045	1017		
		823	833	828	818	829	816	829		
1000		177	167	129	146	228	229	188		

Station:	Wien.	Riva.	Beirut.	Moskwa.	Kola.	Mesen.	Kem.	Arkangelsk.	Valaam.	Povenetz.
Latitude:	48° 15′	45° 53′	33° 54′	55° 49′	68° 53′	65° 50′	64° 57′	64° 33′	61° 23′	62° 51′
Longitude:	16° 21′	10° 50′	35° 29′	37° 34′	33° 1′	44° 16′	34° 39′	40° 32′	30° 57′	34° 49′
Dynamic height:	198	88	33	167	6	14	12	6	36	42
Pressure (m-bars).										
800	1904	2000	1982	1765	1533	1575	1565	1574	1618	1618
	909	935	951	915	909	884	908	894	922	913
900	995	1065	1031	850	624	691	657	680	696	705
	835	847	856	828	825	804	827	815	836	830
1000	160	218	175	22	−201	−113	−170	−135	−140	−125

TABLE S.—*Dynamic heights of standard isobaric surfaces and mutual distances between them, computed by extrapolation from observations taken at the earth's surface, Europe, November 7, 1901*—Continued.

Station:	Kargopol.	Nikolsk.	Vologda.	Pernov.	Velikie Louki.	Vichni Volotchek.	Viatka.	Sarapoul.	Bogoslowsk.	Ekaterinburg.
Latitude:	61° 30′	59° 32′	59° 14′	58° 23′	56° 21′	57° 35′	58° 36′	56° 28′	59° 45′	56° 50′
Longitude:	38° 57′	45° 27′	39° 53′	24° 30′	30° 31′	34° 34′	49° 41′	53° 49′	60° 1′	60° 38′
Dynamic height:	123	148	119	9	102	164	158	116	186	280
Pressure (m-bars).										
800	1619	1683	1676	1671	1712	1704	1709	1731	1653	1725
	909	899	909	920	916	916	881	851	827	852
900	710	784	767	751	796	788	828	880	826	873
	825	814	826	837	830	830	799	768	749	770
1000	—115	—30	—59	—86	—34	—42	29	112	77	103

Station:	Vilno.	Smolensk.	Nischni Novgorod.	Zlatooust.	Oufa.	Orenburg.	Vlotslavsk.	Novaja Alexandria.	Vasilevitchi.	Pinsk.
Latitude:	54° 41′	54° 47′	56° 20′	55° 10′	54° 43′	51° 45′	52° 40°	51° 25′	52° 16′	52° 7′
Longitude:	25° 18′	32° 4′	44° 0′	59° 41′	55° 56′	55° 6′	19° 4′	21° 57′	29° 48′	26° 6′
Dynamic height:	145	216	155	449	171	111	64	144	135	139
Pressure (m-bars).										
700				2749						
				973						
800	1738	1763	1765	1776	1767	1843	1818	1846	1792	1811
	925	914	912	872	857	891	925	927	912	914
900	813	849	853	904	910	952	893	919	880	897
	831	825	824	790	776	807	837	838	825	827
1000	—18	24	29	114	134	145	56	81	55	70

Station:	Orel.	Elatma.	Penza.	Polibino.	Ploti.	Ounan.	Loubny.	Koursk.	Kharkow.	Sagouny.
Latitude:	52° 58′	54° 58′	53° 11′	53° 44′	47° 57′	48° 45′	50° 1′	51° 45′	50° 0′	50° 36′
Longitude:	36° 4′	41° 45′	45° 1′	52° 56′	29° 10′	30° 13′	33° 2′	36° 12′	36° 14′	39° 43′
Dynamic height:	179	137	214	106	140	212	162	231	137	202
Pressure (m-bars).										
800	1819	1776	1846	1780	1847	1874	1903	1845	1895	1865
	907	903	907	867	886	901	923	905	913	898
900	912	873	939	913	961	973	980	940	982	967
	820	818	822	784	804	818	830	821	820	807
1000	92	55	117	129	157	155	150	119	162	160

Station:	Saratow.	Rostrow am Don.	Akktouba.	Astrakhan.	Totaikoi.	Magaratch.	Obdorsk.	Sourgout.	Tioumen	Ouralsk.
Latitude:	51° 32′	47° 13′	48° 18′	46° 21′	44° 54′	44° 32′	66° 31′	61° 17′	57° 10′	51° 12′
Longitude:	46° 3′	39° 43′	46° 9′	48° 2′	34° 11′	34° 13′	66° 35′	73° 20′	65° 32′	51° 22′
Dynamic height:	58	47	11	—14	297	78	26	43	81	37
Pressure (m-bars).										
800	1900	1922	1908	1929	1903	1982	1581	1619	1710	1882
	920	901	895	899	892	933	814	839	856	906
900	980	1021	1013	1030	1011	1049	767	780	854	976
	827	811	809	811	799	839	740	760	776	818
1000	153	210	204	219	212	210	27	20	78	158

Station:	Omsk.	Stavropol.	Novorossisk.	Goudaour.	Tiflis.	Novo Bajazet.	Choucha.	Leukoran.	Askhabad.	Taschkent.
Latitude:	54° 58′	45° 3′	44° 44′	42° 28′	41° 43′	40° 20′	39° 46′	38° 46′	37° 57′	41° 20′
Longitude:	73° 23′	41° 59′	37° 49′	44° 28′	44° 48′	45° 7′	46° 45′	48° 52′	58° 23′	69° 18′
Dynamic height:	88	563	36	2160	396	1940	1340	—20	221	469
Pressure (m-bars).										
600				4168		4212				
				1170		1185				
700		2929		2998		3027	3039			
		1004		1022		1035	1044			
800	1688	1925	1941	1976	1987	1992	1995	2014	2018	1999
	854	899	914		927	920	923	942	944	969
900	834	1026	1027		1060	1072	1072	1072	1074	1030
	773	805	822		829			848	847	849
1000	61	221	205		231			224	227	181

TABLE S.—*Dynamic heights of standard isobaric surfaces and mutual distances between them, computed by extrapolation from observations taken at the earth's surface, Europe, November 7, 1901*—Continued.

Station:	Samarkand.	Derkoulskoe Verderie.	Marionpolskoe.	Kobi.	Krestovaja.	Madrid.	Coimbra.	San Fernando.	Turin.	Riposto.
Latitude:	39° 39′	49° 3′	47° 39′	42° 34′	42° 30′	40° 28′	40° 12′	36° 28′	45° 5′	37° 41′
Longitude:	66° 57′	39° 48′	37° 30′	44° 31′	44° 27′	3° 41′ W.	8° 25′ W.	6° 12′ W.	7° 42′	15° 14′
Dynamic height:	704	152	274	1957	2332	729	138	28	271	14
Pressure (m-bars).										
600				4173	4143					
600–700				1171	1171					
700	3084			3002	2972	3062				
700–800	1071			1025	1019	1041				
800	2013	1893	1915	1977	1953	2021	1956	1932	1951	2069
800–900	964	899	892	900		930	954	955	910	971
900	1049	994	1023	1077		1091	1002	977	1041	1098
900–1000	858	811	808			849	862	864	828	876
1000	191	183	215			242	140	113	213	222

Station:	Aetna.	Prag.	Trieste.	Lizza.	Punta d'Ostro.	Mostar II.	Bjelasnica.	Sarajevo II.	Bihac.	Kupres.
Latitude:	37° 41′	50° 5′	45° 39′	43° 5′	42° 7′	43° 20′	43° 42′	43° 52′	44° 49′	44° 0′
Longitude:	15° 0′	14° 25′	13° 46′	16° 14′	18° 34′	17° 29′	18° 15′	18° 26′	15° 52′	17° 17′
Dynamic height:	2890	193	25	23	63	58	2026	548	222	1166
Pressure (m-bars).										
600	4272						4179			
600–700	1206						1177			
700	3066						3002			3018
700–800	1048						923			1047
800	2018	1908	1996	2027	2029	1986	2079	1959	1913	1971
800–900		923	945	963	958	932		906	915	926
900		985	1051	1064	1071	1054		1053	998	1045
900–1000		837	850	867	862	840		826	821	
1000		148	201	197	209	214		227	177	

Station:	Kolozvar.	O'Gyalla.	Sepsi Szt-Gyorgy.	Turkeve.	Ungvár.	Zsombolya.	Pola.	Jerusalem.	Oran.	Alger.
Latitude:	46° 46′	47° 52′	45° 53′	47° 7′	48° 36′	45° 47′	44° 51′	31° 48′	35° 42′	36° 47′
Longitude:	23° 36′	18° 12′	25° 48′	20° 45′	22° 18′	20° 43′	13° 51′	35° 11′	0° 39′ W.	3° 4′
Dynamic height:	331	117	517	86	116	80	31	733	59	38
Pressure (m-bars).										
700			2979					3056		
700–800			1033					1082		
800	1934	1907	1946	1932	1943	1930	1968	1974	1921	1926
800–900	910	912	925	914	930	910	920	964	964	967
900	1024	995	1021	1018	1013	1020	1048	1010	957	959
900–1000	823	822	826	824	835	820	843	865	871	875
1000	201	173	195	194	178	200	205	145	86	84

Station:	Bizerte.	El-Djem.	Saida.	Fort National.	Geryville.	Laghouat.	Ouargla.	Le Krey.	Ismailia.
Latitude:	37° 17′	35° 21′	34° 51′	36° 38′	33° 41′	33° 48′	31° 55′	33° 49′	30° 36′
Longitude:	9° 50′	10° 38′	0° 10′	4° 12′	1° 00′	2° 53′	5° 10′	35° 40′	32° 16′
Dynamic height:	9	162	848	898	1278	737	153	995	9
Pressure (m-bars).									
700			2985	2982	2990	2967		3087	
700–800			1061	1074	1063	1068		1077	
800	1991	2028	1924	1908	1927	1899	2009	2010	2034
800–900	972	988	958	960	950	948	990	956	980
900	1019	1040	966	948	977	951	1019	1054	1054
900–1000	880	889	863	870		862	879	859	883
1000	139	151	103	78		89	140	195	171

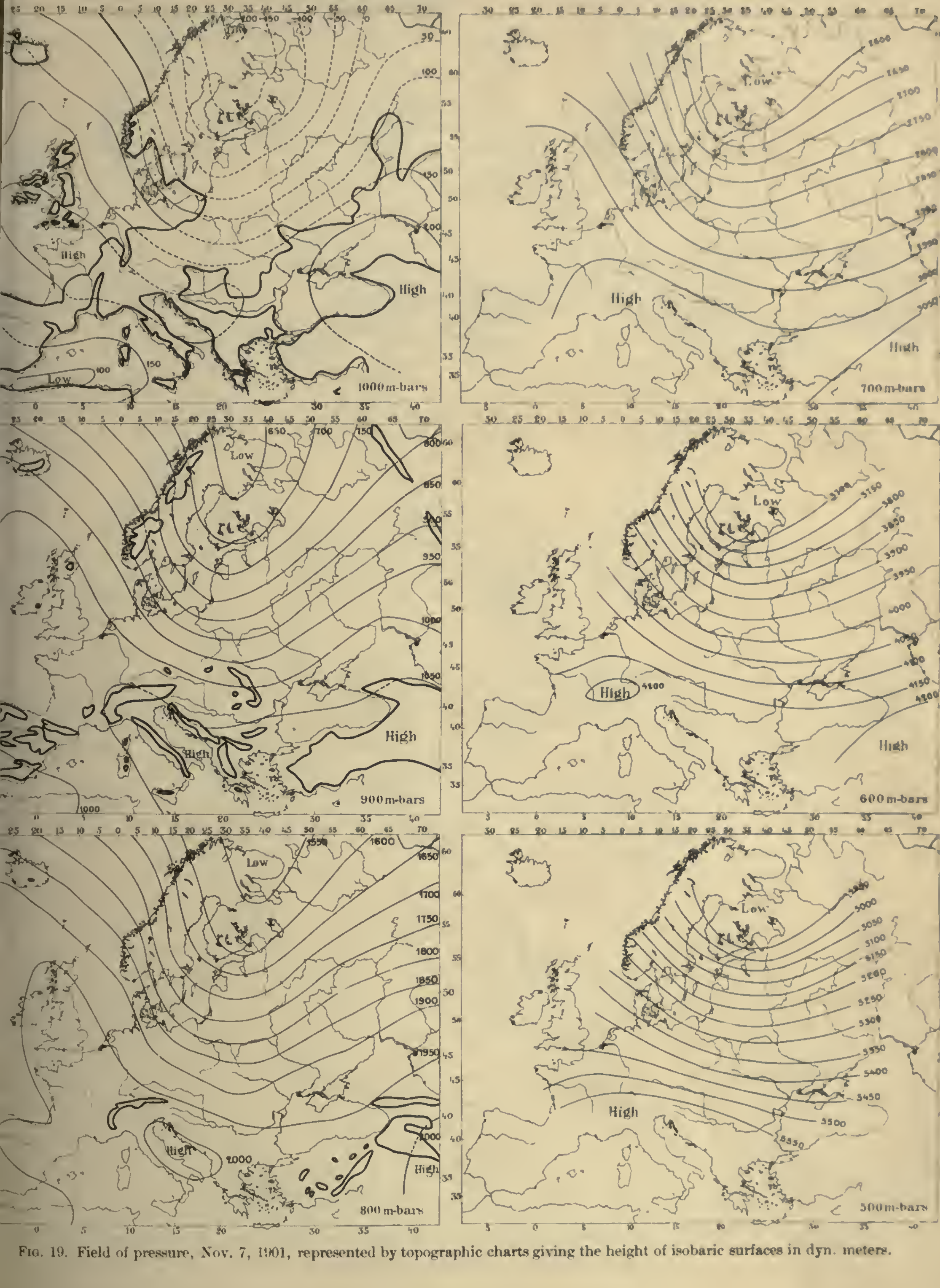


FIG. 19. Field of pressure, Nov. 7, 1901, represented by topographic charts giving the height of isobaric surfaces in dyn. meters.

FIG. 19. (Continued). Field of pressure, Nov. 7, 1901. The last chart gives the situation of the meteorological stations.

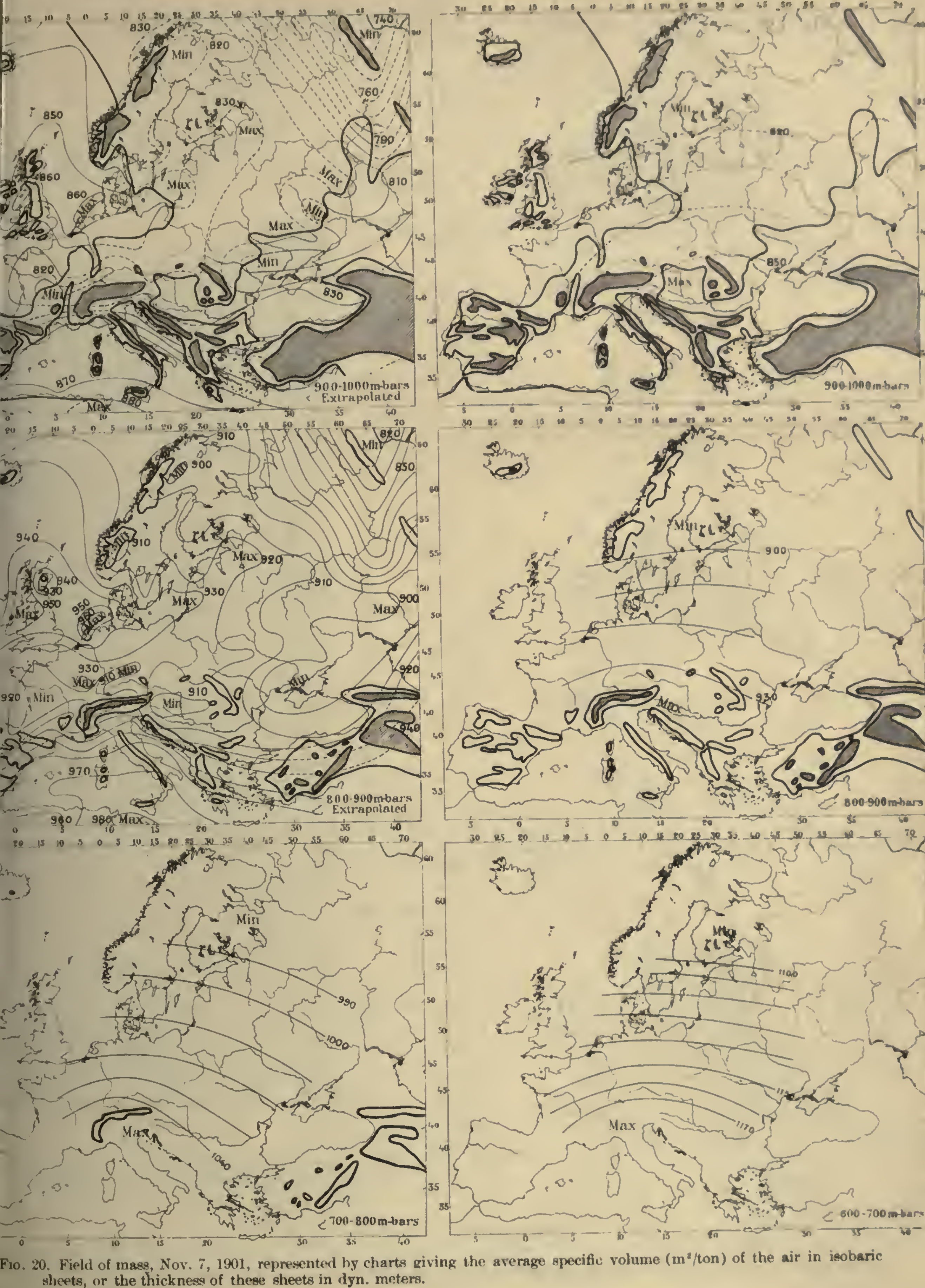


FIG. 20. Field of mass, Nov. 7, 1901, represented by charts giving the average specific volume (m^3/ton) of the air in isobaric sheets, or the thickness of these sheets in dyn. meters.

FIG. 20. (Continued). Field of mass, Nov. 7, 1901

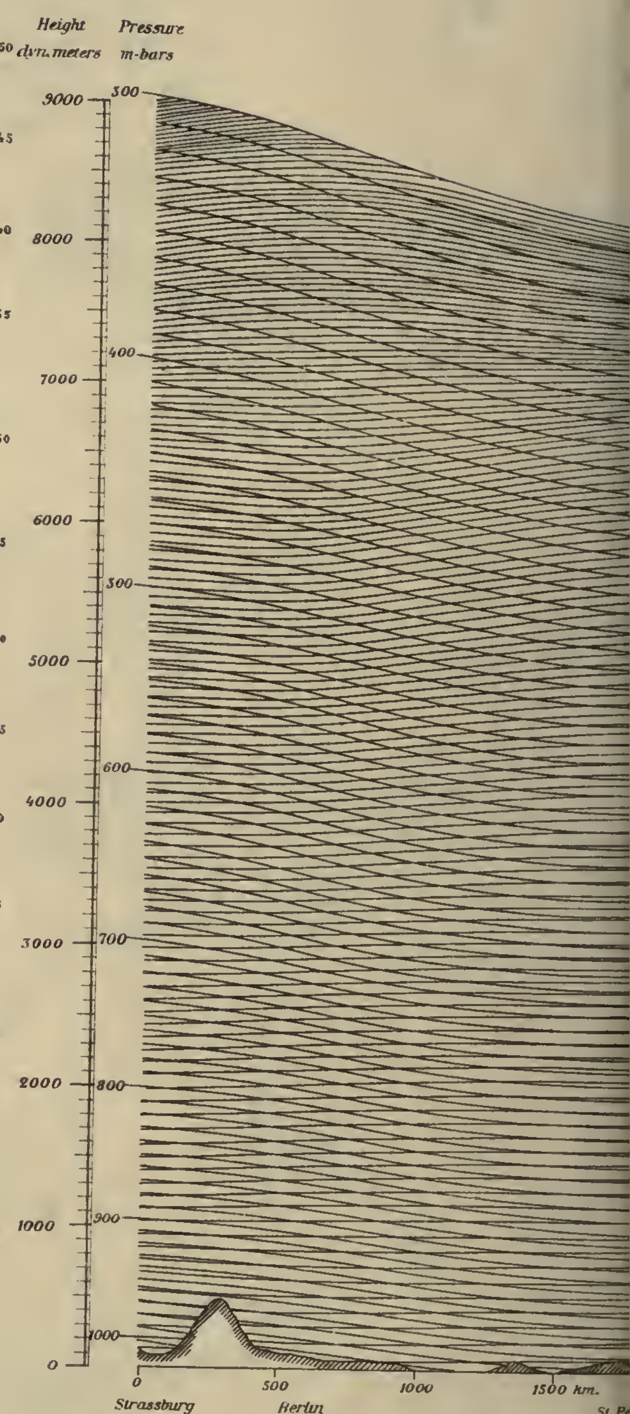


FIG. 21. Profile curves of isobaric and of isosteric surfaces. parallelogram represents 10 m.t.s. isobaric-isosteri unit tubes.

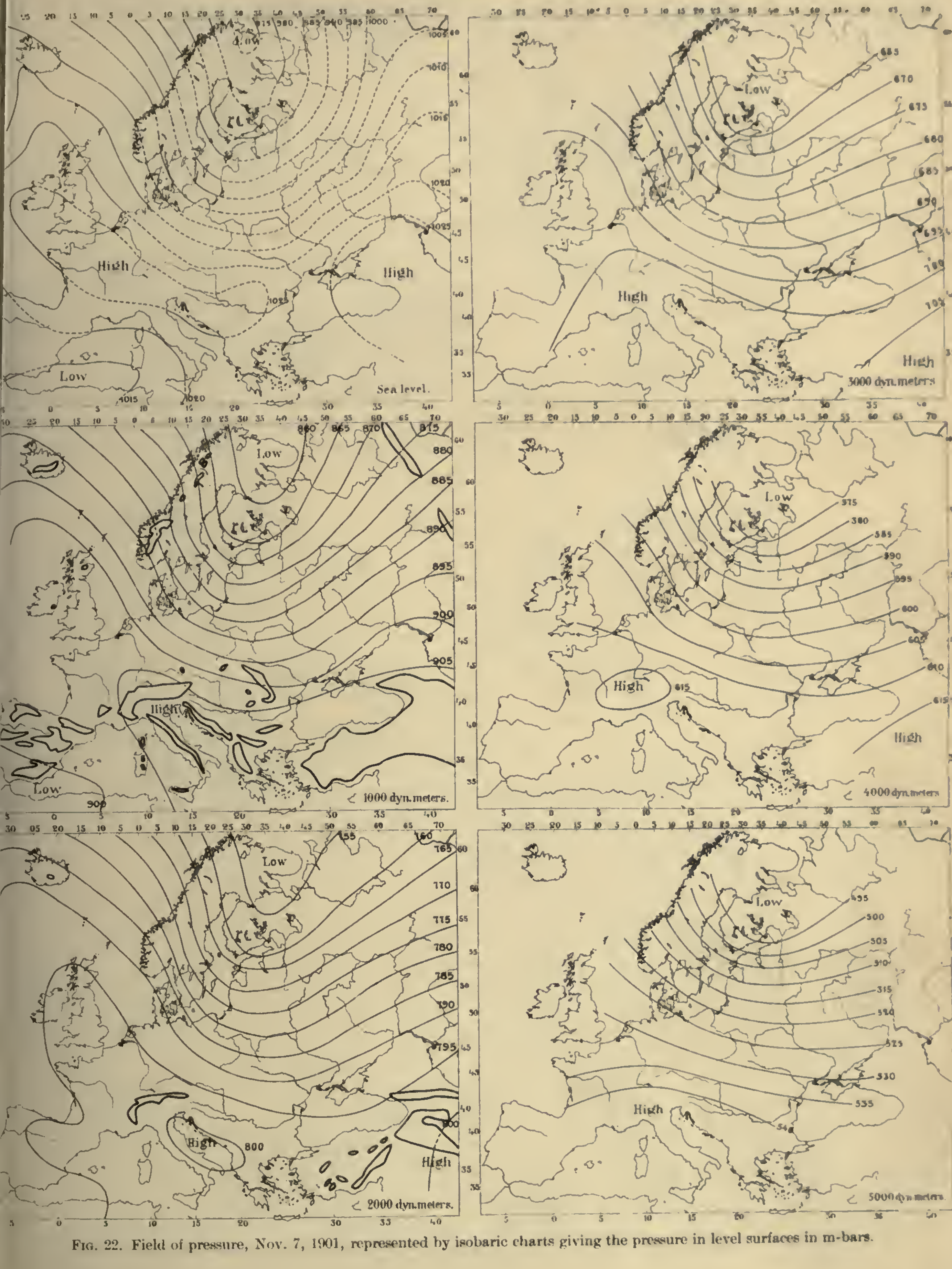


FIG. 22. Field of pressure, Nov. 7, 1901, represented by isobaric charts giving the pressure in level surfaces in m-bars.

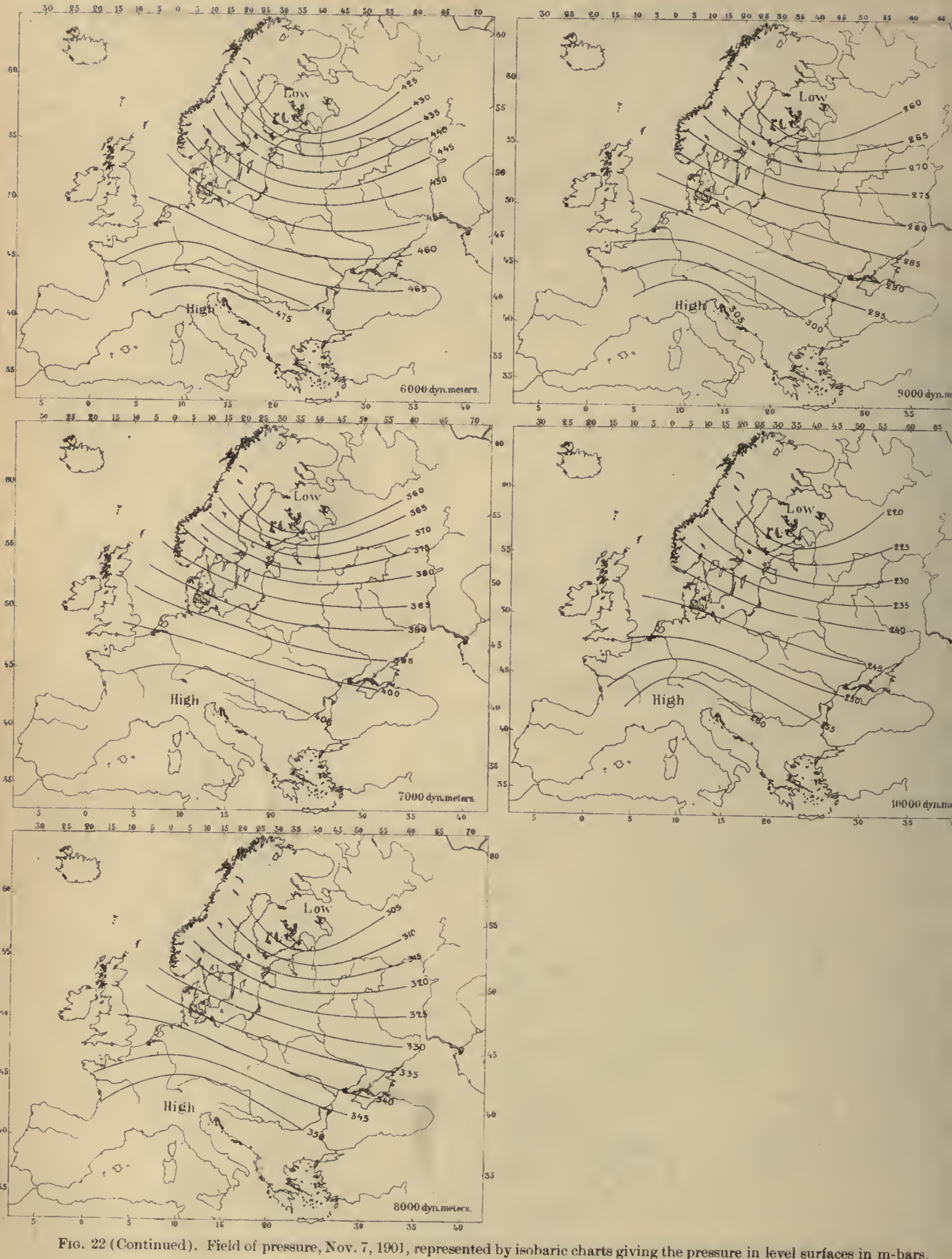


FIG. 22 (Continued). Field of pressure, Nov. 7, 1901, represented by isobaric charts giving the pressure in level surfaces in m-bars.

FIG. 23. Field of mass, Nov. 7, 1901, represented by charts giving the average density (10^{-5}ton/m^3) of the air between level surfaces, or the difference of pressure between these surfaces in m-bars.

Fig. 23. (Continued). Field of mass, Nov. 7, 1901

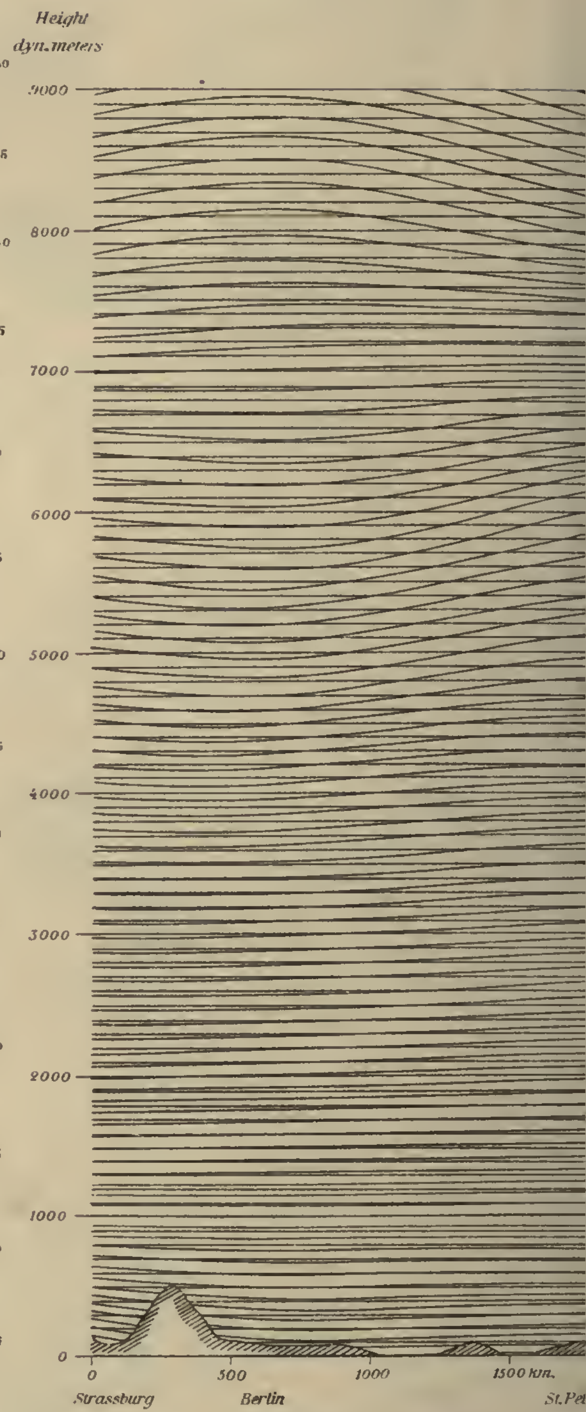


Fig. 24. Profile curves of the level and of the isopycni
Each parallelogram represents 0.01 equip
isopycnic unit tube.

Table S contains the absolute heights of the lowest standard isobaric surfaces, as well as the heights from surface to surface as obtained by the method of extrapolation from the common meteorological stations.* Among those from which observations have been available 219 have been chosen. Their situation is seen from the last chart of fig. 19. The principle in choosing has been to get as many stations as possible on different levels. The chart therefore contains a great number of stations in mountainous regions, and relatively few in low land.

By means of the figures contained in tables R and S the charts of absolute and of mutual topography (figs. 19 and 20) of the standard isobaric surfaces have been drawn in full accordance with the directions given in sections 66 and 68. The two lowest charts of mutual topography obtained by extrapolation from the 219 common stations and the two corresponding obtained from the ascents from the five aeronautical stations are given side by side in fig. 20, no attempt having been made to mold the corresponding charts into one. (Compare section 68.) The charts of absolute pressure in the four lowest levels (fig. 22) have been derived from the corresponding topographic charts of fig. 19 by the graphic method described in section 67. The charts of pressure differences (fig. 23) are drawn exclusively from the pressure differences contained in table T.

As in the preceding example, we have dotted all isobaric or level curves running below the earth's surface, the lines of intersection of the isobaric or the level surfaces with the earth being marked as thick curves. Thus the half of the 1000 m-bar surface is below the earth, while the 900 m-bar surface passes below the earth only in the mountainous parts of Scandinavia, southern Europe, and adjacent parts of Asia. Above the 800 m-bar surface only the higher parts of the Alps and of the Caucasus rise as small islands.

Figs. 21 and 24 are vertical sections containing profile curves, the first of isobaric and isosteric, the second of equipotential and isopycnic surfaces. These sections are worked out from ascents from Strassburg, Berlin, and St. Petersburg.

Figs. 19 to 24 thus described give the distribution of pressure and mass in a cyclone having its center above Finland. Here the isobaric charts show a minimum of pressure, and the topographic charts deep depressions of the isobaric surfaces. A striking feature of the topography of the isobaric surfaces is their inclination as we proceed upwards. It is characteristic of the other method of representation that the isobaric charts do not show noticeably greater difference of pressure in one level than in another. Thus the topography of the isobaric surfaces is in some sense a more sensitive indicator of the distribution of pressure in higher levels than the isobaric charts for given levels. A most striking feature of the charts of mutual topography is that they indicate decreasing specific volume towards the cyclone center. In the same manner the charts of pressure-differences show an increasing density of the air as we approach this center. Thus both indicate a

*The observations are taken from the meteorological year books published by the different countries. Unfortunately some of them (Italy, Spain) only contain different average values, not original observations, and are therefore of no use for investigations in atmospheric dynamics. As is well known, the simultaneity of the observations from the different countries is not very satisfactory. This will be a great difficulty for dynamical investigations. For our present purpose we may treat them as if they were true simultaneous observations.

concentration of mass in the center of the cyclone. We indicate this only to point out what the charts tell, not to discuss the fact in itself. For we have refrained from discussing the reliability of the observations from which charts have been deduced, our only aim being at present to illustrate our methods, the observations being given and considered as trustworthy.

TABLE T.—*Pressure (m-bars) in standard level surfaces and differences of pressure between them, computed from ascents, Europe, November 7, 1901.*

Station: Latitude: Longitude: Dynamic height:	Paris. 48° 48′ 2° 29′ 48		Strassburg. 48° 35′ 7° 46′ 141		Berlin. 52° 30′ 13° 23′ 48		Wien. 48° 15′ 16° 21′ 198		Petersburg. 59° 56′ 30° 16′ 5	
Level (dynamic meters).										
9000	297.0		302.6		290.5		297.5		260.1	
		49.8		48.8		50.4		50.5		46.7
8000	346.8		351.4		340.9		348.0		306.8	
		55.9		55.3		56.2		55.7		54.8
7000	402.7		406.7		397.1		403.7		361.6	
		62.2		62.3		61.7		62.5		63.2
6000	464.9		469.0		458.8		466.2		424.8	
		69.7		69.7		68.7		69.4		71.4
5000	534.6		538.7		527.5		535.6		496.2	
		77.3		77.5		76.8		76.9		78.5
4000	611.9		616.2		604.3		612.5		574.7	
		87.0		86.3		86.3		85.6		87.1
3000	698.9		702.5		690.6		698.1		661.8	
		97.1		95.5		96.7		96.2		96.6
2000	796.0		798.0		787.3		794.3		758.4	
		108.0		106.9		107.3		106.9		107.1
1000	904.0		904.9		894.6		901.2		865.5	
		121.1		121.0		119.3		118.9		119.3
0	1025.1		1025.9		1013.9		1020.1		984.8	

72. Unit-Tubes.—The two sets of curves in figs. 17, 18, 21, and 24 divide the vertical plane into a set of curvilinear parallelograms. These parallelograms are evidently the cross-sections of a set of tubes formed by the intersection of the two sets of surfaces. We may denote them as isobaric-isosteric tubes when they are formed by intersection of the isobaric and the isosteric surfaces (figs. 17 and 21), and as equipotential-isopycnic tubes when formed by intersection of the equipotential and the isopycnic surfaces (figs. 18 and 24). They may further be called unit-tubes if the intersecting surfaces are drawn for unit-differences of the scalar quantities whose fields they represent. The name unit-tubes may be retained also in case of the one set of surfaces being drawn for intervals of a certain multiple of the unit, while the other set is drawn for intervals equal to the corresponding fraction of the unit. In figs. 17 and 21 are isobaric curves drawn for every centibar and isosteric for every 10 m^3/tons. Every parallelogram therefore represents 10 m. t. s. unit isobaric-isosteric tubes. In figs. 18 and 24 a level line is drawn for every 100 dynamic meters, *i. e.*, for every 1000 dynamic decimeters, while the isopycnic curves are drawn for every hundred-thousandth ton/m^3. Every parallelogram in these figures thus represents one-hundredth of a m. t. s. equipotential-isopycnic unit-tube.

In case of true equilibrium there will be no intersection of the surfaces and therefore no tubes. On the other hand, as the angle of intersection increases, the number of unit-tubes will increase. This number can therefore be taken as a measure for the departure from the state of true equilibrium. For this reason it will be useful to develop some simple relations involving the number of unit-tubes.

Proceeding along an isobaric unit-sheet, we get unit-change of specific volume, and consequently unit-change of thickness of the sheet for every isosteric surface met with. Instead of counting the isosteric surfaces, we may also count the unit-tubes. Introducing the ascendant (section 17) of the specific volume, we see that the projection of this vector on the isobaric surface points in the direction of increasing thickness of the sheet. We can therefore count algebraically, reckoning a tube positive when the projection of the ascendant points in the direction in which we proceed, otherwise negative. By this mode of counting we get a measure for the increase of thickness of the unit-sheet. From the unit-sheet we may pass to any sheet composed of any number of unit-sheets; the increase of thickness of the sheet from one vertical to another will be equal to the number of isobaric-isosteric unit-tubes contained between them, counted algebraically in the defined manner. The increase of height comes out in dynamic decimeters if the m. t. s. units be used.

Counting in the same way the number of equipotential-isopycnic unit-tubes contained in an equipotential sheet, we find the variations in the difference of pressure between the upper and the lower limiting surface of the sheet. The rule of signs is formally the same as in the preceding case, the projection of the ascendant of the density pointing in the direction where the difference of pressure increases. Thus, in order to find the increase in the difference of pressure between foot and top of two verticals having their end-points in the same two level surfaces, we have simply to count algebraically the number of equipotential-isopycnic unit-tubes contained within the closed curve formed by the two verticals and two level curves joining their end-points.

73. Relation between Sections and Charts. — These rules lead to a new view of the charts representing the mutual topographies or the differences of pressures. The curves of these charts may be considered as the horizontal projections of vertical walls, dividing the sheets into a set of tubes. These tubes with vertical walls are easily seen to have a close relation to the unit-tubes with oblique walls.

To consider first the charts of mutual topography, each vertical wall has a constant dynamic height. Two different walls therefore have a constant difference of dynamic height. From the numerical relation developed in the preceding article we therefore conclude that every tube with vertical walls must contain a constant number of isobaric-isosteric unit-tubes. If m. t. s. units be used, this number will be equal to the difference of height between the vertical walls, expressed in dynamic decimeters. Thus every section of the tube, it being plane or curved, normal or oblique, contains this constant number of unit-parallelograms. This does not mean that the course of the unit-tubes with their parallelogram-section is exactly the same as that of the tubes with vertical walls. But the latter give the *average* course of the first. Thus, if a unit-tube passes out through the vertical wall, for instance at its base, a corresponding tube will enter through the *same* wall at its top. We thus arrive at this result: The charts of mutual topography of isobaric surfaces show the average course and the number of the isobaric-isosteric unit-tubes in the sheet between the surfaces.

Passing to the charts of pressure differences, we get this perfectly analogous result: The charts for pressure differences between successive level surfaces show the course and the number of equipotential-isopycnic unit-tubes in the sheet between two levels.

On the charts of mutual topography (figs. 14 and 20) the curves are drawn for differences of height of 10 dynamic meters, *i. e.*, for 100 dynamic decimeters. Thus between the vertical walls represented by the curves there run 100 isobaric-isosteric unit-tubes. On the charts of pressure differences (figs. 16 and 23) the curves are drawn for the intervals of pressure of 1 m-bar, *i. e.*, 0.1 c-bar. Between the vertical walls represented by these curves there will consequently run 0.1 equipotential-isopycnic unit-tubes, if the m. t. s. units be used.

74. Complete Representation of the Fields of Moving Forces and Moved Masses in the Atmosphere. — Our aim has been to arrive at a complete representation of the fields of pressure and of mass. But it is worth while mentioning that in reality we have attained more than this.

For the investigation of atmospheric equilibrium and motion a third field, that of gravitational force, is of fundamental importance. Being invariable, this field need not, like the changing fields of pressure and mass, be specially represented. But it merits attention that in our representations of the variable field of pressure is implied also that of the invariable gravitational field.

The charts giving the dynamic topography of the isobaric surfaces are representations of the gravitational field of force tangentially to these surfaces. Mentioning the charts of dynamic topography of the earth's surface and of the bottom of the sea, we have already developed the idea of these charts as representing two-dimensional fields of force (section 18). Evidently a combination of the two-dimensional fields for the succession of isobaric surfaces will give a complete representation of the three-dimensional field in space.

The other representation of the field of pressure is by isobaric curves drawn on level surfaces. Now, the level or equipotential surfaces give themselves a direct representation of the gravity potential and thus of the gravitational field of force. It is the field of pressure, which is represented in the more indirect way, as the field of gravity potential in the preceding case. We have here a perfect parallelism. The isobaric charts in level surfaces represent the two-dimensional fields of the pressure gradient in these surfaces, just as the topographic charts of the isobaric surfaces represented the two-dimensional fields of the potential gradients in these surfaces. The comprehension of these isobaric charts for the successive levels give the representation of the three-dimensional field of the pressure gradient in space.

Whichever of the two methods we choose, we thus get simultaneously a representation of the fields of force due to gravity and to pressure. At the same time, the charts of relative topography or of relative pressure represent the field of mass. We have thus obtained a complete representation of the fields both of the moving forces and of the masses moved.

CHAPTER VIII.

PRACTICAL SOLUTION OF THE HYDROSTATIC PROBLEM FOR THE SEA.

75. Normal Equilibrium Relation and Small Deviations from this Relation. — In order to illustrate the principle of unit-sheets, we have already calculated the depth corresponding to a given pressure (section 36) and the pressure at a given depth (section 39) of the sea having a constant salinity of 35 $^0/_{00}$ and a constant temperature of 0° C. This calculation gave us the fundamental tables 7 H and 15 H of our Hydrographic Tables. In the ideal case of a sea with these constant values of temperature and salinity we have thus fully solved the hydrostatic problem in both its forms.

The treatment of the problem generally is very much simplified by the circumstance that the variations in temperature and salinity only produce minute changes in the equilibrium relation between depth and pressure. We can therefore consider the equilibrium relation represented by tables 7 H or 15 H as the "normal" one. The problem is then reduced to the determination of the small deviations from this relation produced by the variations of temperature and salinity, or, as we may call it, the "anomalies" of the equilibrium relation.

To find the expressions for these anomalies, we have to start with the hydrostatic equation in either of its integral forms, section 40 (*a*) or (*b*). Instead of gravity potential ϕ we introduce the dynamic depth D, measured in dynamic meters and counted positive downwards (section 10). Simultaneously we count the pressure only as sea-pressure (section 27), expressed in decibars. Choosing the lower limit of the integrals in the sea's surface, we then get as expression for the depth D corresponding to a given pressure p,

(*a*) $$D = \int_0^p \alpha dp$$

and as expression for the pressure p at the given depth D,

(*b*) $$p = \int_0^D \rho dD$$

Applying our notations from sections 27 and 29, and introducing

(*c*) $$\alpha = \alpha_{35,0,p} + \delta$$

(*d*) $$\rho = \rho_{35,0,D} + \epsilon$$

we separate the specific volume and the density of the sea-water into their normal values $\alpha_{35,0,p}$, $\rho_{35,0,D}$ and their anomalies δ and ϵ. Substituting this in the equations (*a*) and (*b*), we get D and p separated in two terms,

(*e*) $$D = D_{35,0,p} + \Delta D$$

(*f*) $$p = p_{35,0,D} + \Delta p$$

Where

(g) $$D_{35,0,p} = \int_0^p \alpha_{35,0,p} dp$$

(h) $$p_{35,0,D} = \int_0^D \rho_{35,0,D} dD$$

and

(i) $$\Delta D = \int_0^p \delta dp$$

(j) $$\Delta p = \int_0^D \varepsilon dD$$

Here (g) represents the normal depth corresponding to a given pressure, *i. e.*, the depth tabulated in table 7 H; (h) the normal pressure at a given depth, *i. e.*, the pressure registered in table 15 H. We have therefore henceforth to occupy ourselves only with equations (i) and (j), the first of which gives the anomaly of depth for a given pressure, while the second gives the anomaly of pressure at a given depth.

76. Fundamental Approximation Rules. — The anomalies of depth or of pressure should be determined in accordance with the observed values of salinity and temperature. Generally the values of these quantities are obtained for known values of depth, measured in meters by means of the sounding-line. In other cases a manometer is used, giving the pressure at the places from whence the samples of water are taken, the temperature and salinity of which are determined.

Between the depths of a certain number of common and the same number of dynamic meters there is a difference of about 2 per cent. Between the depths represented by a sea-pressure of a certain number of decibars and that represented by the same number of dynamic meters there is a variable difference not exceeding 3 per cent in the upper layers and 5 per cent in the greatest depths of the sea, as seen from table 7 H. Between the depth represented by a sea-pressure of a certain number of decibars and that represented by the same number of common meters there will finally be a variable difference not exceeding 1 per cent in the smaller and 3 per cent in the greatest depths of the sea. To these differences (from 1 to 5 per cent) of the total depth there will correspond only very small differences of temperature and salinity. For in the upper sheets, where relatively great differences of temperature and salinity may occur, this difference of depth will be very small, and lower down the variations of temperature and salinity will be exceedingly gradual. Thus, these small differences of temperature and salinity will have no appreciable influence upon the small corrections ΔD and Δp. Suppose, therefore, a sample of water to be taken up from a depth of a certain number of common meters. If it be convenient for the calculation we can, without restricting the accuracy of the final result, consider it as taken from the depth of the same number of dynamic meters, or from the isobaric surface of the same number of decibars. Or, suppose the sample to be taken from a place where the manometer has shown a sea-pressure of a certain number of decibars. If it be convenient for the calculations we may consider it as taken from the depth expressed by the same number of dynamic meters.

As a consequence of these approximation rules, it remains indifferent whether depths or pressures have been observed. The four forms of the problem met with in the atmosphere (section 49) are, therefore, in the case of the sea, reduced practically to two, the calculation of the depth corresponding to a given pressure and the calculation of the pressure at a given depth, it being immaterial whether the temperature is registered as functions of pressure or of depth.

77. Calculation of the Anomalies of Depth and of Pressure.—These approximation rules being accepted, the calculation of the integrals (a) and (b) can be made immediately. Taking first the anomaly of depth of the isobaric surfaces, we remember (section 27) that we can write for the anomaly δ of the specific volume

$$\delta = \delta_s + \delta_\tau + \delta_{s\tau} + \delta_{sp} + \delta_{\tau p} + \delta_{s\tau p}$$

the quantities δ_s, δ_τ, $\delta_{s\tau}$, δ_{sp}, $\delta_{\tau p}$, $\delta_{s\tau p}$ being tabulated in tables 9 H, 10 H, 11 H, 12 H, 13 H, and 14 H, respectively, for all occurring values of temperature, salinity, and pressure. By means of these tables and the observed temperatures and salinities we find the values of these quantities and by adding them the values of δ corresponding to a set of known pressures. Then the value of the integral (i), section 75, is found by a regular process of integration; *i. e.*, we take the average of the successive values of δ, multiply by the corresponding difference of pressure, and form the sum from the pressure 0 at sea-level down to the pressure p. This sum represents the anomaly ΔD of the dynamic depth of the isobaric surface of pressure p.

We find the anomaly of pressure Δp in the given dynamic depth D in exactly the same way, writing

$$\epsilon = \epsilon_s + \epsilon_\tau + \epsilon_{s\tau} + \epsilon_{sD} + \epsilon_{\tau D} + \epsilon_{s\tau D}$$

using tables 17 H, 18 H, 19 H, 20 H, 21 H, and 22 H and performing the integration in the same regular way.

The systematic performance of the calculation is easily understood by examination of the examples worked out below.

Adding the anomaly of depth ΔD to the normal value $D_{35,0,p}$ we get the equilibrium relation in form of depth for a given pressure. Adding the anomalies of specific volume δ to the normal values $\alpha_{35,0,p}$ we get the actual specific volumes α for given values of the pressure, *i. e.*, the equilibrium relation between pressure and specific volume.

In the same way the addition of the anomalies of pressure Δp to the normal values $p_{35,0,D}$ gives the equilibrium relation in form of pressures in given dynamic depths, and the addition of the anomalies of density ϵ to the normal densities $\rho_{35,0,D}$ gives the actual densities ρ at given depths, *i. e.*, the equilibrium relation between density and depth.

Example 1.—Norwegian Sea, Station N. 36, June 7, 1904. 64° 55′ N. lat.; 2° 52′ W. long.; 1830 meters, no bottom.

TABLE U̇.—*Depth corresponding to a given pressure.*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Observed depths, meters (in first approximation identical with pressure in decibars).	Observed temperatures, °C.	Observed salinities, ‰.	$10^5 \cdot \delta_s$ found by table 9 H.	$10^5 \cdot \delta_T$ found by table 10 H.	$10^5 \cdot \delta_{ST}$ found by table 11 H.	$10^5 \cdot \delta_{Sp}$ found by table 12 H.	$10^5 \cdot \delta_{Tp}$ found by table 13 H.	$10^5 \cdot \delta_{STp}$ found by table 14 H.	$10^5 \cdot \delta = 10^5(\delta_s + \delta_T + \delta_{ST} + \delta_{Sp} + \delta_{Tp} + \delta_{STp})$ δ = anomaly of specific volume.	Standard pressures, d-bars.	$10^5 \cdot \delta$ interpolated from column 10.	$10^5 \cdot$ anomalies of thickness of isobaric sheets, obtained by multiplication of successive averages of $10^5 \cdot \delta$ (column 12) by corresponding differences of pressure.	$10^5 \cdot \Delta D$, obtained as sums of anomalies of thickness, column 13. ΔD = anomaly of depth of isobaric surfaces.	Depth in dynamic meters of isobaric surfaces, $D = D_{35,0,p} + \Delta D$, $D_{35,0,p}$ being found from table 7 H, the corresponding anomaly ΔD from column 14.	Specific volume in m³/ton, $\alpha = \alpha_{35,0,p} + \delta$, $\alpha_{35,0,p}$ being found from table 8 H, corresponding anomaly δ from column 12.
0	7.49	34.93	6	72	0	0	0	0	78	0	78	785	0	0	0.97342
10	7.33	34.88	9	70	0	0	0	0	79	10	79	740	785	9.7340	0.97339
20	6.70	34.90	8	61	0	0	0	0	69	20	69	630	1525	19.4672	0.97324
30	6.11	34.96	3	54	0	0	0	0	57	30	57	545	2155	29.1988	0.97308
50	5.39	34.99	1	45	0	0	1	0	47	40	52	495	2700	38.9291	0.97298
75	3.88	34.88	9	29	0	0	1	0	39	50	47	455	3195	48.6584	0.97289
100	3.10	34.87	10	22	0	0	1	0	33	60	44	425	3650	58.3869	0.97281
150	2.42	34.85	12	16	0	0	1	0	29	70	41	395	4075	68.1146	0.97274
200	1.77	34.88	9	11	0	0	1	0	21	80	38	365	4470	77.8417	0.97266
250	1.94	34.96	3	12	0	0	1	0	16	90	35	340	4835	87.5679	0.97259
300	1.65	34.96	3	10	0	0	2	0	15	100	33	2700	5175	97.293	0.97252
400	0.77	34.93	6	4	0	0	1	0	11	200	21	1800	7875	194.517	0.97195
500	0.27	34.91	7	2	0	0	0	0	9	300	15	1300	9675	291.687	0.97144
										400	11	1000	10975	388.806	0.97095
										500	9	----	11975	485.878	0.97049

TABLE V.—*Pressure at a given depth.*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Observed depths, meters (in first approximation identical with depths in dynamic meters).	Observed temperatures, °C.	Observed salinities, $^0/_{00}$.	$10^5 \cdot \varepsilon_s$ found by table 17 H.	$10^5 \cdot \varepsilon_\tau$ found by table 18 H.	$10^5 \cdot \varepsilon_{s\tau}$ found by table 19 H.	$10^5 \cdot \varepsilon_{sD}$ found by table 20 H.	$10^5 \cdot \varepsilon_{\tau D}$ found by table 21 H.	$10^5 \cdot \varepsilon_{s\tau D}$ found by table 22 H.	$10^5 \cdot \varepsilon = 10^5(\varepsilon_s + \varepsilon_\tau + \varepsilon_{s\tau} + \varepsilon_{sD} + \varepsilon_{\tau D} + \varepsilon_{s\tau D})$ ε = anomaly of density.	Standard dynamic depths, dynamic meters.	$10^5 \cdot \varepsilon$ interpolated from column 10.	$10^5 \cdot$ anomalies of differences of pressure from level to level, found by multiplication of successive averages of $10^5 \cdot \varepsilon$ (column 10) by corresponding differences of dynamic depth.	$10^5 \cdot \Delta p$ in level surfaces, obtained as sums of the anomalies of differences of pressure, column 13. Δp = anomaly of pressure.	Pressure in d-bars in level surfaces, $p = p_{35,0,D} + \Delta p$, $p_{35,0,D}$ being found from table 15 H, the corresponding anomaly Δp from column 14.	Density in ton/m³, $\rho = \rho_{35,0,D} + \varepsilon$, $\rho_{35,0,D}$ being found from table 16 H, the corresponding anomaly from column 12.
0	7.49	34.93	− 6	−76	0	0	0	0	−82	0	−82		0	0	1.02731
10	7.33	34.88	−10	−73	0	0	0	0	−83	10	−83	825	− 825	10.2733	1.02735
20	6.70	34.90	− 8	−65	0	0	0	0	−73	20	−73	780	− 1605	20.5475	1.02749
30	6.11	34.96	− 3	−57	0	0	−1	0	−61	30	−61	670	− 2275	30.8233	1.02766
50	5.39	34.99	− 1	−48	0	0	−1	0	−50	40	−55	580	− 2855	41.1005	1.02777
75	3.88	34.88	−10	−31	0	0	−1	0	−42	50	−50	525	− 3380	51.3787	1.02787
100	3.10	34.87	−11	−23	0	0	−1	0	−35	60	−47	485	− 3865	61.6578	1.02795
150	2.42	34.85	−12	−17	0	0	−1	0	−30	70	−44	455	− 4320	71.9377	1.02803
200	1.77	34.88	−10	−12	0	0	−1	0	−23	80	−41	425	− 4745	82.2185	1.02811
250	1.94	34.96	− 3	−13	0	0	−1	0	−17	90	−38	395	− 5140	92.4999	1.02819
300	1.65	34.96	− 3	−10	0	0	−2	0	−15	100	−35	365	− 5505	102.782	1.02827
400	0.77	34.93	− 6	− 5	0	0	−1	0	−12	200	−23	2900	− 8405	205.640	1.02888
500	0.27	34.91	− 7	− 2	0	0	0	0	− 9	300	−15	1900	−10305	308.555	1.02945
										400	−12	1350	−11655	411.526	1.02997
										500	− 9	1050	−12705	514.550	1.03049

Example 2.—Baltic, Station F. 64, May 17, 1904. 60° 12.5′ N. lat.; 19° 7′ E. long.; 277 meters.

TABLE W.—*Depth corresponding to a given pressure*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Observed depths, meters (in first approximation identical with pressure in decibars).	Observed temperatures °C.	Observed salinities, ‰.	$10^5 \cdot \delta_s$ found by table 9 H.	$10^5 \cdot \delta_T$ found by table 10 H.	$10^5 \cdot \delta_{sT}$ found by table 11 H.	$10^5 \cdot \delta_{sp}$ found by table 12 H.	$10^5 \cdot \delta_{Tp}$ found by table 13 H.	$10^5 \cdot \delta_{sTp}$ found by table 14 H.	$10^5 \cdot \delta = 10^5(\delta_s + \delta_T + \delta_{sT} + \delta_{sp} + \delta_{Tp} + \delta_{sTp})$. δ= anomaly of specific volume.	Standard pressures, d-bars.	$10^5 \cdot \delta$ interpolated from column 10.	$10^5 \cdot$ anomalies of thickness of isobaric sheets, obtained by multiplication of successive averages of $10^5\delta$ (column 12) by corresponding differences of pressure.	$10^5 \cdot \Delta D$ obtained as sums of the anomalies of thickness $\Delta = D$ anomaly of depth of isobaric surfaces.	Depth in dynamic meters of isobaric surfaces $D = D_{35,0,p} + \Delta D$, $D_{35,0,p}$ being found from table 7 H, the corresponding anomaly ΔD from column 14.	Specific volume in m^3/ton $a = a_{35,0,p} + \delta$, $a_{35,0,p}$ being found from table 8 H, the corresponding anomaly δ from column 12.
0	3.16	5.81	2275	23	−28	0	0	0	2270	0	2270	----	0	0	0.99534
10	3.16	5.81	2275	23	−28	−1	0	0	2269	10	2269	22695	22695	9.9531	0.99529
20	2.82	5.97	2262	19	−25	−1	0	0	2255	20	2255	22620	45315	19.9051	0.99510
30	2.78	5.99	2261	19	−25	−2	0	0	2253	30	2253	22540	67855	29.8558	0.99504
40	2.31	5.99	2261	15	−21	−2	0	0	2253	40	2253	22530	90385	39.8059	0.99499
50	1.69	6.08	2254	10	−15	−3	0	0	2246	50	2246	22495	112880	49.7553	0.99488
75	1.47	6.19	2245	9	−13	−4	0	0	2237	60	2242	22440	135320	59.7036	0.99479
100	1.48	6.22	2242	9	−13	−5	0	0	2233	70	2239	22405	157725	69.6511	0.99472
125	1.88	6.29	2237	12	−17	−6	1	0	2227	80	2236	22375	180100	79.5980	0.99464
150	2.37	6.33	2234	16	−21	−7	1	0	2223	90	2235	22355	202455	89.5440	0.99459
200	2.68	6.38	2230	18	−24	−9	1	0	2216	100	2233	22340	224795	99.4896	0.99452
250	2.76	6.44	2225	19	−25	−11	2	0	2210	200	2216	222400	447195	198.910	0.99390
275	2.73	6.44	2225	18	−25	−13	2	0	2207	300	2206	221100	668295	298.273	0.99335

TABLE X.—*Pressure at a given depth.*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Observed depths, meters (in first approximation identical with depths in dynamic meters).	Observed temperatures °C.	Observed salinities, ‰.	$10^5 \cdot \varepsilon_s$ found by table 17 H.	$10^5 \cdot \varepsilon_\tau$ found by table 18 H.	$10^5 \cdot \varepsilon_{s\tau}$ found by table 19 H.	$10^5 \cdot \varepsilon_{sD}$ found by table 20 H.	$10^5 \cdot \varepsilon_{\tau D}$ found by table 21 H.	$10^5 \cdot \varepsilon_{s\tau D}$ found by table 22 H.	$10^5 \cdot \varepsilon = 10^5(\varepsilon_s + \varepsilon_\tau + \varepsilon_{s\tau} + \varepsilon_{sD} + \varepsilon_{\tau D} + \varepsilon_{s\tau D})$. ε = anomaly of density.	Standard dynamic depths, dynamic meters.	$10^5 \cdot \varepsilon$ interpolated from column 10.	$10^5 \cdot$ anomalies of differences of pressure from level to level, found by multiplication of successive averages of $10^5 \cdot \varepsilon$ (column 12) by corresponding differences of dynamic depth.	$10^5 \cdot \Delta p$ in level surfaces, obtained as sums of the anomalies of differences of pressure, column 13. Δp = anomaly of pressure.	Pressure in d-bars in level surfaces, $p = p_{35, 0, D} + \Delta p$, $p_{35, 0, D}$ being found from table 15 H, the corresponding anomaly Δp from column 14.	Density in ton/m³, $\rho = \rho_{35, 0, D} + \varepsilon$, $\rho_{35, 0, D}$ being found from table 16 H, the corresponding anomaly from column 12.
0	3.16	5.81	−2350	−24	30	0	0	0	−2344	0	−2344	-----	0	0	1.00469
												−23440			
10	3.16	5.81	−2350	−24	30	0	0	0	−2344	10	−2344	-----	− 23440	10.0471	1.00474
												−23370			
20	2.82	5.97	−2337	−20	26	1	0	0	−2330	20	−2330	-----	− 46810	20.0954	1.00492
												−23295			
30	2.78	5.99	−2336	−20	26	1	0	0	−2329	30	−2329	-----	− 70105	30.1449	1.00498
												−23285			
40	2.31	5.99	−2336	−16	22	2	0	0	−2328	40	−2328	-----	− 93390	40.1951	1.00504
												−23250			
50	1.69	6.08	−2329	−11	16	2	0	0	−2322	50	−2322	-----	−116640	50.2461	1.00515
												−23200			
75	1.47	6.19	−2319	−10	14	3	0	0	−2312	60	−2318	-----	−139840	60.2980	1.00524
												−23160			
100	1.48	6.22	−2316	−10	14	4	0	0	−2308	70	−2314	-----	−163000	70.3509	1.00533
												−23120			
125	1.88	6.29	−2311	−13	18	5	−1	0	−2302	80	−2310	-----	−186120	80.4047	1.00542
												−23095			
150	2.37	6.33	−2308	−17	22	6	−1	0	−2298	90	−2309	-----	−209215	90.4591	1.00548
												−23085			
200	2.68	6.38	−2304	−19	25	8	−2	0	−2292	100	−2308	-----	−232300	100.514	1.00554
												−230000			
250	2.76	6.44	−2299	−20	25	10	−2	0	−2286	200	−2292	-----	−462300	201.101	1.00619
												−228850			
275	2.73	6.44	−2299	−19	25	11	−3	0	−2285	300	−2285	-----	−691150	301.747	1.00675

78. Example of the Hydrostatic Results of Soundings in the Sea. — On pages 126–129 are given the schemes for the hydrostatic derivation of the results of soundings in the sea. The examples are chosen from soundings executed by the northern European states, one (Norwegian Expedition, May–June, 1904) being chosen from the Norwegian sea, and one (Finnish Expedition, May, 1904) from the inner Baltic.*

FIG. 25. — State of equilibrium in the Atlantic, 64° 55′ N. lat., 2° 52′ W. long., June 7, 1904.

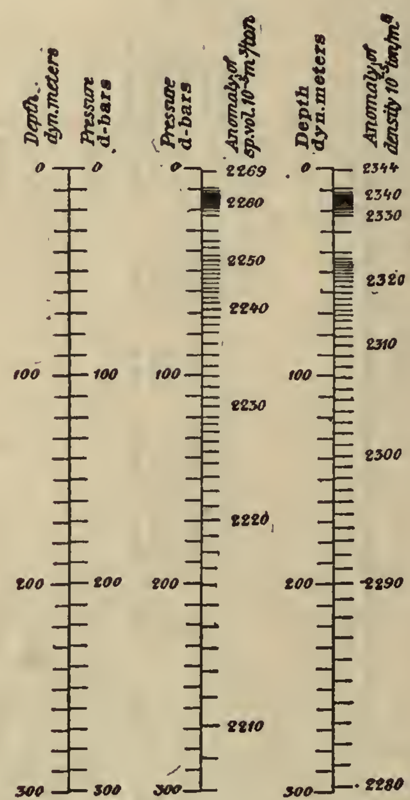


FIG. 26 — State of equilibrium in the Baltic, 60° 12.5′ N. lat., 19° 7′ E. long., May 17, 1904.

* Conseil Permanent International pour l'Exploration de la Mer. Bulletin des Resultats Acquis pendent les Courses Periodiques publié par le Bureau du Conseil avec l'assistance de M. Knudsen. Année 1903–1904. No. 4: Mai, 1904, pp. 96 and 77. Copenhague, 1904.

The soundings are as seen to have been taken with increasing intervals downwards, corresponding to the decreasing variations of temperature and salinity in the greater depths. This inconstancy of intervals, though unavoidable practically, is irrational from a theoretical point of view. Therefore the final results are interpolated for two sets of constant intervals given in column 11 of each scheme. These intervals are 10 dynamic meters or 10 decibars in the upper sheets, and 100 decibars or 100 dynamic meters in the greater depths.

The data for each sounding are treated according to the two different methods, that of depth corresponding to a given pressure (tables U and W), and that of pressure at a given depth (tables V and X).

79. Graphic Representation. — The results worked out in these examples are represented graphically in figs. 25 and 26. The first vertical in each figure gives the depth of the isobaric surfaces exactly as the first of fig. 1 (p. 45) gives these depths for sea-water of 35 ‰ salinity and temperature 0° C. The comparison shows perspicuously a greater depth of the isobaric surfaces in the brackish water of the Baltic than in that of the Atlantic, while the Atlantic vertical would have shown only microscopical differences from that of the normal sea-water (fig. 1), both figures being reduced to the same scale.

On the second vertical of figs. 25 and 26 the first division gives the situation of the isobaric surfaces, transferred from the first vertical. The second division does not, however, give the true specific volume as in fig. 1, but the *anomaly* of the specific volume taken from column 10 of the schemes (pp. 126, 128). In this way the difference from one vertical to another is made much more perspicuous. A vertical would have no anomalous divisions if the water had the "normal" salinity of 35 ‰ and the "normal" temperature of 0° C. The anomalous divisions therefore show the deviation from this normal state. As is seen, these anomalies are of relatively great numerical value in the brackish water of the Baltic, but of much smaller value in the Atlantic. Otherwise the anomaly varies rapidly near the surface and slower as we proceed downward, the variations with the depth being of the same order of magnitude along the vertical in the Baltic as along that in the Atlantic.

The third vertical in figs. 25 and 26 gives in exactly the same way the anomaly of density in different dynamic depths.

CHAPTER IX.

SYNOPTICAL REPRESENTATION OF THE FIELDS OF FORCE AND OF MASS IN THE SEA.

80. Quasi Static State. — The motion of the sea being generally much slower than that of the atmosphere, we may characterize sea-motion with still greater reason than that of the air as slow, and going on near a state of equilibrium. Excepting local phenomena, such as the formation of whirlpools in narrow straits or waves on the surface, we find the conditions of equilibrium apparently fulfilled to a great extent during the motion.

On the other hand we have, still more than was the case in the atmosphere, small distances in a vertical and great in a lateral direction. Consequently the conditions of the quasi static state (section 60) are fulfilled in the sea and with still greater approximation than in the atmosphere. We therefore state this principle, forming the basis of all practical investigations in oceanic dynamics:

The condition of equilibrium is apparently fulfilled along every vertical or quasi vertical line. But as we proceed in a horizontal direction, there is a gradual change in this apparent state of equilibrium from vertical to vertical.

81. Topography of Isobaric Surfaces. — Owing to this principle we can proceed formally as in the case of the atmosphere. Let us suppose first the depth of a given sea-pressure to have been determined by a set of simultaneous soundings. Then on a chart containing the situation of the hydrographic stations, we can note these depths and draw topographic charts, exactly as in the corresponding case of the atmosphere. Fig. 27 below gives examples of such charts.

It is important, however, to get a clear perception of these charts. For an important difference enters between the results attained in the case of the atmosphere and in that of the sea.

First, the surfaces whose topography is represented are surfaces of equal value of the *sea-pressure*, not of the total pressure. Secondly, the topography is given *relatively to the physical sea-level*, from which the measurements are made, and not from the ideal sea-level of the gravity potential zero. To be able to draw charts of absolute topography we want to know the topography of the physical sea-level, and this can not be found from the results of the soundings. This is of great importance to keep in mind. For owing to the motions of the sea and to the varying atmospheric pressure, the distance of physical from ideal sea-level will be of the same order of magnitude as that of isobaric from corresponding level surfaces. Thus, not only theoretically, but also practically, the topographic charts of the isobaric surfaces given in fig. 27 are charts of *relative topography*, *relatively to the unknown topography of physical sea-level.* This is an important restriction on the completeness of the result, making the discussion of sea-motion much less direct than that of atmospheric motions.

It may be useful in this state of indetermination of the results to remark that we can give a slightly changed interpretation to these charts. Let us, instead of *sea-pressure*, consider *total* pressure, obtained by addition of the atmospheric pressure upon the sea's surface. Let us further, to simplify the conditions, imagine the atmosphere to be removed and be replaced by a layer of sea-water of the proper thickness to exert the actual atmospheric pressure. In this case the isobaric surface of absolute pressure 10 decibars will always very nearly coincide with the physical sea-level, passing a little below it in places where the atmospheric pressure upon the physical sea-level is smaller than 10 decibars, and a little above it in the artificially introduced water-layer, where the atmospheric pressure is of smaller value. Now, the working out of the soundings gave the distance from the physical sea-level to a surface of the constant sea-pressure p. But this distance will be essentially the same as the distance from the defined ideal isobaric surface of the total pressure 10 decibars to the isobaric surface where the total pressure is $p + 10$ decibars.

We can thus also interpret the charts of fig. 27 as representing the topography of true isobaric surfaces of a total pressure of $p + 10$ decibars, taken *relatively to the unknown topography of the ideal 10-decibar surface.*

Whichever view we take of the chart representing the distribution of pressure in the sea, the representation remains incomplete. But of the distribution of mass, on the other hand, we are able to give as complete a representation as in the case of the atmosphere.

Forming the differences of depth between two isobaric surfaces of the standard pressures of p and $p + 1$ decibars, we get the numbers representing the specific volume of the water in the standard sheet between the two surfaces. But the thickness of these sheets being only about 1 meter, we would get too many charts by taking every sheet. We have therefore introduced for this purpose dynamic decameters as units of dynamic depth in the upper sheets of the sea, to a depth of 6 dynamic decameters. Simultaneously we use the bar as unit-pressure. For the deeper strata, where the changes with the depth are slower, we have used dynamic hectometers as units of dynamic depth and the decabar as corresponding unit of pressure. The charts representing the specific volume in the corresponding sheets are given in fig. 28.

82. Pressure along Level Surfaces. — Suppose us, on the other hand, to have determined the sea-pressure at a given dynamic depth. We are then able to draw a chart representing the distribution of sea-pressure in this depth. But it must be remembered that this depth is measured from the physical sea-level. The chart thus gives the distribution of the sea-pressure not along a true level surface, but along *a surface of constant dynamic depth below the physical sea-level.*

As in the preceding case, we may take a different view of the chart, giving another definition of the indeterminate element. We then imagine the atmosphere to be replaced by a layer of water having the density of the water at the sea's surface and of the proper thickness to exert the pressure of the atmosphere against the sea's surface. This layer will then fill out the deepenings where the physical surface of the sea is lower than the ideal sea-level. This being done, we can transport all verticals so as to begin at the ideal sea-level. The charts will then represent *a certain pressure along true level surfaces, namely, that which added to the pressure along the ideal sea-level would give the true pressure.* The indeterminate element, then, will be the pressure existing along the ideal sea-level under the defined conditions.

Whichever interpretation we choose for the isobaric charts, the indetermination due to our ignorance of the true topography of physical sea-level will remain. But quite independently of this, the charts giving the differences of pressure from one surface to another will give a full representation of the distribution of density in the standard layers.

83. Change of Topographic into Isobaric Charts. — An isobaric chart for the depth of a certain number of dynamic meters will be exceedingly like the topographic chart of an isobaric surface of the same number of decibars. In the same manner, the chart of pressure differences between two level surfaces will be exceedingly like the chart of mutual topography of the two corresponding isobaric surfaces. If, therefore, the topographic charts be drawn, we can derive the corresponding isobaric charts from them, no independent calculation of pressures at given depths being required.

To change a chart of mutual topography of isobaric surfaces into one of pressure differences between the corresponding levels, table 23 H, which changes densities into corresponding specific volumes, can be used with satisfactory exactitude. This is evident at once if we remember that the charts of mutual topography represent the average specific volume in the isobaric sheets, and those of pressure differences the average density in the level sheets. If, therefore, the water were under exactly the same pressure in the isobaric and the corresponding level sheets, this table would change with perfect exactitude the required differences of pressure between the level surfaces into the corresponding vertical distances between the isobaric surfaces, or *vice versa*. Now, corresponding isobaric and level sheets are not exactly at the same depth, and, therefore, not exactly under the same pressure. But the difference is too small to produce any visible error on the charts drawn according to the directions appended to the table.

We then consider the problem of changing the topographic chart of an isobaric surface into the isobaric chart in the corresponding level surface. Of course, the method of doing this will be independent of the question whether the given chart represents true topography or only relative topography referred to an initial surface of an unknown topography. But in the latter case the resulting isobaric chart will be one of corresponding incompleteness, as explained above (section 81). For simplicity we make our developments as if always true topographies, true level surfaces, and true pressures were under consideration.

Given the isobaric surface of pressure p, represented topographically by the level curves of depth, $D_1, D_2, \ldots D_n$; further, the level D, in which the corresponding isobaric chart should be drawn; and the series of pressures, $p_1, p_2, \ldots p_n$, for which the isobaric curves should be drawn. The problem is to find the situations of these curves from the known situations of the level curves $D_1, D_2, \ldots$ That is, we shall find the depth D_x of a point on the isobaric surface p, vertically below which we have the given pressure p_n at the level surface D.

In the first approximation we may consider the water as homogeneous along every vertical, while its density may vary from vertical to vertical. Under this supposition we have simple proportionality along every vertical between dynamic depth and pressure, thus

$$D_x = D\frac{p}{p_n}$$

But now D and p represent corresponding pressures and depths in the sense defined. Thus *numerically* we may write D instead of p, and consequently

$$(a) \qquad D_x = \frac{D^2}{p_n}$$

This formula is easily tabulated for all depths D at which we wish to draw isobaric charts, and for all pressures p_n which may occur at these depths.

The numbers thus tabulated will, however, be slightly erroneous, because the water between the isobaric and the corresponding level surface is not homogeneous, there being a slight increase of density downward as a consequence of the compression. The amount of this error is easily found in the case of sea-water of 35 $^0/_{00}$ salinity and 0° C. For in this case we have tabulated both the depths at given pressures and the pressures in given depths (tables 7 H and 15 H). From these we find the true D_x, and thus the error involved in the use of the formula (a) in the case of "normal" sea-water. From this the correction in all other cases is easily found. For evidently the error will be proportional to the distance between the isobaric and the corresponding level surface. This distance is zero for water of unit density, and otherwise proportional to the excess of the density above unity.

Table 24 H of our Hydrographic Tables is calculated in this way by the formula (a), with addition of the always very small corrections obtained in the manner described from tables 7 H and 15 H. The practical use of the tables is easily seen from the appended examples.

Evidently these tables also enable us to solve the inverse problem — to change isobaric charts for given levels into topographic charts for the corresponding isobaric surfaces.

84. Vertical Sections. — As we did in the case with the atmosphere, so we can draw diagrams containing the profile curves either of isobaric and isosteric surfaces or of equipotential and isopycnic surfaces. This will, however, present a practical difficulty. Since the deviations of the different profile-curves from the horizontal course are so minute, extreme exaggeration of the vertical dimensions in comparison with the horizontal would be required to make them visible.

FIG. 27. Field of sea-pressure, May, 1904, represented by charts of relative topography, giving the depth of surfaces of equal sea-pressure below physical sea-level in dyn. meters.

Fig. 27 (Continued). Field of sea-pressure, May, 1904, represented by charts of relative topography, giving the depth of surfa
of equal sea-pressure below physical sea-level in

FIG. 28. Field of mass, May 1904, represented by charts giving the average specific volume ($10^{-1}m^3$/ton) of the sea-water in isobaric sheets, or the thickness of these sheets in dyn. meters.

FIG. 28 (Continued). Field of mass, May, 1904, represented by charts giving the average specific volume ($10^{-2} m^3$/ton) of sea-water in isobaric sheets, or the thickness of these sheets in dyn. meters.

But, then, instead of drawing profile curves of true specific volume, we may draw profile curves for constant values of the anomaly of the specific volume. In order to draw sections of this kind we have to place at proper mutual distances verticals like the second of figs. 25 and 26 and join points of the same value of pressure and points of the same value of the anomaly of the specific volume. In the same way, by taking verticals like the third of figs. 25 and 26, we might draw curves through points of the same potential values and through points of the same value of the density anomaly.

These curves for equal values of the anomaly show on an exaggerated scale the deviation of the isosteric or of the isopycnic curves from the horizontal course.

85. Example:—Northern European Waters, May, 1904. — Since November, 1901, hydrographic expeditions have been sent out four times a year from most northern European states for the exploration of the northern European waters. The chart (fig. 29) shows the places where soundings were made by the expeditions in May–June, 1904.* The soundings are very far from being simultaneous. But having no data from true simultaneous soundings, and our object here being mainly to exemplify our methods, and not as yet to discuss the actual states of the sea, we have treated the soundings as if they were simultaneous. How great errors consequently may be introduced it is not possible to find out before more is known of oceanic motion.

The charts of fig. 27 show the topography of surfaces of equal sea-pressure relatively to the physical sea-level, or, according to the other interpretation, the topography of the true isobaric surfaces relatively to the ideal initial surface of the pressure of 10 decibars. The curves are drawn for every dynamic centimeter of depth. The most direct method of obtaining them is to note on the chart the depths calculated from the different soundings and to draw the curves by means of these numbers. But it facilitates the work to note not the absolute depths (column 15 of tables U and W, pp. 126, 128), but the anomalies of depth (column 14 of same tables). For the course and the mutual distance between the curves is determined by the anomalies, while the addition of the constant normal depth of the surface is required only to determine the situation of one of the curves representing an integer value of the total depth. The heavy curves show the intersection of the isobaric surfaces with the bottom of the sea. As the isobaric surfaces are practically level, these limiting curves are obtained at once from a bathymetric chart.

The first six charts of fig. 27 show the topography of six isobaric surfaces with the interval of pressure of 10 decibars, the next six that of six others with the interval of pressure of 100 decibars. A general view of the charts shows that they contain a greater number of lines the more we proceed downward. This does not mean, of course, that the deeper surfaces are necessarily less level than the higher ones. On the contrary, at a certain depth, differing according to circumstances, the isobaric surfaces will show a minimum of deviation from the absolute level surfaces. The greater depths of the isobaric surfaces below the physical sea-level along the

* Bulletin des Resultats Acquis pendent les Courses Periodiques, Année 1903–1904. No. 4. Mai, 1904, pp. 74–98. Copenhague, 1904.

Norwegian coast or in the Baltic, therefore, tell us rather that the sea's surface is higher here than in the open sea. But to what degree this may be the case can not be decided merely from the hydrostatic treatment of sea-soundings. The topography represented by the charts is otherwise a complicated one, showing maxima and minima of distance from the physical sea-level. As a rule there is an increasing distance between physical sea-level and the different isobaric surfaces as we proceed from east to west in the Norwegian Sea, and more especially so as we

Fig. 29.—Hydrographic stations, May, 1904.

continue into the Baltic. In general, the distances are greater along the coasts than in the open sea, but even there both maxima and minima are found, in some cases side by side in a most striking manner. It is important to observe also that, as we proceed downward, the continuity of the isobaric surfaces is soon broken. Only the 10-decibar surface stretches continuously from the Atlantic into the Baltic. Already the 20-decibar surface is broken in the Belts, and for greater depths the Atlantic and the Baltic belong to different systems hydrostatically. As we proceed downward the different deep pools are separated from each other. Finally, also, the Norwegian Sea and the open Atlantic are separated by the Shetland-Faroë-Iceland submarine ridge.

The charts of fig. 28 show the mutual topography of the successive isobaric surfaces, the first six for the interval of pressure of 10, and the last six for the interval of pressure of 100 decibars. The curves on these charts are drawn for intervals 1 dynamic millimeter. The curves along which the upper and the lower surface of

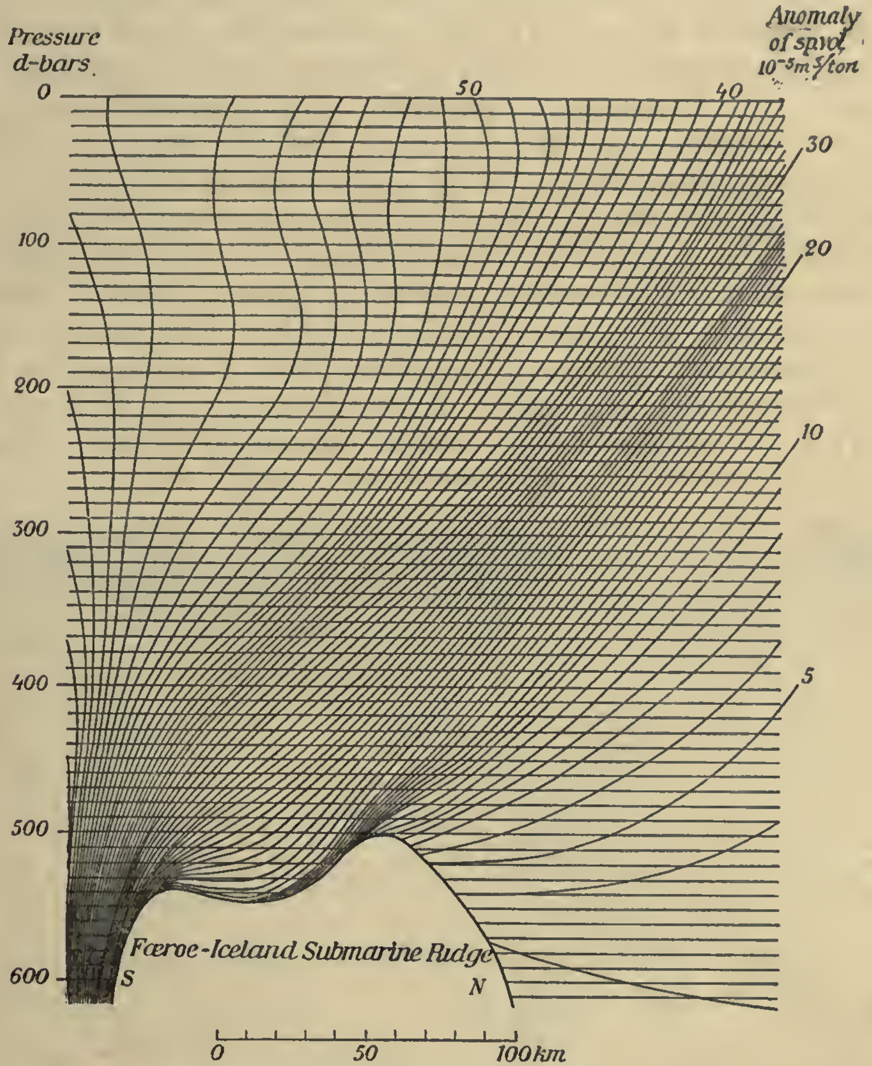


FIG. 30.—Profile curves of isobaric surfaces and surfaces of equal volume anomaly.

the sheets cut the bottom of the sea are drawn heavy. These charts show mainly the same feature as those of fig. 27. In every sheet we find a general increase of thickness as we proceed from Iceland toward Norway, and especially as we get into the Baltic. Otherwise we recognize the same maxima and minima as in the charts of fig. 27. Considered as representing the mass distribution, the charts indicate greater concentration of mass where the sheets have their minimum of thickness and less concentration of mass where they have their maximum of thickness. On the first six charts representing sheets of 10 decibars the figures added to the curves represent the average specific volumes in the sheets after a division by 10, and in the six charts for the sheets of 100 decibars the average specific volume of the water in the sheet after a division by 100.

Besides charts representing the topography of isobaric surfaces, we might also have drawn charts representing the pressures at level surfaces. But these would have been so like the topographic ones that it would have been of no interest to draw them. A glance at table 24 H shows, for instance, that in the Baltic, where the density of the water is so near unity, we have in the upper sheets only to change the numbers added to the curves, 9.95 into 10.05, 19.94 into 20.06 and so on. Then the charts would at once be the isobaric charts for the depth of 10, 20 . . . dynamic meters below sea-level. In the greater depths also a slight change in the situation of the curves representing the integer values would be required. Outside the belts the change would have been a little greater. In the upper layers the isobaric curves would follow each other with 5.5 per cent smaller intervals than the corresponding level curves drawn in fig. 28. This percentage would increase gradually downward with the increasing density due to the compression reaching 6 at the depth of 600 dynamic meters. As, however, the course of the curves is unchanged, the two kinds of charts would be extremely like each other, the most striking difference being that maxima on the one would have been minima on the

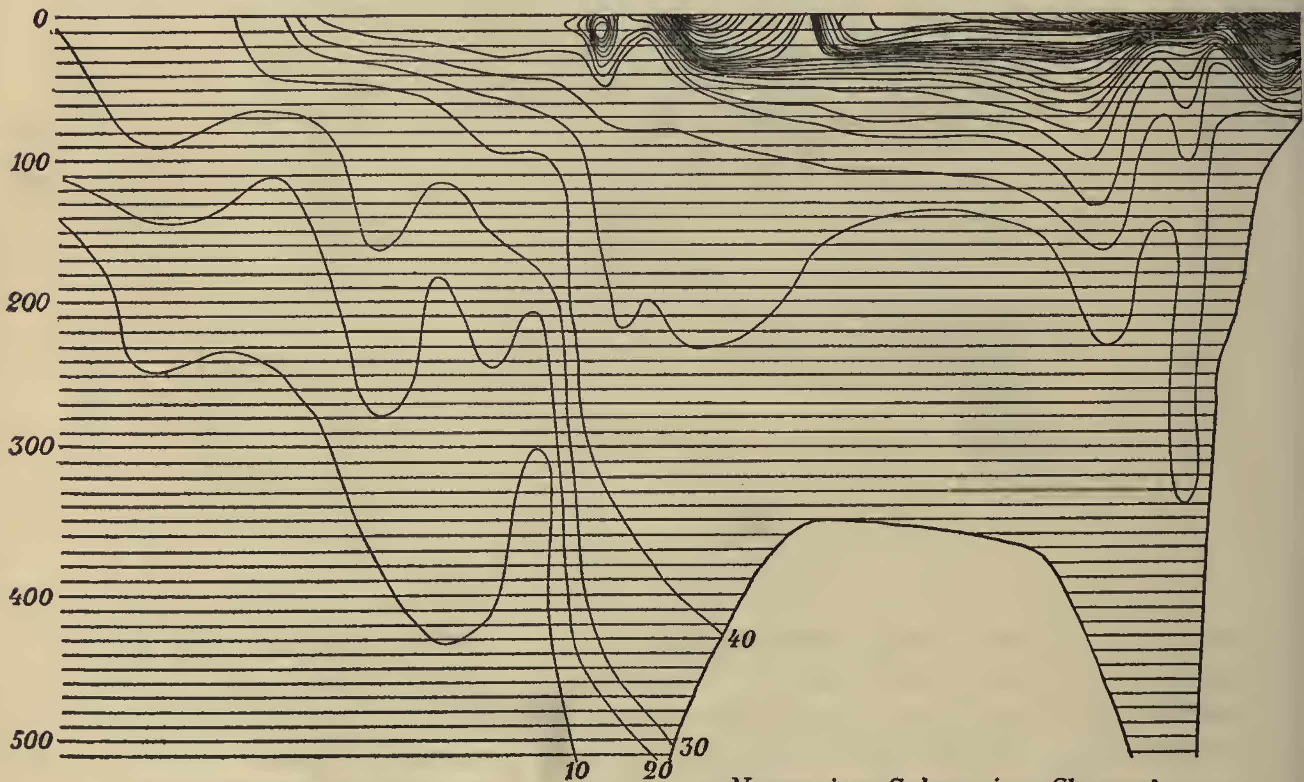
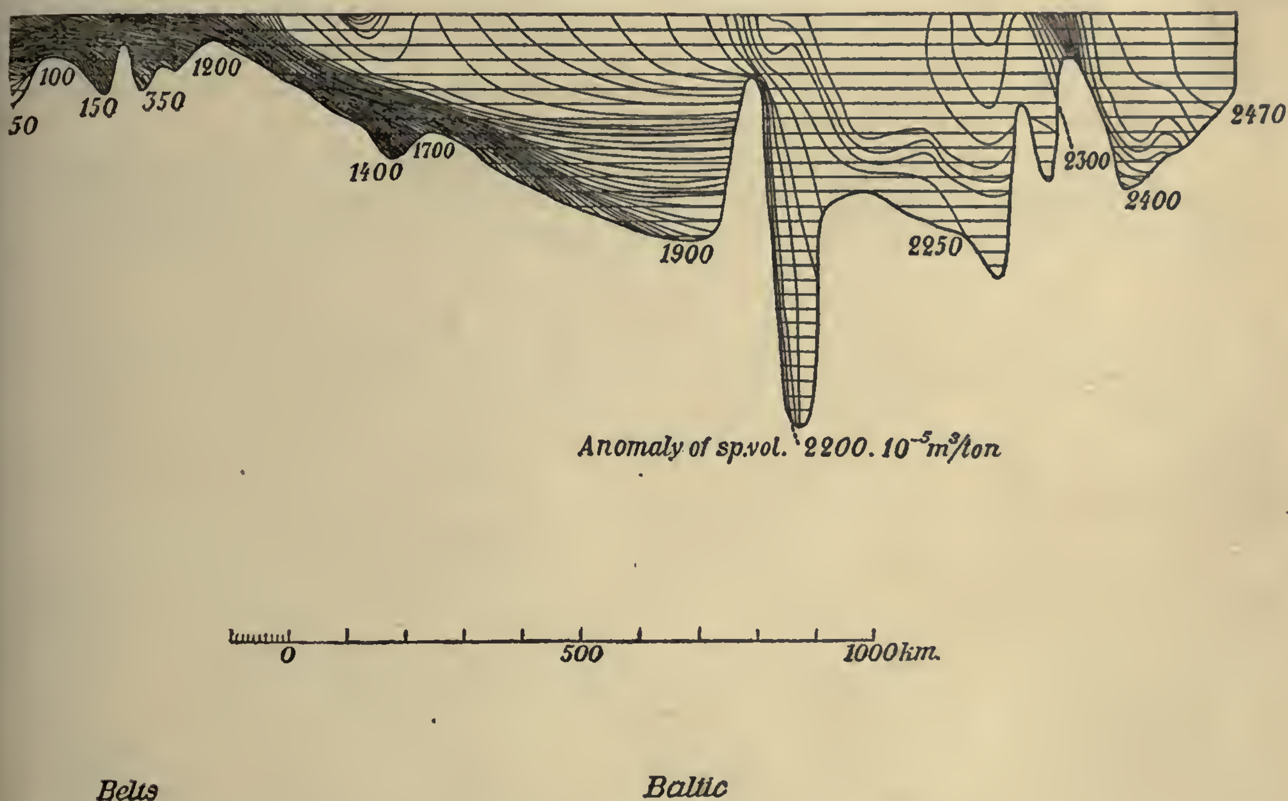


FIG. 31.—Profile curves of isobaric surfaces and surfaces of equal-volume

other, and *vice versa.* If both kinds of charts be drawn, care must be taken that they be not interchanged. The best distinction between them will be this: The figures added to the curves on the topographic charts are all a little below a certain decimal number, thus 9.74, 19.46, 29.20 . . . , while the figures on the isobaric charts are always a little above the same decimal numbers, as 10.26, 20.57, 30.91 . . . .

In figs. 30 and 31 we have, finally, two sections containing the profile curves of the isobaric surfaces drawn simply as horizontal lines, and those of the surfaces of equal anomaly of the specific volume. These give, as we have developed on an exaggerated scale, the elevations and depressions of the true isosteric curves, making the intersection with the isobaric curves more conspicuous. The first is taken across the Faroë Island bank, the second passes from the Baltic through the Belts, along the Norwegian submarine channel across the Norwegian Sea, as shown by the two lines on the station chart (fig. 29). The great density of lines of equal-volume anomaly in the Belts is especially conspicuous. Here we have the change from the brackish Baltic waters to those of the greater salinity of the open sea, and at the same time the greatest deviation from the true equilibrium conditions.

Corresponding sections containing the profile curves of the equipotential surfaces and the surfaces of equal anomaly of density would have had the same appearance, the only difference being that the curves of equal-density anomaly would run a little closer together than those of equal-volume anomaly.



anomaly. Every parallelogram re presents 0.0001 isobaric-isosteric unit-tubes.

86. Remark on Unit-Tubes. — In sections 72 and 73 we have developed some properties of the isobaric-isosteric unit-tubes, formed by the intersection of the isobaric and the isosteric surfaces. It is important to remark that these properties are retained by the tubes whose cross-section is seen in figs. 30 and 31, nothing being changed by the fact that surfaces of equal-volume anomaly have been used instead of the true isosteric surfaces to define the tubes. To show this we remark that the "normal" specific volume is constant all along an isobaric sheet, the anomaly only varying. Consequently we get unit-change of specific volume and unit-change of thickness of the sheet for every volume anomaly met with, these surfaces being drawn for unit-differences of the specific volume. Instead of counting the surfaces we can count the tubes. Further, the variation of the total specific volume and the anomaly going always in the same direction, we can use the same rule for the signs of the tubes, based upon the direction of the projection on the isobaric surfaces of the ascendant of the true specific volume or of its anomaly.

We can therefore use the expression isobaric-isosteric unit-tubes irrespectively of their being defined by true isosteric surfaces or surfaces of equal-volume anomaly. In both cases the algebraic counting of the tubes will give the change of thickness from place to place in an isobaric sheet and the horizontal course of the tubes within the sheets will be given by charts like those of fig. 28, giving the topography of the surfaces limiting the sheet relatively to each other.

The isobaric curves in figs. 30 and 31 being drawn for the interval of 1 centibar, and the curves of equal-volume anomaly for intervals of 0.0001 m^3/ton, each parallelogram in the figure will represent 0.0001 unit-tube. The curves on the charts of fig. 28 being drawn for intervals of 1 dynamic millimeter, the interval between the successive curves will represent 0.01 unit-tube.

What we have thus said of the isobaric-isosteric unit-tubes may, the terms being properly changed, be applied to the equipotential-isopycnic tubes, whether they be defined by the true isopycnic surfaces or by surfaces of equal anomaly of density.

DYNAMIC METEOROLOGY AND HYDROGRAPHY

By V. BJERKNES

AND DIFFERENT COLLABORATORS

HYDROGRAPHIC TABLES

HYDROGRAPHIC TABLES.

Table 1 H.—*Normal value of the acceleration of gravity at sea-level.*

Latitude (degrees).	0	1	2	3	4	5	6	7	8	9
80	9.8306	9.8309	9.8312	9.8314	9.8316	9.8318	9.8319	9.8320	9.8321	9.8321
70	9.8261	9.8266	9.8272	9.8277	9.8282	9.8287	9.8291	9.8295	9.8299	9.8303
60	9.8191	9.8199	9.8207	9.8214	9.8222	9.8229	9.8235	9.8242	9.8249	9.8255
50	9.8107	9.8116	9.8124	9.8133	9.8142	9.8150	9.8159	9.8167	9.8176	9.8184
40	9.8017	9.8026	9.8035	9.8044	9.8053	9.8062	9.8071	9.8080	9.8089	9.8098
30	9.7932	9.7940	9.7948	9.7956	9.7965	9.7973	9.7982	9.7990	9.7999	9.8008
20	9.7864	9.7869	9.7876	9.7882	9.7889	9.7895	9.7902	9.7910	9.7917	9.7925
10	9.7819	9.7822	9.7825	9.7829	9.7833	9.7838	9.7842	9.7847	9.7852	9.7858
0	9.7803	9.7803	9.7804	9.7805	9.7806	9.7807	9.7809	9.7811	9.7813	9.7816

Table 2 H.—*Normal increase of the acceleration of gravity with the depth.*

Depth (meters).	0	100	200	300	400	500	600	700	800	900
0	0.0000	0.0002	0.0004	0.0007	0.0009	0.0011	0.0013	0.0015	0.0018	0.0020
1000	.0022	.0024	.0026	.0029	.0031	.0033	.0035	.0037	.0040	.0042
2000	.0044	.0046	.0048	.0051	.0053	.0055	.0057	.0059	.0062	.0064
3000	.0066	.0068	.0070	.0073	.0075	.0077	.0079	.0081	.0084	.0086
4000	.0088	.0090	.0092	.0095	.0097	.0099	.0101	.0103	.0106	.0108
5000	.0110	.0112	.0115	.0117	.0119	.0121	.0123	.0126	.0128	.0130
6000	.0132	.0134	.0137	.0139	.0141	.0143	.0145	.0148	.0150	.0152
7000	.0154	.0156	.0159	.0161	.0163	.0165	.0167	.0170	.0172	.0174
8000	.0176	.0178	.0181	.0183	.0185	.0187	.0189	.0192	.0194	.0196
9000	.0198	.0200	.0203	.0205	.0207	.0209	.0211	.0214	.0216	.0218

Example:

Latitude, 42° 27′, table 1 H gives............ 9.8039
Depth, 4260 meters, table 2 H gives......... 0.0094

9.8133

Table 3 H.—*Depths reduced from meters to dynamic meters, the acceleration of gravity at sea-level being 9.80.*

Depth (meters).	0	100	200	300	400	500	600	700	800	900
0	0	98	196	294	392	490	588	686	784	882
1000	980	1078	1176	1274	1372	1470	1568	1666	1764	1862
2000	1960	2058	2157	2255	2353	2451	2549	2647	2745	2843
3000	2941	3039	3137	3235	3333	3431	3529	3628	3726	3824
4000	3922	4020	4118	4216	4314	4412	4510	4608	4707	4805
5000	4903	5001	5099	5197	5295	5393	5491	5590	5688	5786
6000	5884	5982	6080	6178	6277	6375	6473	6571	6669	6767
7000	6865	6964	7062	7160	7258	7356	7454	7553	7651	7749
8000	7847	7945	8043	8142	8240	8338	8436	8534	8633	8731
9000	8829	8927	9025	9124	9222	9320	9418	9516	9615	9713

PROPORTIONALITY TABLE.

Meters.	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
10	10	11	12	13	14	15	16	17	18	19
20	20	21	22	23	24	25	25	26	27	28
30	29	30	31	32	33	34	35	36	37	38
40	39	40	41	42	43	44	45	46	47	48
50	49	50	51	52	53	54	55	56	57	58
60	59	60	61	62	63	64	65	66	67	68
70	69	70	71	72	73	74	74	75	76	77
80	78	79	80	81	82	83	84	85	86	87
90	88	89	90	91	92	93	94	95	96	97

Table 4 H.—*Corrections to table 3 H for values of the acceleration of gravity different from 9.80*

Depth (meters).	Acceleration of gravity at sea-level.						
	9.78	9.79	9.80	9.81	9.82	9.83	9.84
0	0	0	0	0	0	0	0
1000	− 2	−1	0	1	2	3	4
2000	− 4	−2	0	2	4	6	8
3000	− 6	−3	0	3	6	9	12
4000	− 8	−4	0	4	8	12	16
5000	−10	−5	0	5	10	15	20
6000	−12	−6	0	6	12	18	24
7000	−14	−7	0	7	14	21	28
8000	−16	−8	0	8	16	24	32
9000	−18	−9	0	9	18	27	36

	Dynamic meters.
Example—Depth 2734 meters, gravity at sea-level 9.7828 :	
Table 3 H gives for the depth 2700 meters	2647
Proportionality table gives for the depth 34 meters	+33
Table 4 H gives for gravity 9.7828 at sea-level and depth 2734 meters	− 4
Dynamic depth corresponding to the geometric depth 2734	2676

Table 5 H.—*Depths reduced from dynamic meters to meters, the acceleration of gravity at sea-level being 9.80.*

Depth (dynamic meters).	0	100	200	300	400	500	600	700	800	900
0	0	102	204	306	408	510	612	714	816	918
1000	1020	1122	1224	1326	1428	1530	1632	1734	1836	1938
2000	2040	2142	2244	2346	2448	2550	2652	2754	2856	2958
3000	3060	3162	3264	3366	3468	3570	3672	3774	3876	3978
4000	4080	4182	4284	4386	4488	4589	4691	4793	4895	4997
5000	5099	5201	5303	5405	5507	5609	5711	5813	5914	6016
6000	6118	6220	6322	6424	6526	6628	6730	6831	6933	7035
7000	7137	7239	7341	7443	7545	7647	7748	7850	7952	8054
8000	8156	8258	8359	8461	8563	8665	8767	8869	8971	9072
9000	9174	9276	9378	9480	9581	9683	9785	9887	9989	10091

PROPORTIONALITY TABLE.

Dynamic meters.	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
10	10	11	12	13	14	15	16	17	18	19
20	20	21	22	23	24	26	27	28	29	30
30	31	32	33	34	35	36	37	38	39	40
40	41	42	43	44	45	46	47	48	49	50
50	51	52	53	54	55	56	57	58	59	60
60	61	62	63	64	65	66	67	68	69	70
70	71	72	73	74	75	77	78	79	80	81
80	82	83	84	85	86	87	88	89	90	91
90	92	93	94	95	96	97	98	99	100	101

Table 6 H.—*Corrections to table 5 H for values of the acceleration of gravity different from 9.80.*

Depth (dy-namic meters).	Acceleration of gravity at sea-level.						
	9.78	9.79	9.80	9.81	9.82	9.83	9.84
0	0	0	0	0	0	0	0
1000	2	1	0	−1	−2	−3	−4
2000	4	2	0	−2	−4	−6	−8
3000	6	3	0	−3	−6	−9	−12
4000	8	4	0	−4	−8	−12	−17
5000	10	5	0	−5	−10	−16	−21
6000	12	6	0	−6	−12	−19	−25
7000	15	7	0	−7	−15	−22	−29
8000	17	8	0	−8	−17	−25	−33
9000	19	9	0	−9	−19	−28	−37

Example—Depth 2676 dynamic meters: Meters.

Table 5 H gives for the depth of 2600 dynamic meters.. 2652

Proportionality table gives for the depth 76 dynamic meters................................ +78

Table 6 H gives for gravity 9.7828 at sea-level and depth 2676 dynamic meters....... + 4

Geometric depth corresponding to the dynamic depth 2676........................ 2734

Table 7 H.—$D_{35, 0, p}$ *(depth in dynamic meters of standard isobaric surfaces in sea-water of 35 °/₀₀ salinity and 0° C.).*

Sea-pressure (decibars).	0	10	20	30	40	50	60	70	80	90
0	0	9.7262	19.4520	29.1773	38.9021	48.6265	58.3504	68.0739	77.7970	87.5195
100	97.2417	106.963	116.685	126.405	136.126	145.846	155.565	165.284	175.003	184.721
200	194.438	204.155	213.872	223.588	233.304	243.020	252.735	262.449	272.163	281.877
300	291.590	301.302	311.015	320.726	330.438	340.149	349.859	359.569	369.279	378.988
400	388.696	398.404	408.112	417.820	427.527	437.233	446.939	456.644	466.350	476.054
500	485.758	495.462	505.165	514.868	524.571	534.273	543.974	553.675	563.376	573.076
600	582.776	592.475	602.174	611.873	621.571	631.268	640.965	650.662	660.358	670.054
700	679.749	689.444	699.138	708.832	718.526	728.219	737.912	747.604	757.296	766.987
800	776.678	786.368	796.058	805.748	815.437	825.126	834.814	844.502	854.189	863.876
900	873.563	883.249	892.934	902.619	912.304	921.988	931.672	941.356	951.039	960.721
1000	970.403	980.09	989.77	999.45	1009.13	1018.81	1028.49	1038.16	1047.84	1057.52
1100	1067.20	1076.88	1086.55	1096.23	1105.91	1115.58	1125.26	1134.93	1144.61	1154.28
1200	1163.95	1173.63	1183.30	1192.97	1202.64	1212.31	1221.98	1231.65	1241.32	1250.99
1300	1260.66	1270.33	1280.00	1289.67	1299.34	1309.00	1318.67	1328.33	1338.00	1347.67
1400	1357.33	1366.99	1376.66	1386.32	1395.98	1405.65	1415.31	1424.97	1434.63	1444.29
1500	1453.95	1463.61	1473.27	1482.93	1492.59	1502.25	1511.91	1521.56	1531.22	1540.88
1600	1550.53	1560.19	1569.84	1579.50	1589.15	1598.81	1608.46	1618.11	1627.77	1637.42
1700	1647.07	1656.72	1666.37	1676.02	1685.67	1695.32	1704.97	1714.62	1724.27	1733.92
1800	1743.56	1753.21	1762.86	1772.50	1782.15	1791.80	1801.44	1811.08	1820.73	1830.37
1900	1840.02	1849.66	1859.30	1868.94	1878.58	1888.23	1897.87	1907.51	1917.15	1926.79
2000	1936.42	1946.06	1955.70	1965.34	1974.98	1984.61	1994.25	2003.89	2013.52	2023.16
2100	2032.79	2042.43	2052.06	2061.69	2071.33	2080.96	2090.59	2100.22	2109.85	2119.49
2200	2129.12	2138.75	2148.38	2158.01	2167.63	2177.26	2186.89	2196.52	2206.15	2215.77
2300	2225.40	2235.02	2244.65	2254.27	2263.90	2273.52	2283.15	2292.77	2302.39	2312.02
2400	2321.64	2331.26	2340.88	2350.50	2360.12	2369.74	2379.36	2388.98	2398.60	2408.22
2500	2417.84	2427.45	2437.07	2446.69	2456.30	2465.92	2475.54	2485.15	2494.77	2504.38
2600	2513.99	2523.61	2533.22	2542.83	2552.44	2562.06	2571.67	2581.28	2590.89	2600.50
2700	2610.11	2619.72	2629.33	2638.94	2648.54	2658.15	2667.76	2677.36	2686.97	2696.58
2800	2706.18	2715.79	2725.39	2735.00	2744.60	2754.20	2763.81	2773.41	2783.01	2792.61
2900	2802.21	2811.81	2821.42	2831.02	2840.61	2850.21	2859.81	2869.41	2879.01	2888.61
3000	2898.20	2907.80	2917.40	2926.99	2936.59	2946.18	2955.78	2965.37	2974.97	2984.56
3100	2994.15	3003.75	3013.34	3022.93	3032.52	3042.11	3051.70	3061.29	3070.88	3080.47
3200	3090.06	3099.65	3109.24	3118.83	3128.42	3138.00	3147.59	3157.18	3166.76	3176.35
3300	3185.93	3195.52	3205.10	3214.68	3224.27	3233.85	3243.43	3253.01	3262.60	3272.18
3400	3281.76	3291.34	3300.92	3310.50	3320.08	3329.66	3339.24	3348.81	3358.39	3367.97
3500	3377.54	3387.12	3396.70	3406.27	3415.85	3425.42	3435.00	3444.57	3454.15	3463.72
3600	3473.29	3482.86	3492.44	3502.01	3511.58	3521.15	3530.72	3540.29	3549.86	3559.43
3700	3569.00	3578.57	3588.13	3597.70	3607.27	3616.84	3626.40	3635.97	3645.53	3655.10
3800	3664.66	3674.23	3683.79	3693.35	3702.92	3712.48	3722.04	3731.60	3741.17	3750.73
3900	3760.29	3769.85	3779.41	3788.97	3798.53	3808.09	3817.64	3827.20	3836.76	3846.32
4000	3855.87	3865.43	3874.99	3884.54	3894.10	3903.65	3913.21	3922.76	3932.31	3941.87
4100	3951.42	3960.97	3970.52	3980.08	3989.63	3999.18	4008.73	4018.28	4027.83	4037.38
4200	4046.92	4056.47	4066.02	4075.57	4085.12	4094.66	4104.21	4113.76	4123.30	4132.85
4300	4142.39	4151.94	4161.48	4171.02	4180.57	4190.11	4199.65	4209.19	4218.74	4228.28
4400	4237.82	4247.36	4256.90	4266.44	4275.98	4285.52	4295.06	4304.59	4314.13	4323.67
4500	4333.21	4342.74	4352.28	4361.81	4371.35	4380.88	4390.42	4399.95	4409.49	4419.02
4600	4428.55	4438.09	4447.62	4457.15	4466.68	4476.21	4485.74	4495.27	4504.80	4514.33
4700	4523.86	4533.39	4542.92	4552.45	4561.98	4571.51	4581.03	4590.56	4600.08	4609.61
4800	4619.13	4628.66	4638.18	4647.71	4657.23	4666.75	4676.28	4685.80	4695.32	4704.84
4900	4714.36	4723.88	4733.41	4742.93	4752.45	4761.96	4771.48	4781.00	4790.52	4800.04

This table is continued on p. 8A.

	Dynamic meters.
Example—Given pressure 4824 d-bars :	
Table 7 H gives for 4820 d-bars	4638.18
Table 8 H gives for 4820 d-bars the increase of depth 0.95243, or with three decimals 0.952, per decibar; thus for 4 d-bars	3.81
Depth of the isobaric surface of 4824 d-bars........	4641.99

Table 8 H.—$10^5 a_{35,0,p}$ ($a_{35,0,p}$ = *specific volume of sea-water of 35 ‰ salinity and 0° C. at standard pressure, expressed in m³/tons*).

Sea-pressure (decibars).	0	10	20	30	40	50	60	70	80	90
0	97264	97260	97255	97251	97246	97242	97237	97233	97228	97224
100	97219	97215	97210	97206	97201	97197	97192	97188	97183	97179
200	97174	97170	97165	97161	97156	97152	97147	97143	97138	97134
300	97129	97125	97120	97116	97111	97107	97102	97098	97093	97089
400	97084	97080	97075	97071	97066	97062	97058	97053	97049	97044
500	97040	97035	97031	97026	97022	97017	97013	97009	97004	97000
600	96995	96991	96986	96982	96978	96973	96969	96964	96960	96955
700	96951	96947	96942	96938	96933	96929	96924	96920	96916	96911
800	96907	96902	96898	96894	96889	96885	96880	96876	96872	96867
900	96863	96858	96854	96850	96845	96841	96836	96832	96828	96823
1000	96819	96814	96810	96806	96801	96797	96793	96788	96784	96779
1100	96775	96771	96766	96762	96758	96753	96749	96745	96740	96736
1200	96732	96727	96723	96718	96714	96710	96705	96701	96697	96692
1300	96688	96684	96679	96675	96671	96666	96662	96658	96653	96649
1400	96645	96640	96636	96632	96627	96623	96619	96615	96610	96606
1500	96602	96597	96593	96589	96584	96580	96576	96571	96567	96563
1600	96559	96554	96550	96546	96541	96537	96533	96529	96524	96520
1700	96516	96511	96507	96503	96499	96494	96490	96486	96481	96477
1800	96473	96469	96464	96460	96456	96452	96447	96443	96439	96435
1900	96430	96426	96422	96418	96413	96409	96405	96401	96396	96392
2000	96388	96384	96379	96375	96371	96367	96362	96358	96354	96350
2100	96345	96341	96337	96333	96329	96324	96320	96316	96312	96307
2200	96303	96299	96295	96291	96286	96282	96278	96274	96270	96265
2300	96261	96257	96253	96249	96244	96240	96236	96232	96228	96223
2400	96219	96215	96211	96207	96203	96198	96194	96190	96186	96182
2500	96177	96173	96169	96165	96161	96157	96152	96148	96144	96140
2600	96136	96132	96127	96123	96119	96115	96111	96107	96103	96098
2700	96094	96090	96086	96082	96078	96074	96069	96065	96061	96057
2800	96053	96049	96045	96040	96036	96032	96028	96024	96020	96016
2900	96011	96007	96003	95999	95995	95991	95987	95983	95979	95974
3000	95970	95966	95962	95958	95954	95950	95946	95942	95938	95933
3100	95929	95925	95921	95917	95913	95909	95905	95901	95897	95893
3200	95888	95884	95880	95876	95872	95868	95864	95860	95856	95852
3300	95848	95844	95840	95835	95831	95827	95823	95819	95815	95811
3400	95807	95803	95799	95795	95791	95787	95783	95779	95775	95770
3500	95766	95762	95758	95754	95750	95746	95742	95738	95734	95730
3600	95726	95722	95718	95714	95710	95706	95702	95698	95694	95690
3700	95686	95682	95678	95674	95670	95666	95662	95658	95654	95650
3800	95646	95642	95638	95634	95630	95626	95622	95618	95614	95610
3900	95606	95602	95598	95594	95590	95586	95582	95578	95574	95570
4000	95566	95562	95558	95554	95550	95546	95542	95538	95534	95530
4100	95526	95522	95518	95514	95510	95506	95502	95498	95494	95490
4200	95486	95482	95478	95474	95470	95466	95462	95458	95454	95451
4300	95447	95443	95439	95435	95431	95427	95423	95419	95415	95411
4400	95407	95403	95399	95395	95391	95387	95384	95380	95376	95372
4500	95368	95364	95360	95356	95352	95348	95344	95340	95336	95333
4600	95329	95325	95321	95317	95313	95309	95305	95301	95297	95293
4700	95289	95286	95282	95278	95274	95270	95266	95262	95258	95254
4800	95251	95247	95243	95239	95235	95231	95227	95223	95219	95216
4900	95212	95208	95204	95200	95196	95192	95188	95185	95181	95177

This table is continued on p. 9A.

Example: $p = 4824$ d-bars. $a_{35,0,p} = 0.95241$.

Table 7 H (continued from p. 6A).—$D_{35,0,p}$ *(depth in dynamic meters of standard isobaric surfaces in sea-water of 35 °/₀₀ salinity and 0° C.).*

Sea-pressure (decibars).	0	10	20	30	40	50	60	70	80	90
5000	4809.56	4819.07	4828.59	4838.11	4847.62	4857.14	4866.65	4876.17	4885.68	4895.20
5100	4904.71	4914.22	4923.74	4933.25	4942.76	4952.27	4961.78	4971.29	4980.80	4990.31
5200	4999.82	5009.33	5018.84	5028.35	5037.86	5047.37	5056.87	5066.38	5075.89	5085.39
5300	5094.90	5104.41	5113.91	5123.42	5132.92	5142.43	5151.93	5151.43	5170.93	5180.44
5400	5189.94	5199.44	5208.94	5218.44	5227.94	5237.44	5246.94	5256.44	5265.94	5275.44
5500	5284.94	5294.44	5303.94	5313.43	5322.93	5332.43	5341.92	5351.42	5360.91	5370.41
5600	5379.90	5389.40	5398.89	5408.38	5417.88	5427.37	5436.86	5446.35	5455.84	5465.33
5700	5474.82	5484.32	5493.80	5503.29	5512.78	5522.27	5531.76	5541.25	5550.74	5560.22
5800	5569.71	5579.20	5588.68	5598.17	5607.65	5617.14	5626.62	5636.11	5645.59	5655.08
5900	5664.56	5674.04	5683.52	5693.00	5702.49	5711.97	5721.45	5730.93	5740.41	5749.89
6000	5759.37	5768.85	5778.33	5787.80	5797.28	5806.76	5816.24	5825.71	5835.19	5844.67
6100	5854.14	5863.62	5873.09	5882.57	5892.04	5901.51	5910.99	5920.46	5929.93	5929.41
6200	5948.88	5958.35	5967.82	5977.29	5986.76	5996.23	6005.70	6015.17	6024.64	6034.11
6300	6043.57	6053.04	6062.51	6071.98	6081.44	6090.91	6100.38	6109.84	6119.31	6128.77
6400	6138.23	6147.70	6157.16	6166.63	6176.09	6185.55	6195.01	6204.48	6213.94	6223.40
6500	6232.86	6242.32	6251.78	6261.24	6270.70	6280.16	6289.61	6299.07	6308.53	6317.99
6600	6327.44	6336.90	6346.36	6355.81	6365.27	6374.72	6384.18	6393.63	6403.09	6412.54
6700	6421.99	6431.45	6440.90	6450.35	6459.80	6469.25	6478.71	6488.16	6497.61	6507.06
6800	6516.51	6525.96	6535.40	6544.85	6554.30	6563.75	6573.20	6582.64	6592.09	6601.54
6900	6610.98	6620.43	6629.87	6639.32	6648.76	6658.21	6667.65	6677.09	6686.54	6695.98
7000	6705.42	6714.86	6724.31	6733.75	6743.19	6752.63	6762.07	6771.51	6780.95	6790.39
7100	6799.82	6809.26	6818.70	6828.14	6837.58	6847.01	6856.45	6865.88	6875.32	6884.76
7200	6894.19	6903.63	6913.06	6922.49	6931.93	6941.36	6950.79	6960.22	6969.66	6979.09
7300	6988.52	6997.95	7007.38	7016.81	7026.24	7035.67	7045.10	7054.53	7063.96	7073.39
7400	7082.81	7092.24	7101.67	7111.09	7120.52	7129.95	7139.37	7148.80	7158.22	7167.65
7500	7177.07	7186.49	7195.92	7205.34	7214.76	7224.19	7233.61	7243.03	7252.45	7261.87
7600	7271.29	7280.71	7290.13	7299.55	7308.97	7318.39	9327.81	7337.22	7346.64	7356.06
7700	7365.48	7374.89	7384.31	7393.73	7403.14	7412.56	7421.97	7431.38	7440.80	7450.21
7800	7459.63	7469.04	7478.45	7487.86	7497.28	7506.69	7516.10	7525.51	7534.92	7544.33
7900	7553.74	7563.15	7572.56	7581.97	7591.38	7600.78	7610.19	7619.60	7629.00	7638.41
8000	7647.82	7657.22	7666.63	7676.03	7685.44	7694.84	7704.25	7713.65	7723.05	7732.46
8100	7741.86	7751.26	7760.66	7770.07	7779.47	7788.87	7798.27	7807.67	7817.07	7826.47
8200	7835.87	7845.27	7854.66	7864.06	7873.46	7882.86	7892.25	7901.65	7911.05	7920.44
8300	7929.84	7939.23	7948.63	7958.02	7967.42	7976.81	7986.20	7995.60	8004.99	8014.38
8400	8023.77	8033.17	8042.56	8051.95	8061.34	8070.73	8080.12	8089.51	8098.90	8108.29
8500	8117.67	8127.06	8136.45	8145.84	8155.23	8164.61	8174.00	8189.39	8192.77	8202.16
8600	8211.54	8220.93	8230.31	8239.69	8249.08	8258.46	8267.84	8277.23	8286.61	8295.99
8700	8305.37	8314.75	8324.14	8333.51	8342.89	8352.27	8361.65	8371.03	8380.41	8389.79
8800	8399.17	8408.55	8417.92	8427.30	8436.68	8446.05	8455.43	8464.80	8474.18	8483.55
8900	8492.93	8502.30	8511.68	8521.05	8530.42	8539.80	8549.17	8558.54	8567.91	8577.28
9000	8586.65	8596.03	8605.40	8614.77	8624.14	8633.51	8642.87	8652.24	8661.61	8670.98
9100	8680.35	8689.71	8699.08	8708.45	8717.81	8727.18	8736.55	8745.91	8755.27	8764.64
9200	8774.00	8783.37	8792.73	8802.09	8811.46	8820.82	8830.18	8839.54	8848.90	8858.27
9300	8867.63	8876.99	8886.35	8895.71	8905.07	8914.42	8923.78	8933.14	8942.50	8951.86
9400	8961.21	8970.57	8979.93	8989.28	8998.64	9008.00	9017.35	9026.71	9036.06	9045.41
9500	9054.77	9064.12	9073.48	9082.83	9092.18	9101.53	9110.88	9120.24	9129.59	9138.94
9600	9148.29	9157.64	9166.99	9176.34	9185.69	9195.03	9204.38	9213.73	9223.08	9232.43
9700	9241.77	9251.12	9260.47	9269.81	9279.16	9288.50	9297.85	9307.19	9316.54	9325.88
9800	9335.22	9344.57	9353.91	9363.25	9372.60	9381.94	9391.28	9400.62	9409.96	9419.30
9900	9428.64	9437.98	9447.32	9456.66	9466.00	9475.34	9484.68	9494.01	9503.35	9512.69

	Dynamic meters.
Example—Given pressure 7512 d-bars :	
Table 7 H gives for 7510 d-bars	7186.49
Table 8 H gives for 7510 d-bars the increase of depth 0.94236, or with three decimals, 0.942, per decibar; thus for 2 decibars	1.88
Depth of the isobaric surface of 7512 d-bars	7188.37

Table 8 H (continued from p. 7A).—$10^5 a_{35,\,0,\,p}$ (*$a_{35,\,0,\,p}$ = specific volume of sea-water of 35 °/₀₀ salinity and 0° C. at standard pressure, expressed in m^3/tons*).

Sea-pressure (decibars).	0	10	20	30	40	50	60	70	80	90
5000	95173	95169	95165	95161	95157	95154	95150	95146	95142	95138
5100	95134	95130	95127	95123	95119	95115	95111	95107	95103	95100
5200	95096	95092	95088	95084	95080	95077	95073	95069	95065	95061
5300	95057	95054	95050	95046	95042	95038	95034	95031	95027	95023
5400	95019	95015	95011	95008	95004	95000	94996	94992	94988	94985
5500	94981	94977	94973	94969	94966	94962	94958	94954	94950	94947
5600	94943	94939	94935	94931	94928	94924	94920	94916	94912	94909
5700	94905	94901	94897	94893	94890	94886	94882	94878	94874	94871
5800	94867	94863	94859	94856	94852	94848	94844	94840	94837	94833
5900	94829	94825	94822	94818	94814	94810	94807	94803	94799	94795
6000	94791	94788	94784	94780	94776	94773	94769	94765	94761	94758
6100	94754	94750	94746	94743	94739	94735	94731	94728	94724	94720
6200	94717	94713	94709	94705	94702	94698	94694	94690	94687	94683
6300	94679	94675	94672	94668	94664	94661	94657	94653	94649	94646
6400	94642	94638	94635	94631	94627	94623	94620	94616	94612	94609
6500	94605	94601	94597	94594	94590	94586	94583	94579	94575	94572
6600	94568	94564	94560	94557	94553	94549	94546	94542	94538	94535
6700	94531	94527	94524	94520	94516	94513	94509	94505	94501	94498
6800	94494	94490	94487	94483	94479	94476	94472	94468	94465	94461
6900	94457	94454	94450	94446	94443	94439	94435	94432	94428	94424
7000	94421	94417	94414	94410	94406	94403	94399	94395	94392	94388
7100	94384	94381	94377	94373	94370	94366	94362	94359	94355	94351
7200	94348	94344	94341	94337	94333	94330	94326	94322	94319	94315
7300	94312	94308	94304	94301	94297	94293	94290	94286	94283	94279
7400	94275	94272	94268	94265	94261	94257	94254	94250	94246	94243
7500	94239	94236	94232	94228	94225	94221	94218	94214	94210	94207
7600	94203	94200	94196	94192	94189	94185	94182	94178	94174	94171
7700	94167	94164	94160	94157	94153	94149	94146	94142	94139	94135
7800	94132	94128	94124	94121	94117	94114	94110	94106	94103	94099
7900	94096	94092	94089	94085	94081	94078	94074	94071	94067	94064
8000	94060	94057	94053	94049	94046	94042	94039	94035	94032	94028
8100	94025	94021	94018	94014	94010	94007	94003	94000	93996	93993
8200	93989	93986	93982	93979	93975	93971	93968	93964	93961	93957
8300	93954	93950	93947	93943	93940	93936	93933	93929	93926	93922
8400	93919	93915	93912	93908	93905	93901	93897	93894	93890	93887
8500	93883	93880	93876	93873	93869	93866	93862	93859	93855	93852
8600	93848	93845	93841	93838	93834	93831	93827	93824	93820	93817
8700	93813	93810	93806	93803	93799	93796	93792	93789	93785	93782
8800	93778	93775	93771	93768	93765	93761	93758	93754	93751	93747
8900	93744	93740	93737	93733	93730	93726	93723	93719	93716	93712
9000	93709	93705	93702	93699	93695	93692	93688	93685	93681	93678
9100	93674	93671	93667	93664	93661	93657	93654	93650	93647	93643
9200	93640	93636	93633	93629	93626	93623	93619	93616	93612	93609
9300	93605	93602	93598	93595	93592	93588	93585	93581	93578	93574
9400	93571	93568	93564	93561	93557	93554	93550	93547	93544	93540
9500	93537	93533	93530	93526	93523	93520	93516	93513	93509	93506
9600	93503	93499	93496	93492	93489	93486	93482	93479	93475	93472
9700	93469	93465	93462	93458	93455	93452	93448	93445	93441	93438
9800	93434	93431	93428	93424	93421	93418	93414	93411	93407	93404
9900	93401	93397	93394	93390	93387	93384	93380	93377	93374	93370

Example: $p = 7512$. $a_{35,\,0,\,p} = 0.94235$.

Table 9 H.—$10^5\delta_s$ (δ_s = *salinity correction in* m^3/*ton to the specific volume of sea-water*).

Salinity (‰).	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	2749	2740	2732	2724	2715	2707	2698	2690	2682	2673
1	2665	2657	2648	2640	2632	2624	2616	2607	2599	2591
2	2583	2574	2566	2558	2550	2542	2534	2526	2518	2510
3	2502	2493	2485	2477	2469	2461	2453	2445	2437	2429
4	2421	2413	2405	2397	2389	2381	2372	2364	2356	2348
5	2340	2332	2324	2316	2308	2300	2292	2284	2276	2268
6	2260	2252	2244	2236	2228	2220	2212	2204	2196	2188
7	2180	2172	2164	2156	2148	2140	2132	2124	2116	2108
8	2100	2092	2084	2076	2068	2060	2052	2044	2036	2028
9	2020	2012	2004	1996	1988	1980	1973	1965	1957	1949
10	1941	1933	1925	1917	1909	1901	1893	1885	1877	1869
11	1861	1853	1845	1838	1830	1822	1814	1806	1798	1790
12	1782	1774	1766	1758	1751	1743	1735	1727	1719	1711
13	1703	1695	1687	1679	1672	1664	1656	1648	1640	1632
14	1624	1616	1609	1601	1593	1585	1577	1569	1561	1553
15	1546	1538	1530	1522	1514	1506	1498	1491	1483	1475
16	1467	1459	1451	1444	1436	1428	1420	1412	1404	1397
17	1389	1381	1373	1365	1357	1350	1342	1334	1326	1318
18	1311	1303	1295	1287	1279	1271	1264	1256	1248	1240
19	1232	1225	1217	1209	1201	1193	1186	1178	1170	1162
20	1155	1147	1139	1131	1123	1116	1108	1100	1092	1085
21	1077	1069	1061	1053	1046	1038	1030	1022	1015	1007
22	999	991	984	976	968	960	953	945	937	929
23	922	914	906	898	891	883	875	867	860	852
24	844	837	829	821	813	806	798	790	782	775
25	767	759	751	744	736	728	721	713	705	697
26	690	682	674	667	659	651	644	636	628	620
27	613	605	597	590	582	574	567	559	551	544
28	536	528	520	513	505	497	490	482	474	467
29	459	451	444	436	428	421	413	405	398	390
30	382	375	367	359	352	344	336	329	321	313
31	306	298	290	283	275	267	260	252	244	237
32	229	221	214	206	198	191	183	176	168	160
33	153	145	137	130	122	114	107	99	92	84
34	76	69	61	53	46	38	31	23	15	8
35	0	− 8	− 15	− 23	− 30	− 38	− 46	− 53	− 61	− 69
36	− 76	− 84	− 91	− 99	− 107	− 114	− 122	− 129	− 137	− 145
37	− 152	− 160	− 168	− 175	− 183	− 190	− 198	− 206	− 213	− 221
38	− 228	− 236	− 244	− 251	− 259	− 266	− 274	− 282	− 289	− 297
39	− 304	− 312	− 320	− 327	− 335	− 342	− 350	− 358	− 365	− 373

Example: Given: $s = 23.12$ ‰. Found: $\delta_s = +0.00912$.

Table 10 H.—$10^5\delta_\tau$ (δ_τ = *temperature correction in* m^3/*ton to the specific volume of sea-water*).

Temperature (° C.).	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
−1	−4	−5	−5	−5	−6	−6	−6	−7	−7	−7
−0	0	−1	−1	−2	−2	−2	−3	−3	−4	−4
0	0	1	1	2	2	3	3	4	4	5
1	6	6	7	8	8	9	10	10	11	12
2	13	13	14	15	16	17	17	18	19	20
3	21	22	23	24	25	25	26	27	28	29
4	30	31	32	33	34	35	37	38	39	40
5	41	42	43	44	45	47	48	49	50	51
6	52	54	55	56	57	59	60	61	63	64
7	65	66	68	69	71	72	73	75	76	77
8	79	80	82	83	85	86	88	89	91	92
9	94	95	97	98	100	102	103	105	106	108
10	109	111	113	114	116	118	119	121	123	125
11	126	128	130	132	133	135	137	139	140	142
12	144	146	148	149	151	153	155	157	159	161
13	163	165	167	168	170	172	174	176	178	180
14	182	184	186	188	190	192	194	196	198	200
15	202	205	207	209	211	213	215	217	220	222
16	224	226	228	230	233	235	237	239	242	244
17	246	248	251	253	255	258	260	262	264	267
18	269	271	274	276	279	281	283	286	288	291
19	293	295	298	300	303	305	308	310	313	315
20	318	320	323	325	328	330	333	335	338	341
21	343	346	348	351	353	356	359	361	364	367
22	369	372	375	377	380	383	385	388	391	394
23	396	399	402	405	407	410	413	416	419	421
24	424	427	430	433	436	438	441	444	447	450
25	453	456	459	462	464	467	470	473	476	479
26	482	485	488	491	494	497	500	503	506	509
27	512	515	519	522	525	528	531	534	537	540
28	543	546	550	553	556	559	562	565	569	572
29	575	578	581	585	588	591	594	598	601	604

Example: Given: $\tau = 11.61°$ C. Found: $\delta_\tau = 0.00137$.

Table 11 H.—$10^5\delta_{s\tau}$ ($\delta_{s\tau}$ = *combined salinity-temperature correction in m^3/ton to the specific volume of sea-water*).

Salinity (‰).	Temperature (° C.).																
	−2	−1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	25	12	0	−12	−23	−33	−43	−53	−62	−71	−80	−88	−95	−103	−110	−116	−122
1	24	12	0	−11	−22	−32	−42	−51	−60	−69	−77	−85	−92	−99	−106	−112	−118
2	23	11	0	−11	−21	−31	−40	−50	−58	−66	−74	−82	−89	−96	−102	−109	−114
3	22	11	0	−10	−20	−30	−39	−48	−56	−64	−72	−79	−86	−92	−99	−105	−110
4	21	10	0	−10	−20	−29	−38	−46	−54	−62	−69	−76	−83	−89	−95	−101	−106
5	20	10	0	−10	−19	−28	−36	−44	−52	−60	−67	−73	−80	−86	−92	−97	−102
6	20	10	0	−9	−18	−27	−35	−43	−50	−57	−64	−71	−77	−83	−88	−94	−99
7	19	9	0	−9	−18	−26	−33	−41	−48	−55	−62	−68	−74	−79	−85	−90	−95
8	18	9	0	−9	−17	−25	−32	−39	−46	−53	−59	−65	−71	−76	−82	−86	−91
9	17	9	0	−8	−16	−24	−31	−38	−44	−51	−57	−63	−68	−73	−78	−83	−87
10	17	8	0	−8	−15	−23	−29	−36	−42	−48	−54	−60	−65	−70	−75	−79	−83
11	16	8	0	−8	−15	−22	−28	−35	−41	−46	−52	−57	−62	−67	−72	−76	−80
12	15	7	0	−7	−14	−21	−27	−33	−39	−44	−49	−55	−59	−64	−68	−72	−76
13	14	7	0	−7	−13	−20	−25	−31	−37	−42	−47	−52	−56	−61	−65	−69	−72
14	14	7	0	−7	−13	−19	−24	−30	−35	−40	−45	−49	−53	−58	−62	−65	−69
15	13	6	0	−6	−12	−18	−23	−28	−33	−38	−42	−47	−51	−55	−59	−62	−65
16	12	6	0	−6	−11	−17	−22	−27	−31	−36	−40	−44	−48	−52	−55	−59	−62
17	12	6	0	−6	−11	−16	−20	−25	−29	−34	−38	−42	−45	−49	−52	−55	−58
18	11	5	0	−5	−10	−15	−19	−24	−28	−32	−36	−39	−43	−46	−49	−52	−55
19	10	5	0	−5	−9	−14	−18	−22	−26	−30	−33	−37	−40	−43	−46	−49	−51
20	9	5	0	−5	−9	−13	−17	−21	−24	−28	−31	−34	−37	−40	−43	−46	−48
21	9	4	0	−4	−8	−12	−16	−19	−23	−26	−29	−32	−35	−37	−40	−42	−44
22	8	4	0	−4	−8	−11	−14	−18	−21	−24	−27	−30	−32	−35	−37	−39	−41
23	7	4	0	−4	−7	−10	−13	−16	−19	−22	−25	−27	−29	−32	−34	−36	−38
24	7	3	0	−3	−6	−9	−12	−15	−17	−20	−22	−25	−27	−29	−31	−33	−35
25	6	3	0	−3	−6	−8	−11	−13	−16	−18	−20	−22	−24	−26	−28	−30	−31
26	5	3	0	−3	−5	−8	−10	−12	−14	−16	−18	−20	−22	−23	−25	−27	−28
27	5	2	0	−2	−5	−7	−9	−11	−12	−14	−16	−18	−19	−21	−22	−24	−25
28	4	2	0	−2	−4	−6	−8	−9	−11	−13	−14	−15	−17	−18	−20	−21	−22
29	3	2	0	−2	−3	−5	−6	−8	−9	−11	−12	−13	−14	−15	−17	−18	−18
30	3	2	0	−2	−3	−4	−5	−7	−8	−9	−10	−11	−12	−13	−14	−15	−15
31	2	1	0	−1	−2	−3	−4	−5	−6	−7	−8	−9	−9	−10	−11	−12	−12
32	2	1	0	−1	−2	−3	−3	−4	−5	−5	−6	−7	−7	−8	−8	−9	−9
33	1	1	0	−1	−1	−2	−2	−3	−3	−4	−4	−4	−5	−5	−6	−6	−6
34	1	0	0	0	−1	−1	−1	−1	−1	−2	−2	−2	−2	−3	−3	−3	−3
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	−1	0	0	0	0	1	1	1	1	2	2	2	2	3	3	3	3
37	−1	−1	0	0	1	1	2	2	3	3	4	4	5	5	5	6	6
38	−2	−1	0	1	2	2	3	4	4	5	6	6	7	7	8	8	9
39	−2	−1	0	1	2	3	4	5	6	7	7	8	9	10	10	11	12
40	−3	−2	0	1	3	4	5	6	7	8	9	10	11	12	13	14	15

Example: Given: $s = 23.12$ ‰, $\tau = 11.61°$ C. Found: $\delta_{s\tau} = -0.00033$.

Table 11 H (continued).—$10^5\delta_{sT}$ (δ_{sT} = *combined salinity-temperature correction in m³/ton to the specific volume of sea-water*).

Salinity (‰).	Temperature (°C.).															
	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0	−128	−134	−139	−144	−149	−153	−158	−162	−164	−169	−172	−175	−178	−181	−184	−186
1	−124	−130	−135	−140	−144	−148	−153	−156	−159	−163	−166	−169	−172	−175	−177	−180
2	−120	−125	−130	−135	−139	−143	−147	−151	−154	−158	−161	−164	−166	−169	−171	−174
3	−116	−121	−126	−130	−135	−138	−142	−146	−149	−152	−155	−158	−161	−163	−165	−167
4	−112	−116	−121	−126	−130	−133	−137	−141	−144	−147	−150	−152	−155	−157	−159	−161
5	−108	−112	−117	−121	−125	−129	−132	−135	−138	−141	−144	−147	−149	−151	−153	−155
6	−103	−108	−112	−116	−120	−124	−127	−130	−133	−136	−139	−141	−143	−145	−148	−149
7	− 99	−104	−108	−112	−116	−119	−122	−125	−128	−131	−133	−136	−138	−140	−142	−143
8	− 95	−100	−104	−107	−111	−114	−118	−120	−123	−126	−128	−130	−132	−134	−136	−138
9	− 91	− 96	− 99	−103	−106	−110	−113	−115	−118	−120	−123	−125	−127	−129	−130	−132
10	− 88	− 91	− 95	− 99	−102	−105	−108	−110	−113	−115	−117	−119	−121	−123	−125	−126
11	− 84	− 87	− 91	− 94	− 97	−100	−103	−106	−108	−110	−112	−114	−116	−118	−119	−120
12	− 80	− 83	− 87	− 90	− 93	− 96	− 98	−101	−103	−105	−107	−109	−111	−112	−114	−115
13	− 76	− 79	− 83	− 86	− 89	− 91	− 94	− 96	− 98	−100	−102	−104	−105	−107	−108	−109
14	− 72	− 75	− 78	− 81	− 84	− 87	− 89	− 91	− 93	− 95	− 97	− 98	−100	−101	−103	−104
15	− 69	− 72	− 74	− 77	− 80	− 82	− 84	− 86	− 88	− 90	− 92	− 93	− 95	− 96	− 97	− 98
16	− 65	− 68	− 70	− 73	− 76	− 78	− 80	− 82	− 84	− 85	− 87	− 88	− 90	− 91	− 92	− 93
17	− 61	− 64	− 66	− 69	− 71	− 73	− 75	− 77	− 79	− 81	− 82	− 83	− 85	− 86	− 87	− 88
18	− 58	− 60	− 63	− 65	− 67	− 69	− 71	− 73	− 74	− 76	− 77	− 78	− 79	− 81	− 82	− 82
19	− 54	− 56	− 59	− 61	− 63	− 65	− 66	− 68	− 70	− 71	− 72	− 73	− 74	− 76	− 76	− 77
20	− 50	− 53	− 55	− 57	− 59	− 60	− 62	− 63	− 65	− 66	− 67	− 68	− 69	− 70	− 71	− 72
21	− 47	− 49	− 51	− 53	− 55	− 56	− 58	− 59	− 60	− 62	− 63	− 64	− 65	− 66	− 66	− 67
22	− 43	− 45	− 47	− 49	− 50	− 52	− 53	− 55	− 56	− 57	− 58	− 59	− 60	− 61	− 61	− 62
23	− 40	− 42	− 43	− 45	− 46	− 48	− 49	− 50	− 51	− 52	− 53	− 54	− 55	− 56	− 56	− 57
24	− 36	− 38	− 39	− 41	− 42	− 43	− 45	− 46	− 47	− 48	− 49	− 49	− 50	− 51	− 51	− 52
25	− 33	− 34	− 36	− 37	− 38	− 39	− 40	− 41	− 42	− 43	− 44	− 45	− 45	− 46	− 46	− 47
26	− 29	− 31	− 32	− 33	− 34	− 35	− 36	− 37	− 38	− 39	− 39	− 40	− 41	− 41	− 42	− 42
27	− 26	− 27	− 28	− 30	− 30	− 31	− 32	− 33	− 34	− 34	− 35	− 35	− 36	− 36	− 37	− 37
28	− 23	− 24	− 25	− 26	− 27	− 27	− 28	− 29	− 29	− 30	− 30	− 31	− 31	− 32	− 32	− 32
29	− 19	− 20	− 21	− 22	− 23	− 23	− 24	− 25	− 25	− 26	− 26	− 26	− 27	− 27	− 27	− 28
30	− 16	− 17	− 18	− 18	− 19	− 19	− 20	− 20	− 21	− 21	− 21	− 22	− 22	− 22	− 23	− 23
31	− 13	− 13	− 14	− 15	− 15	− 15	− 16	− 16	− 17	− 17	− 17	− 17	− 18	− 18	− 18	− 18
32	− 10	− 10	− 10	− 11	− 11	− 12	− 12	− 12	− 12	− 13	− 13	− 13	− 13	− 13	− 13	− 14
33	− 6	− 7	− 7	− 7	− 7	− 8	− 8	− 8	− 8	− 8	− 9	− 9	− 9	− 9	− 9	− 9
34	− 3	− 3	− 3	− 4	− 4	− 4	− 4	− 4	− 4	− 4	− 4	− 4	− 4	− 4	− 4	− 4
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	3	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4
37	6	6	7	7	7	7	8	8	8	8	8	8	9	9	9	9
38	9	10	10	10	11	11	11	12	12	12	12	13	13	13	13	13
39	12	13	13	14	14	15	15	16	16	16	16	17	17	17	17	17
40	15	16	17	17	18	19	19	19	20	20	21	21	21	21	22	22

Example to tables 8 H to 11 H:

Table 8 H gives for sea-pressure zero 97264
Table 9 H gives for salinity 23.12 ‰ .... +912
Table 10 H gives for temperature 11.61° C .... +137
Table 11 H gives for salinity 23.12 ‰; temperature 11.61° C .... − 33

Specific volume of sea-water, salinity 23.12 ‰, temperature 11.61° C. under atmospheric pressure 0.98280

Table 12 H.—$10^5\delta_{sp}$ (δ_{sp} = *combined salinity-pressure correction in m^3/ton to the specific volume of sea-water*).

Salinity (‰).																															Sea-pressure (decibars).
10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-4	-4	-4	-4	-4	-3	-3	-3	-3	-3	-2	-2	-2	-2	-2	-2	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	1	1	100
-8	-8	-7	-7	-7	-6	-6	-6	-5	-5	-5	-4	-4	-4	-3	-3	-3	-2	-2	-2	-2	-1	-1	-1	0	0	0	1	1	1	2	200
-12	-12	-11	-11	-10	-9	-9	-8	-8	-7	-7	-6	-6	-6	-5	-5	-4	-4	-3	-3	-2	-2	-1	-1	0	0	0	1	1	2	2	300
-16	-15	-15	-14	-13	-13	-12	-11	-11	-10	-9	-9	-8	-7	-7	-6	-6	-5	-4	-4	-3	-2	-2	-1	1	0	1	1	2	2	3	400
-20	-19	-18	-17	-16	-16	-15	-14	-13	-12	-12	-11	-10	-9	-8	-8	-7	-6	-5	-5	-4	-3	-2	-2	-1	0	1	2	2	3	4	500
-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-6	-5	-4	-3	-2	-1	0	1	2	3	4	4	600
-28	-27	-25	-24	-23	-22	-21	-20	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	700
-32	-30	-29	-28	-26	-25	-24	-23	-21	-20	-19	-17	-16	-15	-14	-12	-11	-10	-9	-7	-6	-5	-4	-2	-1	0	1	2	4	5	6	800
-36	-34	-33	-31	-29	-28	-27	-25	-24	-22	-21	-19	-18	-16	-15	-14	-12	-11	-10	-8	-7	-5	-4	-3	-1	0	1	3	4	5	7	900
-39	-38	-36	-34	-33	-31	-29	-28	-26	-25	-23	-21	-20	-18	-17	-15	-14	-12	-11	-9	-7	-6	-5	-3	-1	0	1	3	4	6	7	1000
-43	-41	-40	-38	-36	-34	-32	-31	-29	-27	-25	-24	-22	-20	-18	-17	-15	-13	-12	-10	-8	-7	-5	-3	-2	0	2	3	5	7	8	1100
-47	-45	-43	-41	-39	-37	-35	-33	-32	-30	-28	-26	-24	-22	-20	-18	-16	-14	-13	-11	-9	-7	-5	-4	-2	0	2	4	5	7	9	1200
-51	-49	-47	-44	-42	-40	-38	-36	-34	-32	-30	-28	-26	-24	-22	-20	-18	-16	-14	-12	-10	-8	-6	-4	-2	0	2	4	6	8	9	1300
-55	-52	-50	-48	-45	-43	-41	-39	-37	-34	-32	-30	-28	-25	-23	-21	-19	-17	-15	-13	-10	-8	-6	-4	-2	0	2	4	6	8	10	1400
-58	-56	-53	-51	-48	-46	-44	-41	-39	-37	-34	-32	-29	-27	-25	-23	-20	-18	-16	-13	-11	-9	-7	-4	-2	0	2	4	7	9	11	1500
-63	-59	-57	-54	-52	-49	-47	-44	-41	-39	-36	-34	-31	-29	-26	-24	-22	-19	-17	-14	-12	-9	-7	-5	-2	0	2	5	7	9	12	1600
-66	-63	-60	-58	-55	-52	-49	-47	-44	-41	-39	-36	-33	-31	-28	-25	-23	-20	-18	-15	-12	-10	-7	-5	-2	0	2	5	7	10	12	1700
-70	-67	-64	-61	-58	-55	-52	-49	-46	-44	-41	-38	-35	-33	-30	-27	-24	-21	-19	-16	-13	-11	-8	-5	-3	0	3	5	8	10	13	1800
-73	-70	-67	-64	-61	-58	-55	-52	-49	-46	-43	-40	-37	-34	-31	-28	-25	-23	-20	-17	-14	-11	-8	-5	-3	0	3	5	8	11	14	1900
-77	-74	-71	-67	-64	-61	-58	-54	-51	-48	-45	-42	-39	-36	-33	-30	-27	-24	-21	-18	-15	-12	-9	-6	-3	0	3	6	9	12	14	2000
										-47	44	-41	-37	-34	-31	-28	-25	-22	-18	-15	-12	-9	-6	-3	0	3	6	9	12	15	2100
										-49	-46	-43	-39	-36	-32	-29	-26	-23	-19	-16	-13	-10	-6	-3	0	3	6	10	13	16	2200
										-51	-48	-44	-41	-37	-34	-30	-27	-24	-20	-17	-13	-10	-7	-3	0	3	7	10	13	16	2300
										-54	-50	-46	-42	-39	-35	-32	-28	-25	-21	-17	-14	-10	-7	-3	0	3	7	10	14	17	2400
										-56	-52	-48	-44	-40	-37	-33	-29	-26	-22	-18	-14	-11	-7	-4	0	4	7	11	14	18	2500
										-58	-54	-50	-46	-42	-38	-34	-30	-26	-23	-19	-15	-11	-7	-4	0	4	7	11	15	18	2600
										-60	-56	-52	-48	-44	-39	-35	-32	-27	-23	-20	-16	-12	-8	-4	0	4	8	12	15	19	2700
										-62	58	-54	-49	-45	-41	-37	-33	-28	-24	-20	-16	-12	-8	-4	0	4	8	12	16	20	2800
										-64	-60	-55	-51	-47	-42	-38	-34	-29	-25	-21	-17	-12	-8	-4	0	4	8	12	16	20	2900
										-66	62	-57	-52	-48	-44	-39	-35	-30	-26	-22	-17	-13	-9	-4	0	4	9	13	17	21	3000
																				-22	-18	-13	-9	-4	0	4	9	13	17	22	3100
																				-23	-18	-14	-9	-5	0	5	9	14	18	22	3200
																				-24	-19	-14	-9	-5	0	5	9	14	18	23	3300
																				-24	-19	-14	-10	-5	0	5	10	14	19	24	3400
																				-25	-20	-15	-10	-5	0	5	10	15	19	24	3500
																				-25	-20	-15	-10	-5	0	5	10	15	20	25	3600
																				-26	-21	-16	-10	-5	0	5	10	15	20	26	3700
																				-27	-21	-16	-11	-5	0	5	11	16	21	26	3800
																				-27	-22	-16	-11	-5	0	5	11	16	22	27	3900
																				-28	-22	-17	-11	-6	0	6	11	17	22	27	4000
																				-29	-23	-17	-11	-6	0	6	11	17	23	28	4100
																				-29	-23	-18	-12	-6	0	6	12	17	23	29	4200
																				-30	-24	-18	-12	-6	0	6	12	18	24	29	4300
																				-31	-24	-18	-12	-6	0	6	12	18	24	30	4400
																				-31	-25	-19	-12	-6	0	6	12	18	25	31	4500
																				-32	-25	-19	-13	-6	0	6	13	19	25	31	4600
																				-32	-26	-19	-13	-6	0	6	13	19	25	32	4700
																				-33	-26	-20	-13	-7	0	7	13	20	26	32	4800
																				-34	-27	-20	-13	-7	0	7	13	20	26	33	4900
																				-34	-27	-21	-14	-7	0	7	14	20	27	34	5000

Salinity (‰).											Sea-pressure (decibars).
0	1	2	3	4	5	6	7	8	9	10	
0	0	0	0	0	0	0	0	0	0	0	0
-6	-6	-5	-5	-5	-5	-5	-5	-4	-4	-4	100
-12	-11	-11	-11	-10	-10	-9	-9	-9	-9	-8	200
-17	-17	-16	-16	-15	-15	-14	-14	-13	-13	-12	300
-23	-22	-22	-21	-20	-19	-19	-18	-17	-17	-16	400
-29	-28	-27	-26	-25	-24	-23	-22	-22	-21	-20	500
-34	-33	-32	-31	-30	-29	-28	-27	-26	-25	-24	600
-40	-39	-38	-36	-35	-34	-33	-31	-30	-29	-28	700
-46	-44	-43	-41	-40	-39	-37	-36	-34	-33	-32	800
-51	-50	-48	-46	-45	-43	-42	-40	-39	-37	-36	900
-57	-55	-53	-51	-50	-48	-46	-45	-43	-41	-39	1000

This table is continued on p. 16A

Example: Given: $s = 35.17\,^0/_{00}$, $p = 4824$ *d*-bars. Found: $\delta_{sp} = +0.00001$.

Table 13 H.—$10^5 \cdot \delta_{\tau p}$ (*$\delta_{\tau p}$ = combined temperature-pressure correction in m^3/ton to the specific volume of sea-water*).

Sea-pressure (decibars).	Temperature (° C.).																																
	−2	−1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	− 1	0	0	0	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4	4
200	− 1	− 1	0	1	1	1	2	2	3	3	4	4	4	5	5	5	5	6	6	6	6	7	7	7	7	7	8	8	8	8	8	8	8
300	− 2	− 1	0	1	2	2	3	4	4	5	5	6	7	7	8	8	8	9	9	9	10	10	10	11	11	11	11	12	12	12	12	12	13
400	− 2	− 1	0	1	2	3	4	5	6	6	7	8	9	9	10	11	11	12	12	13	13	14	14	14	15	15	15	16	16	16	16	17	17
500	− 3	− 1	0	1	3	4	5	6	7	8	9	10	11	12	12	13	14	15	15	16	16	17	17	18	18	19	19	19	20	20	21	21	21
600	− 3	− 2	0	2	3	5	6	7	8	10	11	12	13	14	15	16	17	17	18	19	20	20	21	21	22	22	23	23	24	24	24	25	25
700	− 4	− 2	0	2	4	5	7	8	10	11	13	14	15	16	17	18	19	20	21	22	23	23	24	25	25	26	26	27	27	28	28	29	29
800	− 4	− 2	0	2	4	6	8	10	11	13	14	16	17	18	20	21	22	23	24	25	26	27	27	28	29	30	30	31	31	32	32	33	33
900	− 5	− 2	0	2	5	7	9	11	13	14	16	18	19	21	22	23	25	26	27	28	29	30	31	32	32	33	34	35	35	36	36	37	37
1000	− 6	− 3	0	3	5	7	10	12	14	16	18	20	21	23	24	26	27	29	30	31	32	33	34	35	36	37	38	38	39	40	40	41	41
1100	− 6	− 3	0	3	6	8	11	13	15	17	19	21	23	25	27	28	30	31	33	34	35	36	37										
1200	− 7	− 3	0	3	6	9	12	14	17	19	21	23	25	27	29	31	33	34	36	37	38	40	41										
1300	− 7	− 3	0	3	6	9	12	15	18	21	23	25	27	29	31	33	35	37	39	40	41	43	44										
1400	− 8	− 4	0	4	7	10	13	16	19	22	25	27	29	32	34	36	38	40	41	43	44	46	47										
1500	− 8	− 4	0	4	7	11	14	17	21	23	26	29	31	34	36	38	40	42	44	46	47	49	50										
1600	− 9	− 4	0	4	8	12	15	19	22	25	28	31	33	36	38	41	43	45	47	49	50	52	54										
1700	− 9	− 4	0	4	8	12	16	20	23	26	30	33	35	38	41	43	45	48	50	52	54	55	57										
1800	−10	− 5	0	5	9	13	17	21	24	28	31	34	37	40	43	45	48	50	52	54	56	58	60										
1900	−10	− 5	0	5	9	14	18	22	26	29	33	36	39	42	45	48	51	53	55	57	59	61	63										
2000	−11	− 5	0	5	10	14	19	23	27	31	34	38	41	44	47	50	53	55	58	60	62	64	66										
2100	−11	− 5	0	5	10	15	20	24	28	32	36	40	43	46	49	52	55	58															
2200	−12	− 6	0	5	11	16	21	25	29	34	38	42	45	48	52	55	58	61															
2300	−12	− 6	0	6	11	16	21	26	31	35	39	43	47	50	54	57	60	63															
2400	−13	− 6	0	6	12	17	22	27	32	37	41	45	49	53	56	60	63	66															
2500	−13	− 6	0	6	12	18	23	28	33	38	42	47	51	55	58	62	65	68															
2600	−14	− 7	0	6	12	18	24	29	34	39	44	48	52	56	60	64	67	71															
2700	−14	− 7	0	7	13	19	25	30	35	40	45	50	54	58	62	66	70	73															
2800	−14	− 7	0	7	13	20	26	31	37	42	47	52	56	60	64	68	72	75															
2900	−15	− 7	0	7	14	20	26	32	38	43	48	53	58	62	67	71	74	78															
3000	−15	− 8	0	7	14	21	27	33	39	45	50	55	60	64	69	73	77	80															
3100	−16	− 8	0	7	14	21	28	34	40	46	51	57	62	66	71	75	79	83															
3200	−16	− 8	0	8	15	22	29	35	41	47	53	58	63	68	73	77	81	85															
3300	−17	− 8	0	8	15	23	30	36	43	49	54	60	65	70	75	79	83	87															
3400	−17	− 8	0	8	16	23	30	37	44	50	56	62	67	72	77	81	86	90															
3500	−18	− 9	0	8	16	24	31	38	45	51	57	63	69	74	79	83	88	92															
3600	−19	− 9	0	9	17	25	32	39	46	53	59	65	70	76	81	85	90	95															
3700	−19	− 9	0	9	17	25	33	40	47	54	60	66	72	77	83	88	92	97															
3800	−19	− 9	0	9	17	26	34	41	48	55	62	68	74	79	84	90	95	99															
3900	−19	− 9	0	9	18	26	34	42	49	56	63	69	75	81	86	91	97	101															
4000	−20	−10	0	9	18	27	35	43	50	58	64	71	77	83	89	94	99	104															
4100	−20	−10	0	10	19	27	36	44																									
4200	−21	−10	0	10	19	28	37	45																									
4300	−21	−10	0	10	19	29	37	46																									
4400	−22	−11	0	10	20	29	38	47																									
4500	−22	−11	0	10	20	30	39	48																									
4600	−22	−11	0	11	21	30	40	48																									
4700	−23	−11	0	11	21	31	40	49																									
4800	−23	−11	0	11	21	31	41	50																									
4900	−24	−12	0	11	22	32	42	51																									
5000	−24	−12	0	11	22	32	42	52																									

Example: Given: $\tau = 1.36$, $p = 4824$. Found: $\delta_{\tau p} = +0.00015$.

This table is continued on p. 16A.

Table 12H (continued from p. 14A).—$10^5\delta_{sp}$ (δ_{sp} = *combined salinity-pressure correction in m^3/ton to the specific volume of sea-water*).

Salinity (‰) 33	34	35	36	37	Sea-pressure (decibars).
−14	− 7	0	7	14	5000
−14	− 7	0	7	14	5100
−14	− 7	0	7	14	5200
−14	− 7	0	7	14	5300
−15	− 7	0	7	14	5400
−15	− 7	0	7	15	5500
−15	− 8	0	8	15	5600
−15	− 8	0	8	15	5700
−16	− 8	0	8	15	5800
−16	− 8	0	8	16	5900
−16	− 8	0	8	16	6000
−16	− 8	0	8	16	6100
−16	− 8	0	8	16	6200
−17	− 8	0	8	17	6300
−17	− 8	0	8	17	6400
−17	− 9	0	9	17	6500
−17	− 9	0	9	17	6600
−18	− 9	0	9	17	6700
−18	− 9	0	9	18	6800
−18	− 9	0	9	18	6900
−18	− 9	0	9	18	7000
−18	− 9	0	9	18	7100
−19	− 9	0	9	19	7200
−19	− 9	0	9	19	7300
−19	−10	0	10	19	7400
−19	−10	0	10	19	7500
−20	−10	0	10	19	7600
−20	−10	0	10	20	7700
−20	−10	0	10	20	7800
−20	−10	0	10	20	7900
−20	−10	0	10	20	8000
−21	−10	0	10	20	8100
−21	−10	0	10	21	8200
−21	−10	0	10	21	8300
−21	−11	0	11	21	8400
−21	−11	0	11	21	8500
−22	−11	0	11	21	8600
−22	−11	0	11	22	8700
−22	−11	0	11	22	8800
−22	−11	0	11	22	8900
−22	−11	0	11	22	9000
−23	−11	0	11	22	9100
−23	−11	0	11	23	9200
−23	−11	0	11	23	9300
−23	−12	0	12	23	9400
−23	−12	0	12	23	9500
−24	−12	0	12	23	9600
−24	−12	0	12	23	9700
−24	−12	0	12	24	9800
−24	−12	0	12	24	9900
−24	−12	0	12	24	10000

Example: Given: $s = 35.63$; $p = 7154$.
Found: $\delta_{sp} = +0.00006$.

Table 13H (continued from p. 15A).—$10^5\delta_{tp}$ (δ_{tp} = *combined temperature-pressure correction in m^3/ton to the specific volume of sea-water*).

Sea-pressure (decibars).	Temperature (° C.) −2	−1	0	1	2
5000	−24	−12	0	11	22
5100	−24	−12	0	11	22
5200	−25	−12	0	12	23
5300	−25	−12	0	12	23
5400	−26	−12	0	12	24
5500	−26	−13	0	12	24
5600	−26	−13	0	12	24
5700	−27	−13	0	13	25
5800	−27	−13	0	13	25
5900	−28	−13	0	13	25
6000	−28	−14	0	13	26
6100	−28	−14	0	13	26
6200	−29	−14	0	13	26
6300	−29	−14	0	14	27
6400	−29	−14	0	14	27
6500	−30	−14	0	14	27
6600	−30	−15	0	14	28
6700	−30	−15	0	14	28
6800	−31	−15	0	14	28
6900	−31	−15	0	15	29
7000	−31	−15	0	15	29
7100	−32	−16	0	15	29
7200	−32	−16	0	15	30
7300	−32	−16	0	15	30
7400	−33	−16	0	15	30
7500	−33	−16	0	16	30
7600	−33	−16	0	16	31
7700	−34	−16	0	16	31
7800	−34	−17	0	16	31
7900	−34	−17	0	16	32
8000	−35	−17	0	16	32
8100	−35	−17	0	16	32
8200	−35	−17	0	17	33
8300	−36	−17	0	17	33
8400	−36	−18	0	17	33
8500	−36	−18	0	17	33
8600	−37	−18	0	17	34
8700	−37	−18	0	17	34
8800	−37	−18	0	17	34
8900	−38	−18	0	18	35
9000	−38	−19	0	18	35
9100	−38	−19	0	18	35
9200	−38	−19	0	18	35
9300	−39	−19	0	18	36
9400	−39	−19	0	18	36
9500	−39	−19	0	18	36
9600	−40	−19	0	19	36
9700	−40	−19	0	19	37
9800	−40	−20	0	19	37
9900	−40	−20	0	19	37
10000	−41	−20	0	19	37

Example: Given: $\tau = 1.26$; $p = 7154$.
Found: $\delta_{\tau p} = +0.00019$.

Table 14 H.—$10^5\delta_{s\tau p}$ ($\delta_{s\tau p}$ = *combined salinity-temperature-pressure correction to the specific volume of sea-water, expressed in m³/ton*).

Sea-pressure (decibars).	Temperature (° C.) -2	-1	0	1	2	3	4	5	6	7	8	9	10	15	20
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1000	-2	-1	0	1	2	3	4	4	5	6	7	7	8	11	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1000	-2	-1	0	1	1	2	3	4	4	5	6	6	7	9	11
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1000	-1	-1	0	1	1	2	2	3	4	4	5	5	6	8	9
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1000	-1	0	0	0	1	1	2	2	3	3	3	4	4	6	7
2000	-2	-1	0	1	2	3	4	4	5	6	7	7	8	11	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1000	-1	0	0	0	1	1	1	2	2	2	3	3	3	4	5
2000	-2	-1	0	1	1	2	3	3	4	4	5	5	6	8	10
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1000	-1	0	0	0	0	1	1	1	1	1	2	2	2	3	3
2000	-1	-1	0	1	1	1	2	2	3	3	3	4	4	5	7
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1000	0	0	0	0	0	0	0	1	1	1	1	1	1	1	2
2000	-1	0	0	0	0	1	1	1	1	1	2	2	2	3	3
3000	-1	0	0	0	1	1	1	1	2	2	2	3	3	4	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1000	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
2000	0	0	0	0	0	1	1	1	1	1	1	1	2	2	3
3000	-1	0	0	0	1	1	1	1	1	1	2	2	2	3	
4000	-1	0	0	0	1	1	1	2	2	2	2	3	3		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1000	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
2000	0	0	0	0	0	0	0	1	1	1	1	1	1	2	2
3000	0	0	0	0	0	0	1	1	1	1	1	1	2	2	
4000	-1	0	0	0	0	1	1	1	1	1	2	2	2		
5000	-1	0	0	0	1	1	1	1							
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1000	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
2000	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
3000	0	0	0	0	0	0	0	1	1	1	1	1	1	1	
4000	0	0	0	0	0	0	1	1	1	1	1	1	1		
5000	0	0	0	0	0	1	1	1							
6000	0	0	0	0	0										
7000	0	0	0	0	1										
8000	-1	0	0	0	1										
9000	-1	0	0	0	1										
10000	-1	0	0	0	1										

Salinity (‰).	Sea-pressure (decibars).	Temperature (° C.) -2	-1	0	1	2	3	4	5	6	7	8	9	10	15	20
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	2000	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
	3000	0	0	0	0	0	0	0	0	0	0	0	1	1	1	
	4000	0	0	0	0	0	0	0	0	0	1	1	1	1		
	5000	0	0	0	0	0	0	0	0							
	6000	0	0	0	0	0										
	7000	0	0	0	0	0										
	8000	0	0	0	0	0										
	9000	0	0	0	0	0										
	10000	0	0	0	0	0										
35		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2000	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1
	3000	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	
	4000	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1		
	5000	0	0	0	0	0	0	0	-1							
	6000	0	0	0	0	0										
	7000	0	0	0	0	0										
	8000	0	0	0	0	0										
	9000	0	0	0	0	0										
	10000	0	0	0	0	0										
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1
	2000	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1
	3000	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	
	4000	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-2	
	5000	0	0	0	0	0	-1	-1	-1							
	6000	0	0	0	0	0										
	7000	0	0	0	0	-1										
	8000	1	0	0	0	-1										
	9000	1	0	0	0	-1										
	10000	1	0	0	0	-1										
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1
	2000	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-2	-2
	3000	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-2	-2	
	4000	1	0	0	0	0	-1	-1	-1	-1	-1	-2	-2	-2	-3	
	5000	1	0	0	0	-1	-1	-1	-2							
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1
	2000	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-2	-2	-3
	3000	1	0	0	0	-1	-1	-1	-1	-1	-1	-2	-2	-2	-3	
	4000	1	0	0	0	-1	-1	-1	-2	-2	-2	-2	-2	-3	-4	
	5000	1	0	0	0	-1	-1	-2	-2							
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-2
	2000	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-2	-2	-3	-3
	3000	1	0	0	0	-1	-1	-1	-2	-2	-2	-2	-3	-3	-4	
	4000	1	0	0	0	-1	-1	-2	-2	-2	-3	-3	-3	-4	-5	
	5000	1	1	0	-1	-1	-2	-2	-2							

Example:
Given: $s = 38.74$ ‰; $\tau = 13.73°$ C.; $p = 2700$. Found: $\delta_{s\tau p} = -0.00002$.

Example to tables 8 H to 14 H.

Table 8 H gives for pressure 4824 d-bar	95241
Table 9 H gives for salinity 35.17 ‰	−13
Table 10 H gives for temperature 1.36° C.	+ 8
Table 11 H gives for salinity 35.17 ‰; temperature 1.36° C.	0
Table 12 H gives for salinity 35.17 ‰; pressure 4824 d-bar	+ 1
Table 13 H gives for temperature 1.36° C.; pressure 4824 d-bar	+15
Table 14 H gives for salinity 35.17 ‰; temperature 1.36° C.; pressure 4824 d-bar	0
Specific volume of sea-water of salinity 35.17 ‰; temperature 1.36° C.; pressure 4824 d-bar	0.95252

Table 15 H.—$p_{35, 0, D}$ *(sea-pressure in decibars at standard dynamic depths in sea-water of 35 °/₀₀ salinity and 0° C.).*

Depth (dynamic meters).	0	10	20	30	40	50	60	70	80	90
0	0	10.2815	20.5635	30.8460	41.1290	51.4125	61.6964	71.9809	82.2659	92.5513
100	102.837	113.124	123.411	133.698	143.986	154.274	164.563	174.853	185.143	195.432
200	205.724	216.015	226.307	236.599	246.892	257.185	267.479	277.773	288.068	298.363
300	308.659	318.955	329.252	339.549	349.847	360.145	370.444	380.743	391.043	401.343
400	411.643	421.945	432.246	442.548	452.851	463.154	473.458	483.762	494.066	504.371
500	514.677	524.983	535.289	545.596	555.904	566.212	576.520	586.829	597.138	607.448
600	617.758	628.069	638.381	648.692	659.005	669.318	679.631	689.945	700.259	710.574
700	720.889	731.205	741.521	751.837	762.155	772.472	782.791	793.109	803.428	813.748
800	824.068	834.389	844.710	855.031	865.353	875.676	885.999	896.322	906.646	916.971
900	927.296	937.621	947.947	958.273	968.600	978.927	989.255	999.583	1009.912	1020.241
1000	1030.57	1040.90	1051.23	1061.56	1071.90	1082.23	1092.56	1102.89	1113.23	1123.56
1100	1133.90	1144.23	1154.57	1164.90	1175.24	1185.58	1195.91	1206.25	1216.59	1226.93
1200	1237.27	1247.61	1257.95	1268.29	1278.63	1288.97	1299.30	1309.66	1320.00	1330.34
1300	1340.64	1351.03	1361.38	1371.72	1382.07	1392.42	1402.76	1413.11	1423.46	1433.81
1400	1444.16	1454.51	1464.86	1475.21	1485.56	1495.91	1506.26	1516.61	1526.96	1537.32
1500	1547.67	1558.03	1568.38	1578.74	1589.09	1599.45	1609.80	1620.16	1630.52	1640.88
1600	1651.24	1661.59	1671.95	1682.31	1692.67	1703.04	1713.40	1723.76	1734.12	1744.48
1700	1754.85	1765.21	1775.58	1785.94	1796.31	1806.67	1817.04	1827.40	1837.77	1848.14
1800	1858.51	1868.87	1879.24	1889.61	1899.98	1910.35	1920.72	1931.11	1941.47	1951.84
1900	1962.21	1972.59	1982.96	1993.33	2003.71	2014.08	2024.46	2034.83	2045.21	2055.59
2000	2065.97	2076.34	2086.72	2097.10	2107.48	2117.86	2128.24	2138.62	2149.00	2159.39
2100	2169.77	2180.15	2190.53	2200.92	2211.30	2221.69	2232.07	2242.46	2252.84	2263.23
2200	2273.62	2284.00	2294.39	2304.78	2315.17	2325.56	2335.95	2346.34	2356.73	2367.12
2300	2377.51	2387.90	2398.30	2408.69	2419.08	2429.48	2439.87	2450.27	2460.66	2471.06
2400	2481.45	2491.85	2502.25	2516.25	2523.05	2533.44	2543.84	2554.24	2564.64	2575.04
2500	2585.44	2595.85	2606.25	2616.65	2627.05	2637.46	2647.86	2658.27	2668.67	2679.08
2600	2689.48	2699.89	2710.29	2720.70	2731.11	2741.52	2751.93	2762.33	2772.74	2783.15
2700	2793.56	2803.98	2814.39	2824.80	2835.21	2845.62	2856.04	2866.45	2876.86	2887.28
2800	2897.69	2908.11	2918.53	2928.94	2939.36	2949.78	2960.19	2970.61	2981.03	2991.45
2900	3001.87	3012.29	3022.71	3033.13	3043.56	3053.98	3064.40	3074.82	3085.25	3095.67
3000	3106.09	3116.52	3126.95	3137.37	3147.80	3158.22	3168.65	3179.08	3189.51	3199.93
3100	3210.36	3220.79	3231.22	3241.65	3252.08	3262.52	3272.95	3283.38	3293.81	3304.24
3200	3314.68	3325.11	3335.55	3345.98	3356.42	3366.85	3377.29	3387.73	3398.16	3408.60
3300	3419.04	3429.48	3439.92	3450.36	3460.80	3471.24	3481.68	3492.12	3502.56	3513.01
3400	3523.45	3533.89	3544.34	3554.78	3565.23	3575.67	3586.12	3596.56	3607.01	3617.45
3500	3627.90	3638.35	3648.80	3659.25	3669.70	3680.15	3690.60	3701.05	3711.50	3721.95
3600	3732.40	3742.86	3753.31	3763.76	3774.22	3784.67	3795.13	3805.58	3816.04	3826.49
3700	3836.95	3847.41	3857.86	3868.32	3878.78	3889.24	3899.70	3910.16	3920.62	3931.08
3800	3941.54	3952 00	3962.46	3972.93	3983.39	3993.85	4004.32	4014.78	4025.25	4035.71
3900	4046.18	4056.64	4067.11	4077.58	4088.05	4098.51	4108.98	4119.45	4129.92	4140.39
4000	4150.86	4161.33	4171.80	4182.28	4192.75	4203.22	4213.69	4224.17	4234.64	4245.12
4100	4255.59	4266.07	4276.54	4287.02	4297.49	4307.97	4318.45	4328.93	4339.41	4349.88
4200	4360.36	4370.84	4381.32	4391.81	4402.29	4412.77	4423.25	4433.73	4444.22	4454.70
4300	4465.18	4475.67	4486.15	4496.64	4507.12	4517.61	4528.10	4538.58	4549.07	4559.56
4400	4570.05	4580.54	4591.03	4601.52	4612.01	4622.50	4632.99	4643.48	4653.97	4664.47
4500	4674.96	4685.45	4695.95	4706.44	4716.94	4727.43	4737.93	4748.42	4758.92	4769.42
4600	4779.91	4790.41	4800.91	4811.41	4821.91	4832.41	4842.91	4853.41	4863.91	4874.41
4700	4884.91	4895.42	4905.92	4916.42	4926.93	4937.43	4947.94	4958.44	4968.95	4979.45
4800	4989.96	5000.47	5010.97	5021.48	5031.99	5042.50	5053.01	5063.52	5074.03	5084.54
4900	5095.05	5105.56	5116.07	5126.59	5137.10	5147.61	5158.13	5168.64	5179.15	5189.67

This table is continued on p. 20A

Example: Given depth 4641.99 dynamic meters : d-bars.

Table 15 H gives for 4640 dynamic meters 4821.91

Table 16 H gives for 4640 dynamic meters the increase of pressure 1.04996, or with three decimals 1.050 per dynamic meter ; thus for 1.99 dynamic meters 2.09

Pressure in the depth of 4642 dynamic meters 4824.00

Table 16 H.—$10^5 \cdot \rho_{35,\,0,\,D}$ ($\rho_{35,\,0,\,D}$ = *density in ton/m³ of sea-water of 35 ‰ salinity and 0° C. at standard dynamic depths*).

Depth (dynamic meters).	0	10	20	30	40	50	60	70	80	90
0	102813	102818	102822	102827	102832	102837	102842	102847	102852	102857
100	102862	102867	102872	102877	102881	102886	102891	102896	102901	102906
200	102911	102916	102821	102926	102931	102935	102940	102945	102950	102955
300	102960	102965	102970	102975	102979	102984	102989	102994	102999	103004
400	103009	103014	103019	103023	103028	103033	103038	103043	103048	103053
500	103058	103062	103067	103072	103077	103082	103087	103092	103097	103101
600	103106	103111	103116	103121	103126	103131	103135	103140	103145	103150
700	103155	103160	103165	103169	103174	103179	103184	103189	103194	103198
800	103203	103208	103213	103218	103223	103228	103232	103237	103242	103247
900	103252	103257	103261	103266	103271	103276	103281	103286	103290	103295
1000	103300	103305	103310	103314	103319	103324	103329	103334	103339	103343
1100	103348	103353	103358	103363	103367	103372	103377	103382	103387	103391
1200	103396	103401	103406	103411	103416	103420	103425	103430	103435	103440
1300	103444	103449	103454	103459	103463	103468	103473	103478	103483	103487
1400	103492	103497	103502	103506	103511	103516	103521	103526	103530	103535
1500	103540	103545	103550	103554	103559	103564	103569	103573	103578	103583
1600	103588	103593	103597	103602	103607	103612	103616	103621	103626	103631
1700	103635	103640	103645	103650	103654	103659	103664	103669	103673	103678
1800	103683	103688	103692	103697	103702	103707	103711	103716	103721	103725
1900	103730	103735	103740	103744	103749	103754	103759	103763	103768	103773
2000	103778	103782	103787	103792	103797	103801	103806	103811	103815	103820
2100	103825	103830	103834	103839	103844	103848	103853	103858	103863	103867
2200	103872	103877	103881	103886	103891	103896	103900	103905	103910	103914
2300	103919	103924	103929	103933	103938	103943	103947	103952	103957	103961
2400	103966	103971	103975	103980	103985	103990	103994	103999	104004	104008
2500	104013	104018	104022	104027	104032	104036	104041	104046	104050	104055
2600	104060	104064	104069	104074	104078	104083	104088	104092	104097	104102
2700	104107	104111	104116	104121	104125	104130	104134	104139	104144	104148
2800	104153	104158	104162	104167	104172	104176	104181	104186	104190	104195
2900	104200	104204	104209	104214	104218	104223	104228	104232	104237	104242
3000	104246	104251	104255	104260	104265	104269	104274	104279	104283	104288
3100	104292	104297	104302	104306	104311	104316	104320	104325	104330	104334
3200	104339	104343	104348	104353	104357	104362	104366	104371	104376	104380
3300	104385	104390	104394	104399	104403	104408	104413	104417	104422	104426
3400	104431	104436	104440	104445	104449	104454	104459	104463	104468	104472
3500	104477	104482	104486	104491	104495	104500	104505	104509	104514	104518
3600	104523	104528	104532	104537	104541	104546	104551	104555	104560	104564
3700	104569	104573	104578	104583	104587	104592	104596	104601	104606	104610
3800	104615	104619	104624	104628	104633	104637	104642	104647	104651	104656
3900	104660	104665	104669	104674	104679	104683	104688	104692	104697	104701
4000	104706	104710	104715	104720	104724	104729	104733	104738	104742	104747
4100	104751	104756	104760	104765	104770	104774	104779	104783	104788	104702
4200	104797	104801	104806	104810	104815	104819	104824	104829	104833	104838
4300	104842	104847	104851	104856	104860	104865	104869	104874	104878	104883
4400	104887	104892	104896	104901	104905	104910	104915	104919	104924	104928
4500	104933	104937	104942	104046	104951	104955	104960	104664	104969	104973
4600	104978	104982	104987	104891	104996	105000	105005	105009	105014	105018
4700	105023	105027	105032	105036	105041	105045	105050	105054	105059	105063
4800	105068	105072	105077	105081	105086	105090	105095	105099	105104	105108
4900	105113	105117	105121	105126	105130	105135	105139	105144	105148	105153

This table is continued on p. 21A.

Example: $D = 4642$ dynamic meters, $\rho_{35,\,0,\,D} = 1.04997$.

Table 15 H (continued from p. 18A).—$p_{35, 0, D}$ *(sea-pressure in decibars at standard dynamic depths in sea-water of 35 °/₀₀ salinity and 0° C.)*

Depth (dynamic meters).	0	10	20	30	40	50	60	70	80	90
5000	5200.18	5210.70	5221.22	5231.73	5242.25	5252.77	5263.29	5273.81	5284.32	5294.84
5100	5305.36	5315.88	5326.40	5336.93	5347.45	5357.97	5368.49	5379.02	5389.54	5400.06
5200	5410.59	5421.11	5431.64	5442.16	5452.69	5463.22	5473.74	5484.27	5494.80	5505.33
5300	5515.86	5526.39	5536.92	5547.45	5557.98	5568.51	5579.04	5589.57	5600.10	5610.64
5400	5621.17	5631.70	5642.24	5652.77	5663.31	6573.84	5684.38	5694.92	5705.45	5715.99
5500	5726.53	5737.07	5747.61	5758.14	5768.68	5779.22	5789.76	5800.31	5810.85	5821.39
5600	5831.93	5842.47	5853.02	5863.56	5874.10	5884.65	5895.19	5905.74	5916.28	5926.83
5700	5937.38	5947.92	5958.47	5969.02	5979.57	5990.12	6000.67	6011.22	6021.77	6032.32
5800	6042.87	6053.42	6063.97	6074.52	6085.08	6095.63	6106.18	6116.74	6127.29	6137.85
5900	6148.40	6158.96	6169.51	6180.07	6190.63	6201.19	6211.74	6222.30	6232.86	6243.42
6000	6253.98	6264.54	6275.10	6285.66	6296.22	6306.79	6317.35	6327.91	6338.48	6349.04
6100	6359.60	6370.17	6380.73	6391.30	6401.86	6412.43	6423.00	6433.57	6444.13	6454.70
6200	6465.27	6475.84	6486.41	6496.98	6507.55	6518.12	6528.69	6539.26	6549.83	6560.41
6300	6570.98	6581.55	6592.13	6602.70	6613.28	6623.85	6634.43	6645.00	6655.58	6666.16
6400	6676.73	6687.31	6697.89	6708.47	6719.05	6729.63	6740.21	6750.79	6761.37	6771.95
6500	6782.53	6793.11	6803.70	6814.28	6824.86	6835.45	6846.03	6856.62	6867.20	6877.79
6600	6888.37	6898.96	6909.55	6920.13	6930.72	6941.31	6951.90	6962.49	6973.08	6983.67
6700	6994.26	7004.85	7015.44	7026.03	7036.62	7047.22	7057.81	7068.40	7079.00	7089.59
6800	7100.19	7110.78	7121.38	7131.97	7142.57	7153.17	7163.76	7174.36	7184.96	7195.56
6900	7206.16	7216.76	7227.36	7237.96	7259.16	7248.56	7269.76	7280.36	7290.97	7301.57
7000	7312.17	7322.78	7333.38	7343.99	7354.59	7365.20	7375.80	7386.41	7397.02	7407.62
7100	7418.23	7428.84	7439.45	7450.06	7460.67	7471.28	7481.89	7492.50	7503.11	7513.72
7200	7524.33	7534.94	7545.56	7556.17	7566.78	7577.39	7588.01	7598.63	7609.24	7619.86
7300	7630.48	7641.09	7651.71	7662.33	7672.95	7683.57	7694.18	7704.80	7715.42	7726.04
7400	7736.66	7747.29	7757.91	7768.53	7779.15	7789.77	7800.40	7811.02	7821.65	7832.27
7500	7842.89	7853.52	7864.15	7874.77	7885.40	7896.03	7906.65	7917.28	7927.91	7938.54
7600	7949.17	7959.80	7940.43	7981.06	7991.69	8002.32	8012.95	8023.58	8034.22	8044.85
7700	8055.48	8066.12	8076.75	8087.39	8098.02	8108.66	8119.29	8129.93	8140.57	8151.20
7800	8161.84	8172.48	8183.12	8193.76	8204.40	8215.04	8225.68	8236.32	8246.96	8257.60
7900	8268.24	8278.89	8289.53	8300.17	8310.82	8321.46	8332.10	8342.75	8353.40	8364.04
8000	8374.69	8385.33	8395.98	8406.63	8417.28	8427.93	8438.57	8449.22	8459.87	8470.52
8100	8481.17	8491.82	8502.48	8513.13	8523.78	8534.43	8545.09	8555.74	8566.39	8577.05
8200	8587.70	8598.36	8609.01	8619.67	8630.33	8640.98	8651.64	8662.30	8672.96	8683.61
8300	8694.27	8704.93	8715.59	8726.25	8736.91	8747.58	8758.24	8768.90	8779.56	8790.22
8400	8800.89	8811.55	8822.22	8832.88	8843.54	8854.21	8864.88	8875.54	8886.21	8896.88
8500	8907.54	8918.21	8928.88	8939.55	8950.22	8960.89	8971.56	8982.23	8992.90	9003.57
8600	9014.24	9024.91	9035.59	9046.26	9056.93	9067.61	9078.28	9088.95	9099.63	9110.31
8700	9120.98	9131.66	9142.33	9153.01	9163.69	9174.37	9185.05	9195.72	9206.40	9217.08
8800	9227.76	9238.44	9249.12	9259.81	9270.49	9281.17	9291.85	9302.54	9313.22	9323.90
8900	9334.59	9345.27	9355.96	9366.64	9377.33	9388.02	9398.70	9409.39	9420.08	9430.76
9000	9441.45	9452.14	9462.83	9473.52	9484.21	9494.90	9505.59	9516.28	9526.98	9537.67
9100	9548.36	9559.05	9569.75	9580.44	9591.14	9601.83	9612.53	9623.22	9633.92	9644.61
9200	9655.31	9666.01	9676.71	9687.40	9698.10	9708.80	9719.50	9730.20	9740.90	9751.60
9300	9762.30	9773.00	9783.71	9794.41	9805.11	9815.81	9826.52	9837.22	9847.93	9858.63
9400	9869.34	9880.04	9890.75	9901.45	9912.16	9922.87	9933.58	9944.28	9954.99	9965.70
9500	9976.41	9987.12	9997.83	10008.54	10019.25	10029.96	10040.68	10051.39	10062.10	10072.81
9600	10083.53	10094.24	10104.95	10115.67	10126.38	10137.10	10147.82	10158.53	10169.25	10179.97
9700	10190.68	10201.40	10212.12	10222.84	10233.56	10244.28	10255.00	10265.72	10276.44	10287.16
9800	10297.88	10308.61	10319.33	10330.05	10340.77	10351.50	10362.22	10372.95	10383.67	10394.40
9900	10405.12	10415.84	10426.58	10437.30	10448.03	10458.76	10469.49	10480.22	10490.95	10501.68

Example: Given depth 7188.37 dynamic meters: d-bars.

Table 15 H gives for 7190 dynamic meters 7513.72

Table 16 H gives for 7190 dynamic meters the increase of pressure 1.06119 or with three decimals 1.061 d-bars per dynamic meter. Thus per 1.63 dynamic meter .... — 1.73

Pressure in the depth of 7188.37 dynamic meters 7511.99

Table 16 H (continued from p. 19 A).—$10^5 \cdot \rho_{35,0,D}$ ($\rho_{35,0,D}$ = *density in ton/m³ of sea-water of 35 °/₀₀ salinity and 0° C. at standard dynamic depths*).

Depth (dynamic meters).	0	10	20	30	40	50	60	70	80	90
5000	105157	105162	105166	105171	105175	105180	105184	105189	105193	105197
5100	105202	105206	105211	105215	105220	105224	105229	105233	105238	105242
5200	105246	105251	105255	105260	105264	105269	105273	105278	105282	105287
5300	105291	105296	105300	105304	105309	105313	105318	105322	105327	105331
5400	105336	105340	105344	105349	105353	105358	105362	105367	105371	105376
5500	105380	105384	105389	105393	105398	105402	105407	105411	105415	105420
5600	105424	105429	105433	105438	105442	105446	105451	105455	105460	105464
5700	105469	105473	105477	105482	105486	105491	105495	105499	105504	105508
5800	105513	105517	105521	105526	105530	105535	105539	105543	105548	105552
5900	105557	105561	105565	105570	105574	105579	105583	105587	105592	105596
6000	105601	105605	105609	105614	105618	105623	105627	105631	105636	105640
6100	105645	105649	105653	105658	105662	105667	105671	105675	105680	105684
6200	105688	105693	105697	105702	105706	105710	105715	105719	105724	105728
6300	105732	105737	105741	105745	105750	105754	105758	105763	105767	105772
6400	105776	105780	105785	105789	105793	105798	105802	105806	105811	105815
6500	105820	105824	105828	105833	105837	105841	105846	105850	105854	105859
6600	105863	105867	105872	105876	105880	105885	105889	105893	105898	105902
6700	105907	105911	105915	105920	105924	105928	105933	105937	105941	105946
6800	105950	105954	105959	105963	105967	105972	105976	105980	105985	105989
6900	105993	105998	106002	106006	106011	106015	106019	106024	106028	106032
7000	106037	106041	106045	106049	106054	106058	106062	106067	106071	106075
7100	106080	106084	106088	106093	106097	106101	106106	106110	106114	106119
7200	106123	106127	106131	106136	106140	106144	106149	106153	106157	106162
7300	106166	106170	106174	106179	106183	106187	106192	106196	106200	106205
7400	106209	106213	106217	106222	106226	106230	106235	106239	106243	106247
7500	106252	106256	106260	106265	106269	106273	106277	106282	106286	106290
7600	106295	106299	106303	106307	106312	106316	106320	106324	106329	106333
7700	106337	106342	106346	106350	106354	106359	106363	106367	106371	106376
7800	106380	106384	106388	106393	106397	106401	106406	106410	106414	106418
7900	106423	106427	106431	106435	106440	106444	106448	106452	106457	106461
8000	106465	106469	106474	106478	106482	106486	106491	106495	106499	106503
8100	106508	106512	106516	106520	106525	106529	106533	106537	106542	106546
8200	106550	106554	106559	106563	106567	106571	106575	106580	106584	106588
8300	106592	106597	106601	106605	106609	106614	106618	106622	106626	106630
8400	106635	106639	106643	106647	106652	106656	106660	106664	106668	106673
8500	106677	106681	106685	106690	106694	106698	106702	106706	106711	106715
8600	106719	106723	106727	106732	106736	106740	106744	106749	106753	106757
8700	106761	106765	106770	106774	106778	106782	106786	106791	106795	106799
8800	106803	106807	106812	106816	106820	106824	106828	106833	106837	106841
8900	106845	106849	106853	106858	106862	106866	106870	106874	106879	106883
9000	106887	106891	106895	106899	106904	106908	106912	106916	106920	106925
9100	106929	106933	106937	106941	106945	106950	106954	106958	106962	106966
9200	106971	106975	106979	106983	106987	106991	106996	107000	107004	107008
9300	107012	107016	107021	107025	107029	107033	107037	107041	107046	107050
9400	107054	107058	107062	107066	107070	107075	107079	107083	107087	107091
9500	107095	107099	107104	107108	107112	107116	107120	107124	107129	107133
9600	107137	107141	107145	107149	107153	107158	107162	107166	107170	107174
9700	107178	107182	107187	107191	107195	107199	107203	107207	107211	107216
9800	107220	107224	107228	107232	107236	107240	107244	107249	107253	107257
9900	107261	107265	107269	107273	107277	107282	107286	107290	107294	107298

Example: Given: $D = 7188.38$. Found: $\rho_{35,0,D} = 1.06118$.

Table 17 H.—$10^5 \cdot \varepsilon_s$ (*ε_s = salinity correction in ton/m^3 to the density of sea-water*).

Salinity (‰).	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	−2826	−2817	−2809	−2800	−2792	−2783	−2775	−2766	−2758	−2749
1	−2741	−2732	−2724	−2716	−2708	−2700	−2692	−2684	−2675	−2667
2	−2659	−2651	−2643	−2635	−2627	−2619	−2610	−2602	−2594	−2586
3	−2578	−2570	−2562	−2554	−2545	−2537	−2529	−2521	−2513	−2505
4	−2497	−2489	−2480	−2472	−2464	−2456	−2448	−2440	−2432	−2424
5	−2416	−2408	−2399	−2391	−2383	−2375	−2367	−2359	−2351	−2343
6	−2335	−2327	−2318	−2310	−2302	−2294	−2286	−2278	−2270	−2262
7	−2254	−2246	−2238	−2229	−2221	−2213	−2205	−2197	−2189	−2181
8	−2173	−2165	−2157	−2149	−2140	−2132	−2124	−2116	−2108	−2100
9	−2092	−2084	−2076	−2068	−2060	−2052	−2044	−2035	−2027	−2019
10	−2011	−2003	−1995	−1987	−1979	−1971	−1963	−1955	−1947	−1939
11	−1931	−1922	−1914	−1906	−1898	−1890	−1882	−1874	−1866	−1858
12	−1850	−1842	−1834	−1826	−1818	−1810	−1802	−1793	−1785	−1777
13	−1769	−1761	−1753	−1745	−.1737	−1729	−1721	−1713	−1705	−1697
14	−1689	−1681	−1673	−1665	−1657	−1648	−1640	−1632	−1624	−1616
15	−1608	−1600	−1592	−1584	−1576	−1568	−1560	−1552	−1544	−1536
16	−1528	−1520	−1512	−1504	−1496	−1488	−1479	−1471	−1463	−1455
17	−1447	−1439	−1431	−1423	−1415	−1407	−1399	−1391	−1383	−1375
18	−1367	−1359	−1351	−1343	−1335	−1327	−1319	−1311	−1302	−1294
19	−1286	−1278	−1270	−1262	−1254	−1246	−1238	−1230	−1222	−1214
20	−1206	−1198	−1190	−1182	−1174	−1166	−1158	−1150	−1142	−1134
21	−1126	−1118	−1110	−1102	−1094	−1086	−1078	−1070	−1061	−1053
22	−1045	−1037	−1029	−1021	−1013	−1005	− 997	− 989	− 981	− 973
23	− 965	− 957	− 949	− 941	− 933	− 925	− 917	− 909	− 901	− 893
24	− 885	− 877	− 869	− 861	− 853	− 845	− 836	− 828	− 820	− 812
25	− 804	− 796	− 788	− 780	− 772	− 764	− 756	− 748	− 740	− 732
26	− 724	− 716	− 708	− 700	− 692	− 684	− 676	− 668	− 660	− 652
27	− 644	− 636	− 628	− 620	− 611	− 603	− 595	− 587	− 579	− 571
28	− 563	− 555	− 547	− 539	− 531	− 523	− 515	− 507	− 499	− 491
29	− 483	− 475	− 467	− 459	− 451	− 443	− 435	− 427	− 419	− 411
30	− 403	− 394	− 386	− 378	− 370	− 362	− 354	− 346	− 338	− 330
31	− 322	− 314	− 306	− 298	− 290	− 282	− 274	− 266	− 258	− 250
32	− 242	− 234	− 226	− 217	− 209	− 201	− 193	− 185	− 177	− 169
33	− 161	− 153	− 145	− 137	− 129	− 121	− 113	− 105	− 97	− 89
34	− 81	− 72	− 64	− 56	− 48	− 40	− 33	− 24	− 16	− 8
35	0	8	16	24	32	40	48	56	65	73
36	81	89	97	105	113	121	129	137	145	153
37	161	169	177	186	194	202	210	218	226	234
38	242	250	258	266	274	282	291	299	307	315
39	323	331	339	347	355	363	371	379	388	396

Example: Given: $s = 23.12$ ‰. Found: $\varepsilon_s = -0.00955$.

Table 18 H.—$10^5 \cdot \varepsilon_\tau$ (ε_τ = *temperature correction in ton/m³ to the density of sea-water*).

Temperature (° C.).	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
−1	5	5	5	6	6	6	7	7	7	8
−0	0	1	1	2	2	3	3	3	4	4
0	0	−1	−1	−2	−2	−3	−3	−4	−5	−5
1	−6	−7	−7	−8	−9	−10	−10	−11	−12	−13
2	−13	−14	−15	−16	−17	−17	−18	−19	−20	−21
3	−22	−23	−24	−25	−26	−27	−28	−29	−30	−31
4	−32	−33	−34	−35	−36	−37	−38	−40	−41	−42
5	−43	−44	−45	−47	−48	−49	−50	−52	−53	−54
6	−55	−57	−58	−59	−61	−62	−63	−65	−66	−67
7	−69	−70	−72	−73	−74	−76	−77	−79	−80	−82
8	−83	−85	−86	−88	−89	−91	−93	−94	−96	−97
9	−99	−101	−102	−104	−105	−107	−109	−110	−112	−114
10	−115	−117	−119	−121	−122	−124	−126	−128	−130	−131
11	−133	−135	−137	−139	−141	−142	−144	−146	−148	−150
12	−152	−154	−156	−158	−160	−162	−164	−166	−168	−169
13	−171	−173	−176	−178	−180	−182	−184	−186	−188	−190
14	−192	−194	−196	−198	−201	−203	−205	−207	−209	−211
15	−214	−216	−218	−220	−222	−225	−227	−229	−231	−234
16	−236	−238	−241	−243	−245	−248	−250	−252	−255	−257
17	−259	−262	−264	−267	−269	−271	−274	−276	−279	−281
18	−284	−286	−289	−291	−294	−296	−299	−301	−304	−306
19	−309	−311	−314	−316	−319	−322	−324	−327	−329	−332
20	−334	−337	−340	−342	−345	−348	−351	−353	−356	−359
21	−361	−364	−367	−369	−372	−375	−378	−381	−383	−386
22	−389	−392	−394	−397	−400	−403	−406	−409	−412	−414
23	−417	−420	−423	−426	−429	−432	−435	−438	−441	−443
24	−446	−449	−452	−455	−458	−461	−464	−467	−470	−473
25	−476	−479	−482	−486	−489	−492	−495	−498	−501	−504
26	−507	−510	−513	−517	−520	−523	−526	−529	−532	−535
27	−539	−542	−545	−548	−552	−555	−558	−561	−565	−568
28	−571	−574	−578	−581	−584	−588	−591	−594	−598	−601
29	−604	−608	−611	−614	−618	−621	−624	−628	−631	−635

Example: Given : $\tau = 11.61°$ C. Found : $\varepsilon_\tau = -0.00144$.

Table 19 H.—$10^5 \cdot \varepsilon_{s\tau}$ ($\varepsilon_{s\tau}$ = *combined salinity-temperature correction in ton/m³ to the density of sea-water*).

Salinity (‰).	Temperature (° C.).															
	−2	−1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	−25	−13	0	12	23	34	45	55	65	75	84	93	101	110	118	125	132
1	−24	−12	0	12	22	33	44	54	63	72	81	90	98	106	114	121	128
2	−24	−12	0	11	22	32	42	52	61	70	79	87	95	103	110	117	124
3	−23	−11	0	11	21	31	41	50	59	68	76	84	91	99	107	113	120
4	−22	−11	0	10	20	30	39	48	57	65	73	81	89	96	103	109	116
5	−21	−10	0	10	20	29	38	47	55	63	71	78	86	92	99	106	112
6	−20	−10	0	10	19	28	37	45	53	61	68	76	82	89	96	102	108
7	−20	−10	0	9	18	27	35	43	51	59	66	73	79	86	92	98	104
8	−19	− 9	0	9	17	26	34	42	49	56	63	70	76	82	89	94	100
9	−18	− 9	0	9	17	25	32	40	47	54	61	67	73	79	85	91	96
10	−17	− 9	0	8	16	24	31	38	45	52	58	65	70	76	82	87	92
11	−17	− 8	0	8	15	23	30	37	43	50	56	62	67	73	78	83	88
12	−16	− 8	0	7	15	22	29	35	41	47	53	59	64	70	75	79	84
13	−15	− 8	0	7	14	21	27	33	39	45	51	56	61	66	71	76	80
14	−14	− 7	0	7	13	20	26	32	37	43	48	54	58	63	68	72	77
15	−14	− 7	0	6	13	19	25	30	36	41	46	51	55	60	64	69	73
16	−13	− 6	0	6	12	18	23	29	34	39	43	48	52	57	61	65	69
17	−12	− 6	0	6	11	17	22	27	32	37	41	45	50	54	58	61	65
18	−11	− 6	0	5	11	16	21	25	30	34	39	43	47	51	54	58	61
19	−11	− 5	0	5	10	15	19	24	28	32	36	40	44	47	51	54	57
20	−10	− 5	0	5	9	14	18	22	26	30	34	38	41	44	48	51	54
21	− 9	− 5	0	4	9	13	17	21	24	28	31	35	38	41	44	47	50
22	− 9	− 4	0	4	8	12	16	19	23	26	29	32	35	38	41	44	46
23	− 8	− 4	0	4	7	11	14	18	21	24	27	30	33	35	38	40	43
24	− 7	− 4	0	3	7	10	13	16	19	22	25	27	30	32	35	37	39
25	− 7	− 3	0	3	6	9	12	15	17	20	22	25	27	29	31	33	35
26	− 6	− 3	0	3	5	8	11	13	15	18	20	22	24	26	28	30	32
27	− 5	− 3	0	2	5	7	10	12	14	16	18	20	21	23	25	27	28
28	− 5	− 2	0	2	4	6	8	10	12	14	15	17	19	20	22	23	25
29	− 4	− 2	0	2	4	5	7	9	10	12	13	15	16	17	19	20	21
30	− 3	− 2	0	2	3	4	6	7	8	10	11	12	13	14	16	16	18
31	− 3	− 1	0	1	2	4	5	6	7	8	9	10	11	11	12	13	14
32	− 2	− 1	0	1	2	3	4	4	5	6	6	7	8	9	9	10	10
33	− 1	− 1	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7
34	− 1	0	0	0	1	1	1	1	2	2	2	2	3	3	3	3	3
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	1	0	0	0	−1	−1	−1	−1	−2	−2	− 2	− 2	− 3	− 3	− 3	− 3	− 3
37	1	1	0	−1	−1	−2	−2	−3	−3	−4	− 4	− 5	− 5	− 6	− 6	− 7	− 7
38	2	1	0	−1	−2	−3	−3	−4	−5	−6	− 7	− 7	− 8	− 9	− 9	−10	−10
39	2	1	0	−1	−2	−3	−4	−6	−7	−8	− 9	− 9	−10	−11	−12	−13	−14
40	3	1	0	−1	−3	−4	−6	−7	−8	−9	−11	−12	−13	−14	−15	−16	−17

Example: Given: $s = 23.12$ ‰; $\tau = 11.61°$ C. Found: $\varepsilon_{s\tau} = + 0.00036$.

Table 19 H (continued).—$10^5 \cdot \varepsilon_{sT}$ (ε_{sT} = *combined salinity-temperature correction in ton/m³ to the density of sea-water*).

Salinity (‰).	Temperature (° C.). 15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0	140	146	152	159	165	171	176	182	186	192	197	201	206	210	214	219
1	135	142	148	154	160	165	171	176	181	186	190	195	199	203	208	212
2	131	137	143	149	155	160	165	170	175	180	184	189	193	197	201	205
3	127	132	138	144	150	155	160	165	169	174	178	182	187	190	194	198
4	122	128	134	139	145	149	154	159	163	168	172	176	180	184	188	191
5	118	124	129	134	140	144	149	154	158	162	166	170	174	177	181	184
6	114	119	124	130	135	139	144	148	152	156	160	164	168	171	174	178
7	110	115	120	125	130	134	138	143	147	150	154	158	161	165	168	171
8	105	110	115	120	125	129	133	137	141	145	148	152	155	158	161	164
9	101	106	111	115	120	124	128	132	135	139	142	146	149	152	155	158
10	97	102	106	111	115	119	123	126	130	133	137	140	143	146	149	151
11	93	97	102	106	110	114	117	121	125	128	131	134	137	140	142	145
12	89	93	97	101	105	109	112	116	119	122	125	128	131	134	136	139
13	85	89	93	97	100	104	107	110	114	116	119	122	125	127	130	132
14	81	85	88	92	96	99	102	105	108	111	114	116	119	121	124	126
15	77	80	84	87	91	94	97	100	103	105	108	110	113	115	117	119
16	73	76	79	83	86	89	92	95	97	100	102	105	107	109	111	113
17	69	72	75	78	81	84	87	90	92	94	97	99	101	103	105	107
18	65	68	71	74	77	79	82	84	87	89	91	93	95	97	99	101
19	61	64	66	69	72	74	77	79	81	83	85	87	89	91	93	94
20	57	59	62	65	67	70	72	74	76	78	80	82	84	85	87	88
21	53	55	58	60	63	65	67	69	71	73	74	76	78	79	81	82
22	49	51	54	56	58	60	62	64	66	67	69	70	72	73	75	76
23	45	47	49	52	54	55	57	59	61	62	64	65	66	68	69	70
24	41	43	45	47	49	50	52	54	55	57	58	59	61	62	63	64
25	37	39	41	43	44	46	47	49	50	51	53	54	55	56	57	58
26	34	35	37	38	40	41	42	44	45	46	47	48	49	50	51	52
27	30	31	33	34	35	36	38	39	40	41	42	43	44	45	45	46
28	26	27	28	30	31	32	33	34	35	36	37	37	38	39	40	40
29	22	23	24	25	26	27	28	29	30	31	31	32	33	33	34	34
30	18	19	20	21	22	23	23	24	25	25	26	26	27	28	28	29
31	15	15	16	17	18	18	19	19	20	20	21	21	22	22	23	23
32	11	12	12	13	13	13	14	14	15	15	15	16	16	17	17	17
33	7	8	8	8	9	9	9	10	10	10	10	10	11	11	11	11
34	4	4	4	4	4	4	5	5	5	5	5	5	5	5	6	6
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	−4	−4	−4	−4	−4	−4	−5	−5	−5	−5	−5	−5	−5	−5	−6	−6
37	−7	−8	−8	−8	−9	−9	−9	−9	−10	−10	−10	−10	−11	−11	−11	−11
38	−11	−11	−12	−12	−13	−13	−14	−14	−15	−15	−15	−16	−16	−16	−17	−17
39	−14	−15	−16	−16	−17	−18	−18	−19	−20	−20	−20	−21	−21	−22	−22	−22
40	−18	−19	−20	−21	−21	−22	−23	−24	−24	−25	−26	−26	−27	−27	−27	−28

Example to tables 16 H to 19 H:

Table 16 H gives for the sea-pressure zero	1.02813
Table 17 H gives for salinity 23.12 ‰	−955
Table 18 H gives for temperature 11.61° C.	−144
Table 19 H gives for salinity 23.12 ‰; temperature 11.61° C.	+36
Density of sea-water of salinity 23.12 ‰, temperature 11.61° C. under atmospheric pressure	1.01750

Table 20 H.—$10^5 \cdot \varepsilon_{sD}$ (ε_{sD} = *combined salinity-depth correction in ton/m³ to the density of sea-water*).

Salinity (‰).																															Depth (dyn. meters).
10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	4	4	4	3	3	3	3	3	3	2	2	2	2	2	2	1	1	1	1	1	1	0	0	0	0	0	0	0	−1	−1	100
8	8	8	7	7	7	6	6	6	5	5	5	4	4	4	3	3	3	2	2	2	1	1	1	0	0	0	−1	−1	−1	−2	200
13	12	12	11	10	10	9	9	8	8	7	7	6	6	5	5	4	4	3	3	2	2	1	1	0	0	0	−1	−1	−2	−2	300
17	16	15	15	14	13	13	12	11	11	10	9	9	8	7	7	6	5	5	4	3	3	2	1	1	0	−1	−1	−2	−3	−3	400
21	20	19	18	17	17	16	15	14	13	12	12	11	10	9	8	7	7	6	5	4	3	3	2	1	0	−1	−2	−2	−3	−4	500
25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	−1	−2	−3	−4	−5	600
29	28	27	26	24	23	22	21	20	18	17	16	15	14	13	11	10	9	8	7	6	5	3	2	1	0	−1	−2	−3	−4	−6	700
33	32	30	29	28	26	25	24	22	21	20	18	17	16	14	13	12	10	9	8	7	5	4	3	1	0	−1	−3	−4	−5	−6	800
37	36	34	33	31	30	28	27	25	24	22	21	19	18	16	15	13	12	10	9	7	6	4	3	1	0	−1	−3	−4	−6	−7	900
41	40	38	36	35	33	31	30	28	26	25	23	21	20	18	16	15	13	11	10	8	7	5	3	2	0	−2	−3	−5	−6	−8	1000
45	43	42	40	38	36	34	32	31	29	27	25	23	21	20	18	16	14	12	11	9	7	5	4	2	0	−2	−4	−5	−7	−9	1100
49	47	45	43	41	39	37	35	33	31	29	27	25	23	21	19	17	16	14	12	10	8	6	4	2	0	−2	−4	−6	−8	−9	1200
53	51	49	47	45	42	40	38	36	34	32	29	27	25	23	21	19	17	15	13	10	8	6	4	2	0	−2	−4	−6	−8	−10	1300
57	55	53	50	48	46	43	41	39	36	34	32	29	27	25	23	20	18	16	13	11	9	7	4	2	0	−2	−4	−7	−9	−11	1400
										36	34	32	29	27	24	22	19	17	14	12	10	7	5	2	0	−2	−5	−7	−9	−12	1500
										39	36	34	31	28	26	23	21	18	15	13	10	8	5	3	0	−3	−5	−8	−10	−13	1600
										41	38	36	33	30	27	25	22	19	16	14	11	8	5	3	0	−3	−5	−8	−11	−13	1700
										43	40	38	35	32	29	26	23	20	17	14	11	9	6	3	0	−3	−6	−8	−11	−14	1800
										46	43	40	36	33	30	27	24	21	18	15	12	9	6	3	0	−3	−6	−9	−12	−15	1900
										48	45	42	38	35	32	29	25	22	19	16	13	10	6	3	0	−3	−6	−9	−13	−16	2000
										50	47	43	40	37	33	30	27	23	20	17	13	10	7	3	0	−3	−7	−10	−13	−16	2100
										52	49	45	42	38	35	31	28	24	21	17	14	10	7	3	0	−3	−7	−10	−14	−17	2200
										55	51	47	44	40	36	33	29	25	22	18	14	11	7	4	0	−4	−7	−11	−14	−18	2300
										57	53	49	46	42	38	34	30	26	23	19	15	11	8	4	0	−4	−8	−11	−15	−19	2400
																				20	16	12	8	4	0	−4	−8	−12	−16	−20	2500
																				20	16	12	8	4	0	−4	−8	−12	−16	−20	2600
																				21	17	13	8	4	0	−4	−8	−13	−17	−21	2700
																				22	17	13	9	4	0	−4	−9	−13	−17	−22	2800
																				23	18	14	9	5	0	−4	−9	−13	−18	−22	2900
																				23	19	14	9	5	0	−5	−9	−14	−19	−23	3000
																				24	19	14	10	5	0	−5	−10	−14	−19	−24	3100
																				25	20	15	10	5	0	−5	−10	−15	−20	−25	3200
																				25	20	15	10	5	0	−5	−10	−15	−20	−25	3300
																				26	21	16	11	5	0	−5	−10	−16	−21	−26	3400
																				27	22	16	11	5	0	−5	−11	−16	−21	−27	3500
																				28	22	17	11	6	0	−5	−11	−16	−22	−27	3600
																				28	23	17	11	6	0	−6	−11	−17	−22	−28	3700
																				29	23	17	12	6	0	−6	−11	−17	−23	−29	3800
																				30	24	18	12	6	0	−6	−12	−18	−24	−29	3900
																				30	24	18	12	6	0	−6	−12	−18	−24	−30	4000
																				31	25	19	12	6	0	−6	−12	−19	−25	−31	4100
																				32	25	19	13	6	0	−6	−13	−19	−25	−31	4200
																				33	26	19	13	7	0	−6	−13	−19	−26	−32	4300
																				33	27	20	13	7	0	−7	−13	−20	−26	−33	4400
																				34	27	20	14	7	0	−7	−13	−20	−27	−34	4500
																				35	28	21	14	7	0	−7	−14	−21	−27	−34	4600
																				35	28	21	14	7	0	−7	−14	−21	−28	−35	4700
																				36	29	22	14	7	0	−7	−14	−21	−28	−36	4800
																				37	29	22	15	7	0	−7	−15	−22	−29	−36	4900
																				37	30	22	15	8	0	−7	−15	−22	−29	−37	5000

Salinity (‰).										Depth (dyn. meters).
0	1	2	3	4	5	6	7	8	9	
0	0	0	0	0	0	0	0	0	0	0
6	6	6	5	5	5	5	5	5	4	100
12	11	11	11	10	10	10	9	9	9	200
18	17	17	16	16	15	15	14	14	13	300
24	23	22	22	21	20	19	19	18	17	400
30	29	28	27	26	25	24	23	22	22	500
Average salinity (‰).										**Depth (dyn. meters.)**
0	1	2	3	4	5	6	7	8	9	
0	0	0	0	0	0	0	0	0	0	0
−15	−14	−14	−13	−13	−12	−12	−11	−11	−11	1000

Average salinity (‰).																															Depth (dyn. meters).
10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
−10	−10	−9	−9	−8	−8	−8	−7	−7	−6	−6	−5	−5	−4	−4	−4	−3	−3	−3	−2	−2	−2	−1	−1	0	0	0	1	1	2	2	1000
−20	−19	−18	−17	−16	−15	−15	−14	−13	−12	−11	−10	−10	−9	−8	−7	−7	−6	−5	−4	−4	−3	−2	−1	−1	0	1	1	2	3	4	2000
										−16	−15	−14	−13	−12	−11	−10	−9	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	3000
																				−7	−6	−4	−3	−1	0	1	3	4	6	7	4000
																				−8	−7	−5	−3	−2	0	2	3	5	7	8	5000

This table is continued on p. 28A.

Example: Given: $s = 38.74$ ‰; $D = 2700$; average salinity above this level, 38.5 ‰.
The main table gives for $s = 38.74$, $D = 2700$ .... −0.00016
The additional table gives for average salinity = 38.5, $D = 2700$ .... +0.00003
Found, ε_{sD} .... −0.00013

Table 21 H.—$10^5\epsilon_{\tau D}$ (*$\epsilon_{\tau D}$ = combined temperature-depth correction in ton/m³ to the density of sea-water*).

Depth (dyn. meters).	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Temperature (°C.).																																
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	1	0	0	0	-1	-1	-1	-1	-2	-2	-2	-2	-2	-3	-3	-3	-3	-3	-3	-3	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-5	-5	-5
200	1	1	0	-1	-1	-2	-2	-3	3	-4	-4	-4	-5	-5	-5	-6	-6	-6	-7	-7	-7	-7	-8	-8	-8	-8	-8	-9	-9	-9	-9	-9	-9
300	2	1	0	-1	-2	-3	-3	-4	-5	-5	-6	-7	-7	-8	-8	-9	-9	-10	-10	-10	-11	-11	-11	-12	-12	-12	-12	-13	-13	-13	-13	-13	-14
400	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-9	-10	-11	-11	-12	-13	-13	-14	-14	-15	-15	-16	-16	-16	-17	-17	-17	-17	-18	-18	-18
500	3	1	0	-1	-3	-4	-5	-7	-8	-9	-10	-11	-12	-13	-13	-14	-15	-16	-16	-17	-18	-18	-19	-19	-20	-20	-21	-21	-21	-22	-22	-22	-23
600	4	2	0	-2	-3	-5	-7	-8	-9	-11	-12	-13	-14	-15	-16	-17	-18	-19	-20	-20	-21	-22	-23	-23	-24	-24	-25	-25	-26	-26	-26	-27	-27
700	4	2	0	-2	-4	-6	-8	-9	-11	-12	-14	-15	-16	-18	-19	-20	-21	-22	-23	-24	-25	-25	-26	-27	-27	-28	-29	-29	-30	-30	-31	-31	-31
800	5	2	0	-2	-4	-6	-8	-10	-12	-14	-16	-17	-19	-20	-21	-23	-24	-25	-26	-27	-28	-29	-30	-31	-31	-32	-33	-33	-34	-34	-35	-35	-36
900	5	3	0	-3	-5	-7	-10	-12	-14	-16	-17	-19	-21	-22	-24	-25	-27	-28	-29	-30	-31	-32	-33	-34	-35	-36	-37	-37	-38	-39	-39	-40	-40
1000	6	3	0	-3	-5	-8	-11	-13	-15	-17	-19	-21	-23	-25	-27	-28	-30	-31	-32	-34	-35	-36	-37	-38	-39	-40	-41	-41	-42	-43	-43	-44	-44
1100	7	3	0	-3	-6	-9	-12	-14	-17	-19	-21	-23	-25	-27	-29	-31	-33	-34	-36	-37	-38	-40	-41										
1200	7	3	0	-3	-6	-10	-13	-15	-18	-21	-23	-25	-28	-30	-32	-34	-35	-37	-39	-40	-42	-43	-44										
1300	8	4	0	-4	-7	-10	-14	-17	-20	-22	-25	-27	-30	-32	-34	-36	-38	-40	-42	-43	-45	-46	-48										
1400	8	4	0	-4	-8	-11	-15	-18	-21	-24	-27	-29	-32	-34	-37	-39	-41	-43	-45	-47	-48	-50	-51										
1500	9	4	0	-4	-8	-12	-16	-19	-22	-26	-29	-31	-34	-37	-39	-42	-44	-46	-48	-50	-52	-53	-55										
1600	9	4	0	-4	-9	-13	-17	-20	-24	-27	-30	-33	-36	-39	-42	-44	-47	-49	-51	-53	-55	-57	-58										
1700	10	5	0	-5	-9	-13	-17	-21	-25	-29	-32	-35	-38	-41	-44	-47	-49	-52	-54	-56	-58	-60	-62										
1800	10	5	0	-5	-10	-14	-18	-23	-27	-30	-34	-37	-41	-44	-47	-49	-52	-55	-57	-59	-61	-63	-65										
1900	11	5	0	-5	-10	-15	-19	-24	-28	-32	-36	-39	-43	-46	-49	-52	-55	-57	-60	-62	-64	-66	-68										
2000	12	6	0	-5	-11	-16	-20	-25	-29	-34	-38	-41	-45	-48	-52	-55	-57	-60	-63	-65	-68	-70	-72										
2100	12	6	0	-6	-11	-16	-21	-26	-31	-35	-39	-43	-47	-51	-54	-57	-60	-63															
2200	13	6	0	-6	-12	-17	-22	-27	-32	-37	-41	-45	-49	-53	-56	-60	-63	-66															
2300	13	6	0	-6	-12	-18	-23	-28	-33	-38	-43	-47	-51	-55	-59	-62	-65	-68															
2400	14	7	0	-6	-13	-19	-24	-30	-35	-40	-44	-49	-53	-57	-61	-65	-68	-71															
2500	14	7	0	-7	-13	-19	-25	-31	-36	-41	-46	-51	-55	-59	-63	-67	-71	-74															
2600	15	7	0	-7	-14	-20	-26	-32	-37	-43	-48	-53	-57	-62	-66	-70	-73	-77															
2700	15	7	0	-7	-14	-21	-27	-33	-39	-44	-50	-55	-59	-64	-68	-72	-76	-79															
2800	16	8	0	-7	-14	-21	-28	-34	-40	-46	-51	-56	-61	-66	-70	-74	-78	-82															
2900	16	8	0	-8	-15	-22	-29	-35	-41	-47	-53	-58	-63	-68	-73	-77	-81	-85															
3000	17	8	0	-8	-15	-23	-30	-36	-43	-49	-55	-60	-65	-70	-75	-79	-83	-87															
3100	17	8	0	-8	-16	-23	-30	-37	-44	-50	-56	-62	-67	-72	-77	-82	-86	-90															
3200	18	9	0	-8	-16	-24	-31	-38	-45	-52	-58	-64	-69	-74	-79	-84	-89	-93															
3300	18	9	0	-9	-17	-25	-32	-40	-47	-53	-59	-65	-71	-76	-81	-86	-91	-95															
3400	19	9	0	-9	-17	-25	-33	-41	-48	-54	-61	-67	-73	-78	-84	-89	-93	-98															
3500	19	9	0	-9	-18	-26	-34	-42	-49	-56	-63	-69	-75	-81	-86	-91	-96	-100															
3600	20	10	0	-9	-18	-27	-35	-43	-50	-57	-64	-71	-77	-83	-88	-93	-98	-103															
3700	20	10	0	-10	-19	-27	-36	-44	-51	-59	-66	-72	-79	-85	-90	-96	-101	-106															
3800	21	10	0	-10	-19	-28	-37	-45	-53	-60	-67	-74	-80	-86	-92	-98	-103	-108															
3900	21	10	0	-10	-20	-29	-37	-46	-54	-61	-69	-76	-82	-89	-95	-100	-106	-111															
4000	22	10	0	-10	-20	-29	-38	-47	-55	-63	-70	-77	-84	-91	-97	-102	-108	-113															
4100	22	11	0	-10	-20	-30	-39	-48																									
4200	23	11	0	-11	-21	-31	-40	-49																									
4300	23	11	0	-11	-21	-31	-41	-50																									
4400	24	11	0	-11	-22	-32	-42	-51																									
4500	24	12	0	-11	-22	-32	-42	-52																									
4600	25	12	0	-12	-23	-33	-43	-53																									
4700	25	12	0	-12	-23	-34	-44	-54																									
4800	25	12	0	-12	-23	-34	-45	-55																									
4900	26	12	0	-12	-24	-35	-46	-56																									
5000	26	13	0	-12	-24	-36	-46	-57																									

Depth (dyn. meters).	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Average temperature (°C.).																																
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1000	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-2	-2	-2	-2	-2	-2	-2	-2	-3	-3	-3	-3	-3
2000	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-2	-2	-2	-2	-2	-3	-3	-3										
3000	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-2	-2	-2	-3	-3	-3															
4000	0	0	0	0	0	-1	-1	-1	-1	-2	-2	-2	-3	-3	-3	-4	-4	-5															
5000	0	0	0	0	0	-1	-1	-1	-2	-2	-2	-3	-3																				

This table is continued on p. 28A.

Example: Given: $\tau = 13.73°$ C.; $D = 2700$; average temperature above this level, 14.2° C.

The main table gives for $\tau = 13.73°$ C.; $D = 2700$ .... −0.00075

The additional table gives for average temperature = 14.2; $D = 2700$ .... −0.00002

Found: $\epsilon_{\tau D}$ .... −0.00077

Table 20 H (continued from p. 26A).—$10^5\varepsilon_{sD}$ (ε_{sD} = *combined salinity-depth correction in ton/m³ to the density of sea-water.*

Salinity (°/oo). 33	34	35	36	37	Depth (dynamic meters).
15	8	0	− 7	−15	5000
15	8	0	− 8	−15	5100
15	8	0	− 8	−15	5200
16	8	0	− 8	−16	5300
16	8	0	− 8	−16	5400
16	8	0	− 8	−16	5500
16	8	0	− 8	−16	5600
17	8	0	− 8	−17	5700
17	8	0	− 8	−17	5800
17	9	0	− 9	−17	5900
17	9	0	− 9	−17	6000
18	9	0	− 9	−18	6100
18	9	0	− 9	−18	6200
18	9	0	− 9	−18	6300
18	9	0	− 9	−18	6400
19	9	0	− 9	−19	6500
19	9	0	− 9	−19	6600
19	10	0	−10	−19	6700
19	10	0	−10	−19	6800
20	10	0	−10	−20	6900
20	10	0	−10	−20	7000
20	10	0	−10	−20	7100
20	10	0	−10	−20	7200
21	10	0	−10	−21	7300
21	10	0	−10	−21	7400
21	11	0	−11	−21	7500
21	11	0	−11	−21	7600
22	11	0	−11	−22	7700
22	11	0	−11	−22	7800
22	11	0	−11	−22	7900
22	11	0	−11	−22	8000
22	11	0	−11	−22	8100
23	11	0	−11	−23	8200
23	11	0	−11	−23	8300
23	12	0	−12	−23	8400
23	12	0	−12	−23	8500
24	12	0	−12	−24	8600
24	12	0	−12	−24	8700
24	12	0	−12	−24	8800
24	12	0	−12	−24	8900
25	12	0	−12	−24	9000
25	12	0	−12	−25	9100
25	12	0	−12	−25	9200
25	13	0	−13	−25	9300
25	13	0	−13	−25	9400
26	13	0	−13	−25	9500
26	13	0	−13	−26	9600
26	13	0	−13	−26	9700
26	13	0	−13	−26	9800
26	13	0	−13	−26	9900

Average salinity (°/oo). 33	34	35	36	37	Depth (dynamic meters).
−3	−2	0	2	3	5000
−4	−2	0	2	4	6000
−5	−2	0	2	5	7000
−5	−2	0	2	5	8000
−6	−3	0	3	6	9000
−6	−3	0	3	6	10000

Table 21 H (continued from p. 27A).—$10^5\varepsilon_{\tau D}$ ($\varepsilon_{\tau D}$ = *combined temperature-depth correction in ton/m³ to the density of sea-water*).

Depth (dynamic meters).	Temperature (° C.). −2	−1	0	1	2
5000	26	13	0	−12	−24
5100	27	13	0	−13	−25
5200	27	13	0	−13	−25
5300	28	13	0	−13	−25
5400	28	13	0	−13	−26
5500	29	14	0	−14	−26
5600	29	14	0	−14	−27
5700	29	14	0	−14	−27
5800	30	14	0	−14	−27
5900	30	14	0	−14	−28
6000	31	15	0	−15	−28
6100	31	15	0	−15	−28
6200	31	15	0	−15	−29
6300	32	15	0	−15	−29
6400	32	16	0	−15	−30
6500	33	16	0	−16	−30
6600	33	16	0	−16	−30
6700	33	16	0	−16	−31
6800	34	16	0	−16	−31
6900	34	16	0	−16	−31
7000	34	17	0	−17	−32
7100	35	17	0	−17	−32
7200	35	17	0	−17	−32
7300	36	17	0	−17	−33
7400	36	17	0	−17	−33
7500	36	17	0	−17	−33
7600	37	18	0	−18	−34
7700	37	18	0	−18	−34
7800	37	18	0	−18	−34
7900	38	18	0	−18	−35
8000	38	18	0	−18	−35
8100	38	18	0	−18	−35
8200	39	19	0	−18	−36
8300	39	19	0	−19	−36
8400	40	19	0	−19	−36
8500	40	19	0	−19	−37
8600	40	19	0	−19	−37
8700	41	19	0	−19	−37
8800	41	20	0	−19	−37
8900	41	20	0	−20	−38
9000	42	20	0	−20	−38
9100	42	20	0	−20	−38
9200	42	20	0	−20	−39
9300	43	20	0	−20	−39
9400	43	21	0	−20	−39
9500	43	21	0	−21	−40
9600	43	21	0	−21	−40
9700	44	21	0	−21	−40
9800	44	21	0	−21	−40
9900	44	21	0	−21	−41

Depth (dynamic meters).	Average temperature (° C.). −2	−1	0	1	2	3	4	5
5000	0	0	0	0	0	−1	−1	−1
6000	0	0	0	0	−1	−1	−1	−1
7000	0	0	0	0	−1	−1	−1	−1
8000	1	0	0	0	−1	−1	−1	−1
9000	1	0	0	0	−1	−1	−1	−2
10000	1	0	0	0	−1	−1	−1	−2

Example to table 20 H: Given: $s = 35.63\ ^0/_{00}$, $D = 7100$, average salinity above this level $= 36.2\ ^0/_{00}$.

The main table 20 H gives for $s = 35.63$, $D = 7100$ .... — 0.00006
The additional table 20 H gives for average salinity $= 36.2$, $D = 7100$ .... + 0.00002
Found ε_{sD} .... — 0.00004

Example to table 21 H: Given: $\tau = 1.26°$ C, $D = 7100$, average temperature above this level $= 3.1°$ C.

The main table 21 H gives for $\tau = 1.26$, $D = 7100$ .... — 0.00021
The additional table 21 H gives for average temperature $= 3.1$, $D = 7100$ .... — 0.00001
Found $\varepsilon_{\tau D}$ .... — 0.00022

Table 22 H.—$10^5 \varepsilon_{s\tau D}$ (*$\varepsilon_{s\tau D}$ = combined salinity-temperature-depth correction in ton/m³ to the density of sea-water*).

Salinity (‰).	Depth (dynamic meters).	Temperature (° C.).														
		-2	-1	0	1	2	3	4	5	6	7	8	9	10	15	20
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	2	1	0	-1	-2	-3	-4	-4	-5	-6	-7	-7	-8	-11	-14
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	2	1	0	-1	-1	-2	-3	-4	-4	-5	-6	-6	-7	-9	-11
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	1	1	0	-1	-1	-2	-2	-3	-4	-4	-5	-5	-6	-8	-9
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	1	0	0	0	-1	-1	-2	-2	-3	-3	-3	-4	-4	-6	-7
	2000	2	1	0	-1	-2	-3	-4	-4	-5	-6	-7	-7	-8	-11	-14
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	1	0	0	0	-1	-1	-1	-2	-2	-2	-3	-3	-3	-4	-5
	2000	2	1	0	-1	-1	-2	-3	-3	-4	-4	-5	-5	-6	-8	-10
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	1	0	0	0	0	-1	-1	-1	-1	-1	-2	-2	-2	-3	-3
	2000	1	1	0	-1	-1	-1	-2	-2	-3	-3	-3	-4	-4	-5	-7
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-2
	2000	1	0	0	0	0	-1	-1	-1	-1	-1	-2	-2	-2	-3	-3
	3000	1	0	0	0	-1	-1	-1	-1	-2	-2	-2	-3	-3	-4	
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1
	2000	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-2	-2	-3
	3000	1	0	0	0	-1	-1	-1	-1	-1	-1	-2	-2	-2	-3	
	4000	1	0	0	0	-1	-1	-1	-2	-2	-2	-2	-3	-3		
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1
	2000	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-2	-2
	3000	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-2	-2	
	4000	1	0	0	0	0	-1	-1	-1	-1	-1	-2	-2	-2		
	5000	1	0	0	0	-1	-1	-1	-1							
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1
	2000	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1
	3000	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	
	4000	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1		
	5000	0	0	0	0	0	-1	-1	-1							
	6000	0	0	0	0	0										
	7000	0	0	0	0	-1										
	8000	1	0	0	0	-1										
	9000	1	0	0	0	-1										
	10000	1	0	0	0	-1										
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1
	2000	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1
	3000	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	
	4000	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1		
	5000	0	0	0	0	0	0	0	0							
	6000	0	0	0	0	0										
	7000	0	0	0	0	0										
	8000	0	0	0	0	0										
	9000	0	0	0	0	0										
	10000	0	0	0	0	0										
35		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2000	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
	3000	0	0	0	0	0	0	0	0	0	0	1	1	1	1	
	4000	0	0	0	0	0	0	0	0	1	1	1	1	1		
	5000	0	0	0	0	0	0	0	1							
	6000	0	0	0	0	0										
	7000	0	0	0	0	0										
	8000	0	0	0	0	0										
	9000	0	0	0	0	0										
	10000	0	0	0	0	0										
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
	2000	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
	3000	0	0	0	0	0	0	0	1	1	1	1	1	1	1	
	4000	0	0	0	0	0	0	1	1	1	1	1	1	1	2	
	5000	0	0	0	0	0	1	1	1							
	6000	0	0	0	0	0										
	7000	-1	0	0	0	1										
	8000	-1	0	0	0	1										
	9000	-1	0	0	0	1										
	10000	-1	0	0	0	1										
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
	2000	0	0	0	0	0	0	0	1	1	1	1	1	1	2	2
	3000	0	0	0	0	0	1	1	1	1	1	1	1	2	2	
	4000	-1	0	0	0	0	1	1	1	1	1	2	2	2	3	
	5000	-1	0	0	0	1	1	1	2							
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
	2000	0	0	0	0	0	1	1	1	1	1	1	1	2	2	3
	3000	-1	0	0	0	1	1	1	1	1	1	2	2	2	3	
	4000	-1	0	0	0	1	1	1	2	2	2	2	2	3	4	
	5000	-1	0	0	0	1	1	2	2							
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1000	0	0	0	0	0	0	0	0	1	1	1	1	1	1	2
	2000	0	0	0	0	0	1	1	1	1	1	1	2	2	3	3
	3000	-1	0	0	0	1	1	1	2	2	2	2	3	3	4	
	4000	-1	0	0	0	1	1	2	2	2	3	3	3	4	5	
	5000	-1	-1	0	1	1	2	2	2							

Example:
Given: $s = 38.74$ ‰; $\tau = 13.73°$ C.; $D = 2700$. Found: $\varepsilon_{s\tau D} = 0.00002$.

Example to tables 16 H to 22 H.

Table 16 H gives for depth 4642 dynamic meters 1.04997
Table 17 H gives for salinity 35.17 ‰ .... +14
Table 18 H gives for temperature 1.36° C. .... − 9
Table 19 H gives for salinity, 35.17 ‰; temperature, 1.36° C. 0
Table 20 H gives for salinity, 35.17 ‰; average salinity 34.85 ‰; depth 4642 dynamic meters .... − 1
Table 21 H gives for temperature 1.36° C.; average temperature 2.7° C.; depth 4642 dynamic meters .... −16
Table 22 H gives for salinity 35.17 ‰; temperature 1.36° C.; depth 4642 dynamic meters 0

Density of sea-water (salinity 35.17 ‰; temperature 1.36° C.; depth 4642 dynamic meters; average salinity and average temperature above this level being 34.85 ‰ and 2.7° C., respectively) 1.04985

Table 23 H.—*Inversion table to pass from densities to specific volumes, and vice versa.*

Density (ton/m³).	0	1	2	3	4	5	6	7	8	9
1.000	1.00000	0.99990	0.99980	0.99970	0.99960	0.99950	.099940	0.99930	0.99920	0.99910
1.001	.99900	.99890	.99880	.99870	.99860	.99850	.99840	.99830	.99820	.99810
1.002	.99800	.99790	.99780	.99771	.99761	.99751	.99741	.99731	.99721	.99711
1.003	.99701	.99691	.99681	.99671	.99661	.99651	.99641	.99631	.99621	.99612
1.004	.99602	.99592	.99582	.99572	.99562	.99552	.99542	.99532	.99522	.99512
1.005	.99502	.99493	.99483	.99473	.99463	.99453	.99443	.99433	.99423	.99413
1.006	.99404	.99394	.99384	.99374	.99364	.99354	.99344	.99334	.99325	.99315
1.007	.99305	.99295	.99285	.99275	.99265	.99256	.99246	.99236	.99226	.99216
1.008	.99206	.99196	.99187	.99177	.99167	.99157	.99147	.99137	.99128	.99118
1.009	.99108	.99098	.99088	.99079	.99069	.99059	.99049	.99039	.99030	.99020
1.010	.99010	.99000	.98990	.98981	.98971	.98961	.98951	.98941	.98932	.98922
1.011	.98912	.98902	.98892	.98883	.98873	.98863	.98853	.98844	.98834	.98824
1.012	.98814	.98804	.98795	.98785	.98775	.98765	.98756	.98746	.98736	.98726
1.013	.98717	.98707	.98697	.98687	.98678	.98668	.98658	.98649	.98639	.98629
1.014	.98619	.98610	.98600	.98590	.98580	.98571	.98561	.98551	.98542	.98532
1.015	.98522	.98512	.98503	.98493	.98483	.98474	.98464	.98454	.98445	.98435
1.016	.98425	.98416	.98406	.98396	.98386	.98377	.98367	.98357	.98348	.98338
1.017	.98328	.98319	.98309	.98299	.98290	.98280	.98270	.98261	.98251	.98241
1.018	.98232	.98222	.98213	.98203	.98193	.98184	.98174	.98164	.98155	.98145
1.019	.98135	.98126	.98116	.98107	.98097	.98087	.98078	.98068	.98058	.98049
1.020	.98039	.98030	.98020	.98010	.98001	.97991	.97982	.97972	.97962	.97953
1.021	.97943	.97934	.97924	.97914	.97905	.97895	.97886	.97876	.97867	.97857
1.022	.97847	.97838	.97828	.97819	.97809	.97800	.97790	.97780	.97771	.97761
1.023	.97752	.97742	.97733	.97723	.97714	.97704	.97694	.97685	.97675	.97666
1.024	.97656	.97647	.97637	.97628	.97618	.97609	.97599	.97590	.97580	.97570
1.025	.97561	.97551	.97542	.97532	.97523	.97513	.97504	.97494	.97485	.97475
1.026	.97466	.97456	.97447	.97437	.97428	.97418	.97409	.97399	.97390	.97380
1.027	.97371	.97362	.97352	.97343	.97333	.97324	.97314	.97305	.97295	.97286
1.028	.97276	.97267	.97257	.97248	.97238	.97229	.97220	.97210	.97200	.97191
1.029	.97182	.97172	.97163	.97153	.97144	.97135	.97125	.97116	.97106	.97096
1.030	.97087	.97078	.97069	.97059	.97050	.97040	.97031	.97021	.97012	.97003
1.031	.96993	.96984	.96974	.96965	.96956	.96946	.96937	.96927	.96918	.96909
1.032	.96899	.96890	.96880	.96871	.96862	.96852	.96843	.96834	.96824	.96815
1.033	.96805	.96796	.96787	.96777	.96768	.96759	.96749	.96740	.96731	.96721
1.034	.96712	.96702	.96693	.96684	.96674	.96665	.96656	.96646	.96637	.96628
1.035	.96618	.96609	.96600	.96590	.96581	.96572	.96562	.96553	.96544	.96534
1.036	.96525	.96516	.96506	.96497	.96488	.96479	.96469	.96460	.96451	.96441
1.037	.96432	.96423	.96413	.96404	.96395	.96386	.96376	.96367	.96358	.96348
1.038	.96339	.96330	.96321	.96311	.96302	.96293	.96283	.96274	.96265	.96256
1.039	.96246	.96237	.96228	.96219	.96209	.96200	.96191	.96182	.96172	.96163
1.040	.96154	.96145	.96135	.96126	.96117	.96108	.96098	.96089	.96080	.96071
1.041	.96061	.96052	.96043	.96034	.96025	.96015	.96006	.95997	.95988	.95979
1.042	.95969	.95960	.95951	.95942	.95932	.95923	.95914	.95905	.95896	.95886
1.043	.95877	.95868	.95859	.95850	.95841	.95831	.95822	.95813	.95804	.95795
1.044	.95785	.95776	.95767	.95758	.95749	.95740	.95730	.95721	.95712	.95703
1.045	.95694	.95685	.95675	.95666	.95657	.95648	.95639	.95630	.95621	.95611
1.046	.95602	.95593	.95584	.95575	.95566	.95557	.95548	.95538	.95529	.95520
1.047	.95511	.95502	.95493	.95484	.95475	.95465	.95456	.95447	.95438	.95429
1.048	.95420	.95411	.95402	.95393	.95383	.95374	.95365	.95356	.95347	.95338
1.049	.95329	.95320	.95311	.95302	.95293	.95283	.95274	.95265	.95256	.95247

Table 23 H (continued).—*Inversion table to pass from density to specific volumes, and vice versa.*

Density (ton/m³).	0	1	2	3	4	5	6	7	8	9
1.050	0.95238	0.95229	0.95220	0.95211	0.95202	0.95193	0.95184	0.95175	0.95166	0.95157
1.051	.95147	.95138	.95129	.95120	.95111	.95102	.95093	.95084	.95075	.95066
1.052	.95057	.95048	.95039	.95030	.95021	.95012	.95003	.94994	.94985	.94976
1.053	.94967	.94958	.94949	.94940	.94931	.94922	.94913	.94904	.94895	.94886
1.054	.94877	.94868	.94859	.94850	.94841	.94832	.94823	.94814	.94805	.94796
1.055	.94787	.94778	.94769	.94760	.94751	.94742	.94733	.94724	.94715	.94706
1.056	.94697	.94688	.94679	.94670	.94661	.94652	.94643	.94634	.94625	.94616
1.057	.94607	.94598	.94589	.94581	.94572	.94563	.94554	.94545	.94536	.94527
1.058	.94518	.94509	.94500	.94491	.94482	.94473	.94464	.94455	.94447	.94438
1.059	.94429	.94420	.94411	.94402	.94393	.94384	.94375	.94366	.94357	.94349
1.060	.94340	.94331	.94322	.94313	.94304	.94295	.94286	.94277	.94268	.94260
1.061	.94251	.94242	.94233	.94224	.94215	.94206	.94197	.94189	.94180	.94171
1.062	.94162	.94153	.94144	.94135	.94127	.94118	.94109	.94100	.94091	.94082
1.063	.94073	.94065	.94056	.94047	.94038	.94029	.94020	.94011	.94003	.93994
1.064	.93985	.93976	.93967	.93958	.93950	.93941	.93932	.93923	.93914	.93906
1.065	.93897	.93888	.93879	.93870	.93861	.93853	.93844	.93835	.93826	.93817
1.066	.93809	.93800	.93791	.93782	.93773	.93765	.93756	.93747	.93738	.93729
1.067	.93721	.93712	.93703	.93694	.93686	.93677	.93668	.93659	.93650	.93642
1.068	.93633	.93624	.93615	.93607	.93598	.93589	.93580	.93572	.93563	.93554
1.069	.93545	.93537	.93528	.93519	.93510	.93502	.93493	.93484	.93475	.93467
1.070	.93458	.93449	.93440	.93432	.93423	.93414	.93406	.93397	.93388	.93379
1.071	.93371	.93362	.93353	.93345	.93336	.93327	.93318	.93310	.93301	.93292
1.072	.93284	.93275	.93266	.93257	.93249	.93240	.93231	.93223	.93214	.93205
1.073	.93197	.93188	.93179	.93171	.93162	.93153	.93145	.93136	.93127	.93119
1.074	.93110	.93101	.93093	.93084	.93075	.93067	.93058	.93049	.93041	.93032
1.075	.93023	.93015	.93006	.92997	.92989	.92980	.92971	.92963	.92954	.92945
1.076	.92937	.92928	.92920	.92911	.92902	.92894	.92885	.92876	.92868	.92859
1.077	.92851	.92842	.92833	.92825	.92816	.92807	.92799	.92790	.92782	.92773
1.078	.92764	.92756	.92747	.92739	.92730	.92721	.92713	.92704	.92696	.92687
1.079	.92678	.92670	.92661	.92653	.92644	.92635	.92627	.92618	.92610	.92601
1.080	.92593	.92584	.92575	.92567	.92558	.92550	.92541	.92533	.92524	.92515
1.081	.92507	.92498	.92490	.92481	.92473	.92464	.92456	.92447	.92439	.92430

Example 1: Given density....................1.02742
The table gives specific volume......0.97331

Example 2: Given specific volume......0.96857
The table gives the density...........1.03245

Special Application. — Change of charts of mutual topography of isobaric surfaces (*i. e.*, charts of specific volume in isobaric sheets) into charts of pressure differences between corresponding level surfaces (*i. e.*, density in corresponding level sheets).

Example 1: Given the chart of mutual topography of the isobaric surfaces of 5 and 6 d-bars sea-pressure. Required the chart of pressure differences between the level surfaces of 5 and 6 dynamic meters of depth. The table shows the curve of pressure difference 1.0120 d-bars to coincide with the 0.98814 dynamic meters curve, the curve of pressure difference 1.0121 d-bars to coincide with the 0.98804 dynamic meters curve, etc.

Example 2: Given the chart of mutual topography of the isobaric surfaces of 50 and 60 d-bars sea-pressure. Required the chart of pressure differences between the level surfaces of 50 and 60 dynamic meters depth. The decimal in the table is displaced one step to the right. The table, then, shows the curve of pressure difference 10.120 d-bars to coincide with the 9.8814 dynamic meters curve, the curve of pressure difference 10.121 d-bars to coincide with the 9.8804 dynamic meters curve, etc.

Example 3: Given the chart of mutual topography of the isobaric surfaces of 500 and 600 d-bars sea-pressure. Required the chart of pressure differences between the level surfaces of 500 and 600 dynamic meters depth. The decimal in the table is displaced two steps to the right. The table then shows the curve of pressure difference 101.20 d-bars to coincide with the 98.814 dynamic meters curve, the curve of pressure difference 101.21 d-bars to coincide with the 98.804 dynamic meters curve, etc.

Displacing the decimal three steps the table may be used in the same way for the case of isobaric sheets of 1000 d-bars and the corresponding level sheets of 1000 dynamic meters.

Table 24 H.—*Change of topographic charts of isobaric surfaces into isobaric charts in level surfaces.*

Pressure at standard dynamic depths (d-bars).	0	1	2	3	4	5	6	7	8	9
I A. Depth of 10 d-bars Pressure (Dynamic Meters).										
10.0	10.000	9.990	9.980	9.970	9.960	9.950	9.940	9.931	9.921	9.911
10.1	9.901	9.891	9.881	9.872	9.862	9.852	9.843	9.833	9.823	9.814
10.2	9.804	9.794	9.785	9.775	9.766	9.756	9.747	9.737	9.728	9.718
II A. Depth of 20 d-bars Pressure (Dynamic Meters).										
20.0	20.000	19.990	19.980	19.970	19.960	19.950	19.940	19.930	19.920	19.910
20.1	19.901	19.891	19.881	19.871	19.861	19.851	19.841	19.832	19.822	19.812
20.2	19.802	19.792	19.782	19.773	19.763	19.753	19.743	19.734	19.724	19.714
20.3	19.704	19.695	19.685	19.675	19.666	19.656	19.646	19.637	19.627	19.617
20.4	19.608	19.598	19.589	19.579	19.570	19.560	19.550	19.541	19.531	19.522
20.5	19.512	19.503	19.493	19.484	19.474	19.465	19.455	19.446	19.436	19.427
III A. Depth of 30 d-bars Pressure (Dynamic Meters).										
30.0	30.000	29.990	29.980	29.970	29.960	29.950	29.940	29.930	29.920	29.910
30.1	29.900	29.890	29.880	29.870	29.861	29.851	29.841	29.831	29.821	29.811
30.2	29.801	29.791	29.782	29.772	29.762	29.752	29.742	29.732	29.723	29.713
30.3	29.703	29.693	29.683	29.674	29.664	29.654	29.644	29.634	29.625	29.615
30.4	29.605	29.595	29.586	29.576	29.566	29.557	29.547	29.537	29.527	29.518
30.5	29.508	29.498	29.489	29.479	29.470	29.460	29.450	29.441	29.431	29.422
30.6	29.412	29.402	29.393	29.383	29.374	29.364	29.354	29.345	29.335	29.326
30.7	29.316	29.307	29.297	29.288	29.278	29.268	29.259	29.249	29.240	29.230
30.8	29.221	29.211	29.202	29.192	29.183	29.174	29.164	29.155	29.145	29.136
IV A. Depth of 40 d-bars Pressure (Dynamic Meters).										
40.0	40.000	39.990	39.980	39.970	39.960	39.950	39.940	39.930	39.920	39.910
40.1	39.900	39.890	39.880	39.870	39.860	39.851	39.841	39.831	39.821	39.811
40.2	39.801	39.791	39.781	39.771	39.761	39.752	39.742	39.732	39.722	39.712
40.3	39.702	39.692	39.682	39.673	39.663	39.653	39.643	39.633	39.624	39.614
40.4	39.604	39.594	39.584	39.575	39.565	39.555	39.545	39.535	39.526	39.516
40.5	39.506	39.496	39.487	39.477	39.467	39.458	39.448	39.438	39.428	39.419
40.6	39.409	39.399	39.390	39.380	39.370	39.360	39.351	39.341	39.331	39.322
40.7	39.312	39.302	39.293	39.283	39.274	39.264	39.254	39.245	39.235	39.225
40.8	39.216	39.206	39.197	39.187	39.177	39.168	39.158	39.149	39.139	39.129
40.9	39.120	39.110	39.101	39.091	39.082	39.072	39.063	39.053	39.044	39.034
41.0	39.025	39.015	39.005	38.996	38.986	38.977	38.967	38.958	38.948	38.939
41.1	38.929	38.920	38.911	38.901	38.892	38.882	38.873	38.863	38.854	38.845

Table 24 H (continued).—*Change of topographic charts of isobaric surfaces into isobaric charts in level surfaces.*

Pressure at standard dynamic depths (d-bars).	0	1	2	3	4	5	6	7	8	9
VA. DEPTH OF 50 D-BARS PRESSURE (DYNAMIC METERS).										
50.0	50.000	49.990	49.980	49.970	49.960	49.950	49.940	49.930	49.920	49.910
50.1	49.900	49.890	49.880	49.870	49.860	49.851	49.841	49.831	49.821	49.811
50.2	49.801	49.791	49.781	49.771	49.761	49.751	49.741	49.732	49.722	49.712
50.3	49.702	49.692	49.682	49.672	49.662	49.653	49.643	49.633	49.623	49.613
50.4	49.603	49.593	49.584	49.574	49.564	49.554	49.544	49.534	49.525	49.515
50.5	49.505	49.495	49.485	49.476	49.466	49.456	49.446	49.436	49.427	49.417
50.6	49.407	49.397	49.388	49.378	49.368	49.359	49.349	49.339	49.329	49.320
50.7	49.310	49.300	49.290	49.281	49.271	49.261	49.252	49.242	49.232	49.222
50.8	49.213	49.203	49.193	49.184	49.174	49.164	49.155	49.145	49.135	49.126
50.9	49.116	49.106	49.097	49.087	49.077	49.068	49.058	49.049	49.039	49.029
51.0	49.020	49.010	49.001	48.991	48.981	48.972	48.962	48.953	48.943	48.933
51.1	48.924	48.914	48.905	48.895	48.886	48.876	48.866	48.857	48.847	48.838
51.2	48.828	48.819	48.809	48.800	48.790	48.781	48.771	48.762	48.752	48.743
51.3	48.733	48.724	48.714	48.705	48.695	48.686	48.676	48.667	48.657	48.648
51.4	48.638	48.629	48.619	48.610	48.600	48.591	48.582	48.572	48.563	48.553
VIA. DEPTH OF 60 D-BARS PRESSURE (DYNAMIC METERS).										
60.0	60.000	59.990	59.980	59.970	59.960	59.950	59.940	59.930	59.920	59.910
60.1	59.900	59.890	59.880	59.870	59.860	59.850	59.841	59.831	59.821	59.811
60.2	59.801	59.791	59.781	59.771	59.761	59.751	59.741	59.731	59.721	59.711
60.3	59.702	59.692	59.682	59.672	59.662	59.652	59.642	59.632	59.622	59.613
60.4	59.603	59.593	59.583	59.573	59.563	59.553	59.544	59.534	59.524	59.514
60.5	59.504	59.494	59.484	59.475	59.465	59.455	59.445	59.435	59.426	59.416
60.6	59.406	59.396	59.386	59.377	59.367	59.357	59.347	59.337	59.328	59.318
60.7	59.308	59.298	59.289	59.279	59.269	59.259	59.250	59.240	59.230	59.220
60.8	59.211	59.201	59.191	59.181	59.172	59.162	59.152	59.142	59.133	59.123
60.9	59.113	59.104	59.094	59.084	59.075	59.065	59.055	59.045	59.036	59.026
61.0	59.016	59.007	58.997	58.987	58.978	58.968	58.958	58.949	58.939	58.930
61.1	58.920	58.910	58.901	58.891	58.881	58.872	58.862	58.853	58.843	58.833
61.2	58.824	58.814	58.804	58.795	58.785	58.776	58.766	58.757	58.747	58.737
61.3	58.728	58.718	58.709	58.699	58.690	58.680	58.670	58.661	58.651	58.642
61.4	58.632	58.623	58.613	58.604	58.594	58.584	58.575	58.565	58.556	58.546
61.5	58.537	58.527	58.518	58.508	58.499	58.489	58.480	58.470	58.461	58.451
61.6	58.442	58.432	58.423	58.413	58.404	58.394	58.385	58.375	58.366	58.357
61.7	58.347	58.338	58.328	58.319	58.309	58.300	58.290	58.281	58.271	58.262

Example: Given the topographical chart of the 40 d-bars surface. Required the isobaric chart in 40 dynamic meters depth.

Table IV A shows the curve of 40.60 d-bars to be identical with the 39.409 dynamic meters curve, the curve of 40.61 d-bars to be identical with the 39.399 dynamic meters curve, and so on.

Table 24 H (continued).—*Change of topographic charts of isobaric surfaces into isobaric charts in level surfaces.*

Pressure at standard dynamic depths (d-bars).	0	1	2	3	4	5	6	7	8	9
I B. DEPTH OF 100 D-BARS PRESSURE (DYNAMIC METERS).										
100	100.000	99.900	99.800	99.701	99.602	99.503	99.404	99.305	99.207	99.108
101	99.010	98.912	98.815	98.717	98.620	98.523	98.425	98.329	98.232	98.136
102	98.040	97.944	97.848	97.752	97.657	97.562	97.466	97.372	97.277	97.184
103	97.088	96.994	96.900	96.806	96.713	96.619	96.526	96.433	96.340	96.247
102.7	97.372	97.362	97.353	97.343	97.334	97.324	97.315	97.305	97.296	97.286
102.8	97.277	97.267	97.258	97.249	97.239	97.230	97.220	97.211	97.201	97.192
102.9	97.182	97.173	97.163	97.154	97.145	97.135	97.126	97.116	97.107	97.097
103.0	97.088	97.079	97.069	97.060	97.050	97.041	97.032	97.022	97.013	97.003
II B. DEPTH OF 200 D-BARS PRESSURE (DYNAMIC METERS).										
200	200.000	199.900	199.800	199.701	199.601	199.501	199.402	199.303	199.204	199.105
201	199.005	198.907	198.808	198.709	198.610	198.512	198.414	198.315	198.217	198.119
202	198.021	197.923	197.825	197.727	197.630	197.532	197.435	197.337	197.240	197.143
203	197.046	196.949	196.852	196.755	196.658	196.562	196.465	196.369	196.273	196.176
204	196.080	195.984	195.888	195.793	195.697	195.601	195.506	195.410	195.315	195.220
205	195.124	195.029	194.934	194.839	194.745	194.650	194.555	194.461	194.366	194.272
206	194.178	194.083	193.989	193.895	193.802	193.708	193.614	193.520	193.427	193.333
205.5	194.650	194.640	194.631	194.621	194.612	194.602	194.593	194.583	194.574	194.565
205.6	194.555	194.546	194.536	194.527	194.517	194.508	194.498	194.489	194.479	194.470
205.7	194.461	194.451	194.442	194.432	194.423	194.413	194.404	194.395	194.385	194.376
205.8	194.366	194.357	194.347	194.338	194.328	194.319	194.310	194.300	194.291	194.281
205.9	194.272	194.262	194.253	194.244	194.234	194.225	194.215	194.206	194.196	194.187
III B. DEPTH OF 300 D-BARS PRESSURE (DYNAMIC METERS).										
300	300.000	299.900	299.800	299.701	299.601	299.501	299.402	299.302	299.203	299.103
301	299.004	298.905	298.806	298.707	298.608	298.509	298.410	298.311	298.212	298.113
302	298.015	297.916	297.818	297.719	297.621	297.523	297.424	297.326	297.228	297.130
303	297.032	296.934	296.836	296.738	296.641	296.543	296.445	296.348	296.250	296.153
304	296.055	295.958	295.861	295.764	295.667	295.570	295.473	295.376	295.279	295.182
305	295.086	294.989	294.892	294.796	294.699	294.603	294.507	294.410	294.314	294.218
306	294.122	294.026	293.930	293.834	293.738	293.642	293.547	293.451	293.355	293.260
307	293.165	293.069	292.974	292.879	292.783	292.688	292.593	292.498	292.403	292.308
308	292.213	292.119	292.024	291.929	291.835	291.740	291.646	291.551	291.457	291.363
309	291.268	291.174	291.080	290.986	290.892	290.798	290.704	290.611	290.517	290.423
308.4	291.835	291.825	291.816	291.806	291.797	291.787	291.778	291.768	291.759	291.750
308.5	291.740	291.731	291.721	291.712	291.702	291.693	291.683	291.674	291.665	291.655
308.6	291.646	291.636	291.627	291.617	291.608	291.598	291.589	291.580	291.570	291.561
308.7	291.551	291.542	291.532	291.523	291.514	291.504	291.495	291.485	291.476	291.466
308.8	291.457	291.447	291.438	291.429	291.419	291.410	291.400	291.391	291.381	291.372
IV B. DEPTH OF 400 D-BARS PRESSURE (DYNAMIC METERS).										
400	400.000	399.900	399.800	399.701	399.601	399.501	399.402	399.302	399.202	399.103
401	399.003	398.904	398.805	398.705	398.606	398.507	398.408	398.309	398.210	398.111
402	398.012	397.913	397.814	397.715	397.617	397.518	397.419	397.321	397.222	397.124
403	397.025	396.927	396.828	396.730	396.632	396.534	396.436	396.337	396.239	396.141
404	396.043	395.945	395.848	395.750	395.652	395.554	395.457	395.359	395.261	395.164
405	395.066	394.969	394.872	394.774	394.677	394.580	394.483	394.385	394.288	394.191
406	394.094	393.997	393.900	393.803	393.707	393.610	393.513	393.417	393.320	393.223
407	393.127	393.030	392.934	392.838	392.741	392.645	392.549	392.453	392.356	392.260
408	392.164	392.068	391.972	391.876	391.781	391.685	391.589	391.493	391.398	391.302
409	391.206	391.111	391.015	390.920	390.825	390.729	390.634	390.539	390.443	390.348
410	390.253	390.158	390.063	389.968	380.873	389.778	389.683	389.589	389.494	389.399
411	389.304	389.210	389.115	389.021	388.926	388.832	388.738	388.643	388.549	388.455
412	388.360	388.266	388.172	388.078	387.984	387.890	387.796	387.702	387.609	387.515
411.4	388.926	388.917	388.907	388.898	388.889	388.879	388.870	388.860	388.851	388.841
411.5	388.832	388.822	388.813	388.804	388.794	388.785	388.775	388.766	388.756	388.747
411.6	388.738	388.728	388.719	388.709	388.700	388.690	388.681	388.671	388.662	388.653
411.7	388.643	388.634	388.624	388.615	388.605	388.596	388.587	388.577	388.568	388.558
411.8	388.549	388.539	388.530	388.521	388.511	388.502	388.492	388.483	388.473	388.464

Table 24 H (continued).—*Change of topographic charts of isobaric surfaces into isobaric charts in level surfaces.*

V B. DEPTH OF 500 D-BARS PRESSURE (DYNAMIC METERS).

Pressure at standard dynamic depths (d-bars).	0	1	2	3	4	5	6	7	8	9
500	500.000	499.900	499.800	499.701	499.601	499.501	499.401	499.302	499.202	499.103
501	499.003	498.904	498.804	498.705	498.606	498.506	498.407	498.308	498.209	498.109
502	498.010	497.911	497.812	497.713	497.614	497.515	497.416	497.318	497.219	497.120
503	497.021	496.923	496.824	496.725	496.627	496.528	496.430	496.331	496.233	496.135
504	496.036	495.938	495.840	495.742	495.643	495.545	495.447	495.349	495.251	495.153
505	495.055	494.957	494.860	494.762	494.664	494.566	494.468	494.371	494.273	494.176
506	494.078	493.980	493.883	493.786	493.688	493.591	493.493	493.396	493.299	493.202
507	493.105	493.008	492.910	492.813	492.716	492.619	492.522	492.426	492.329	492.232
508	492.135	492.038	491.942	491.845	491.748	491.652	491.555	491.459	491.362	491.266
509	491.169	491.073	490.977	490.880	490.784	490.688	490.592	490.496	490.400	490.304
510	490.208	490.112	490.016	489.920	489.824	489.728	489.632	489.536	489.441	489.345
511	489.249	489.154	489.058	488.963	488.867	488.772	488.676	488.581	488.486	488.390
512	488.295	488.200	488.104	488.090	487.914	487.819	487.724	487.629	487.534	487.439
513	487.344	487.249	487.154	487.060	486.965	486.870	486.775	486.681	486.586	486.492
514	486.397	486.303	486.208	486.114	486.019	485.925	485.831	485.736	485.642	485.548
515	485.454	485.360	485.266	485.172	485.077	484.983	484.890	484.796	484.702	484.608
514.5	485.925	485.915	485.906	485.897	485.887	485.878	485.868	485.859	485.849	485.840
514.6	485.831	485.821	485.812	485.802	485.793	485.783	485.774	485.765	485.755	485.746
514.7	485.736	485.727	485.718	485.708	485.699	485.689	485.680	485.670	485.661	485.652
514.8	485.642	485.632	485.623	485.614	485.605	485.595	485.586	485.576	485.567	485.558
514.9	485.548	485.539	485.529	485.520	485.510	485.501	485.492	485.482	485.473	485.463

VI B. DEPTH OF 600 D-BARS PRESSURE (DYNAMIC METERS).

Pressure at standard dynamic depths (d-bars).	0	1	2	3	4	5	6	7	8	9
600	600.000	599.900	599.800	599.701	599.601	599.501	599.401	599.302	599.202	599.103
601	599.003	598.904	598.804	598.705	598.605	598.506	598.406	598.307	598.208	598.109
602	598.009	597.910	597.811	597.712	597.613	597.514	597.415	597.316	597.217	597.118
603	597.019	596.920	596.821	596.723	596.624	596.525	596.426	596.328	596.229	596.131
604	596.032	595.934	595.835	595.737	595.638	595.540	595.441	595.343	595.245	595.146
605	595.048	594.950	594.852	594.754	594.656	594.558	594.460	594.361	594.263	594.166
606	594.068	593.970	593.872	593.774	593.676	593.579	593.481	593.383	593.286	593.188
607	593.090	592.993	592.895	592.798	592.700	592.603	592.505	592.408	592.311	592.214
608	592.116	592.019	591.922	591.825	591.727	591.630	591.533	591.436	591.339	591.242
609	591.145	591.048	590.952	590.855	590.758	590.661	590.564	590.468	590.371	590.274
610	590.178	590.081	589.984	589.888	589.791	589.695	589.598	589.502	589.406	589.309
611	589.213	589.117	589.020	588.924	588.828	588.732	588.636	588.540	588.444	588.348
612	588.252	588.156	588.060	587.964	587.868	587.772	587.676	587.580	587.485	587.389
613	587.293	587.198	587.102	587.006	586.911	586.815	586.720	586.624	586.529	586.433
614	586.338	586.243	586.147	586.052	585.957	585.862	585.767	585.671	585.576	585.481
615	585.386	585.291	585.196	585.101	585.006	584.911	584.816	584.721	584.627	584.532
616	584.437	584.342	584.248	584.152	584.058	583.964	583.869	583.775	583.680	583.586
617	583.491	583.397	583.302	583.208	583.114	583.020	582.925	582.831	582.737	582.643
618	582.548	582.454	582.360	582.266	582.172	582.078	581.984	581.890	581.796	581.702
617.5	583.020	583.010	583.001	582.991	582.982	582.972	582.963	582.953	582.944	582.935
617.6	582.925	582.916	582.906	582.897	582.888	582.878	582.869	582.859	582.850	582.840
617.7	582.831	582.822	582.812	582.803	582.793	582.784	582.774	582.765	582.756	582.746
617.8	582.737	582.727	582.718	582.709	582.699	582.690	582.680	582.671	582.662	582.652
617.9	582.643	582.633	582.624	582.614	582.605	582.596	582.586	582.577	582.567	582.558

Example: Given the topographical chart of the 600 d-bars surface. Required the isobaric chart in 600 dynamic meters depth.

Table VI B shows the curve of 618 d-bars to be identical with the 582.548 dynamic meters curve, the curve of 618.1 d-bars to be identical with the 582.454 dynamic meters curve, and so on.

Table 24 H (continued).—*Change of topographic charts of isobaric surfaces into isobaric charts in level surfaces.*

I c. Depth of 1000 d-bars Pressure (Dynamic Meters).

Pressure at standard dynamic depths (d-bars).	0	1	2	3	4	5	6	7	8	9
1030.4	970.56	970.55	970.55	970.54	970.53	970.52	970.51	970.50	970.49	970.48
1030.5	970.47	970.46	970.45	970.44	970.43	970.42	970.41	970.40	970.40	970.39
1030.6	970.38	970.37	970.36	970.35	970.34	970.33	970.32	970.31	970.30	970.29
1030.7	970.28	970.27	970.26	970.25	970.24	970.24	970.23	970.22	970.21	970.20
1030.8	970.19	970.18	970.17	970.16	970.15	970.14	970.13	970.12	970.11	970.10

II c. Depth of 2000 d-bars Pressure (Dynamic Meters).

Pressure at standard dynamic depths (d-bars).	0	1	2	3	4	5	6	7	8	9
2065.8	1936.58	1936.57	1936.56	1936.55	1936.54	1936.54	1936.53	1936.52	1936.51	1936.50
2065.9	1936.49	1936.48	1936.47	1936.46	1936.45	1936.44	1936.43	1936.42	1936.41	1936.40
2066.0	1936.40	1936.39	1936.38	1936.37	1936.36	1936.35	1936.34	1936.33	1936.32	1936.31
2066.1	1936.30	1936.29	1936.28	1936.27	1936.26	1936.26	1936.25	1936.24	1936.23	1936.22
2066.2	1936.21	1936.20	1936.19	1936.18	1936.17	1936.16	1936.15	1936.14	1936.13	1936.12

III c. Depth of 3000 d-bars Pressure (Dynamic Meters).

Pressure at standard dynamic depths (d-bars).	0	1	2	3	4	5	6	7	8	9
3105.9	2898.39	2898.38	2898.37	2898.36	2898.35	2898.34	2898.33	2898.32	2898.31	2898.30
3106.0	2898.30	2898.29	2898.28	2898.27	2898.26	2898.25	2898.24	2898.23	2898.22	2898.21
3106.1	2898.20	2898.19	2898.18	2898.18	2898.17	2898.16	2898.15	2898.14	2898.13	2898.12
3106.2	2898.11	2898.10	2898.09	2898.08	2898.07	2898.06	2898.05	2898.05	2898.04	2898.03
3106.3	2898.02	2898.01	2898.00	2897.99	2897.98	2897.97	2897.96	2897.95	2897.94	2897.93

IV c. Depth of 4000 d-bars Pressure (Dynamic Meters).

Pressure at standard dynamic depths (d-bars).	0	1	2	3	4	5	6	7	8	9
4150.7	3856.18	3856.17	3856.16	3856.15	3856.14	3856.13	3856.12	3856.11	3856.10	3856.10
4150.8	3856.09	3856.08	3856.07	3856.06	3856.05	3856.04	3856.03	3856.02	3856.01	3856.00
4150.9	3855.99	3855.99	3855.98	3855.97	3855.96	3855.95	3855.94	3855.93	3855.92	3855.91
4151.0	3855.90	3855.89	3855.88	3855.87	3855.87	3855.86	3855.85	3855.84	3855.83	3855.82
4151.1	3855.81	3855.80	3855.79	3855.78	3855.77	3855.76	3855.76	3855.75	3855.74	3855.73

V c. Depth of 5000 d-bars Pressure (Dynamic Meters).

Pressure at standard dynamic depths (d-bars).	0	1	2	3	4	5	6	7	8	9
5200.0	4809.75	4809.74	4809.73	4809.72	4809.71	4809.70	4809.69	4809.68	4809.67	4809.66
5200.1	4809.66	4809.65	4809.64	4809.63	4809.62	4809.61	4809.60	4809.59	4809.58	4809.57
5200.2	4809.56	4809.55	4809.55	4809.54	4809.53	4809.52	4809.51	4809.50	4809.49	4809.48
5200.3	4809.47	4809.46	4809.45	4809.44	4809.44	4809.43	4809.42	4809.41	4809.40	4809.39
5200.4	4809.38	4809.37	4809.36	4809.35	4809.34	4809.33	4809.33	4809.32	4809.31	4809.30

VI c. Depth of 6000 d-bars Pressure (Dynamic Meters).

Pressure at standard dynamic depths (d-bars).	0	1	2	3	4	5	6	7	8	9
6253.8	5759.57	5759.56	5759.55	5759.54	5759.53	5759.52	5759.51	5759.51	5759.50	5759.49
6253.9	5759.48	5759.47	5759.46	5759.45	5759.44	5759.43	5759.42	5759.41	5759.41	5759.40
6254.0	5759.39	5759.38	5759.37	5759.36	5759.35	5759.34	5759.33	5759.32	5759.31	5759.31
6254.1	5759.30	5759.29	5759.28	5759.27	5759.26	5759.25	5759.24	5759.23	5759.22	5759.21
6254.2	5759.21	5759.20	5759.19	5759.18	5759.17	5759.16	5759.15	5759.14	5759.13	5759.12

VII c. Depth of 7000 d-bars Pressure (Dynamic Meters).

Pressure at standard dynamic depths (d-bars).	0	1	2	3	4	5	6	7	8	9
7312.0	6705.64	6705.63	6705.62	6705.61	6705.60	6705.59	6705.58	6705.58	6705.57	6705.56
7312.1	6705.55	6705.54	6705.53	6705.52	6705.51	6705.50	6705.49	6705.49	6705.48	6705.47
7312.2	6705.46	6705.45	6705.44	6705.43	6705.42	6705.41	6705.40	6705.39	6705.39	6705.38
7312.3	6705.37	6705.36	6705.35	6705.34	6705.33	6705.32	6705.31	6705.30	6705.30	6705.29
7312.4	6705.28	6705.27	6705.26	6705.25	6705.24	6705.23	6705.22	6705.21	6705.20	6705.20

VIII c. Depth of 8000 d-bars Pressure (Dynamic Meters).

Pressure at standard dynamic depths (d-bars).	0	1	2	3	4	5	6	7	8	9
8374.5	7648.08	7648.07	7648.06	7648.05	7648.04	7648.03	7648.02	7648.02	7648.01	7648.00
8374.6	7647.99	7647.98	7647.97	7647.96	7647.95	7647.94	7647.93	7647.93	7647.92	7647.91
8374.7	7647.90	7647.89	7647.88	7647.87	7647.86	7647.85	7647.84	7647.84	7647.83	7647.82
8374.8	7647.81	7647.80	7647.79	7647.78	7647.77	7647.76	7647.75	7647.75	7647.74	7647.73
8374.9	7647.72	7647.71	7647.70	7647.69	7647.68	7647.67	7647.66	7647.66	7647.65	7647.64

For example see p. 35A.

DYNAMIC METEOROLOGY AND HYDROGRAPHY

BY V. BJERKNES

AND DIFFERENT COLLABORATORS

METEOROLOGICAL TABLES

METEOROLOGICAL TABLES.

Table 1 M.—*Normal decrease of the acceleration of gravity with the height.*

Height (meters).	0	100	200	300	400	500	600	700	800	900
29000	−0.0895	−0.0898	−0.0901	−0.0904	−0.0907	−0.0910	−0.0913	−0.0917	−0.0920	−0.0923
28000	− .0864	− .0867	− .0870	− .0873	− .0876	− .0880	− .0883	− .0886	− .0889	− .0892
27000	− .0833	− .0836	− .0839	− .0842	− .0846	− .0849	− .0852	− .0855	− .0858	− .0861
26000	− .0802	− .0805	− .0809	− .0812	− .0815	− .0818	− .0821	− .0824	− .0827	− .0830
25000	− .0772	− .0775	− .0778	− .0781	− .0784	− .0787	− .0790	− .0793	− .0796	− .0799
24000	− .0741	− .0744	− .0747	− .0750	− .0753	− .0756	− .0759	− .0762	− .0765	− .0768
23000	− .0710	− .0713	− .0716	− .0719	− .0722	− .0725	− .0728	− .0731	− .0734	− .0738
22000	− .0679	− .0682	− .0685	− .0688	− .0691	− .0694	− .0697	− .0701	− .0704	− .0707
21000	− .0648	− .0651	− .0654	− .0657	− .0660	− .0663	− .0667	− .0670	− .0673	− .0676
20000	− .0617	− .0620	− .0623	− .0626	− .0630	− .0633	− .0636	− .0639	− .0642	− .0645
19000	− .0586	− .0589	− .0593	− .0596	− .0599	− .0602	− .0605	− .0608	− .0611	− .0614
18000	− .0555	− .0559	− .0562	− .0565	− .0568	− .0571	− .0574	− .0577	− .0580	− .0583
17000	− .0525	− .0528	− .0531	− .0534	− .0537	− .0540	− .0543	− .0546	− .0549	− .0552
16000	− .0494	− .0497	− .0500	− .0503	− .0506	− .0509	− .0512	− .0515	− .0518	− .0522
15000	− .0463	− .0466	− .0469	− .0472	− .0475	− .0478	− .0481	− .0485	− .0488	− .0491
14000	− .0432	− .0435	− .0438	− .0441	− .0444	− .0447	− .0451	− .0454	− .0457	− .0460
13000	− .0401	− .0404	− .0407	− .0410	− .0414	− .0417	− .0420	− .0423	− .0426	− .0429
12000	− .0370	− .0373	− .0376	− .0380	− .0383	− .0386	− .0389	− .0392	− .0395	− .0398
11000	− .0339	− .0343	− .0346	− .0349	− .0352	− .0355	− .0358	− .0361	− .0364	− .0367
10000	− .0309	− .0312	− .0315	− .0318	− .0321	− .0324	− .0327	− .0330	− .0333	− .0336
9000	− .0278	− .0281	− .0284	− .0287	− .0290	− .0293	− .0296	− .0299	− .0302	− .0306
8000	− .0247	− .0250	− .0253	− .0256	− .0259	− .0262	− .0265	− .0268	− .0272	− .0275
7000	− .0216	− .0219	− .0222	− .0225	− .0228	− .0231	− .0235	− .0238	− .0241	− .0244
6000	− .0185	− .0188	− .0191	− .0194	− .0198	− .0201	− .0204	− .0207	− .0210	− .0213
5000	− .0154	− .0157	− .0160	− .0164	− .0167	− .0170	− .0173	− .0176	− .0179	− .0182
4000	− .0123	− .0127	− .0130	− .0133	− .0136	− .0139	− .0142	− .0145	− .0148	− .0151
3000	− .0093	− .0096	− .0099	− .0102	− .0105	− .0108	− .0111	− .0114	− .0117	− .0120
2000	− .0062	− .0065	− .0068	− .0071	− .0074	− .0077	− .0080	− .0083	− .0086	− .0089
1000	− .0031	− .0034	− .0037	− .0040	− .0043	− .0046	− .0049	− .0052	− .0056	− .0059
0	− .0000	− .0003	− .0006	− .0009	− .0012	− .0015	− .0019	− .0022	− .0025	− .0028
	0	100	200	300	400	500	600	700	800	900

Table 2 M.—*Normal value of the acceleration of gravity at sea-level.*

Latitude (degrees).	0	1	2	3	4	5	6	7	8	9
80	9.8306	9.8309	9.8312	9.8314	9.8316	9.8318	9.8319	9.8320	9.8321	9.8321
70	9.8261	9.8266	9.8272	9.8277	9.8282	9.8287	9.8291	9.8295	9.8299	9.8303
60	9.8191	9.8199	9.8207	9.8214	9.8222	9.8229	9.8235	9.8242	9.8249	9.8255
50	9.8107	9.8116	9.8124	9.8133	9.8142	9.8150	9.8159	9.8167	9.8176	9.8184
40	9.8017	9.8026	9.8035	9.8044	9.8053	9.8062	9.8071	9.8080	9.8089	9.8098
30	9.7932	9.7940	9.7948	9.7956	9.7965	9.7973	9.7982	9.7990	9.7999	9.8008
20	9.7864	9.7869	9.7876	9.7882	9.7889	9.7895	9.7902	9.7910	9.7917	9.7925
10	9.7819	9.7822	9.7825	9.7829	9.7833	9.7838	9.7842	9.7847	9.7852	9.7858
0	9.7803	9.7803	9.7804	9.7805	9.7806	9.7807	9.7809	9.7811	9.7813	9.7816

Example 1: Latitude 42° 27′, table 2 M gives 9.8039
Height 18429 meters, table 1 M gives .... −0.0569
Gravity at latitude 42° 27′, height 18429 meters.. 9.7470

Example 2: Measured gravity at the earth's surface. 9.7881
Height of station 1470 meters, table 1 M gives (sign reversed).. +0.0045
Gravity reduced to sea level 9.7926

Table 3 M.—*Heights reduced from meters to dynamic meters, the acceleration of gravity at sea-level being 9.80.*

Height (meters).	0	100	200	300	400	500	600	700	800	900
29000	28290	28387	28484	28582	28679	28776	28873	28970	29067	29164
28000	27319	27416	27513	27610	27708	27805	27902	27999	28096	28193
27000	26347	26445	26542	26639	26736	26833	26930	27028	27125	27222
26000	25376	25473	25570	25667	25764	25862	25959	26056	26153	26250
25000	24404	24501	24598	24695	24792	24890	24987	25084	25181	25279
24000	23431	23528	23626	23723	23820	23917	24015	24112	24209	24306
23000	22458	22556	22653	22750	22847	22945	23042	23139	23237	23334
22000	21485	21583	21680	21777	21875	21972	22069	22166	22264	22361
21000	20512	20609	20707	20804	20901	20999	21096	21193	21291	21388
20000	19538	19636	19733	19830	19928	20025	20122	20220	20317	20415
19000	18564	18662	18759	18856	18954	19051	19149	19246	19344	19441
18000	17590	17687	17785	17882	17980	18077	18175	18272	18369	18467
17000	16615	16713	16810	16908	17005	17103	17200	17298	17395	17493
16000	15640	15738	15835	15933	16030	16128	16225	16323	16420	16518
15000	14665	14763	14860	14958	15055	15153	15250	15348	15446	15543
14000	13690	13787	13885	13982	14080	14178	14275	14373	14470	14568
13000	12714	12811	12909	13007	13104	13202	13299	13397	13495	13592
12000	11738	11835	11933	12031	12128	12226	12323	12421	12519	12616
11000	10761	10859	10957	11054	11152	11250	11347	11445	11543	11640
10000	9785	9882	9980	10078	10175	10273	10371	10468	10566	10664
9000	8807	8905	9003	9101	9198	9296	9394	9492	9589	9687
8000	7830	7928	8026	8123	8221	8319	8417	8514	8612	8710
7000	6852	6950	7048	7146	7244	7341	7439	7537	7635	7732
6000	5874	5972	6070	6168	6266	6363	6461	6559	6657	6755
5000	4896	4994	5092	5190	5287	5385	5483	5581	5679	5777
4000	3918	4015	4113	4211	4309	4407	4505	4603	4700	4798
3000	2939	3037	3134	3232	3330	3428	3526	3624	3722	3820
2000	1959	2057	2155	2253	2351	2449	2547	2645	2743	2841
1000	980	1078	1176	1274	1372	1470	1568	1666	1763	1861
0	0	98	196	294	392	490	588	686	784	882
	0	100	200	300	400	500	600	700	800	900

PROPORTIONALITY TABLE.

Meters.	0	1	2	3	4	5	6	7	8	9
90	88	89	90	91	92	93	94	95	96	97
80	78	79	80	81	82	83	84	85	86	87
70	69	70	71	72	73	74	74	75	76	77
60	59	60	61	62	63	64	65	66	67	68
50	49	50	51	52	53	54	55	56	57	58
40	39	40	41	42	43	44	45	46	47	48
30	29	30	31	32	33	34	35	36	37	38
20	20	21	22	23	24	24	25	26	27	28
10	10	11	12	13	14	15	16	17	18	19
0	0	1	2	3	4	5	6	7	8	9
	0	1	2	3	4	5	6	7	8	9

Table 4 M.—*Corrections to table 3 M for values of the acceleration of gravity at sea-level different from 9.80.*

Height (meters).	Acceleration of gravity at sea-level.								
	9.76	9.77	9.78	9.79	9.80	9.81	9.82	9.83	9.84
29000	−116	−87	−58	−29	0	29	58	87	116
28000	−112	−84	−56	−28	0	28	56	84	112
27000	−108	−81	−54	−27	0	27	54	81	108
26000	−104	−78	−52	−26	0	26	52	78	104
25000	−100	−75	−50	−25	0	25	50	75	100
24000	− 96	−72	−48	−24	0	24	48	72	96
23000	− 92	−69	−46	−23	0	23	46	69	92
22000	− 88	−66	−44	−22	0	22	44	66	88
21000	− 84	−63	−42	−21	0	21	42	63	84
20000	− 80	−60	−40	−20	0	20	40	60	80
19000	− 76	−57	−38	−19	0	19	38	57	76
18000	− 72	−54	−36	−18	0	18	36	54	72
17000	− 68	−51	−34	−17	0	17	34	51	68
16000	− 64	−48	−32	−16	0	16	32	48	64
15000	− 60	−45	−30	−15	0	15	30	45	60
14000	− 56	−42	−28	−14	0	14	28	42	56
13000	− 52	−39	−26	−13	0	13	26	39	52
12000	− 48	−36	−24	−12	0	12	24	36	48
11000	− 44	−33	−22	−11	0	11	22	33	44
10000	− 40	−30	−20	−10	0	10	20	30	40
9000	− 36	−27	−18	− 9	0	9	18	27	36
8000	− 32	−24	−16	− 8	0	8	16	24	32
7000	− 28	−21	−14	− 7	0	7	14	21	28
6000	− 24	−18	−12	− 6	0	6	12	18	24
5000	− 20	−15	−10	− 5	0	5	10	15	20
4000	− 16	−12	− 8	− 4	0	4	8	12	16
3000	− 12	− 9	− 6	− 3	0	3	6	9	12
2000	− 8	− 6	− 4	− 2	0	2	4	6	8
1000	− 4	− 3	− 2	− 1	0	1	2	3	4
0	0	0	0	0	0	0	0	0	0
	9.76	9.77	9.78	9.79	9.80	9.81	9.82	9.83	9.84

Example to tables 3 M and 4 M.

1	2	3	4	5
1649	1568	48	− 2	1614
2865	2743	64	− 3	2804
4810	4700	10	− 6	4704
12428	12128	27	−15	12140

Column 1. Heights above sea-level given in meters.
2. Values of table 3 M for the heights 1600, 2800, 4800, 12400.
3. Values of proportionality table for the heights 49, 65, 10, 28.
4. Corrections from table 4 M for $g = 9.7873$ at sea-level and for the heights of column 1.
5. Sum of numbers in columns 2, 3 and 4, giving the dynamic heights corresponding to the geometrical heights of column 1.

Table 5 M.—*Heights reduced from dynamic meters to meters, the acceleration of gravity at sea-level being 9.80.*

Height (dynamic meters).	0	100	200	300	400	500	600	700	800	900
29000	29729	29832	29935	30038	30141	30244	30347	30451	30554	30657
28000	28700	28803	28906	29009	29112	29215	29318	29420	29523	29626
27000	27670	27773	27876	27979	28082	28185	28288	28391	28494	28597
26000	26641	26744	26847	26950	27053	27156	27259	27362	27464	27567
25000	25612	25715	25818	25921	26024	26127	26230	26333	26435	26538
24000	24584	24687	24790	24893	24995	25098	25201	25304	25407	25510
23000	23556	23659	23762	23864	23967	24070	24173	24276	24378	24481
22000	22528	22631	22734	22836	22939	23042	23145	23248	23350	23453
21000	21501	21603	21706	21809	21912	22014	22117	22220	22323	22425
20000	20474	20576	20679	20782	20884	20987	21090	21193	21295	21398
19000	19447	19549	19652	19755	19858	19960	20063	20166	20268	20371
18000	18420	18523	18626	18728	18831	18934	19036	19139	19242	19344
17000	17394	17497	17599	17702	17805	17907	18010	18112	18215	18318
16000	16368	16471	16574	16676	16779	16881	16984	17086	17189	17292
15000	15343	15445	15548	15651	15753	15856	15958	16061	16163	16266
14000	14318	14420	14523	14625	14728	14830	14933	15035	15138	15240
13000	13293	13395	13498	13600	13703	13805	13908	14010	14113	14215
12000	12268	12371	12473	12576	12678	12781	12883	12986	13088	13190
11000	11244	11347	11449	11552	11654	11756	11859	11961	12064	12166
10000	10220	10323	10425	10528	10630	10732	10835	10937	11040	11142
9000	9197	9299	9402	9504	9606	9709	9811	9913	10016	10118
8000	8174	8276	8378	8481	8583	8685	8788	8890	8992	9095
7000	7151	7253	7355	7458	7560	7662	7765	7867	7969	8071
6000	6128	6231	6333	6435	6537	6640	6742	6844	6946	7049
5000	5106	5208	5311	5413	5515	5617	5719	5822	5924	6026
4000	4084	4186	4289	4391	4493	4595	4697	4800	4902	5004
3000	3063	3165	3267	3369	3471	3573	3676	3778	3880	3982
2000	2042	2144	2246	2348	2450	2552	2654	2756	2858	2961
1000	1021	1123	1225	1327	1429	1531	1633	1735	1837	1939
0	0	102	204	306	408	510	612	714	816	919
	0	100	200	300	400	500	600	700	800	900

PROPORTIONALITY TABLE.

Meters.	0	1	2	3	4	5	6	7	8	9
90	92	93	94	95	96	97	98	99	100	101
80	82	83	84	85	86	87	88	89	90	91
70	71	72	73	74	76	77	78	79	80	81
60	61	62	63	64	65	66	67	68	69	70
50	51	52	53	54	55	56	57	58	59	60
40	41	42	43	44	45	46	47	48	49	50
30	31	32	33	34	35	36	37	38	39	40
20	20	21	22	23	24	26	27	28	29	30
10	10	11	12	13	14	15	16	17	18	19
0	0	1	2	3	4	5	6	7	8	9
	0	1	2	3	4	5	6	7	8	9

Table 6 M.—*Corrections to table 5 M for values of the acceleration of gravity at sea-level different from 9.80.*

Height (dynamic meters).	Acceleration of gravity at sea-level.								
	9.76	9.77	9.78	9.79	9.80	9.81	9.82	9.83	9.84
29000	121	91	60	30	0	−30	−60	−91	−121
28000	117	88	58	29	0	−29	−58	−88	−117
27000	113	84	56	28	0	−28	−56	−84	−113
26000	108	81	54	27	0	−27	−54	−81	−108
25000	104	78	52	26	0	−26	−52	−78	−104
24000	100	75	50	25	0	−25	−50	−75	−100
23000	96	72	48	24	0	−24	−48	−72	− 96
22000	92	69	46	23	0	−23	−46	−69	− 92
21000	87	66	44	22	0	−22	−44	−66	− 87
20000	83	62	42	21	0	−21	−42	−62	− 83
19000	79	59	40	20	0	−20	−40	−59	− 79
18000	75	56	37	19	0	−19	−37	−56	− 75
17000	71	53	35	18	0	−18	−35	−53	− 71
16000	67	50	33	17	0	−17	−33	−50	− 67
15000	62	47	31	16	0	−16	−31	−47	− 62
14000	58	44	29	15	0	−15	−29	−44	− 58
13000	54	41	27	14	0	−14	−27	−41	− 54
12000	50	37	25	13	0	−13	−25	−37	− 50
11000	46	34	23	11	0	−11	−23	−34	− 46
10000	42	31	21	10	0	−10	−21	−31	− 42
9000	37	28	19	9	0	− 9	−19	−28	− 37
8000	33	25	17	8	0	− 8	−17	−25	− 33
7000	29	22	15	7	0	− 7	−15	−22	− 29
6000	25	19	12	6	0	− 6	−12	−19	− 25
5000	21	16	10	5	0	− 5	−10	−16	− 21
4000	17	12	8	4	0	− 4	− 8	−12	− 17
3000	13	9	6	3	0	− 3	− 6	− 9	− 13
2000	8	6	4	2	0	− 2	− 4	− 6	− 8
1000	4	3	2	1	0	− 1	− 2	− 3	− 4
0	0	0	0	0	0	0	0	0	0
	9.76	9.77	9.78	9.79	9.80	9.81	9.82	9.83	9.84

Examples to tables 5 M and 6 M.

1	2	3	4	5
1614	1633	14	+ 2	1649
2804	2858	4	+ 3	2865
4704	4800	4	+ 6	4810
12140	12371	41	+16	12428

Column 1. Heights above sea-level given in dynamic meters.
2. Values of table 5 M for the dynamic heights, 1600, 2800, 4700, 12100.
3. Values of proportionality table for dynamic heights 14, 4, 4, 40.
4. Corrections from table 6 M for $g = 9.7873$ at sea-level and for the heights of column 1.
5. Sum of numbers in columns 2, 3 and 4, giving the geometrical heights corresponding to the dynamic heights of column 1.

Table 7 M.—*Virtual temperature of saturated air for given pressures.*

Pressure (m-bars).	Temperature (°C.).																	
	−50	−40	−30	−20	−15	−10	−5	−2	0	1	2	3	4	5	6	7	8	9
200	0.0	0.1	0.2															
250	0.0	0.0	0.1	0.4														
300	0.0	0.0	0.1	0.3														
350	0.0	0.0	0.1	0.3	0.5													
400	0.0	0.0	0.1	0.2	0.4	0.6												
450	0.0	0.0	0.1	0.2	0.4	0.6	0.9											
500	0.0	0.0	0.1	0.2	0.3	0.5	0.8	1.1	1.3									
550	0.0	0.0	0.1	0.2	0.3	0.5	0.7	1.0	1.1	1.2	1.3	1.4	1.5					
600	0.0	0.0	0.1	0.2	0.3	0.4	0.7	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.9	2.0
650	0.0	0.0	0.1	0.2	0.2	0.4	0.6	0.8	1.0	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.9
700	0.0	0.0	0.1	0.1	0.2	0.4	0.6	0.8	0.9	1.0	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
750	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.7	0.8	0.9	1.0	1.0	1.1	1.2	1.3	1.4	1.5	1.6
800	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.7	0.8	0.8	0.9	1.0	1.1	1.1	1.2	1.3	1.4	1.5
850	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.6	0.7	0.8	0.9	0.9	1.0	1.1	1.2	1.2	1.3	1.4
900	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.6	0.7	0.8	0.8	0.9	0.9	1.0	1.1	1.2	1.3	1.4
950	0.0	0.0	0.0	0.1	0.2	0.3	0.4	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2	1.3
1000	0.0	0.0	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.1	1.1	1.2
1050	0.0	0.0	0.0	0.1	0.2	0.2	0.4	0.5	0.6	0.6	0.7	0.7	0.8	0.9	0.9	1.0	1.1	1.2
1100	0.0	0.0	0.0	0.1	0.1	0.2	0.4	0.5	0.6	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.0	1.1

Pressure (m-bars).	Temperature (°C.).																			
	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
650	2.0	2.2	2.3	2.5	2.7	2.8	3.0	3.2	3.5	3.7										
700	1.9	2.0	2.1	2.3	2.5	2.6	2.8	3.0	3.2	3.4	3.7	3.9	4.2	4.5	4.8					
750	1.7	1.9	2.0	2.1	2.3	2.5	2.6	2.8	3.0	3.2	3.4	3.7	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.1
800	1.6	1.8	1.9	2.0	2.2	2.3	2.5	2.6	2.8	3.0	3.2	3.4	3.7	3.9	4.2	4.5	4.7	5.0	5.4	5.7
850	1.5	1.6	1.8	1.9	2.0	2.2	2.3	2.5	2.7	2.8	3.0	3.2	3.4	3.7	3.9	4.3	4.5	4.7	5.1	5.4
900	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.3	2.5	2.7	2.9	3.0	3.3	3.5	3.7	4.0	4.2	4.5	4.8	5.1
950	1.4	1.5	1.6	1.7	1.8	1.9	2.1	2.2	2.4	2.5	2.7	2.9	3.1	3.3	3.5	3.8	4.0	4.2	4.5	4.8
1000	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.3	2.4	2.6	2.7	2.9	3.1	3.3	3.6	3.8	4.0	4.3	4.6
1050	1.2	1.3	1.4	1.5	1.6	1.8	1.9	2.0	2.1	2.3	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.1	4.3
1100	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.3	2.5	2.7	2.8	3.0	3.3	3.4	3.7	3.9	4.1

Pressure (m-bars).	Temperature (°C.).																			
	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
800	6.1	6.5	6.9	7.3	7.8															
850	5.7	6.1	6.5	6.9	7.3	7.8	8.3	8.8	9.3	9.9										
900	5.4	5.7	6.1	6.5	6.9	7.3	7.8	8.3	8.8	9.3	9.9	10.4	11.1	11.7	12.4					
950	5.1	5.4	5.8	6.1	6.5	6.9	7.4	7.8	8.3	8.8	9.3	9.9	10.5	11.1	11.7	12.4	13.1	13.9	14.7	15.6
1000	4.8	5.2	5.5	5.8	6.2	6.6	7.0	7.4	7.9	8.3	8.8	9.4	9.9	10.5	11.1	11.8	12.5	13.2	14.0	14.8
1050	4.6	4.9	5.2	5.5	5.9	6.3	6.7	7.1	7.5	7.9	8.4	8.9	9.4	10.0	10.6	11.2	11.9	12.5	13.3	14.0
1100	4.4	4.7	5.0	5.3	5.6	6.0	6.3	6.7	7.1	7.6	8.0	8.5	9.0	9.5	10.1	10.7	11.3	11.9	12.6	13.4

Example:

Pressure 871 m-bars, temperature......... 14.2°. Table 7M gives 2.0.
63 per cent. of 2.0 gives...................... 1.3°
Virtual temperature.................. 15.5° for air of 14.2° C. and 63 per cent relative humidity at the pressure of 871 m-bars.

Table 8 M. — *Virtual temperature of saturated air in given heights.*

Height (dynamic meters).	Temperature (°C.).																	
	−50	−40	−30	−20	−15	−10	−5	−2	0	1	2	3	4	5	6	7	8	9
10000	0.0	0.0	0.1	0.4														
9500	0.0	0.0	0.1	0.4														
9000	0.0	0.0	0.1	0.3														
8500	0.0	0.0	0.1	0.3	0.5													
8000	0.0	0.0	0.1	0.3	0.5													
7500	0.0	0.0	0.1	0.3	0.4	0.7												
7000	0.0	0.0	0.1	0.2	0.4	0.6												
6500	0.0	0.0	0.1	0.2	0.4	0.6	1.0											
6000	0.0	0.0	0.1	0.2	0.4	0.6	0.9	1.2										
5500	0.0	0.0	0.1	0.2	0.3	0.5	0.8	1.1	1.3									
5000	0.0	0.0	0.1	0.2	0.3	0.5	0.8	1.0	1.2	1.3	1.4							
4500	0.0	0.0	0.1	0.2	0.3	0.5	0.7	0.9	1.1	1.2	1.3	1.4	1.5	1.6				
4000	0.0	0.0	0.1	0.2	0.3	0.4	0.7	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.8	1.9	2.0
3500	0.0	0.0	0.1	0.2	0.3	0.4	0.6	0.8	1.0	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.8	1.9
3000	0.0	0.0	0.1	0.1	0.2	0.4	0.6	0.8	0.9	1.0	1.1	1.1	1.2	1.3	1.4	1.5	1.6	1.8
2500	0.0	0.0	0.0	0.1	0.2	0.4	0.6	0.7	0.9	0.9	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.7
2000	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.7	0.8	0.9	0.9	1.0	1.1	1.2	1.3	1.3	1.4	1.6
1500	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.6	0.8	0.8	0.9	0.9	1.0	1.1	1.2	1.3	1.4	1.5
1000	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.6	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2	1.3	1.4
500	0.0	0.0	0.0	0.1	0.2	0.3	0.4	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2	1.3
0	0.0	0.0	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2

Height (dynamic meters).	Temperature (°C.).																			
	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
3500	2.0	2.2	2.3	2.5	2.7															
3000	1.9	2.0	2.2	2.3	2.5	2.7	2.9	3.1	3.3	3.5										
2500	1.8	1.9	2.1	2.2	2.4	2.5	2.7	2.9	3.1	3.3	3.5	3.7	4.0	4.3	4.6					
2000	1.7	1.8	1.9	2.1	2.2	2.4	2.5	2.7	2.9	3.1	3.3	3.5	3.8	4.0	4.3	4.6	4.9	5.2	5.5	5.9
1500	1.6	1.7	1.8	1.9	2.1	2.2	2.4	2.5	2.7	2.9	3.1	3.3	3.5	3.8	4.0	4.3	4.6	4.9	5.2	5.5
1000	1.5	1.6	1.7	1.8	2.0	2.1	2.2	2.4	2.6	2.7	2.9	3.1	3.3	3.5	3.8	4.0	4.3	4.6	4.9	5.2
500	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.2	2.4	2.6	2.7	2.9	3.1	3.3	3.5	3.8	4.0	4.3	4.6	4.9
0	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.3	2.4	2.6	2.7	2.9	3.1	3.3	3.6	3.8	4.0	4.3	4.6

Height (dynamic meters).	Temperature (°C.).																			
	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
1500	5.9	6.2	6.7	7.1	7.5															
1000	5.5	5.9	6.2	6.6	7.1	7.5	7.9	8.4	9.0	9.5										
500	5.2	5.5	5.8	6.2	6.6	7.0	7.4	7.9	8.4	8.9	9.4	10.0	10.6	11.2	11.9	12.6	13.3	14.1	14.9	15.8
0	4.8	5.2	5.5	5.8	6.2	6.6	7.0	7.4	7.9	8.3	8.8	9.4	9.9	10.5	11.1	11.8	12.5	13.2	14.0	14.8

Example:

Dynamic height 2637; temperature......... 10.4°. Table 8 M gives 1.9.
40 per cent. of 1.9 gives................. .... 0.8°
Virtual temperature...................... 11.2° for air of 10.4° C. and 46 per cent. relative humidity in the height of 2637 dynamic meters.

Table 9 M.—*Mutual distances in dynamic meters between standard isobaric surfaces.*

Standard isobaric surface (m-bars)	Average temperature of sheet (° C.).	0	1	2	3	4	5	6	7	8	9	
100												100
	−100	3442	3422	3402	3382	3362	3343	3323	3303	3283	3263	
	− 90	3641	3621	3601	3581	3561	3542	3522	3502	3482	3462	
	− 80	3840	3820	3800	3780	3760	3741	3721	3701	3681	3661	
	− 70	4039	4019	3999	3979	3959	3939	3920	3900	3880	3860	
	− 60	4238	4218	4198	4178	4158	4138	4119	4099	4079	4059	
	− 50	4437	4417	4397	4377	4357	4337	4318	4298	4278	4258	
	− 40	4636	4616	4596	4576	4556	4536	4516	4497	4477	4457	
	− 30	4835	4815	4795	4775	4755	4735	4715	4696	4676	4656	
200												200
	− 90	2130	2118	2107	2095	2083	2072	2060	2048	2037	2025	
	− 80	2246	2235	2223	2211	2200	2188	2176	2165	2153	2142	
	− 70	2363	2351	2339	2328	2316	2304	2293	2281	2270	2258	
	− 60	2479	2467	2456	2444	2432	2421	2409	2398	2386	2374	
	− 50	2595	2584	2572	2561	2549	2537	2526	2514	2502	2491	
	− 40	2712	2700	2689	2677	2665	2654	2642	2630	2619	2607	
	− 30	2828	2817	2805	2793	2782	2770	2758	2747	2735	2723	
	− 20	2945	2933	2921	2910	2898	2886	2875	2863	2851	2840	
300												300
	− 80	1594	1585	1577	1569	1561	1552	1544	1536	1528	1519	
	− 70	1676	1668	1660	1652	1643	1635	1627	1619	1610	1602	
	− 60	1759	1751	1742	1734	1726	1718	1709	1701	1693	1685	
	− 50	1841	1833	1825	1817	1808	1800	1792	1784	1775	1767	
	− 40	1924	1916	1908	1899	1891	1883	1874	1866	1858	1850	
	− 30	2007	1998	1990	1982	1974	1965	1957	1949	1941	1932	
	− 20	2089	2081	2073	2064	2056	2048	2040	2031	2023	2015	
	− 10	2172	2164	2155	2147	2139	2130	2122	2114	2106	2097	
400												400
	− 70	1300	1294	1287	1281	1275	1268	1262	1255	1249	1243	
	− 60	1364	1358	1351	1345	1339	1332	1326	1319	1313	1307	
	− 50	1428	1422	1416	1409	1403	1396	1390	1384	1377	1371	
	− 40	1492	1486	1480	1473	1467	1460	1454	1448	1441	1435	
	− 30	1556	1550	1544	1537	1531	1524	1518	1512	1505	1499	
	− 20	1621	1614	1608	1601	1595	1588	1582	1576	1569	1563	
	− 10	1685	1678	1672	1665	1659	1653	1646	1640	1633	1627	
	− 0	1749	1742	1736	1729	1723	1717	1710	1704	1697	1691	
500												500
	− 60	1115	1109	1104	1099	1094	1089	1083	1078	1073	1068	
	− 50	1167	1162	1157	1151	1146	1141	1136	1130	1125	1120	
	− 40	1219	1214	1209	1204	1198	1193	1188	1183	1178	1172	
	− 30	1272	1266	1261	1256	1251	1246	1240	1235	1230	1225	
	− 20	1324	1319	1314	1308	1303	1298	1293	1287	1282	1277	
	− 10	1376	1371	1366	1361	1355	1350	1345	1340	1335	1329	
	− 0	1429	1423	1418	1413	1408	1403	1397	1392	1387	1382	
	+ 0	1429	1434	1439	1444	1450	1455	1460	1465	1471	1476	
600												600
		0	1	2	3	4	5	6	7	8	9	

Table 9 M (continued).—*Mutual distances in dynamic meters between standard isobaric surfaces.*

Standard isobaric surface (m-bars)	Average temperature of sheet (° C.).	0	1	2	3	4	5	6	7	8	9
600											
	− 50	987	982	978	973	969	965	960	956	951	947
	− 40	1031	1027	1022	1018	1013	1009	1004	1000	996	991
	− 30	1075	1071	1066	1062	1058	1053	1049	1044	1040	1035
	− 20	1119	1115	1111	1106	1102	1097	1093	1088	1084	1080
	− 10	1164	1159	1155	1150	1146	1142	1137	1133	1128	1124
	− 0	1208	1204	1199	1195	1190	1186	1181	1177	1173	1168
	+ 0	1208	1212	1217	1221	1226	1230	1235	1239	1243	1248
	+ 10	1252	1257	1261	1265	1270	1274	1279	1283	1288	1292
700											
	− 40	893	889	885	882	878	874	870	866	862	859
	− 30	931	928	924	920	916	912	908	905	901	897
	− 20	970	966	962	958	954	951	947	943	939	935
	− 10	1008	1004	1000	997	993	989	985	981	977	974
	− 0	1046	1043	1039	1035	1031	1027	1023	1020	1016	1012
	+ 0	1046	1050	1054	1058	1062	1066	1069	1073	1077	1081
	+ 10	1085	1089	1092	1096	1100	1104	1108	1112	1115	1119
	+ 20	1123	1127	1131	1135	1138	1142	1146	1150	1154	1158
800											
	− 40	788	784	781	778	774	771	767	764	761	757
	− 30	822	818	815	811	808	805	801	798	795	791
	− 20	855	852	849	845	842	838	835	832	828	825
	− 10	889	886	882	879	876	872	869	866	862	859
	− 0	923	920	916	913	909	906	903	899	896	893
	+ 0	923	926	930	933	937	940	943	947	950	953
	+ 10	957	960	964	967	970	974	977	980	984	987
	+ 20	991	994	997	1001	1004	1008	1011	1014	1018	1021
	+ 30	1024	1028	1031	1035	1038	1041	1045	1048	1051	1055
900											
	− 40	705	702	699	696	693	690	687	684	680	677
	− 30	735	732	729	726	723	720	717	714	711	708
	− 20	765	762	759	756	753	750	747	744	741	738
	− 10	795	792	789	786	783	780	777	774	771	768
	− 0	826	823	820	817	814	811	808	804	801	798
	+ 0	826	829	832	835	838	841	844	847	850	853
	+ 10	856	859	862	865	868	871	874	877	880	883
	+ 20	886	889	892	895	898	901	904	907	910	913
	+ 30	916	919	922	925	928	931	935	938	941	944
	+ 40	947	950	953	956	959	962	965	968	971	974
1000											
		0	1	2	3	4	5	6	7	8	9

Example.

1	2	3	4
700			3107
	+ 8.0°	1077	
800			2030
	+13.0	967	
900			1063
	+21.0	889	
1000			174

Column 1. Standard isobaric surfaces.
2. Average virtual temperature of standard sheets, taken from the virtual temperature diagram.
3. Distances between the isobaric surfaces found from table 9 M for the virtual temperatures of column 2.
4. Heights of standard isobaric surfaces calculated by addition of their mutual distances, 174 being the height of the 1000 m-bar standard surface above sea-level. (See example to table 11 M.)

Table 10 M.—*Distances in dynamic meters from a standard isobaric surface to a surface of given pressure, the average virtual temperature of the sheet between the surfaces being 0° C.*

I. Distances from the 100-millibar Surface.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
0	∞	36087	30656	27478	25224	23475	22047	20839	19792	18869
10	18044	17297	16615	15988	15407	14866	14361	13885	13438	13014
20	12612	12230	11865	11517	11183	10863	10556	10260	9975	9700
30	9435	9178	8929	8688	8454	8227	8006	7791	7582	7379
40	7180	6987	6798	6614	6433	6257	6085	5917	5752	5590
50	5432	5277	5124	4975	4829	4685	4544	4405	4269	4135
60	4003	3873	3746	3621	3497	3376	3256	3138	3022	2908
70	2795	2684	2574	2466	2360	2254	2151	2048	1947	1847
80	1749	1651	1555	1460	1366	1274	1182	1091	1002	913
90	826	739	653	569	485	402	320	239	158	79
100 m-bars.										
100	0	− 78	− 155	− 232	− 307	− 383	− 457	− 530	− 603	− 675
110	− 747	− 818	− 888	− 958	−1027	−1095	−1163	−1231	−1297	−1363
120	−1429	−1494	−1558	−1623	−1686	−1749	−1811	−1873	−1934	−1996
130	−2056	−2116	−2176	−2235	−2293	−2352	−2410	−2467	−2524	−2581
140	−2637	−2693	−2748	−2803	−2857	−2912	−2965	−3019	−3072	−3125
150	−3177	−3230	−3281	−3333	−3384	−3434	−3485	−3535	−3585	−3634
160	−3683	−3732	−3780	−3829	−3876	−3924	−3972	−4019	−4065	−4112
170	−4158	−4204	−4250	−4295	−4341	−4385	−4430	−4474	−4518	−4562
180	−4606	−4650	−4693	−4736	−4778	−4821	−4863	−4905	−4947	−4988
190	−5030	−5071	−5112	−5153	−5193	−5233	−5273	−5313	−5353	−5393

II. Distances from the 200-millibar Surface.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
100	5432	5353	5276	5200	5124	5049	4975	4901	4829	4756
110	4685	4614	4544	4474	4405	4336	4269	4201	4135	4068
120	4003	3938	3873	3809	3746	3683	3621	3558	3497	3436
130	3376	3315	3256	3197	3138	3080	3022	2965	2908	2851
140	2795	2739	2684	2629	2574	2520	2466	2413	2360	2307
150	2254	2202	2151	2099	2048	1997	1947	1897	1847	1798
160	1749	1700	1651	1603	1555	1507	1460	1413	1366	1320
170	1274	1227	1182	1136	1091	1046	1002	957	913	869
180	826	782	739	696	653	611	569	527	485	443
190	402	361	320	279	239	198	158	118	79	39
200 m-bars.										
200	0	− 39	− 78	− 117	− 155	− 194	− 232	− 270	− 307	− 345
210	− 382	− 420	− 457	− 494	− 530	− 567	− 603	− 639	− 675	− 711
220	− 747	− 782	− 818	− 853	− 888	− 923	− 958	− 992	−1027	−1061
230	−1095	−1129	−1163	−1197	−1230	−1264	−1297	−1330	−1363	−1396
240	−1429	−1461	−1494	−1526	−1558	−1590	−1622	−1654	−1686	−1717
250	−1749	−1780	−1811	−1842	−1873	−1904	−1934	−1965	−1995	−2026
260	−2056	−2086	−2116	−2146	−2176	−2205	−2235	−2264	−2293	−2323
270	−2352	−2381	−2410	−2438	−2467	−2496	−2524	−2552	−2580	−2609
280	−2637	−2665	−2692	−2720	−2748	−2775	−2803	−2830	−2857	−2885
290	−2912	−2939	−2965	−2992	−3019	−3046	−3072	−3099	−3125	−3151

Table 10 M (continued).—*Distances in dynamic meters from a standard isobaric surface to a surface of given pressure, the average virtual temperature of the sheet between the surfaces being 0° C.*

III. Distances from the 300-millibar Surface.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
200	3177	3138	3099	3061	3022	2984	2946	2908	2870	2832
210	2795	2758	2721	2684	2647	2611	2574	2538	2502	2466
220	2430	2395	2360	2324	2289	2254	2220	2185	2151	2116
230	2082	2048	2014	1980	1947	1914	1880	1847	1814	1781
240	1749	1716	1683	1651	1619	1587	1555	1523	1492	1460
250	1429	1397	1366	1335	1304	1273	1243	1212	1182	1152
260	1121	1091	1061	1031	1002	972	943	913	884	855
270	826	797	768	739	710	682	653	625	597	569
280	541	513	485	457	429	402	374	347	320	293
290	266	239	212	185	158	132	105	79	52	26
o m-bars.										300 m-bars.
300	0	− 26	− 52	− 78	− 104	− 130	− 155	− 181	− 206	− 232
310	− 257	− 282	− 307	− 332	− 357	− 382	− 407	− 432	− 457	− 481
320	− 506	− 530	− 555	− 579	− 603	− 627	− 651	− 675	− 699	− 723
330	− 747	− 771	− 794	− 818	− 841	− 865	− 888	− 911	− 935	− 958
340	− 981	−1004	−1027	−1050	−1072	−1095	−1118	−1141	−1163	−1186
350	−1208	−1230	−1253	−1275	−1297	−1319	−1341	−1363	−1385	−1407
360	−1429	−1450	−1472	−1494	−1515	−1537	−1558	−1580	−1601	−1622
370	−1643	−1665	−1685	−1707	−1728	−1749	−1769	−1790	−1811	−1832
380	−1852	−1873	−1893	−1914	−1934	−1955	−1975	−1995	−2016	−2036
390	−2056	−2076	−2096	−2116	−2136	−2156	−2176	−2196	−2215	−2235

IV. Distances from the 400-millibar Surface.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
300	2254	2228	2202	2176	2151	2125	2099	2074	2048	2023
310	1997	1972	1947	1922	1897	1872	1847	1822	1798	1773
320	1749	1724	1700	1676	1651	1627	1603	1579	1555	1531
330	1508	1484	1460	1437	1413	1390	1366	1343	1320	1297
340	1274	1251	1228	1205	1182	1159	1136	1114	1091	1069
350	1046	1024	1002	979	957	935	913	891	869	847
360	826	804	782	761	739	718	696	675	653	632
370	611	590	569	548	527	506	485	464	443	423
380	402	381	361	340	320	300	279	259	239	219
390	198	178	158	138	118	99	79	59	39	20
o m-bars.										400 m-bars.
400	0	− 20	− 39	− 59	− 78	− 97	− 117	− 136	− 155	− 174
410	− 193	− 213	− 232	− 251	− 270	− 289	− 307	− 326	− 345	− 364
420	− 382	− 401	− 419	− 438	− 457	− 475	− 493	− 512	− 530	− 549
430	− 567	− 585	− 603	− 621	− 639	− 657	− 675	− 693	− 711	− 729
440	− 747	− 765	− 782	− 800	− 818	− 835	− 853	− 871	− 888	− 906
450	− 923	− 940	− 958	− 975	− 993	−1010	−1027	−1044	−1061	−1078
460	−1095	−1112	−1129	−1146	−1163	−1180	−1197	−1214	−1230	−1247
470	−1264	−1280	−1297	−1314	−1330	−1347	−1363	−1380	−1396	−1412
480	−1429	−1445	−1461	−1478	−1494	−1510	−1526	−1542	−1558	−1574
490	−1590	−1606	−1622	−1638	−1654	−1670	−1686	−1702	−1717	−1733

Table 10 M (continued).—*Distances in dynamic meters from a standard isobaric surface to a surface of given pressure, the average virtual temperature of the sheet between the surfaces being 0° C.*

V. DISTANCES FROM THE 500-MILLIBAR SURFACE.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
400	1749	1729	1709	1690	1671	1651	1632	1613	1593	1574
410	1555	1536	1517	1498	1479	1460	1441	1422	1404	1385
420	1366	1348	1329	1310	1292	1274	1255	1237	1218	1200
430	1182	1164	1146	1127	1109	1091	1073	1055	1037	1020
440	1002	984	966	949	931	913	896	878	861	843
450	826	808	791	774	756	739	722	705	688	670
460	653	636	619	602	586	569	552	535	518	501
470	485	468	452	435	418	402	385	369	353	336
480	320	304	287	271	255	239	222	206	190	174
490	158	142	126	110	95	79	63	47	31	16
500 m-bars.										500 m-bars.
500	0	− 16	− 31	− 47	− 62	− 78	− 93	− 109	− 124	− 140
510	− 155	− 170	− 186	− 201	− 216	− 232	− 247	− 262	− 277	− 292
520	− 307	− 322	− 337	− 352	− 367	− 382	− 397	− 412	− 427	− 442
530	− 457	− 471	− 486	− 501	− 515	− 530	− 545	− 559	− 574	− 588
540	− 603	− 618	− 632	− 646	− 661	− 675	− 690	− 704	− 718	− 733
550	− 747	− 761	− 775	− 789	− 804	− 818	− 832	− 846	− 860	− 874
560	− 888	− 902	− 916	− 930	− 944	− 958	− 972	− 985	− 999	−1013
570	−1027	−1040	−1054	−1068	−1082	−1095	−1109	−1123	−1136	−1150
580	−1163	−1177	−1190	−1204	−1217	−1230	−1244	−1257	−1270	−1284
590	−1297	−1310	−1324	−1337	−1350	−1363	−1376	−1389	−1403	−1416

VI. DISTANCES FROM THE 600-MILLIBAR SURFACE.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
500	1429	1413	1397	1382	1366	1351	1335	1320	1304	1289
510	1274	1258	1243	1228	1212	1197	1182	1167	1152	1136
520	1121	1106	1091	1076	1061	1046	1031	1017	1002	987
530	972	957	943	928	913	899	884	869	855	840
540	826	811	797	782	768	753	739	725	710	696
550	682	668	653	639	625	611	597	583	569	555
560	541	527	513	499	485	471	457	443	430	416
570	402	388	375	361	347	333	320	306	293	279
580	266	252	239	225	212	198	185	172	158	145
590	132	119	105	92	79	66	52	39	26	13
600 m-bars.										600 m-bars.
600	0	− 13	− 26	− 39	− 52	− 65	− 78	− 91	− 104	− 117
610	− 130	− 142	− 155	− 168	− 181	− 194	− 206	− 219	− 232	− 244
620	− 257	− 270	− 282	− 295	− 307	− 320	− 332	− 345	− 357	− 370
630	− 382	− 395	− 407	− 419	− 432	− 444	− 457	− 469	− 481	− 493
640	− 506	− 518	− 530	− 542	− 555	− 567	− 579	− 591	− 603	− 615
650	− 627	− 639	− 651	− 663	− 675	− 687	− 699	− 711	− 723	− 735
660	− 747	− 759	− 771	− 782	− 794	− 806	− 818	− 830	− 841	− 853
670	− 865	− 876	− 888	− 900	− 911	− 923	− 935	− 946	− 958	− 969
680	− 981	− 992	−1004	−1015	−1027	−1038	−1050	−1061	−1073	−1084
690	−1095	−1107	−1118	−1129	−1141	−1152	−1163	−1174	−1186	−1197

Table 10 M (continued).—*Distances in dynamic meters from a standard isobaric surface to a surface of given pressure, the average virtual temperature of the sheet between the surfaces being 0° C.*

VII. Distances from the 700-millibar Surface.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
600	1208	1195	1182	1169	1116	1143	1130	1117	1104	1091
610	1078	1066	1053	1040	1027	1014	1002	989	976	964
620	951	938	926	913	901	888	876	863	851	838
630	826	813	801	788	776	764	751	739	727	715
640	702	690	678	666	653	641	629	617	605	593
650	581	569	557	545	533	521	509	497	485	473
660	461	449	438	426	414	402	390	379	367	355
670	343	332	320	308	297	285	273	262	250	239
680	227	216	204	193	181	170	158	147	136	124
690	113	101	90	79	68	56	45	34	23	11
700 m-bars.										700 m-bars.
700	0	− 11	− 22	− 33	− 45	− 56	− 67	− 78	− 89	− 100
710	− 111	− 122	− 133	− 144	− 155	− 166	− 177	− 188	− 199	− 210
720	− 221	− 232	− 242	− 253	− 264	− 275	− 286	− 297	− 307	− 318
730	− 329	− 340	− 350	− 361	− 372	− 382	− 393	− 404	− 414	− 425
740	− 435	− 446	− 457	− 467	− 478	− 488	− 499	− 509	− 520	− 530
750	− 541	− 551	− 561	− 572	− 582	− 593	− 603	− 613	− 624	− 634
760	− 644	− 655	− 665	− 675	− 685	− 696	− 706	− 716	− 726	− 737
770	− 747	− 757	− 767	− 777	− 787	− 798	− 808	− 818	− 828	− 838
780	− 848	− 858	− 868	− 878	− 888	− 898	− 908	− 918	− 928	− 938
790	− 948	− 958	− 968	− 978	− 987	− 997	−1007	−1017	−1027	− 1037

VIII. Distances from the 800-millibar Surface.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
700	1046	1035	1024	1013	1002	991	980	968	957	946
710	935	924	913	902	891	880	869	858	847	837
720	826	815	804	793	782	771	761	750	739	728
730	718	707	696	685	675	664	653	643	632	622
740	611	600	590	579	569	558	548	537	527	516
750	506	495	485	474	464	454	443	433	423	412
760	402	392	381	371	361	351	340	330	320	310
770	300	289	279	269	259	249	239	229	219	209
780	198	188	178	168	158	148	138	128	118	108
790	99	89	79	69	59	49	39	29	20	10
800 m-bars.										800 m-bars.
800	0	− 10	− 20	− 29	− 39	− 49	− 59	− 68	− 78	− 88
810	− 97	− 107	− 117	− 126	− 136	− 146	− 155	− 165	− 174	− 184
820	− 193	− 203	− 213	− 222	− 232	− 241	− 251	− 260	− 270	− 279
830	− 289	− 298	− 307	− 317	− 326	− 336	− 345	− 354	− 364	− 373
840	− 382	− 392	− 401	− 410	− 420	− 429	− 438	− 447	− 457	− 466
850	− 475	− 484	− 494	− 503	− 512	− 521	− 530	− 539	− 549	− 558
860	− 567	− 576	− 585	− 594	− 603	− 612	− 621	− 630	− 639	− 648
870	− 657	− 666	− 675	− 684	− 693	− 702	− 711	− 720	− 729	− 738
880	− 747	− 756	− 765	− 774	− 782	− 791	− 800	− 809	− 818	− 827
890	− 835	− 844	− 853	− 862	− 871	− 879	− 888	− 897	− 906	− 914

Table 10 M (continued).—*Distances in dynamic meters from a standard isobaric surface to a surface of given pressure, the average virtual temperature of the sheet between the surfaces being 0° C.*

IX. Distances from the 900-millibar Surface.

Pressure (m-bars).	0	1	2	3	4	5	6	7	8	9
800	923	913	903	894	884	874	864	855	845	835
810	826	816	806	797	787	777	768	758	749	739
820	729	720	710	701	691	682	672	663	653	644
830	634	625	616	606	597	587	578	569	559	550
840	541	531	522	513	503	494	485	476	466	457
850	448	439	429	420	411	402	393	384	374	365
860	356	347	338	329	320	311	302	293	284	275
870	266	257	248	239	230	221	212	203	194	185
880	176	167	158	149	141	132	123	114	105	96
890	88	79	70	61	52	44	35	26	17	9
900 m-bars.										
900	0	− 9	− 17	− 26	− 35	− 43	− 52	− 61	− 69	− 78
910	− 87	− 95	− 104	− 112	− 121	− 130	− 138	− 147	− 155	− 164
920	− 172	− 181	− 189	− 198	− 206	− 215	− 223	− 232	− 240	− 249
930	− 257	− 265	− 274	− 282	− 291	− 299	− 307	− 316	− 324	− 333
940	− 341	− 349	− 357	− 366	− 374	− 382	− 391	− 399	− 407	− 415
950	− 424	− 432	− 440	− 448	− 457	− 465	− 473	− 481	− 489	− 498
960	− 506	− 514	− 522	− 530	− 538	− 547	− 555	− 563	− 571	− 579
970	− 587	− 595	− 603	− 611	− 619	− 627	− 635	− 643	− 651	− 659
980	− 667	− 675	− 683	− 691	− 699	− 707	− 715	− 723	− 731	− 739
990	− 747	− 755	− 763	− 771	− 778	− 786	− 794	− 802	− 810	− 818

X. Distances from the 1000-millibar Surface.

Pressure (m-bars)	0	1	2	3	4	5	6	7	8	9
900	826	817	808	800	791	782	774	765	756	748
910	739	730	722	713	705	696	688	679	670	662
920	653	645	636	628	619	611	603	594	586	577
930	569	560	552	543	535	527	518	510	502	493
940	485	477	468	460	452	443	435	427	419	410
950	402	394	386	377	369	361	353	345	336	328
960	320	312	304	295	287	279	271	263	255	247
970	239	231	223	215	207	198	190	182	174	166
980	158	150	142	134	126	118	110	103	95	87
990	79	71	63	55	47	39	31	24	16	8
1000 m-bars.										
1000	0	− 8	− 16	− 23	− 31	− 39	− 47	− 55	− 62	− 70
1010	− 78	− 86	− 93	− 101	− 109	− 117	− 124	− 132	− 140	− 147
1020	− 155	− 163	− 171	− 178	− 186	− 193	− 201	− 209	− 216	− 224
1030	− 232	− 239	− 247	− 254	− 262	− 270	− 277	− 285	− 292	− 300
1040	− 307	− 315	− 322	− 330	− 337	− 345	− 352	− 360	− 367	− 375
1050	− 382	− 390	− 397	− 405	− 412	− 420	− 427	− 434	− 442	− 449
1060	− 457	− 464	− 471	− 479	− 486	− 493	− 501	− 508	− 516	− 523
1070	− 530	− 538	− 545	− 552	− 559	− 567	− 574	− 581	− 589	− 596
1080	− 603	− 610	− 618	− 625	− 632	− 639	− 647	− 654	− 661	− 668
1090	− 675	− 683	− 690	− 697	− 704	− 711	− 718	− 725	− 733	− 740

Example:

	Dynamic meters.
Height of standard surface 300 m-bars found as shown in example to table 9 M	9365
Table 10, III, gives for pressure 257.3 m-bars	+1203
Virtual-temperature diagram giving for the sheet between 300 and 257.3 m-bars the average virtual temperature —40°, table 12 gives for this temperature and the height 1203	— 177
Height of isobaric surface 257.3 m-bars	10391

Table 11 M.—*Distances in dynamic meters from the earth's surface to the nearest standard isobaric surfaces, the average virtual temperature of the sheet being 0° C.*

Pressure at station (m-bars).	Standard surfaces (m-bars).			
	700	600	500	Δ
600	−1208	0	1429	13
601	−1195	13	1442	13
602	−1182	26	1455	13
603	−1169	39	1468	13
604	−1156	52	1481	13
605	−1143	65	1494	13
606	−1130	78	1507	13
607	−1117	91	1520	13
608	−1104	104	1533	13
609	−1091	117	1546	13
610	−1078	130	1559	12
611	−1066	142	1571	13
612	−1053	155	1584	13
613	−1040	168	1597	13
614	−1027	181	1610	13
615	−1014	194	1623	12
616	−1002	206	1635	13
617	− 989	219	1648	13
618	− 976	232	1661	12
619	− 964	244	1673	13
620	− 951	257	1686	13
621	− 938	270	1699	12
622	− 926	282	1711	13
623	− 913	295	1724	12
624	− 901	307	1736	13
625	− 888	320	1749	12
626	− 876	332	1761	13
627	− 863	345	1774	12
628	− 851	357	1786	13
629	− 838	370	1799	12
630	− 826	382	1811	13
631	− 813	395	1824	12
632	− 801	407	1836	12
633	− 788	419	1848	13
634	− 776	432	1861	12
635	− 764	444	1873	13
636	− 751	457	1886	12
637	− 739	469	1898	12
638	− 727	481	1910	12
639	− 715	493	1922	13
640	− 702	506	1935	12
641	− 690	518	1947	12
642	− 678	530	1959	12
643	− 666	542	1971	13
644	− 653	555	1984	12
645	− 641	567	1996	12
646	− 629	579	2008	12
647	− 617	591	2020	12
648	− 605	603	2032	12
649	− 593	615	2044	12
650	− 581	627	2056	12
651	− 569	639	2068	12
652	− 557	651	2080	12
653	− 545	663	2092	12
654	− 533	675	2104	12
655	− 521	687	2116	12
656	− 509	699	2128	12
657	− 497	711	2140	12
658	− 485	723	2152	12
659	− 473	735	2164	12

Pressure at station (m-bars).	Standard surfaces (m-bars).				
	800	700	600	500	Δ
660		−461	747	2176	12
661		−449	759	2188	12
662		−438	771	2200	11
663		−426	782	2211	12
664		−414	794	2223	12
665		−402	806	2235	12
666		−390	818	2247	12
667		−379	830	2259	11
668		−367	841	2270	12
669		−355	853	2282	12
670		−343	865	2294	11
671		−332	876	2305	12
672		−320	888	2317	12
673		−308	900	2329	11
674		−297	911	2340	12
675		−285	923	2352	12
676		−273	935	2364	11
677		−262	946	2375	12
678		−250	958	2387	11
679		−239	969	2398	12
680		−227	981	2410	11
681		−216	992	2421	12
682		−204	1004	2433	11
683		−193	1015	2444	12
684		−181	1027	2456	11
685		−170	1038	2467	12
686		−158	1050	2479	11
687		−147	1061	2490	12
688		−136	1073	2502	11
689		−124	1084	2513	11
690		−113	1095	2524	12
691		−101	1107	2536	11
692		− 90	1118	2547	11
693		− 79	1129	2558	12
694		− 68	1141	2570	11
695		− 56	1152	2581	11
696		− 45	1163	2592	11
697		− 34	1174	2603	12
698		− 23	1186	2615	11
699		− 11	1197	2626	11
700	−1046	0	1208	2637	11
701	−1035	11	1219	2648	11
702	−1024	22	1230	2659	11
703	−1013	33	1241	2670	12
704	−1002	45	1253	2682	11
705	− 991	56	1264	2693	11
706	− 980	67	1275	2704	11
707	− 968	78	1286	2715	11
708	− 957	89	1297	2726	11
709	− 946	100	1308	2737	11
710	− 935	111	1319	2748	11
711	− 924	122	1320	2759	11
712	− 913	133	1341	2770	11
713	− 902	144	1352	2781	11
714	− 891	155	1363	2792	11
715	− 880	166	1374		11
716	− 869	177	1385		11
717	− 858	188	1396		11
718	− 847	199	1407		11
719	− 837	210	1418		11

Pressure at station (m-bars).	Standard surfaces (m-bars).			
	800	700	600	Δ
720	−826	221	1429	11
721	−815	232	1440	10
722	−804	242	1450	11
723	−793	253	1461	11
724	−782	264	1472	11
725	−771	275	1483	11
726	−761	286	1494	11
727	−750	297	1505	10
728	−739	307	1515	11
729	−728	318	1526	11
730	−718	329	1537	11
731	−707	340	1548	10
732	−696	350	1558	11
733	−685	361	1569	11
734	−675	372	1580	10
735	−664	382	1590	11
736	−653	393	1601	11
737	−643	404	1612	10
738	−632	414	1622	11
739	−622	425	1633	10
740	−611	435	1643	11
741	−600	446	1654	11
742	−590	457	1665	10
743	−579	467	1675	11
744	−569	478	1686	10
745	−558	488	1696	11
746	−548	499	1707	10
747	−537	509	1717	11
748	−527	520	1728	10
749	−516	530	1738	11
750	−506	541	1749	10
751	−495	551	1759	10
752	−485	561	1769	11
753	−474	572	1780	10
754	−464	582	1790	11
755	−454	593	1801	10
756	−444	603	1811	10
757	−433	613	1821	11
758	−423	624	1832	10
759	−412	634	1842	10
760	−402	644	1852	11
761	−392	655	1863	10
762	−381	665	1873	10
763	−371	675	1883	10
764	−361	685	1893	11
765	−351	696	1904	10
766	−340	706	1914	10
767	−330	716	1924	10
768	−320	726	1934	11
769	−310	737	1945	10
770	−300	747	1955	10
771	−289	757	1965	10
772	−279	767	1975	10
773	−269	777	1985	10
774	−259	787	1995	11
775	−249	798	2006	10
776	−239	808	2016	10
777	−229	818	2026	10
778	−219	828	2036	10
779	−209	838	2046	10

Table 11 M (continued).—*Distances in dynamic meters from the earth's surface to the nearest standard isobaric surfaces, the average virtual temperature of the sheet being 0° C.*

Pressure at station (m-bars.)	Standard surfaces (m-bars).				
	900	800	700	600	Δ
780		−198	848	2056	10
781		−188	858	2066	10
782		−178	868	2076	10
783		−168	878	2086	10
784		−158	888	2096	10
785		−148	898	2106	10
786		−138	908	2116	10
787		−128	918	2126	10
788		−118	928	2136	10
789		−108	938	2146	10
790		− 99	948	2156	10
791		− 89	958	2166	10
792		− 79	968	2176	10
793		− 69	978	2186	9
794		− 59	987	2195	10
795		− 49	997	2205	10
796		− 39	1007	2215	10
797		− 29	1017	2225	10
798		− 20	1027	2235	10
799		− 10	1037	2245	9
800	−923	0	1046	2254	10
801	−913	10	1056	2264	10
802	−903	20	1066	2274	9
803	−894	29	1075	2283	10
804	−884	39	1085	2293	10
805	−874	49	1095	2303	10
806	−864	59	1105	2313	9
807	−855	68	1114	2322	10
808	−845	78	1124	2332	10
809	−835	88	1134	2342	9
810	−826	97	1143	2351	10
811	−816	107	1153	2361	10
812	−806	117	1163	2371	9
813	−797	126	1172	2380	10
814	−787	136	1182	2390	10
815	−777	146	1192	2400	9
816	−768	155	1201	2409	10
817	−758	165	1211	2419	9
818	−749	174	1220	2428	10
819	−739	184	1230	2438	9
820	−729	193	1239	2447	10
821	−720	203	1249	2457	10
822	−710	213	1259	2467	9
823	−701	222	1268	2476	10
824	−691	232	1278	2486	9
825	−682	241	1287	2495	10
826	−672	251	1297	2505	9
827	−663	260	1306	2514	10
828	−653	270	1316	2524	9
829	−644	279	1325	2533	10
830	−634	289	1335	2543	9
831	−625	298	1344	2552	9
832	−616	307	1353	2561	10
833	−606	317	1363	2571	9
834	−597	326	1372	2580	10
835	−587	336	1382	2590	9
836	−578	345	1391	2599	9
837	−569	354	1400	2608	10
838	−559	364	1410	2618	9
839	−550	373	1419	2627	9

Pressure at station (m-bars).	Standard surfaces (m-bars).			
	900	800	700	Δ
840	−541	382	1428	10
841	−531	392	1438	9
842	−522	401	1447	9
843	−513	410	1456	10
844	−503	420	1466	9
845	−494	429	1475	9
846	−485	438	1484	9
847	−476	447	1493	10
848	−466	457	1503	9
849	−457	466	1512	9
850	−448	475	1521	9
851	−439	484	1530	10
852	−429	494	1540	9
853	−420	503	1549	9
854	−411	512	1558	9
855	−402	521	1567	9
856	−393	530	1576	9
857	−384	539	1585	10
858	−374	549	1595	9
859	−365	558	1604	9
860	−356	567	1613	9
861	−347	576	1622	9
862	−338	585	1631	9
863	−329	594	1640	9
864	−320	603	1649	9
865	−311	612	1658	9
866	−302	621	1667	9
867	−293	630	1676	9
868	−284	639	1685	9
869	−275	648	1694	9
870	−266	657	1703	9
871	−257	666	1712	9
872	−248	675	1721	9
873	−239	684	1730	9
874	−230	693	1739	9
875	−221	702	1748	9
876	−212	711	1757	9
877	−203	720	1766	9
878	−194	729	1775	9
879	−185	738	1784	9
880	−176	747	1793	9
881	−167	756	1802	9
882	−158	765	1811	9
883	−149	774	1820	8
884	−141	782	1828	9
885	−132	791	1837	9
886	−123	800	1846	9
887	−114	809	1855	9
888	−105	818	1864	9
889	− 96	827	1873	8
890	− 88	835	1881	9
891	− 79	844	1890	9
892	− 70	853	1899	9
893	− 61	862	1908	9
894	− 52	871	1917	8
895	− 44	879	1925	9
896	− 35	888	1934	9
897	− 26	897	1943	9
898	− 17	906	1952	8
899	− 9	914	1960	9

Pressure at station (m-bars).	Standard surfaces (m-bars).				
	1000	900	800	700	Δ
900	−826	0	923	1969	9
901	−817	9	932	1978	8
902	−808	17	940	1986	9
903	−800	26	949	1995	9
904	−791	35	958	2004	8
905	−782	43	966	2012	9
906	−774	52	975	2021	9
907	−765	61	984	2030	8
908	−756	69	992	2038	9
909	−748	78	1001	2047	9
910	−739	87	1010	2056	8
911	−730	95	1018	2064	9
912	−722	104	1027	2073	8
913	−713	112	1035	2081	9
914	−705	121	1044	2090	9
915	−696	130	1053	2099	8
916	−688	138	1061	2107	9
917	−679	147	1070	2116	8
918	−670	155	1078	2124	9
919	−662	164	1087	2133	8
920	−653	172	1095	2141	9
921	−645	181	1104	2150	8
922	−636	189	1112	2158	9
923	−628	198	1121	2167	8
924	−619	206	1129	2175	9
925	−611	215	1138	2184	8
926	−603	223	1146	2192	9
927	−594	232	1155	2201	8
928	−586	240	1163	2209	9
929	−577	249	1172	2218	8
930	−569	257	1180	2226	8
931	−560	265	1188	2234	9
932	−552	274	1197	2243	8
933	−543	282	1205	2251	9
934	−535	291	1214	2260	8
935	−527	299	1222	2268	8
936	−518	307	1230	2276	9
937	−510	316	1239	2285	8
938	−502	324	1247	2293	9
939	−493	333	1256	2302	8
940	−485	341	1264	2310	8
941	−477	349	1272	2318	8
942	−468	357	1280	2326	9
943	−460	366	1289	2335	8
944	−452	374	1297	2343	8
945	−443	382	1305	2351	9
946	−435	391	1314	2360	8
947	−427	399	1322	2368	8
948	−419	407	1330	2376	8
949	−410	415	1338	2384	9
950	−402	424	1347	2393	8
951	−394	432	1355	2401	8
952	−386	440	1363	2409	8
953	−377	448	1371	2417	9
954	−369	457	1380	2426	8
955	−361	465	1388	2434	8
956	−353	473	1396	2442	8
957	−345	481	1404	2450	8
958	−336	489	1412	2458	9
959	−328	498	1421	2467	8

Table 11 M (continued).—*Distances in dynamic meters from the earth's surface to the nearest standard isobaric surface, the average virtual temperature of the sheet being 0° C.*

Pressure at station (m-bars).	Standard surfaces (m-bars).				
	1000	900	800	700	Δ
960	−320	506	1429	2475	8
961	−312	514	1437	2483	8
962	−304	522	1445		8
963	−295	530	1453		8
964	−287	538	1461		9
965	−279	547	1470		8
966	−271	555	1478		8
967	−263	563	1486		8
968	−255	571	1494		8
969	−247	579	1502		8
970	−239	587	1510		8
971	−231	595	1518		8
972	−223	603	1526		8
973	−215	611	1534		8
974	−207	619	1542		8
975	−198	627	1550		8
976	−190	635	1558		8
977	−182	643	1566		8
978	−174	651	1574		8
979	−166	659	1582		8
980	−158	667	1590		8
981	−150	675	1598		8
982	−142	683	1606		8
983	−134	691	1614		8
984	−126	699	1622		8
985	−118	707	1630		8
986	−110	715	1638		8
987	−103	723	1646		8
988	− 95	731	1654		8
989	− 87	739	1662		8
990	− 79	747	1670		8
991	− 71	755	1678		8
992	− 63	763	1686		8
993	− 55	771	1694		7
994	− 47	778	1701		8
995	− 39	786	1709		8
996	− 31	794	1717		8
997	− 24	802	1725		8
998	− 16	810	1733		8
999	− 8	818	1741		8
1000	0	826	1749		8
1001	8	834	1757		8
1002	16	842	1765		7
1003	23	849	1772		8
1004	31	857	1780		8
1005	39	865	1788		8
1006	47	873	1796		8
1007	55	881	1804		7
1008	62	888	1811		8
1009	70	896	1819		8

Pressure at station (m-bars).	Standard surfaces (m-bars).			
	1000	900	800	Δ
1010	78	904	1827	8
1011	86	912	1835	7
1012	93	919	1842	8
1013	101	927	1850	8
1014	109	935	1858	8
1015	117	943	1866	7
1016	124	950	1873	8
1017	132	958	1881	8
1018	140	966	1889	7
1019	147	973	1896	8
1020	155	981	1904	8
1021	163	989	1912	8
1022	171	997	1920	7
1023	178	1004	1927	8
1024	186	1012	1935	7
1025	193	1019	1042	8
1026	201	1027	1950	8
1027	209	1035	1958	7
1028	216	1042	1965	8
1029	224	1050	1973	8
1030	232	1058	1981	7
1031	239	1065	1988	8
1032	247	1073	1996	7
1033	254	1080	2003	8
1034	262	1088	2011	8
1035	270	1096	2019	7
1036	277	1103	2026	8
1037	285	1111	2034	7
1038	292	1118	2041	8
1039	300	1126	2049	7
1040	307	1133	2056	8
1041	315	1141	2064	7
1042	322	1148	2071	8
1043	330	1156	2079	7
1044	337	1163	2086	8
1045	345	1171	2094	7
1046	352	1178	2101	8
1047	360	1186	2109	7
1048	367	1193	2116	8
1049	375	1201	2124	7
1050	382	1208	2131	8
1051	390	1216	2139	7
1052	397	1223	2146	8
1053	405	1231	2154	7
1054	412	1238	2161	8

Pressure at station (m-bars).	Standard surfaces (m-bars).			
	1000	900	800	Δ
1055	420	1246	2169	7
1056	427	1253	2176	7
1057	434	1260	2183	8
1058	442	1268	2191	7
1059	449	1275	2198	8
1060	457	1283	2206	7
1061	464	1290	2213	7
1062	471	1297	2220	8
1063	479	1305	2228	7
1064	486	1312	2235	7
1065	493	1319	2242	8
1066	501	1327	2250	7
1067	508	1334	2257	8
1068	516	1342	2265	7
1069	523	1349	2272	7
1070	530	1356	2279	8
1071	538	1364	2287	7
1072	545	1371	2294	7
1073	552	1378	2301	7
1074	559	1385	2308	8
1075	567	1393	2316	7
1076	574	1400	2323	7
1077	581	1407	2330	8
1078	589	1415	2338	7
1079	596	1422	2345	7
1080	603	1429	2352	7
1081	610	1436	2359	8
1082	618	1444	2367	7
1083	625	1451	2374	7
1084	632	1458	2381	7
1085	639	1465	2388	8
1086	647	1473	2396	7
1087	654	1480	2403	7
1088	661	1487	2410	7
1089	668	1494	2417	7
1090	675	1501	2424	8
1091	683	1509	2432	7
1092	690	1516	2439	7
1093	697	1523	2446	7
1094	704	1530	2453	7
1095	711	1537	2460	7
1096	718	1544	2467	7
1097	725	1551	2474	8
1098	733	1559	2482	7
1099	740	1566	2489	7

Example:

	Dynamic meters.
Height of station above sea level	39
Table 11 M gives for the 1000 m-bar surface and the pressure of 1015.9 at station	+123
Virtual-temperature diagram giving for the sheet between the station and the 1000 m-bar surface the average virtual temperature +25°, table 12 M gives for this temperature and the height 123 the correction	+ 11
Height of standard surface 1000 m-bars above sea level	173

Table 12 M.—*Corrections to tables 10 M and 11 M for temperature.*

Height (dynamic meters).	Temperature (°C.).																																		
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
30	0	0	0	0	0	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	4	4	4
40	0	0	0	0	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5	5
50	0	0	0	1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	5	6	6	6	6
60	0	0	0	1	1	1	1	2	2	2	2	2	3	3	3	3	4	4	4	4	4	5	5	5	5	5	6	6	6	6	7	7	7	7	7
70	0	0	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5	6	6	6	6	7	7	7	7	8	8	8	8	9
80	0	0	1	1	1	1	2	2	2	3	3	3	4	4	4	4	5	5	5	6	6	6	6	7	7	7	8	8	8	8	9	9	9	10	10
90	0	0	1	1	1	2	2	2	3	3	3	4	4	4	5	5	5	6	6	6	7	7	7	8	8	8	9	9	9	10	10	10	11	11	11
100	0	0	1	1	1	2	2	3	3	3	4	4	4	5	5	5	6	6	7	7	7	8	8	8	9	9	10	10	10	11	11	11	12	12	12
110	0	0	1	1	2	2	2	3	3	4	4	4	5	5	6	6	6	7	7	8	8	8	9	9	10	10	10	11	11	12	12	12	13	13	14
120	0	0	1	1	2	2	3	3	4	4	4	5	5	6	6	7	7	7	8	8	9	9	10	10	11	11	11	12	12	13	13	14	14	15	15
130	0	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	16
140	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17	17
150	0	1	1	2	2	3	3	4	4	5	5	6	7	7	8	8	9	9	10	10	11	12	12	13	13	14	14	15	15	16	16	17	18	18	19
160	0	1	1	2	2	3	4	4	5	5	6	6	7	8	8	9	9	10	11	11	12	12	13	13	14	15	15	16	16	17	18	18	19	19	20
170	0	1	1	2	2	3	4	4	5	6	6	7	7	8	9	9	10	11	11	12	12	13	14	14	15	16	16	17	17	18	19	19	20	21	21
180	0	1	1	2	3	3	4	5	5	6	7	7	8	9	9	10	11	11	12	13	13	14	15	15	16	16	17	18	18	19	20	20	21	22	22
190	0	1	1	2	3	3	4	5	6	6	7	8	8	9	10	10	11	12	13	13	14	15	15	16	17	17	18	19	19	20	21	22	22	23	24
200	0	1	1	2	3	4	4	5	6	7	7	8	9	10	10	11	12	12	13	14	15	15	16	17	18	18	19	20	21	21	22	23	23	24	25
210	0	1	2	2	3	4	5	5	6	7	8	8	9	10	11	12	12	13	14	15	15	16	17	18	18	19	20	21	22	22	23	24	25	25	26
220	0	1	2	2	3	4	5	6	6	7	8	9	10	10	11	12	13	14	15	15	16	17	18	19	19	20	21	22	23	23	24	25	26	27	27
230	0	1	2	3	3	4	5	6	7	8	8	9	10	11	12	13	13	14	15	16	17	18	19	19	20	21	22	23	24	24	25	26	27	28	29
240	0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	14	15	16	17	18	18	19	20	21	22	23	24	25	25	26	27	28	29	30
250	0	1	2	3	4	5	5	6	7	8	9	10	11	12	13	14	15	16	16	17	18	19	20	21	22	23	24	25	26	27	27	28	29	30	31
260	0	1	2	3	4	5	6	7	8	9	10	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	30	31	32
270	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
280	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
290	0	1	2	3	4	5	6	7	8	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	27	28	29	30	31	32	33	34	35	36
300	0	1	2	3	4	5	7	8	9	10	11	12	13	14	15	16	18	19	20	21	22	23	24	25	26	27	29	30	31	32	33	34	35	36	37
310	0	1	2	3	5	6	7	8	9	10	11	12	14	15	16	17	18	19	20	22	23	24	25	26	27	28	30	31	32	33	34	35	36	37	39
320	0	1	2	4	5	6	7	8	9	11	12	13	14	15	16	18	19	20	21	22	23	25	26	27	28	29	30	32	33	34	35	36	38	39	40
330	0	1	2	4	5	6	7	8	10	11	12	13	15	16	17	18	19	21	22	23	24	25	27	28	29	30	31	33	34	35	36	37	39	40	41
340	0	1	2	4	5	6	7	9	10	11	12	14	15	16	17	19	20	21	22	24	25	26	27	29	30	31	32	34	35	36	37	39	40	41	42
350	0	1	3	4	5	6	8	9	10	12	13	14	15	17	18	19	21	22	23	24	26	27	28	29	31	32	33	35	36	37	38	40	41	42	44
360	0	1	3	4	5	7	8	9	11	12	13	15	16	17	18	20	21	22	24	25	26	28	29	30	32	33	34	36	37	38	40	41	42	44	45
370	0	1	3	4	5	7	8	9	11	12	14	15	16	18	19	20	22	23	24	26	27	28	30	31	33	34	35	37	38	39	41	42	43	45	46
380	0	1	3	4	6	7	8	10	11	13	14	15	17	18	19	21	22	24	25	26	28	29	31	32	33	35	36	38	39	40	42	43	45	46	47
390	0	1	3	4	6	7	9	10	11	13	14	16	17	19	20	21	23	24	26	27	29	30	31	33	34	36	37	39	40	41	43	44	46	47	49

Example 1:

	Dynamic meters.
Given the average virtual temperature of 25.0° C. and the height of.........	123
Table 12 M gives for 123 meters and 25.0° C. the correction......................	11
123 meters at 0° C. reduced to 25.0° C. gives the height of	134

Table 12 M (continued). — *Corrections to tables 10 M and 11 M for temperature.*

Height (dynamic meters).	Temperature (° C.) 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
400	0	1	3	4	6	7	9	10	12	13	15	16	18	19	21	22	23	25	26	28	29	31	32	34	35	37	38	40	41	42	44	45	47	48	50
410	0	2	3	5	6	8	9	11	12	14	15	17	18	20	21	23	24	26	27	29	30	32	33	35	36	38	39	41	42	44	45	47	48	50	51
420	0	2	3	5	6	8	9	11	12	14	15	17	18	20	22	23	25	26	28	29	31	32	34	35	37	38	40	42	43	45	46	48	49	51	52
430	0	2	3	5	6	8	9	11	13	14	16	17	19	20	22	24	25	27	28	30	32	33	35	36	38	39	41	43	44	46	47	49	50	52	54
440	0	2	3	5	6	8	10	11	13	15	16	18	19	21	23	24	26	27	29	31	32	34	35	37	39	40	42	44	45	47	48	50	52	53	55
450	0	2	3	5	7	8	10	12	13	15	16	18	20	21	23	25	26	28	30	31	33	35	36	38	40	41	43	45	46	48	49	51	53	54	56
460	0	2	3	5	7	8	10	12	13	15	17	19	20	22	24	25	27	29	30	32	34	35	37	39	40	42	44	45	47	49	51	52	54	56	57
470	0	2	3	5	7	9	10	12	14	15	17	19	21	22	24	26	28	29	31	33	34	36	38	40	41	43	45	46	48	50	52	53	55	57	59
480	0	2	4	5	7	9	11	12	14	16	18	19	21	23	25	26	28	30	32	33	35	37	39	40	42	44	46	47	49	51	53	55	56	58	60
490	0	2	4	5	7	9	11	13	14	16	18	20	22	23	25	27	29	31	32	34	36	38	39	41	43	45	47	48	50	52	54	56	57	59	61
500	0	2	4	5	7	9	11	13	15	16	18	20	22	24	26	27	29	31	33	35	37	38	40	42	44	46	48	49	51	53	55	57	59	60	62
510	0	2	4	6	7	9	11	13	15	17	19	21	22	24	26	28	30	32	34	35	37	39	41	43	45	47	49	50	52	54	56	58	60	62	64
520	0	2	4	6	8	10	11	13	15	17	19	21	23	25	27	29	30	32	34	36	38	40	42	44	46	48	50	51	53	55	57	59	61	63	65
530	0	2	4	6	8	10	12	14	16	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	50	52	54	56	58	60	62	64	66
540	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	45	47	49	51	53	55	57	59	61	63	65	67
550	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68
560	0	2	4	6	8	10	12	14	16	18	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	62	64	66	68	70
570	0	2	4	6	8	10	13	15	17	19	21	23	25	27	29	31	33	35	38	40	42	44	46	48	50	52	54	56	58	61	63	65	67	69	71
580	0	2	4	6	8	11	13	15	17	19	21	23	25	28	30	32	34	36	38	40	42	45	47	49	51	53	55	57	59	62	64	66	68	70	72
590	0	2	4	6	9	11	13	15	17	19	22	24	26	28	30	32	35	37	39	41	43	45	48	50	52	54	56	58	61	63	65	67	69	71	73
600	0	2	4	7	9	11	13	15	18	20	22	24	26	29	31	33	35	37	40	42	44	46	48	51	53	55	57	59	62	64	66	68	70	73	75
610	0	2	4	7	9	11	13	16	18	20	22	25	27	29	31	34	36	38	40	42	45	47	49	51	54	56	58	60	63	65	67	69	72	74	76
620	0	2	5	7	9	11	14	16	18	20	23	25	27	30	32	34	36	39	41	43	45	48	50	52	55	57	59	61	64	66	68	70	73	75	77
630	0	2	5	7	9	12	14	16	18	21	23	25	28	30	32	35	37	39	42	44	46	48	51	53	55	58	60	62	65	67	69	72	74	76	78
640	0	2	5	7	9	12	14	16	19	21	23	26	28	30	33	35	38	40	42	45	47	49	52	54	56	59	61	63	66	68	70	73	75	77	80
650	0	2	5	7	10	12	14	17	19	21	24	26	29	31	33	36	38	40	43	45	48	50	52	55	57	60	62	64	67	69	71	74	76	79	81
660	0	2	5	7	10	12	15	17	19	22	24	27	29	31	34	36	39	41	44	46	48	51	53	56	58	60	63	65	68	70	73	75	77	80	82
670	0	2	5	7	10	12	15	17	20	22	25	27	29	32	34	37	39	42	44	47	49	52	54	56	59	61	64	66	69	71	74	76	79	81	83
680	0	2	5	7	10	12	15	17	20	22	25	27	30	32	35	37	40	42	45	47	50	52	55	57	60	62	65	67	70	72	75	77	80	82	85
690	0	3	5	8	10	13	15	18	20	23	25	28	30	33	35	38	40	43	45	48	51	53	56	58	61	63	66	68	71	73	76	78	81	83	86
700	0	3	5	8	10	13	15	18	21	23	26	28	31	33	36	38	41	44	46	49	51	54	56	59	62	64	67	69	72	74	77	79	82	85	87
710	0	3	5	8	10	13	16	18	21	23	26	29	31	34	36	39	42	44	47	49	52	55	57	60	62	65	68	70	73	75	78	81	83	86	88
720	0	3	5	8	11	13	16	18	21	24	26	29	32	34	37	40	42	45	47	50	53	55	58	61	63	66	69	71	74	76	79	82	84	87	90
730	0	3	5	8	11	13	16	19	21	24	27	29	32	35	37	40	43	45	48	51	53	56	59	62	64	67	70	72	75	78	80	83	86	88	91
740	0	3	5	8	11	14	16	19	22	24	27	30	33	35	38	41	43	46	49	52	54	57	60	62	65	68	70	73	76	79	81	84	87	89	92
750	0	3	5	8	11	14	16	19	22	25	27	30	33	36	38	41	44	47	49	52	55	58	60	63	66	69	71	74	77	80	82	85	88	91	93
760	0	3	6	8	11	14	17	19	22	25	28	31	33	36	39	42	45	47	50	53	56	58	61	64	67	70	72	75	78	81	84	86	89	92	95
770	0	3	6	8	11	14	17	20	23	25	28	31	34	37	39	42	45	48	51	54	56	59	62	65	68	71	73	76	79	82	85	87	90	93	96
780	0	3	6	9	11	14	17	20	23	26	29	31	34	37	40	43	46	49	51	54	57	60	63	66	69	71	74	77	80	83	86	89	91	94	97
790	0	3	6	9	12	14	17	20	23	26	29	32	35	38	41	43	46	49	52	55	58	61	64	67	69	72	75	78	81	84	87	90	93	95	98

Example 2:

	Dynamic meters.
Given the temperature 24° C. and the height of........................	1462
Table 12 M gives for 1000 meters and 24° C. the correction.........	88
Table 12 M gives for 462 meters and 24° C. the correction.........	40
1462 meters at 0° C. reduced to 24° C. gives......................	1590

Table 12 M (continued).—*Corrections to tables 10 M and 11 M for temperature.*

Height (dynamic meters).	Temperature (° C.).																																	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
800	0	3	6	9	12	15	18	21	23	26	29	32	35	38	41	44	47	50	53	56	59	62	64	67	70	73	76	79	82	85	88	91	94	97	100
810	0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	47	50	53	56	59	62	65	68	71	74	77	80	83	86	89	92	95	98	101
820	0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75	78	81	84	87	90	93	96	99	102
830	0	3	6	9	12	15	18	21	24	27	30	33	36	40	43	46	49	52	55	58	61	64	67	70	73	76	79	82	85	88	91	94	97	100	103
840	0	3	6	9	12	15	18	22	25	28	31	34	37	40	43	46	49	52	55	58	62	65	68	71	74	77	80	83	86	89	92	95	98	102	105
850	0	3	6	9	12	16	19	22	25	28	31	34	37	40	44	47	50	53	56	59	62	65	68	72	75	78	81	84	87	90	93	97	100	103	106
860	0	3	6	9	13	16	19	22	25	28	32	35	38	41	44	47	50	54	57	60	63	66	69	72	76	79	82	85	88	91	95	98	101	104	107
870	0	3	6	10	13	16	19	22	25	29	32	35	38	41	45	48	51	54	57	61	64	67	70	73	76	80	83	86	89	92	96	99	102	105	108
880	0	3	6	10	13	16	19	23	26	29	32	35	39	42	45	48	52	55	58	61	64	68	71	74	77	81	84	87	90	93	97	100	103	106	110
890	0	3	7	10	13	16	20	23	26	29	33	36	39	42	46	49	52	55	59	62	65	68	72	75	78	82	85	88	91	95	98	101	104	108	111
900	0	3	7	10	13	16	20	23	26	30	33	36	40	43	46	49	53	56	59	63	66	69	73	76	79	82	86	89	92	96	99	102	105	109	112
910	0	3	7	10	13	17	20	23	27	30	33	37	40	43	47	50	53	57	60	63	67	70	73	77	80	83	87	90	93	97	100	103	107	110	113
920	0	3	7	10	13	17	20	24	27	30	34	37	40	44	47	51	54	57	61	64	67	71	74	78	81	84	88	91	94	98	101	104	108	111	115
930	0	3	7	10	14	17	20	24	27	31	34	37	41	44	48	51	55	58	61	65	68	72	75	78	82	85	89	92	95	99	102	106	109	112	116
940	0	3	7	10	14	17	21	24	28	31	34	38	41	45	48	52	55	59	62	65	69	72	76	79	83	86	90	93	96	100	103	107	110	114	117
950	0	3	7	10	14	17	21	24	28	31	35	38	42	45	49	52	56	59	63	66	70	73	77	80	84	87	90	94	97	101	104	108	111	115	118
960	0	4	7	11	14	18	21	25	28	32	35	39	42	46	49	53	56	60	63	67	70	74	77	81	84	88	91	95	98	102	105	109	113	116	120
970	0	4	7	11	14	18	21	25	28	32	36	39	43	46	50	53	57	60	64	68	71	75	78	82	85	89	92	96	99	103	107	110	114	117	121
980	0	4	7	11	14	18	22	25	29	32	36	39	43	47	50	54	57	61	65	68	72	75	79	83	86	90	93	97	101	104	108	111	115	118	122
990	0	4	7	11	15	18	22	25	29	33	36	40	44	47	51	54	58	62	65	69	73	76	80	83	87	91	94	98	102	105	109	112	116	120	123
1000	0	4	7	11	15	18	22	26	29	33	37	40	44	48	51	55	59	62	66	70	73	77	81	84	88	92	95	99	103	106	110	114	117	121	125
1010	0	4	7	11	15	18	22	26	30	33	37	41	44	48	52	55	59	63	67	70	74	78	81	85	89	92	96	100	104	107	111	115	118	122	126
1020	0	4	7	11	15	19	22	26	30	34	37	41	45	49	52	56	60	64	67	71	75	78	82	86	90	93	97	101	105	108	112	116	120	123	127
1030	0	4	8	11	15	19	23	26	30	34	38	42	45	49	53	57	60	64	68	72	75	79	83	87	91	94	98	102	106	109	113	117	121	125	128
1040	0	4	8	11	15	19	23	27	30	34	38	42	46	50	53	57	61	65	69	72	76	80	84	88	91	95	99	103	107	110	114	118	122	126	130
1050	0	4	8	12	15	19	23	27	31	35	38	42	46	50	54	58	62	65	69	73	77	81	85	88	92	96	100	104	108	112	115	119	123	127	131
1060	0	4	8	12	16	19	23	27	31	35	39	43	47	50	54	58	62	66	70	74	78	82	85	89	93	97	101	105	109	113	116	120	124	128	132
1070	0	4	8	12	16	20	24	27	31	35	39	43	47	51	55	59	63	67	71	74	78	82	86	90	94	98	102	106	110	114	118	122	125	129	133
1080	0	4	8	12	16	20	24	28	32	36	40	44	47	51	55	59	63	67	71	75	79	83	87	91	95	99	103	107	111	115	119	123	127	131	135
1090	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100	104	108	112	116	120	124	128	132	136
1000	0	4	7	11	15	18	22	26	29	33	37	40	44	48	51	55	59	62	66	70	73	77	81	84	88	92	95	99	103	106	110	114	117	121	125
2000	0	7	15	22	29	37	44	51	59	66	73	81	88	95	103	110	117	125	132	139	147	154	161	168	176	183	100	190	205	212	220	227	234	242	249
3000	0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275	286	297	308	319	330	341	352	363	374
4000	0	15	29	44	59	73	88	103	117	132	147	161	176	190	205	220	234	249	264	278	293	308	322	337	352	366	381	396	410	425	440	454	469	484	498
5000	0	18	37	55	73	92	110	128	147	165	183	201	220	238	256	275	293	311	330	348	366	385	403	421	440	458	476	495	513	531	549	568	586	604	623
6000	0	22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	505	527	549	571	593	615	637	659	681	703	725	747
7000	0	26	51	77	103	128	154	179	205	231	256	282	308	333	359	385	410	436	462	487	513	538	564	590	615	641	667	692	718	744	769	795	821	846	872
8000	0	29	59	88	117	147	176	205	234	264	293	322	352	381	410	440	469	498	527	557	586	615	645	674	703	733	762	791	821	850	879	908	938	967	996
9000	0	33	66	99	132	165	198	231	264	297	330	363	396	429	462	495	527	560	593	626	659	692	725	758	791	824	857	890	923	956	989	1022	1055	1088	1121
10000	0	37	73	110	147	183	220	256	293	330	366	403	440	476	513	549	586	623	659	696	733	769	806	842	879	916	952	989	1026	1062	1099	1136	1172	1209	1245

Example 3:

	Dynamic meters.
Given the temperature —52° C. and the height of..................	769
Table 12 M gives for 769 meters and —32° C. the correction.........	—90
Table 12 M gives for 769 meters and —20° C. the correction.........	—56
769 meters at 0° C. reduced to —52° C. gives......................	623

Table 12 M (continued).—*Corrections to tables 10 M and 11 M for temperature.*

Height (dynamic meters).	Temperature (° C.). 30	40	50	60	70	80	90	100
0	0	0	0	0	0	0	0	0
10	1	1	2	2	3	3	3	4
20	2	3	4	4	5	6	7	7
30	3	4	5	7	8	9	10	11
40	4	6	7	9	10	12	13	15
50	5	7	9	11	13	15	16	18
60	7	9	11	13	15	18	20	22
70	8	10	13	15	18	21	23	26
80	9	12	15	18	21	23	26	29
90	10	13	16	20	23	26	30	33
100	11	15	18	22	26	29	33	37
110	12	16	20	24	28	32	36	40
120	13	18	22	26	31	35	40	44
130	14	19	24	29	33	38	43	48
140	15	21	26	31	36	41	46	51
150	16	22	27	33	38	44	49	55
160	18	23	29	35	41	47	53	59
170	19	25	31	37	44	50	56	62
180	20	26	33	40	46	53	59	66
190	21	28	35	42	49	56	63	70
200	22	29	37	44	51	59	66	73
210	23	31	38	46	54	62	69	77
220	24	32	40	48	56	64	73	81
230	25	34	42	51	59	67	76	84
240	26	35	44	53	62	70	79	88
250	27	37	46	55	64	73	82	92
260	29	38	48	57	67	76	86	95
270	30	40	49	59	69	79	89	99
280	31	41	51	62	72	82	92	103
290	32	42	53	64	74	85	96	106
300	33	44	55	66	77	88	99	110
310	34	45	57	68	79	91	102	114
320	35	47	59	70	82	94	105	117
330	36	48	60	73	85	97	109	121
340	37	50	62	75	87	100	112	125
350	38	51	64	77	90	103	115	128
360	40	53	66	79	92	105	119	132
370	41	54	68	81	95	108	122	136
380	42	56	70	84	97	111	125	139
390	43	57	71	86	100	114	129	143

Height (dynamic meters).	Temperature (° C.). 30	40	50	60	70	80	90	100
400	44	59	73	88	103	117	132	147
410	45	60	75	90	105	120	135	150
420	46	62	77	92	108	123	138	154
430	47	63	79	95	110	126	142	158
440	48	64	81	97	113	129	145	161
450	49	66	82	99	115	132	148	165
460	51	67	84	101	118	135	152	168
470	52	69	86	103	121	138	155	172
480	53	70	88	105	123	141	158	176
490	54	72	90	108	126	144	162	179
500	55	73	92	110	128	147	165	183
510	56	75	93	112	131	149	168	187
520	57	76	95	114	133	152	171	190
530	58	78	97	116	136	155	175	194
540	59	79	99	119	138	158	178	198
550	60	81	101	121	141	161	181	201
560	62	82	103	123	144	164	185	205
570	63	84	104	125	146	167	188	209
580	64	85	106	127	149	170	191	212
590	65	86	108	130	151	173	195	216
600	66	88	110	132	154	176	198	220
610	67	89	112	134	156	179	201	223
620	68	91	114	136	159	182	204	227
630	69	92	115	138	162	185	208	231
640	70	94	117	141	164	188	211	234
650	71	95	119	143	167	190	214	238
660	73	97	121	145	169	193	218	242
670	74	98	123	147	172	196	221	245
680	75	100	125	149	174	199	224	249
690	76	101	126	152	177	202	227	253
700	77	103	128	154	179	205	231	256
710	78	104	130	156	182	208	234	260
720	79	105	132	158	185	211	237	264
730	80	107	134	160	187	214	241	267
740	81	108	136	163	190	217	244	271
750	82	110	137	165	192	220	247	275
760	84	111	139	167	195	223	251	278
770	85	113	141	169	197	226	254	282
780	86	114	143	171	200	229	257	286
790	87	116	145	174	203	232	260	289

Height (dynamic meters).	Temperature (° C.). 30	40	50	60	70	80	90	100
800	88	117	147	176	205	234	264	293
810	89	119	148	178	208	237	267	297
820	90	120	150	180	210	240	270	300
830	91	122	152	182	213	243	274	304
840	92	123	154	185	215	246	277	308
850	93	125	156	187	218	249	280	311
860	95	126	158	189	221	252	284	315
870	96	127	159	191	223	255	287	319
880	97	129	161	193	226	258	290	322
890	98	130	163	196	228	261	293	326
900	99	132	165	198	231	264	297	330
910	100	133	167	200	233	267	300	333
920	101	135	168	202	236	270	303	337
930	102	136	170	204	238	273	307	341
940	103	138	172	207	241	275	310	344
950	104	139	174	209	244	278	313	348
960	105	141	176	211	246	281	316	352
970	107	142	178	213	249	284	320	355
980	108	144	179	215	251	287	323	359
990	109	145	181	218	254	290	326	363
1000	110	147	183	220	256	293	330	366
1010	111	148	185	222	259	296	333	370
1020	112	149	187	224	262	299	336	374
1030	113	151	189	226	264	302	340	377
1040	114	152	190	229	267	305	343	381
1050	115	154	192	231	269	308	346	385
1060	116	155	194	233	272	311	349	388
1070	118	157	196	235	274	314	353	392
1080	119	158	198	237	277	316	356	396
1090	120	160	200	240	279	319	359	399
1000	110	147	183	220	256	293	330	366
2000	220	293	366	440	513	586	659	733
3000	330	440	549	659	769	879	989	1099
4000	440	586	733	879	1026	1172	1310	1465
5000	549	733	916	1099	1282	1465	1648	1832
6000	659	879	1099	1319	1538	1758	1978	2198
7000	769	1026	1282	1538	1795	2051	2308	2564
8000	879	1172	1465	1758	2051	2344	2637	2930
9000	989	1319	1648	1978	2308	2637	2967	3297
10000	1099	1465	1832	2198	2564	2930	3297	3663

Example 4:

	Dynamic meters.
Given temperature —69° C. and the height of..............................	4785
Table 12 M gives for 4000 meters and —60° C. the correction.........	—879
Table 12 M gives for 785 meters and —60° C. the correction...........	—172
Table 12 M gives for 4000 meters and —9° C. the correction...........	—132
Table 12 M gives for 785 meters and —9° C. the correction............	— 26
4785 meters at 0° C. reduced to —69° C. gives......................	3576

Table 13 M.—*Artificial temperature to be used in table 12 M for calculating pressures in given heights.*

Temperature (°C.).	0	1	2	3	4	5	6	7	8	9
−90	134.3	136.5	138.8	141.1	143.4	145.7	148.1	150.5	152.9	155.3
−80	113.2	115.2	117.2	119.3	121.3	123.4	125.6	127.7	129.9	132.1
−70	94.1	96.0	97.8	99.7	101.5	103.4	105.3	107.2	109.2	111.2
−60	76.9	78.5	80.2	81.9	83.6	85.3	87.0	88.8	90.6	92.3
−50	61.2	62.7	64.2	65.8	67.3	68.9	70.5	72.0	73.7	75.3
−40	46.9	48.2	49.6	51.0	52.5	53.9	55.3	56.8	58.2	59.7
−30	33.7	35.0	36.2	37.5	38.8	40.1	41.5	42.8	44.2	45.5
−20	21.6	22.8	23.9	25.1	26.3	27.5	28.7	30.0	31.2	32.4
−10	10.4	11.5	12.6	13.7	14.8	15.9	17.0	18.1	19.3	20.4
−0	0	1.0	2.0	3.0	4.1	5.1	6.1	7.2	8.2	9.3
0	0	− 1.0	− 2.0	− 3.0	− 3.9	− 4.9	− 5.9	− 6.8	− 7.8	− 8.7
10	− 9.6	−10.6	−11.5	−12.4	−13.3	−14.2	−15.1	−16.0	−16.9	−17.8
20	−18.6	−19.5	−20.4	−21.2	−22.1	−22.9	−23.7	−24.6	−25.4	−26.2
30	−27.0	−27.8	−28.6	−29.4	−30.2	−31.0	−31.8	−32.6	−33.4	−34.1
40	−34.9	−35.6	−36.4	−37.2	−37.9	−38.6	−39.4	−40.1	−40.8	−41.5

Example:

	Dynamic meters.
To calculate the pressure at the height of	5000
Height of nearest standard surface, 600 m-bars, found as in example to table 9 M	4323
Difference	+677
Table 10 M (VI) gives for +677 the pressure 550.4 m-bars; the virtual-temperature diagram giving for the sheet between the surfaces 600 and 550.4 the average virtual temperature −5°; table 13 M gives the artificial temperature +5.1; table 12 M gives for temperature 5.1 and height 677 the correction	+ 12
Sum (artificial height)	689

Table 10 M (VI) gives for +689 the pressure 549.5, which is thus the pressure in the given height of 5000 dynamic meters.

Table 14 M.—*Specific volume of the air at the standard pressures.*

Pressure (m-bars).	Virtual temperature (centigrade).															
	−100	−90	−80	−70	−60	−50	−40	−30	−20	−10	0	10	20	30	40	50
0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
100	4966	5253	5540	5827	6114	6401	6688	6975	7262	7549	7836	8123	8410	8697	8984	9271
200	2483	2626	2770	2913	3057	3201	3344	3488	3631	3775	3918	4062	4205	4349	4492	4636
300	1655	1751	1847	1942	2038	2134	2229	2325	2421	2516	2612	2708	2803	2899	2995	3091
400	1241	1313	1385	1457	1529	1600	1672	1744	1816	1887	1959	2031	2103	2174	2246	2318
500	993	1051	1108	1165	1223	1280	1338	1395	1452	1510	1567	1625	1682	1739	1797	1854
600	828	875	923	971	1019	1067	1115	1163	1210	1258	1306	1354	1402	1450	1497	1545
700	709	750	791	832	873	914	955	996	1037	1078	1119	1160	1201	1242	1283	1324
800	621	657	692	728	764	800	836	872	908	944	980	1015	1051	1087	1123	1159
900	552	584	616	647	679	711	743	775	807	839	871	903	934	966	998	1030
1000	497	525	554	583	611	640	669	698	726	755	784	812	841	870	898	927
1100	451	478	504	530	556	582	608	634	660	686	712	738	765	791	817	843

Proportionality Table.

Pressure (m-bars).	Vitrual temperature (centigrade).										
	0	1	2	3	4	5	6	7	8	9	10
0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
100	0	29	57	86	115	144	172	201	230	258	287
200	0	14	29	43	57	72	86	100	115	129	144
300	0	10	19	29	38	48	57	67	77	86	96
400	0	7	14	22	29	36	43	50	57	65	72
500	0	6	11	17	13	29	34	40	46	52	57
600	0	5	10	14	19	24	29	33	38	43	48
700	0	4	8	12	16	21	25	29	33	37	41
800	0	4	7	11	14	18	22	25	29	32	36
900	0	3	6	10	13	16	19	22	26	29	32
1000	0	3	6	9	11	14	17	20	23	26	29
1100	0	3	5	8	10	13	16	18	21	23	26

Example.

1	2	3	4	5
700	+ 5.8	1119	24	1143
800	+10.1	1015	0	1015
900	+16.5	903	20	923

Column 1. Standard isobaric surfaces.
2. Virtual temperature at standard surfaces, taken from the virtual temperature diagram.
3. Numbers found from table 14 M for pressures of column 1 and temperatures, respectively 0, +10, +10, +20.
4. Numbers found from proportionality table for pressures of column 1, and temperatures respectively 5 8, 0.1, 6.5, 4.7.

Table 15 M.—*Temperature correction to be added to the virtual temperature at the earth's surface in order to give the most probable average virtual temperature in the sheet between the earth's surface and the nearest standard isobaric surfaces (based upon statistics).*

Height of standard surfaces above station (dynamic meters).	Temperature Correction.											
	Winter.			Spring.			Summer.			Autumn.		
	High.	Mean.	Low.	High.	Mean.	Low.	High.	Mean.	Low.	High.	Mean.	Low.
	°C.	°C.	°C.	°C.	°C.	°C.	°C.	°C.	°C.	°C.	°C.	°C.
2400	+2.6	+0.9	−1.6	−3.2	−5.6	−5.6	−4.0	−6.3	−6.2	−1.1	−4.6	−6.4
2300	+2.7	+1.0	−1.5	−3.0	−5.4	−5.4	−3.8	−6.1	−5.9	−0.9	−4.5	−6.1
2200	+2.7	+1.2	−1.3	−2.8	−5.1	−5.1	−3.6	−5.8	−5.6	−0.8	−4.2	−5.8
2100	+2.8	+1.3	−1.2	−2.6	−4.9	−4.9	−3.4	−5.6	−5.4	−0.6	−4.1	−5.6
2000	+2.8	+1.4	−1.1	−2.4	−4.6	−4.7	−3.2	−5.4	−5.1	−0.5	−3.8	−5.3
1900	+2.8	+1.5	−1.0	−2.2	−4.4	−4.4	−3.0	−5.2	−4.8	−0.3	−3.7	−5.0
1800	+2.8	+1.6	−0.9	−2.0	−4.2	−4.2	−2.8	−5.0	−4.5	−0.2	−3.4	−4.8
1700	+2.8	+1.6	−0.8	−1.8	−3.9	−3.9	−2.6	−4.7	−4.2	−0.1	−3.3	−4.5
1600	+2.8	+1.7	−0.7	−1.6	−3.7	−3.7	−2.4	−4.5	−4.0	+0.1	−3.1	−4.3
1500	+2.7	+1.7	−0.6	−1.4	−3.5	−3.5	−2.2	−4.3	−3.7	+0.2	−2.9	−4.0
1400	+2.6	+1.7	−0.5	−1.2	−3.2	−3.3	−2.0	−4.1	−3.4	+0.3	−2.7	−3.8
1300	+2.5	+1.7	−0.5	−1.0	−3.0	−3.0	−1.8	−3.8	−3.2	+0.4	−2.5	−3.5
1200	+2.3	+1.6	−0.4	−0.9	−2.8	−2.8	−1.6	−3.6	−2.9	+0.5	−2.3	−3.2
1100	+2.2	+1.6	−0.4	−0.7	−2.6	−2.6	−1.4	−3.4	−2.6	+0.5	−2.1	−3.0
1000	+2.1	+1.5	−0.3	−0.6	−2.3	−2.4	−1.2	−3.1	−2.4	+0.6	−1.9	−2.7
900	+2.0	+1.4	−0.3	−0.5	−2.1	−2.1	−1.0	−2.9	−2.2	+0.6	−1.7	−2.5
800	+1.8	+1.3	−0.2	−0.4	−1.9	−1.9	−0.9	−2.6	−1.9	+0.6	−1.5	−2.2
700	+1.6	+1.1	−0.2	−0.3	−1.6	−1.7	−0.7	−2.3	−1.7	+0.5	−1.4	−1.9
600	+1.4	+1.0	−0.1	−0.2	−1.4	−1.4	−0.6	−2.0	−1.4	+0.5	−1.1	−1.7
500	+1.2	+0.8	−0.1	−0.2	−1.2	−1.2	−0.4	−1.7	−1.2	+0.4	−1.0	−1.4
400	+1.0	+0.7	−0.1	−0.1	−1.0	−1.0	−0.3	−1.4	−0.9	+0.4	−0.8	−1.1
300	+0.7	+0.5	0	−0.1	−0.7	−0.7	−0.2	−1.2	−0.7	+0.3	−0.6	−0.9
200	+0.5	+0.3	0	−0.1	−0.5	−0.5	−0.2	−0.8	−0.4	+0.2	−0.4	−0.6
100	+0.2	+0.2	0	0	−0.2	−0.2	−0.1	−0.4	−0.2	+0.1	−0.2	−0.3
0	0	0	0	0	0	0	0	0	0	0	0	0

Extrapolation below the earth's surface common for all pressures and seasons.

Dynamic meters.	Temperature correction (°C.).	Dynamic meters.	Temperature correction (°C.).	Dynamic meters.	Temperature correction (°C.).
− 0	0	−500	+1.2	−1000	+2.5
−100	+0.2	−600	+1.5	−1100	+2.8
−200	+0.5	−700	+1.8	−1200	+3.0
−300	+0.8	−800	+2.0		
−400	+1.0	−900	+2.2		

Example.—Low pressure, autumn, at the station pressure 984.3 m-bars, virtual temperature 9.4°.

1	2	3	4	5	6
800	1624	−4.3	+5.1	+29	1653
900	701	−1.9	+7.5	+19	720
1000	−124	+0.3	+9.7	− 4	−128

Column 1. Standard surfaces.
2. Approximate height of these surfaces, found from table 11 M for the pressure of 984.3 m-bars at the station.
3. Temperature corrections according to table 15 M for low pressure, autumn, and for the heights of column 2.
4. Most probable average virtual temperature of the sheets between the earth and the standard surfaces of column 1, found by addition of the corrections of column 3 to the virtual temperature at the station 9.4°.
5. Corrections to the heights of column 2, found from table 12 M for the heights of column 2 and the average virtual temperatures of column 4.
6. Heights of the standard surfaces above the station, found by addition of the approximate heights of column 2 and the corrections of column 5.

Table 16 M.—*Temperature corrections for the extrapolation of virtual-temperature diagrams (based upon statistics).*

Height (dynamic meters).	Winter.			Spring.			Summer.			Autumn.		
	High.	Mean.	Low.	High.	Mean.	Low.	High.	Mean.	Low.	High.	Mean.	Low.
	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.	° C.
2400	+0.6	−2.7	−4.9	−8.0	−11.6	−11.7	−8.9	−11.8	−13.3	−5.2	−9.6	−12.8
2300	+1.0	−2.1	−4.5	−7.6	−11.0	−11.1	−8.4	−11.3	−12.6	−4.7	−9.1	−12.2
2200	+1.4	−1.6	−4.2	−7.2	−10.5	−10.5	−8.0	−10.8	−12.0	−4.2	−8.7	−11.7
2100	+1.8	−1.1	−3.8	−6.8	− 9.9	− 9.9	−7.6	−10.3	−11.4	−3.8	−8.2	−11.1
2000	+2.2	−0.7	−3.4	−6.4	− 9.4	− 9.4	−7.2	− 9.8	−10.8	−3.4	−7.8	−10.6
1900	+2.6	−0.2	−3.1	−6.0	− 8.9	− 8.9	−6.9	− 9.4	−10.1	−3.0	−7.4	−10.0
1800	+3.0	+0.3	−2.8	−5.6	− 8.4	− 8.4	−6.5	− 8.9	− 9.5	−2.6	−7.0	− 9.5
1700	+3.3	+0.7	−2.4	−5.2	− 7.9	− 7.9	−6.1	− 8.5	− 8.9	−2.3	−6.6	− 8.9
1600	+3.6	+1.1	−2.1	−4.8	− 7.4	− 7.4	−5.7	− 8.1	− 8.3	−2.0	−6.2	− 8.4
1500	+3.9	+1.4	−1.8	−4.4	− 6.9	− 6.9	−5.3	− 7.7	− 7.7	−1.6	−5.8	− 7.9
1400	+4.2	+1.8	−1.5	−3.8	− 6.4	− 6.4	−4.9	− 7.2	− 7.1	−1.1	−5.4	− 7.4
1300	+4.2	+2.0	−1.3	−3.2	− 6.0	− 5.9	−4.5	− 6.8	− 6.5	−0.7	−5.0	− 6.8
1200	+4.3	+2.3	−1.1	−2.7	− 5.5	− 5.5	−4.0	− 6.4	− 6.0	−0.3	−4.6	− 6.3
1100	+3.6	+2.4	−0.9	−2.2	− 5.0	− 5.0	−3.5	− 6.0	− 5.4	0.0	−4.2	− 5.8
1000	+3.0	+2.5	−0.7	−1.8	− 4.6	− 4.6	−3.1	− 5.6	− 4.9	+0.3	−3.8	− 5.3
900	+3.2	+2.4	−0.6	−1.4	− 4.1	− 4.2	−2.7	− 5.2	− 4.4	+0.6	−3.4	− 4.8
800	+3.4	+2.3	−0.5	−1.2	− 3.7	− 3.8	−2.2	− 4.8	− 3.9	+0.8	−3.0	− 4.3
700	+3.0	+2.1	−0.5	−0.9	− 3.2	− 3.3	−1.8	− 4.4	− 3.4	+0.8	−2.6	− 3.8
600	+2.7	+1.9	−0.5	−0.6	− 2.8	− 2.9	−1.3	− 3.9	− 2.9	+0.8	−2.2	− 3.3
500	+2.3	+1.6	−0.3	−0.4	− 2.3	− 2.4	−1.0	− 3.2	− 2.4	+0.7	−1.8	− 2.7
400	+1.9	+1.4	−0.2	−0.3	− 1.9	− 1.9	−0.7	− 2.4	− 1.9	+0.6	−1.5	− 2.2
300	+1.4	+1.0	−0.1	−0.2	− 1.4	− 1.4	−0.5	− 2.0	− 1.4	+0.5	−1.2	− 1.7
200	+1.0	+0.7	0	−0.1	− 1.0	− 1.0	−0.3	− 1.6	− 0.9	+0.4	−0.8	− 1.2
100	+0.5	+0.3	0	−0.1	− 0.5	− 0.5	−0.2	− 0.8	− 0.4	+0.2	−0.4	− 0.6
0	0	0	0	0	0	0	0	0	0	0	0	0

Below the earth's surface.

Height (dynamic meters).	Temperature correction (° C.).	Height (dynamic meters).	Temperature correction (° C.).	Height (dynamic meters).	Temperature correction (° C.).
0	0				
−100	+0.5	−400	+2	− 800	+4
−200	+1	−500	+2.5	− 900	+4.5
−300	+1.5	−600	+3	−1000	+5
		−700	+3.5		

Examples.

(1) Spring, mean pressure. Required the virtual temperature 1761 dynamic meters above the earth's surface:

Given virtual temperature at the earth's surface............	+12.8
Table 16 gives for spring, mean pressure, 1761 dynamic meters..	— 8.2
Required virtual temperature................................	+ 4.6

(2) Summer, high pressure. Height of station 391 dynamic meters above sea-level. Virtual temperature at the station +7.1. Required the virtual temperatures in standard heights. These are found as shown in the annexed scheme.

Standard heights (dynamic meters).	Corresponding heights above station (dynamic meters).	Corrections from table 16 to be added to virtual temperature +7.1 at station (° C.).	Required virtual temperature in standard heights (° C.).
2500	2109	−7.6	−0.5
2000	1609	−5.7	+1.4
1500	1109	−3.5	+3.6
1000	609	−1.3	+5.8
500	109	−0.2	+6.9
0	−391	+2	+9.1

Table 17 M. —*Change of topographic charts of isobaric surfaces into isobaric charts in level surfaces.*

I. HEIGHT OF THE 1000 M-BARS SURFACE (DYNAMIC METERS).

Pressure at sea-level (m-bars).	Virtual temperature (° C.) in isobaric surface (1000 m-bars).										
	−50	−40	−30	−20	−10	0	10	20	30	40	50
950	−330	−344	−359	−374	−389	−403	−418	−433	−448	−463	−477
955	−296	−309	−322	−336	−349	−362	−375	−389	−402	−415	−428
960	−262	−274	−286	−297	−309	−321	−333	−344	−356	−368	−380
965	−229	−239	−249	−259	−270	−280	−290	−300	−311	−321	−331
970	−195	−204	−213	−222	−230	−239	−248	−257	−265	−274	−283
975	−162	−170	−177	−184	−192	−199	−206	−213	−221	−228	−235
980	−129	−135	−141	−147	−153	−158	−164	−170	−176	−182	−187
985	− 97	−101	−105	−110	−114	−118	−123	−127	−132	−136	−140
990	− 64	− 67	− 70	− 73	− 76	− 79	− 82	− 85	− 87	− 90	− 93
995	− 32	− 34	− 35	− 36	− 38	− 39	− 41	− 42	− 44	− 45	− 46
1000	0	0	0	0	0	0	0	0	0	0	0
1005	32	33	35	36	38	39	41	42	44	45	46
1010	64	67	69	72	75	78	81	84	87	89	92
1015	95	100	104	108	112	117	121	125	130	134	138
1020	127	132	138	144	149	155	161	166	172	178	183
1025	158	165	172	179	186	193	200	207	214	221	228
1030	189	197	206	214	223	231	240	248	257	265	274
1035	220	230	239	249	259	269	279	289	299	308	318
1040	250	261	273	284	295	306	318	329	340	351	363
1045	281	293	306	319	331	344	356	369	382	394	407
1050	311	325	339	353	367	381	395	409	423	437	451

II. HEIGHT OF THE 900 M-BARS SURFACE (DYNAMIC METERS).

Pressure in 1000 dynamic meters height (m-bars).	Virtual temperature (° C.) in isobaric surface (900 m-bars).									
	−50	−40	−30	−20	−10	0	10	20	30	40
830	479	455	432	409	385	362	338	315	292	268
835	518	496	474	453	431	409	388	366	345	323
840	556	536	516	496	477	457	437	417	397	377
845	595	576	558	540	522	504	486	467	449	431
850	633	616	600	583	567	550	534	517	501	484
855	671	656	641	626	611	597	582	567	552	538
860	708	695	682	669	656	643	630	616	603	590
865	745	734	723	711	700	688	677	666	654	643
870	782	773	763	753	743	734	724	714	704	695
875	819	811	803	795	787	779	771	763	755	746
880	856	849	843	837	830	824	817	811	804	798
885	892	887	883	878	873	868	863	858	854	849
890	928	925	922	919	916	912	909	906	903	900
895	964	963	961	960	958	956	955	953	952	950
900	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
905	1036	1037	1039	1040	1042	1043	1045	1047	1048	1050
910	1071	1074	1077	1080	1083	1087	1090	1093	1096	1099
915	1106	1110	1115	1120	1125	1129	1134	1139	1144	1148
920	1141	1147	1153	1159	1166	1172	1178	1185	1181	1197
925	1175	1183	1191	1199	1206	1214	1222	1230	1238	1246
930	1209	1219	1228	1238	1247	1256	1266	1275	1285	1294

Example 1.—Given the topographic and isothermic charts of the 1000 m-bar surface. Required the course of the 1025 m-bar curve in sea-level.

To the pressure 1025 m-bars in table 17 M (1) corresponds height 186 dynamic meters for temperature —10° C.; height 193 dynamic meters for temperature 0° C.; height 200 dynamic meters for temperature +10° C.; and so on. The required curve runs through the points of intersection of the isothermic curve —10° C. with the level curve 186 dynamic meters; of the isothermic curve 0° C. with the level curve 193 dynamic meters; of the isothermic curve +10° C. with the level curve 200 dynamic meters; and so on.

Table 17 M (continued).—*Change of topographic charts of isobaric surfaces into isobaric charts in level surfaces.*

III. Height of the 800 m-bars Surface (Dynamic Meters).

Pressure in 2000 dynamic meters height (m-bars).	Virtual temperature (° C.) in isobaric surface (800 m-bars).								
	−50	−40	−30	−20	−10	0	10	20	30
735	1454	1430	1405	1381	1356	1332	1307	1283	1259
740	1498	1476	1453	1431	1408	1386	1363	1341	1318
745	1542	1521	1501	1480	1460	1439	1418	1398	1377
750	1585	1566	1548	1529	1510	1492	1473	1455	1436
755	1628	1611	1595	1578	1561	1544	1528	1511	1494
760	1670	1656	1641	1626	1611	1596	1582	1567	1552
765	1713	1700	1687	1674	1661	1648	1635	1622	1610
770	1755	1744	1733	1722	1711	1700	1689	1678	1667
775	1796	1787	1778	1769	1760	1751	1741	1732	1723
780	1838	1830	1823	1816	1808	1801	1794	1787	1779
785	1879	1873	1868	1862	1857	1851	1846	1841	1835
790	1919	1916	1912	1909	1905	1901	1898	1894	1891
795	1960	1958	1956	1954	1953	1951	1949	1947	1945
800	2000	2000	2000	2000	2000	2000	2000	2000	2000
805	2040	2042	2044	2045	2047	2049	2051	2052	2054
810	2080	2083	2087	2090	2094	2097	2101	2104	2108
815	2119	2124	2129	2135	2140	2145	2151	2156	2161
820	2158	2165	2172	2179	2186	2193	2200	2207	2214
825	2196	2205	2214	2223	2232	2241	2249	2258	2267
830	2235	2246	2256	2267	2277	2288	2298	2309	2319
835	2273	2286	2298	2310	2322	2335	2347	2359	2371

IV. Height of the 700 m-bars Surface (Dynamic Meters).

Pressure in 3000 dynamic meters height (m-bars).	Virtual temperature (° C.) in isobaric surface (700 m-bars).								
	−60	−50	−40	−30	−20	−10	0	10	20
630	2351	2320	2290	2259	2229	2198	2168	2138	2107
635	2400	2372	2344	2315	2287	2259	2231	2203	2174
640	2449	2423	2397	2371	2345	2319	2293	2267	2241
645	2497	2473	2449	2426	2402	2379	2355	2331	2308
650	2544	2523	2502	2480	2459	2437	2416	2395	2373
655	2592	2573	2553	2534	2515	2496	2477	2458	2438
660	2639	2622	2605	2588	2571	2554	2537	2520	2503
665	2685	2670	2656	2641	2626	2611	2597	2582	2567
670	2731	2719	2706	2693	2681	2668	2656	2643	2630
675	2777	2766	2756	2746	2735	2725	2714	2704	2693
680	2822	2814	2806	2797	2789	2781	2772	2764	2756
685	2867	2861	2855	2849	2842	2836	2830	2824	2817
690	2912	2908	2904	2899	2895	2891	2887	2883	2879
695	2956	2954	2952	2950	2948	2946	2944	2942	2939
700	3000	3000	3000	3000	3000	3000	3000	3000	3000
705	3044	3046	3048	3050	3052	3054	3056	3058	3060
710	3087	3091	3095	3099	3103	3107	3111	3115	3119
715	3129	3136	3142	3148	3154	3160	3166	3172	3178
720	3172	3180	3188	3196	3204	3212	3220	3228	3236
725	3214	3224	3234	3244	3254	3264	3274	3284	3294
730	3256	3268	3280	3292	3304	3316	3328	3340	3352

Example 2.—Given the topographic and the isothermic chart of the 700 m-bars surface. Required the isobaric chart in level 3000 dynamic meters.

Table 17 M (IV) shows that—

The isobaric curve 700 m-bars is identical with the level curve 3000 dynamic meters.

The isobaric curve 695 m-bars goes through the points of intersection of the isothermic curve —60° C. with the level curve 2956 dynamic meters; of the isothermic curve —50° C. with the level curve 2954 dynamic meters; and so on.

The isobaric curve 690 m-bars goes through the points of intersection of the isothermic curve —60° C. with the level curve 2912 dynamic meters; of the isothermic curve —50° C. with the level curve 2908 dynamic meters; and so on.

Table 17 M (continued).—*Change of topographic charts of isobaric surfaces into isobaric charts in level surfaces.*

V. HEIGHT OF THE 600 M-BARS SURFACE (DYNAMIC METERS).

Pressure in 4000 dynamic meters height (m-bars).	Virtual temperature (° C.) at isobaric surface (600 m-bars).									
	−70	−60	−50	−40	−30	−20	−10	0	10	20
550	3490	3465	3439	3414	3389	3364	3339	3314	3289	3264
555	3543	3521	3498	3476	3453	3431	3408	3386	3363	3341
560	3596	3576	3556	3536	3516	3496	3477	3457	3437	3417
565	3648	3631	3614	3596	3579	3562	3544	3527	3510	3492
570	3700	3685	3670	3656	3641	3626	3611	3596	3582	3567
575	3751	3739	3727	3715	3702	3690	3678	3666	3653	3641
380	3802	3792	3783	3773	3763	3753	3743	3734	3724	3714
585	3852	3845	3838	3830	3823	3816	3809	3801	3794	3787
590	3902	3897	3892	3887	3883	3878	3873	3868	3863	3858
595	3951	3949	3946	3944	3942	3939	3937	3934	3932	3930
600	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
605	4048	4051	4053	4056	4058	4060	4063	4065	4067	4070
610	4096	4101	4106	4110	4115	4120	4125	4129	4134	4139
615	4144	4151	4158	4165	4172	4179	4186	4193	4200	4207
620	4191	4200	4209	4219	4228	4238	4247	4256	4266	4275
625	4237	4249	4261	4272	4284	4296	4307	4319	4331	4342
630	4283	4297	4311	4325	4339	4353	4367	4381	4395	4409
635	4329	4345	4361	4378	4394	4410	4426	4442	4459	4475
640	4374	4393	4411	4430	4448	4467	4485	4503	4522	4540
645	4419	4440	4461	4481	4502	4522	4543	4564	4584	4605
650	4464	4487	4509	4532	4555	4578	4601	4624	4646	4669

VI. HEIGHT OF THE 500 M-BARS SURFACE (DYNAMIC METERS).

Pressure in 5000 dynamic meters height (m-bars).	Virtual temperature (° C.) at isobaric surface (500 m-bars).								
	−70	−60	−50	−40	−30	−20	−10	0	10
465	4575	4554	4533	4512	4491	4470	4449	4428	4407
470	4638	4620	4602	4584	4567	4549	4531	4513	4495
475	4700	4685	4670	4656	4641	4626	4611	4597	4582
480	4761	4750	4738	4726	4714	4703	4691	4679	4667
485	4822	4813	4805	4796	4787	4778	4770	4761	4752
490	4882	4876	4871	4865	4859	4853	4847	4842	4836
495	4941	4939	4936	4933	4930	4927	4924	4921	4918
500	5000	5000	5000	5000	5000	5000	5000	5000	5000
505	5058	5061	5064	5066	5069	5072	5075	5078	5081
510	5115	5121	5127	5132	5138	5144	5149	5155	5161
515	5172	5180	5189	5197	5206	5214	5223	5231	5240
520	5228	5239	5250	5262	5273	5284	5295	5306	5318
525	5283	5297	5311	5325	5339	5353	5367	5381	5395
530	5338	5355	5371	5388	5405	5421	5438	5455	5471
535	5392	5412	5431	5450	5470	5489	5508	5528	5547
540	5446	5468	5490	5512	5534	5556	5578	5600	5622
545	5499	5524	5548	5573	5597	5622	5647	5671	5696
550	5551	5579	5606	5633	5660	5687	5714	5742	5769
555	5603	5633	5663	5693	5722	5752	5782	5812	5841
560	5655	5687	5719	5752	5784	5816	5849	5881	5913
565	5706	5741	5775	5810	5845	5880	5914	5949	5984

Table 17 M (continued).—*Change of topographic charts of isobaric surfaces into isobaric charts in level surfaces.*

VII. HEIGHT OF THE 400 M-BARS SURFACE (DYNAMIC METERS).

Pressure in 7000 dynamic meters height (m-bars).	Virtual temperature (° C.) at isobaric surface (400 m-bars).								
	−80	−70	−60	−50	−40	−30	−20	−10	0
350	6253	6214	6176	6137	6098	6060	6021	5982	5944
355	6333	6299	6264	6229	6195	6160	6126	6091	6057
360	6412	6381	6351	6320	6290	6259	6229	6198	6168
365	6489	6463	6436	6410	6383	6357	6331	6304	6278
370	6566	6543	6521	6498	6476	6453	6431	6408	6386
375	6641	6622	6604	6585	6566	6548	6529	6510	6492
380	6715	6700	6685	6670	6656	6641	6626	6611	6596
385	6788	6777	6766	6755	6744	6733	6722	6711	6700
390	6859	6852	6845	6838	6830	6823	6816	6808	6801
395	6930	6927	6923	6919	6916	6912	6909	6905	6901
400	7000	7000	7000	7000	7000	7000	7000	7000	7000
405	7069	7072	7076	7080	7083	7087	7090	7094	7097
410	7137	7144	7151	7158	7165	7172	7179	7186	7193
415	7203	7214	7225	7235	7246	7256	7267	7277	7288
420	7269	7283	7297	7311	7325	7339	7353	7367	7381
425	7334	7352	7369	7386	7404	7421	7438	7456	7473
430	7399	7419	7440	7461	7481	7502	7522	7543	7564
435	7462	7486	7510	7534	7558	7582	7605	7629	7653
440	7524	7551	7579	7606	7633	7660	7687	7715	7742
445	7586	7616	7647	7677	7707	7738	7768	7799	7829
450	7646	7680	7713	7747	7780	7814	7847	7881	7914

VIII. HEIGHT OF THE 300 M-BARS SURFACE (DYNAMIC METERS).

Pressure in 9000 dynamic meters height (m-bars).	Virtual temperature (° C.) at isobaric surface (300 m-bars).								
	−90	−80	−70	−60	−50	−40	−30	−20	−10
250	8030	7977	7924	7871	7818	7765	7711	7658	7605
255	8136	8089	8042	7995	7947	7900	7853	7806	7759
260	8240	8199	8157	8116	8074	8033	7991	7950	7908
265	8343	8307	8271	8235	8199	8163	8127	8091	8055
270	8442	8412	8381	8351	8320	8290	8259	8229	8198
275	8540	8515	8490	8465	8440	8414	8389	8364	8339
280	8636	8616	8596	8576	8556	8536	8516	8497	8477
285	8730	8715	8700	8685	8670	8656	8641	8626	8611
290	8822	8812	8802	8792	8783	8773	8763	8753	8744
295	8912	8907	8902	8897	8892	8887	8883	8878	8873
300	9000	9000	9000	9000	9000	9000	9000	9000	9000
305	9087	9091	9096	9101	9106	9110	9115	9120	9125
310	9172	9181	9191	9200	9209	9219	9228	9238	9247
315	9255	9269	9283	9297	9311	9325	9339	9353	9367
320	9337	9356	9374	9393	9411	9430	9448	9467	9485
325	9418	9441	9464	9487	9509	9532	9555	9578	9601
330	9497	9524	9551	9579	9606	9633	9660	9687	9714
335	9575	9606	9638	9669	9701	9732	9763	9795	9826
340	9652	9687	9723	9758	9794	9830	9865	9901	9936
345	9727	9766	9806	9846	9886	9925	9965	10005	10044
350	9801	9844	9888	9932	9976	10020	10063	10107	10151

DYNAMIC METEOROLOGY AND HYDROGRAPHY

By V. BJERKNES
AND DIFFERENT COLLABORATORS

APPENDIX

TO

METEOROLOGICAL AND HYDROGRAPHIC TABLES

Containing Tables to be Used when the Observations are Given in Units not Belonging to the C.G.S. System

Table 1 A. — *Heights reduced from feet to meters.*

Feet.	0	1000	2000	3000	4000	5000	6000	7000	8000	9000
90000	27432	27736	28041	28346	28651	28955	29260	29565	29870	30175
80000	24384	24688	24993	25298	25603	25908	26212	26517	26822	27127
70000	21336	21640	21945	22250	22555	22860	23164	23469	23774	24079
60000	18288	18592	18897	19202	19507	19812	20116	20421	20726	21031
50000	15240	15545	15849	16154	16459	16764	17068	17373	17678	17983
40000	12192	12497	12801	13106	13411	13716	14021	14325	14630	14935
30000	9144	9449	9753	10058	10363	10668	10973	11277	11582	11887
20000	6096	6401	6705	7010	7315	7620	7925	8229	8534	8839
10000	3048	3353	3658	3962	4267	4572	4877	5182	5486	5791
0	0	305	610	914	1219	1524	1829	2134	2438	2743

Feet.	0	10	20	30	40	50	60	70	80	90
900	274	277	280	283	287	290	293	296	299	302
800	244	247	250	253	256	259	262	265	268	271
700	213	216	219	223	226	229	232	235	238	241
600	183	186	189	192	195	198	201	204	207	210
500	152	155	158	162	165	168	171	174	177	180
400	122	125	128	131	134	137	140	143	146	149
300	91	94	98	101	104	107	110	113	116	119
200	61	64	67	70	73	76	79	82	85	88
100	30	34	37	40	43	46	49	52	55	58
0	0	3	6	9	12	15	18	21	24	27

Example: Given 33562 feet. Meters.

First section of the table gives for 33000 feet.........10058

Second section of the table gives for 562 feet......... 172

10230

Table 2 A.—*Heights reduced from feet to dynamic meters, the acceleration of gravity at sea-level being 9.80.*

Height (feet).	0	1000	2000	3000	4000	5000	6000	7000	8000	9000
90000	26767	27063	27359	27655	27951	28247	28543	28839	29135	29431
80000	23804	24101	24397	24693	24990	25286	25582	25878	26175	26471
70000	20839	21135	21432	21729	22025	22322	22618	22915	23211	23508
60000	17870	18167	18464	18761	19058	19355	19652	19948	20245	20542
50000	14899	15196	15494	15791	16088	16385	16682	16979	17276	17573
40000	11925	12223	12520	12818	13115	13412	13710	14007	14304	14602
30000	8948	9246	9544	9841	10139	10437	10735	11032	11330	11627
20000	5968	6266	6564	6862	7161	7459	7757	8054	8352	8650
10000	2986	3284	3582	3881	4179	4477	4775	5074	5372	5670
0	0	299	597	896	1195	1493	1792	2090	2389	2687

Proportionality Table.

Feet.	0	10	20	30	40	50	60	70	80	90
900	269	272	275	278	281	284	287	290	293	296
800	239	242	245	248	251	254	357	260	263	266
700	209	212	215	218	221	224	227	230	233	236
600	179	182	185	188	191	194	197	200	203	206
500	149	152	155	158	161	164	167	170	173	176
400	119	122	125	128	131	134	137	140	143	146
300	90	93	96	99	102	105	108	111	114	116
200	60	63	66	69	72	75	78	81	84	87
100	30	33	36	39	42	45	48	51	54	57
0	0	3	6	9	12	15	18	21	24	27

Table 3 A.—*Corrections to table 2 A for values of the acceleration of gravity at sea-level different from 9.80.*

Height (feet).	Acceleration of gravity.								
	9.76	9.77	9.78	9.79	9.80	9.81	9.82	9.83	9.84
90000	−110	−82	−55	−27	0	27	55	82	110
80000	− 98	−73	−49	−24	0	24	49	73	98
70000	− 85	−64	−43	−21	0	21	43	64	85
60000	− 73	−55	−37	−18	0	18	37	55	73
50000	− 61	−46	−30	−15	0	15	30	46	61
40000	− 49	−37	−24	−12	0	12	24	37	49
30000	− 37	−27	−18	− 9	0	9	18	27	37
20000	− 24	−18	−12	− 6	0	6	12	18	24
10000	− 12	− 9	− 6	− 3	0	3	6	9	12
0	0	0	0	0	0	0	0	0	0

Example, tables 2 A and 3 A.

(1)	(2)	(3)	(4)	(5)
4871	1195	260	+ 3	1458
11492	3284	147	+ 7	3438
19601	5670	179	+12	5861
32416	9544	124	+19	9687

Column 1. Heights above sea-level.

2. Values of table 2 A for the heights 4000, 11000, 19000, 32000.
3. Values of proportionality table for the heights 871, 492, 601, 416.
4. Corrections from table 3 A for $g = 9.8197$ at sea-level and for the heights of column 1.
5. Sum of numbers in columns 2, 3, and 4, giving the dynamic heights corresponding to the geometrical heights of column 1.

Table 4 A.—*Depths reduced from fathoms to meters.*

Fathoms.	0	100	200	300	400	500	600	700	800	900
0	0	183	366	549	732	914	1097	1280	1463	1646
1000	1829	2012	2195	2377	2560	2743	2926	3109	3292	3475
2000	3658	3840	4023	4206	4389	4572	4755	4938	5121	5303
3000	5486	5669	5852	6035	6218	6401	6584	6766	6949	7132
4000	7315	7498	7681	7864	8047	8229	8412	8595	8778	8961
5000	9144	9327	9510	9692	9875	10058	10241	10424	10607	10790
Fathoms.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	2	4	5	7	9	11	13	15	16
10	18	20	22	24	26	27	29	31	33	35
20	37	38	40	42	44	46	48	49	51	53
30	55	57	59	60	62	64	66	68	69	71
40	73	75	77	79	80	82	84	86	88	90
50	91	93	95	97	99	101	102	104	106	108
60	110	112	113	115	117	119	121	123	124	126
70	128	130	132	133	135	137	139	141	143	144
80	146	148	150	152	154	155	157	159	161	163
90	165	166	168	170	172	174	176	177	179	181

Example: Given 3678 fathoms. Meters.
First section of the table gives for 3600 fathoms......... 6584
Second section of the table gives for 78 fathoms......... 143
6727

Table 5 A.—*Depths reduced from fathoms to dynamic meters, the acceleration of gravity at sea-level being 9.80.*

Depth (fathoms).	0	100	200	300	400	500	600	700	800	900
0	0	179	358	538	717	896	1075	1255	1434	1613
1000	1793	1972	2151	2330	2510	2689	2868	3048	3227	3407
2000	3586	3765	3945	4124	4303	4483	4662	4842	5021	5200
3000	5380	5559	5739	5918	6098	6277	6457	6636	6816	6995
4000	7175	7354	7534	7713	7893	8072	8252	8431	8611	8791
5000	8970	9150	9329	9509	9689	9868	10048	10227	10407	10587

Proportionality Table.

Fathoms.	0	1	2	3	4	5	6	7	8	9
0	0	2	4	5	7	9	11	13	14	16
10	18	20	21	23	25	27	29	30	32	34
20	36	38	39	41	43	45	47	48	50	52
30	54	56	57	59	61	63	65	66	68	70
40	72	73	75	77	79	81	82	84	86	88
50	90	91	93	95	97	99	100	102	104	106
60	108	109	111	113	115	116	118	120	122	124
70	125	127	129	131	133	134	136	138	140	142
80	143	145	147	149	151	152	154	156	158	159
90	161	163	165	167	168	170	172	174	176	177

Table 6 A.—*Corrections to table 5 A for values of the acceleration of gravity different from 9.80.*

Depth (fathoms).	Acceleration of gravity at sea-level.						
	9.78	9.79	9.80	9.81	9.82	9.83	9.84
0	0	0	0	0	0	0	0
1000	− 4	−2	0	2	4	5	7
2000	− 7	−4	0	4	7	11	15
3000	−11	−5	0	5	11	16	22
4000	−15	−7	0	7	15	22	29
5000	−18	−9	0	9	18	27	37

Example: Depth 2769 fathoms.
Value of table 5 A for the depth of 2700 fathoms 4842
Value of proportionality table for the depth of 69 fathoms.......... 124
Correction from table 6 A for gravity 9.8197 at sea-level and for the depth 2769 fathoms......... +10
Depth in dynamic meters.......... 4976

Table 7 A.—*Pressure reduced from millimeters of mercury to millibars.*

Milli-meters of mercury.	0	1	2	3	4	5	6	7	8	9
0	0	1.3	2.7	4.0	5.3	6.7	8.0	9.3	10.7	12.0
10	13.3	14.7	16.0	17.3	18.7	20.0	21.3	22.7	24.0	25.3
20	26.7	28.0	29.3	30.7	32.0	33.3	34.7	36.0	37.3	38.7
30	40.0	41.3	42.7	44.0	45.3	46.7	48.0	49.3	50.7	52.0
40	53.3	54.7	56.0	57.3	58.7	60.0	61.3	62.7	64.0	65.3
50	66.7	68.0	69.3	70.7	72.0	73.3	74.7	76.0	77.3	78.7
60	80.0	81.3	82.7	84.0	85.3	86.7	88.0	89.3	90.7	92.0
70	93.3	94.7	96.0	97.3	98.7	100.0	101.3	102.7	104.0	105.3
80	106.7	108.0	109.3	110.7	112.0	113.3	114.7	116.0	117.3	118.7
90	120.0	121.3	122.7	124.0	125.3	126.7	128.0	129.3	130.7	132.0
100	133.3	134.7	136.0	137.3	138.7	140.0	141.3	142.7	144.0	145.3
110	146.7	148.0	149.3	150.7	152.0	153.3	154.7	156.0	157.3	158.6
120	160.0	161.3	162.6	164.0	165.3	166.6	168.0	169.3	170.6	172.0
130	173.3	174.6	176.0	177.3	178.6	180.0	181.3	182.6	184.0	185.3
140	186.6	188.0	189.3	190.6	192.0	193.3	194.6	196.0	197.3	198.6
150	200.0	201.3	202.6	204.0	205.3	206.6	208.0	209.3	210.6	212.0
160	213.3	214.6	216.0	217.3	218.6	220.0	221.3	222.6	224.0	225.3
170	226.6	228.0	229.3	230.6	232.0	233.3	234.6	236.0	237.3	238.6
180	240.0	241.3	242.6	244.0	245.3	246.6	248.0	249.3	250.6	252.0
190	253.3	254.6	256.0	257.3	258.6	260.0	261.3	262.6	264.0	265.3
200	266.6	268.0	269.3	270.6	272.0	273.3	274.6	276.0	277.3	278.6
210	280.0	281.3	282.6	284.0	285.3	286.6	288.0	289.3	290.6	292.0
220	293.3	294.6	296.0	297.3	298.6	300.0	301.3	302.6	304.0	305.3
230	306.6	308.0	309.3	310.6	312.0	313.3	314.6	316.0	317.3	318.6
240	320.0	321.3	322.6	324.0	325.3	326.6	328.0	329.3	330.6	332.0
250	333.3	334.6	336.0	337.3	338.6	340.0	341.3	342.6	344.0	345.3
260	346.6	348.0	349.3	350.6	352.0	353.3	354.6	356.0	357.3	358.6
270	360.0	361.3	362.6	364.0	365.3	366.6	368.0	369.3	370.6	372.0
280	373.3	374.6	376.0	377.3	378.6	380.0	381.3	382.6	384.0	385.3
290	386.6	388.0	389.3	390.6	392.0	393.3	394.6	396.0	397.3	398.6
300	400.0	401.3	402.6	404.0	405.3	406.6	408.0	409.3	410.6	412.0
310	413.3	414.6	416.0	417.3	418.6	420.0	421.3	422.6	424.0	425.3
320	426.6	428.0	429.3	430.6	432.0	433.3	434.6	436.0	437.3	438.6
330	440.0	441.3	442.6	444.0	445.3	446.6	448.0	449.3	450.6	452.0
340	453.3	454.6	456.0	457.3	458.6	460.0	461.3	462.6	464.0	465.3
350	466.6	468.0	469.3	470.6	472.0	473.3	474.6	475.9	477.3	478.6
360	479.9	481.3	482.6	483.9	485.3	486.6	487.9	489.3	490.6	491.9
370	493.3	494.6	495.9	497.3	498.6	499.9	501.3	502.6	503.9	505.3
380	506.6	507.9	509.3	510.6	511.9	513.3	514.6	515.9	517.3	518.6
390	519.9	521.3	522.6	523.9	525.3	526.6	527.9	529.3	530.6	531.9
400	533.3	534.6	535.9	537.3	538.6	539.9	541.3	542.6	543.9	545.3
410	546.6	547.9	549.3	550.6	551.9	553.3	554.6	555.9	557.3	558.6
420	559.9	561.3	562.6	563.9	565.3	566.6	567.9	569.3	570.6	571.9
430	573.3	574.6	575.9	577.3	578.6	579.9	581.3	582.6	583.9	585.3
440	586.6	587.9	589.3	590.6	591.9	593.3	594.6	595.9	597.3	598.6
450	599.9	601.3	602.6	603.9	605.3	606.6	607.9	609.3	610.6	611.9
460	613.3	614.6	615.9	617.3	618.6	619.9	621.3	622.6	623.9	625.3
470	626.6	627.9	629.3	630.6	631.9	633.3	634.6	635.9	637.3	638.6
480	639.9	641.3	642.6	643.9	645.3	646.6	647.9	649.3	650.6	651.9
490	653.3	654.6	655.9	657.3	658.6	659.9	661.3	662.6	663.9	665.3

Table 7 A (continued).—*Pressure reduced from millimeters of mercury to millibars.*

Millimeters of mercury.	0	1	2	3	4	5	6	7	8	9
500	666.6	667.9	669.3	670.6	671.9	673.3	674.6	675.9	677.3	678.6
510	679.9	681.3	682.6	683.9	685.3	686.6	687.9	689.3	690.6	691.9
520	693.3	694.6	695.9	697.3	698.6	699.9	701.3	702.6	703.9	705.3
530	706.6	707.9	709.3	710.6	711.9	713.3	714.6	715.9	717.3	718.6
540	719.9	721.3	722.6	723.9	725.3	726.6	727.9	729.3	730.6	731.9
550	733.3	734.6	735.9	737.3	738.6	739.9	741.3	742.6	743.9	745.3
560	746.6	747.9	749.3	750.6	751.9	753.3	754.6	755.9	757.3	758.6
570	759.9	761.3	762.6	763.9	765.3	766.6	767.9	769.3	770.6	771.9
580	773.3	774.6	775.9	777.3	778.6	779.9	781.3	782.6	783.9	785.3
590	786.6	787.9	789.3	790.6	791.9	793.2	794.6	795.9	797.2	798.6
600	799.9	801.2	802.6	803.9	805.2	806.6	807.9	809.2	810.6	811.9
610	813.2	814.6	815.9	817.2	818.6	819.9	821.2	822.6	823.9	825.2
620	826.6	827.9	829.2	830.6	831.9	833.2	834.6	835.9	837.2	838.6
630	839.9	841.2	842.6	843.9	845.2	846.6	847.9	849.2	850.6	851.9
640	853.2	854.6	855.9	857.2	858.6	859.9	861.2	862.6	863.9	865.2
650	866.6	867.9	869.2	870.6	871.9	873.2	874.6	875.9	877.2	878.6
660	879.9	881.2	882.6	883.9	885.2	886.6	887.9	889.2	890.6	891.9
670	893.2	894.6	895.9	897.2	898.6	899.9	901.2	902.6	903.9	905.2
680	906.6	907.9	909.2	910.6	911.9	913.2	914.6	915.9	917.2	918.6
690	919.9	921.2	922.6	923.9	925.2	926.6	927.9	929.2	930.6	931.9
700	933.2	934.6	935.9	937.2	938.6	939.9	941.2	942.6	943.9	945.2
710	946.6	947.9	949.2	950.6	951.9	953.2	954.6	955.9	957.2	958.6
720	959.9	961.2	962.6	963.9	965.2	966.6	967.9	969.2	970.6	971.9
730	973.2	974.6	975.9	977.2	978.6	979.9	981.2	982.6	983.9	985.2
740	986.6	987.9	989.2	990.6	991.9	993.2	994.6	995.9	997.2	998.6
750	999.9	1001.2	1002.6	1003.9	1005.2	1006.6	1007.9	1009.2	1010.6	1011.9
760	1013.2	1014.6	1015.9	1017.2	1018.6	1019.9	1021.2	1022.6	1023.9	1025.2
770	1026.6	1027.9	1029.2	1030.6	1031.9	1033.2	1034.6	1035.9	1037.2	1038.6
780	1039.9	1041.2	1042.6	1043.9	1045.2	1046.6	1047.9	1049.2	1050.6	1051.9
790	1053.2	1054.6	1055.9	1057.2	1058.6	1059.9	1061.2	1062.6	1063.9	1065.2
800	1066.6	1067.9	1069.2	1070.6	1071.9	1073.2	1074.6	1075.9	1077.2	1078.6
810	1079.9	1081.2	1082.6	1083.9	1085.2	1086.6	1087.9	1089.2	1090.6	1091.9

Example:

Pressure .... 622.2 mm. of mercury.
Table 7 A gives......... 829.5 m-bar.

Table 8 A.— *Pressure reduced from inches of mercury to millibars.*

Inches of mercury.	0	1	2	3	4	5	6	7	8	9
0	0	3.4	6.8	10.2	13.5	16.9	20.3	23.7	27.1	30.5
1	33.9	37.2	40.6	44.0	47.4	50.8	54.2	57.6	61.0	64.3
2	67.7	71.1	74.5	77.9	81.3	84.7	88.0	91.4	94.8	98.2
3	101.6	105.0	108.4	111.7	115.1	118.5	121.9	125.3	128.7	132.1
4	135.5	138.8	142.2	145.6	149.0	152.4	155.8	159.2	162.5	165.9
5	169 3	172.7	176.1	179.5	182.9	186.2	189.6	193.0	196.4	199.8
6	203.2	206.6	209.9	213.3	216.7	220.1	223.5	226.9	230.3	233.7
7	237.0	240.4	243.8	247.2	250.6	254.0	257.4	260.7	264.1	267.5
8	270.9	274.3	277.7	281.1	284.4	287.8	291.2	294.6	298.0	301.4
9	304.8	308.1	311.5	314.9	318.3	321.7	325.1	328.5	331.9	335.2
10	338.6	342.0	345.4	348.8	352.2	355.6	358.9	362.3	365.7	369.1
11	372.5	375.9	379.3	382.6	386.0	389.4	392.8	396.2	399.6	403.0
12	406.3	409.7	413.1	416.5	419.9	423.3	426.7	430.1	433.4	436.8
13	440.2	443.6	447.0	450.4	453.8	457.1	460.5	463.9	467.3	470.7
14	474.1	477.5	480.8	484.2	487.6	491.0	494.4	497.8	501.2	504.6
15	507.9	511.3	514.7	518.1	521.5	524.9	528.3	531.6	535.0	538.4
16	541.8	545.2	548.6	552.0	555.3	558.7	562.1	565.5	568.9	572.3
17	575.7	579.0	582.4	585.8	589.2	592.6	596.0	599.4	602.8	606.1
18	609.5	612.9	616.3	619.7	623.1	626.5	629.8	633.2	636.6	640.0
19	643.4	646.8	650.2	653.5	656.9	660.3	663.7	667.1	670.5	673.9
20	677.2	680.6	684.0	687.4	690.8	694.2	697.6	701.0	704.3	707.7
21	711.1	714.5	717.9	721.3	724.7	728.0	731.4	734.8	738.2	741.6
22	745.0	748.4	751.7	755.1	758.5	761.9	765.3	768.7	772.1	775.5
23	778.8	782.2	785.6	789.0	792.4	795.8	799.2	802.5	805.9	809.3
24	812.7	816.1	819.5	822.9	826.2	829.6	833.0	836.4	839.8	843.2
25.0	846.6	846.9	847.2	847.6	847.9	848.3	848.6	848.9	849.3	849.6
25.1	849.9	850.3	850.6	851.0	851.3	851.6	852.0	852.3	852.7	853.0
25.2	853.3	853.7	854.0	854.4	854.7	855.0	855.4	855.7	856.0	856.4
25.3	856.7	857.1	857.4	857.7	858.1	858.4	858.8	859.1	859.4	859.8
25.4	860.1	860.4	860.8	861.1	861.5	861.8	862.1	862.5	862.8	863.2
25.5	863.5	863.8	864.2	864.5	864.8	865.2	865.5	865.9	866.2	866.5
25.6	866.9	867.2	867.6	867.9	868.2	868.6	868.9	869.2	869.6	869.9
25.7	870.3	870.6	870.9	871.3	871.6	872.0	872.3	872.6	873.0	873.3
25.8	873.7	874.0	874.3	874.7	875.0	875.3	875.7	876.0	876.4	876.7
25.9	877.0	877.4	877.7	878.1	878.4	878.7	879.1	879.4	879.7	880.1
26.0	880.4	880.8	881.1	881.4	881.8	882.1	882.5	882.8	883.1	883.5
26.1	883.8	884.1	884.5	884.8	885.2	885.5	885.8	886.2	886.5	886.9
26.2	887.2	887.5	887.9	888.2	888.6	888.9	889.2	889.6	889.9	890.2
26.3	890.6	890.9	891.3	891.6	891.9	892.3	892.6	893.0	893.3	893.6
26.4	894.0	894.3	894.6	895.0	895.3	895.7	896.0	896.3	896.7	897.0
26.5	897.4	897.7	898.0	898.4	898.7	899.0	899.4	899.7	900.1	900.4
26.6	900.7	901.1	901.4	901.8	902.1	902.4	902.8	903.1	903.5	903.8
26.7	904.1	904.5	904.8	905.1	905.5	905.8	906.2	906.5	906.8	907.2
26.8	907.5	907.9	908.2	908.5	908.9	909.2	909.5	909.9	910.2	910.6
26.9	910.9	911.2	911.6	911.9	912.3	912.6	912.9	913.3	913.6	913.9
27.0	914.3	914.6	915.0	915.3	915.6	916.0	916.3	916.7	917.0	917.3
27.1	917.7	918.0	918.4	918.7	919.0	919.4	919.7	920.0	920.4	920.7
27.2	921.1	921.4	921.7	922.1	922.4	922.8	923.1	923.4	923.8	924.1
27.3	924.4	924.8	925.1	925.5	925.8	926.1	926.5	926.8	927.2	927.5
27.4	927.8	928.2	928.5	928.8	929.2	929.5	929.9	930.2	930.5	930.9

Table 8A (continued).—*Pressure reduced from inches of mercury to millibars.*

Inches of mercury.	0	1	2	3	4	5	6	7	8	9
27.5	931.2	931.6	931.9	932.2	932.6	932.9	933.2	933.6	933.9	934.3
27.6	934.6	934.9	935.3	935.6	936.0	936.3	936.6	937.0	937.3	937.7
27.7	938.0	938.3	938.7	939.0	939.3	939.7	940.0	940.4	940.7	941.0
27.8	941.4	941.7	942.1	942.4	942.7	943.1	943.4	943.7	944.1	944.4
27.9	944.8	945.1	945.4	945.8	946.1	946.5	946.8	947.1	947.5	947.8
28.0	948.1	948.5	948.8	949.2	949.5	949.8	950.2	950.5	950.9	951.2
28.1	951.5	951.9	952.2	952.6	952.9	953.2	953.6	953.9	954.2	954.6
28.2	954.9	955.3	955.6	955.9	956.3	956.6	957.0	957.3	957.6	958.0
28.3	958.3	958.6	959.0	959.3	959.7	960.0	960.3	960.7	961.0	961.4
28.4	961.7	962.0	962.4	962.7	963.0	963.4	963.7	964.1	964.4	964.7
28.5	965.1	965.4	965.8	966.1	966.4	966.8	967.1	967.5	967.8	968.1
28.6	968.5	968.8	969.1	969.5	969.8	970.2	970.5	970.8	971.2	971.5
28.7	971.9	972.2	972.5	972.9	973.2	973.5	973.9	974.2	974.6	974.9
28.8	975.2	975.6	975.9	976.3	976.6	976.9	977.3	977.6	977.9	978.3
28.9	978.6	979.0	979.3	979.6	980.0	980.3	980.7	981.0	981.3	981.7
29.0	982.0	982.3	982.7	983.0	983.4	983.7	984.0	984.4	984.7	985.1
29.1	985.4	985.7	986.1	986.4	986.8	987.1	987.4	987.8	988.1	988.4
29.2	988.8	989.1	989.5	989.8	990.1	990.5	990.8	991.2	991.5	991.8
29.3	992.2	992.5	992.8	993.2	993.5	993.9	994.2	994.5	994.9	995.2
29.4	995.6	995.9	996.2	996.6	996.9	997.2	997.6	997.9	998.3	998.6
29.5	998.9	999.3	999.6	1000.0	1000.3	1000.6	1001.0	1001.3	1001.7	1002.0
29.6	1002.3	1002.7	1003.0	1003.3	1003.7	1004.0	1004.4	1004.7	1005.0	1005.4
29.7	1005.7	1006.1	1006.4	1006.7	1007.1	1007.4	1007.7	1008.1	1008.4	1008.8
29.8	1009.1	1009.4	1009.8	1010.1	1010.5	1010.8	1011.1	1011.5	1011.8	1012.1
29.9	1012.5	1012.8	1013.2	1013.5	1013.8	1014.2	1014.5	1014.9	1015.2	1015.5
30.0	1015.9	1016.2	1016.6	1016.9	1017.2	1017.6	1017.9	1018.2	1018.6	1018.9
30.1	1019.3	1019.6	1019.9	1020.3	1020.6	1021.0	1021.3	1021.6	1022.0	1022.3
30.2	1022.6	1023.0	1023.3	1023.7	1024.0	1024.3	1024.7	1025.0	1025.4	1025.7
30.3	1026.0	1026.4	1026.7	1027.0	1027.4	1027.7	1028.1	1028.4	1028.7	1029.1
30.4	1029.4	1029.8	1030.1	1030.4	1030.8	1031.1	1031.5	1031.8	1032.1	1032.5
30.5	1032.8	1033.1	1033.5	1033.8	1034.2	1034.5	1034.8	1035.2	1035.5	1035.9
30.6	1036.2	1036.5	1036.9	1037.2	1037.5	1037.9	1038.2	1038.6	1038.9	1039.2
30.7	1039.6	1039.9	1040.3	1040.6	1040.9	1041.3	1041.6	1041.9	1042.3	1042.6
30.8	1043.0	1043.3	1043.6	1044.0	1044.3	1044.7	1045.0	1045.3	1045.7	1046.0
30.9	1046.4	1046.7	1047.0	1047.4	1047.7	1048.0	1048.4	1048.7	1049.1	1049.4
31.0	1049.7	1050.1	1050.4	1050.8	1051.1	1051.4	1051.8	1052.1	1052.4	1052.8
31.1	1053.1	1053.5	1053.8	1054.1	1054.5	1054.8	1055.2	1055.5	1055.8	1056.2
31.2	1056.5	1056.8	1057.2	1057.5	1057.9	1058.2	1058.5	1058.9	1059.2	1059.6
31.3	1059.9	1060.2	1060.6	1060.9	1061.2	1061.6	1061.9	1062.3	1062.6	1062.9
31.4	1063.3	1063.6	1064.0	1064.3	1064.6	1065.0	1065.3	1065.7	1066.0	1066.3
31.5	1066.7	1067.0	1067.3	1067.7	1068.0	1068.4	1068.7	1069.0	1069.4	1069.7
31.6	1070.1	1070.4	1070.7	1071.1	1071.4	1071.7	1072.1	1072.4	1072.8	1073.1
31.7	1073.4	1073.8	1074.1	1074.5	1074.8	1075.1	1075.5	1075.8	1076.1	1076.5
31.8	1076.8	1077.2	1077.5	1077.8	1078.2	1078.5	1078.9	1079.2	1079.5	1079.9
31.9	1080.2	1080.6	1080.9	1081.2	1081.6	1081.9	1082.2	1082.6	1082.9	1083.3

Example:

Pressure.................. 28.52 inches of mercury.
Table 8A gives........... 965.8 m-bars.

Table 9 A.—*Air-temperatures reduced from Fahrenheit to centigrade.*

Degrees Fahrenheit.	0	1	2	3	4	5	6	7	8	9
−140	−95.6	−96.1	−96.7	−97.2	−97.8	−98.3	−98.9	−99.4	−100.0	−100.6
−130	−90.0	−90.6	−91.1	−91.7	−92.2	−92.8	−93.3	−93.9	− 94.4	− 95.0
−120	−84.4	−85.0	−85.6	−86.1	−86.7	−87.2	−87.8	−88.3	− 88.9	− 89.4
−110	−78.9	−79.4	−80.0	−80.6	−81.1	−81.7	−82.2	−82.8	− 83.3	− 83.9
−100	−73.3	−73.9	−74.4	−75.0	−75.6	−76.1	−76.7	−77.2	− 77.8	− 78.3
− 90	−67.8	−68.3	−68.9	−69.4	−70.0	−70.6	−71.1	−71.7	− 72.2	− 72.8
− 80	−62.2	−62.8	−63.3	−63.9	−64.4	−65.0	−65.6	−66.1	− 66.7	− 67.2
− 70	−56.7	−57.2	−57.8	−58.3	−58.9	−59.4	−60.0	−60.6	− 61.1	− 61.7
− 60	−51.1	−51.7	−52.2	−52.8	−53.3	−53.9	−54.4	−55.0	− 55.6	− 56.1
− 50	−45.6	−46.1	−46.7	−47.2	−47.8	−48.3	−48.9	−49.4	− 50.0	− 50.6
− 40	−40.0	−40.6	−41.1	−41.7	−42.2	−42.8	−43.3	−43.9	− 44.4	− 45.0
− 30	−34.4	−35.0	−35.6	−36.1	−36.7	−37.2	−37.8	−38.3	− 38.9	− 39.4
− 20	−28.9	−29.4	−30.0	−30.6	−31.1	−31.7	−32.2	−32.8	− 33.3	− 33.9
− 10	−23.3	−23.9	−24.4	−25.0	−25.6	−26.1	−26.7	−27.2	− 27.8	− 28.3
− 0	−17.8	−18.3	−18.9	−19.4	−20.0	−20.6	−21.1	−21.7	− 22.2	− 22.8
0	−17.8	−17.2	−16.7	−16.1	−15.6	−15.0	−14.4	−13.9	− 13.3	− 12.8
10	−12.2	−11.7	−11.1	−10.6	−10.0	− 9.4	− 8.9	− 8.3	− 7.8	− 7.2
20	− 6.7	− 6.1	− 5.6	− 5.0	− 4.4	− 3.9	− 3.3	− 2.8	− 2.2	− 1.7
30	− 1.1	− 0.6	0	0.6	1.1	1.7	2.2	2.8	3.3	3.9
40	4.4	5.0	5.6	6.1	6.7	7.2	7.8	8.3	8.9	9.4
50	10.0	10.6	11.1	11.7	12.2	12.8	13.3	13.9	14.4	15.0
60	15.6	16.1	16.7	17.2	17.8	18.3	18.9	19.4	20.0	20.6
70	21.1	21.7	22.2	22.8	23.3	23.9	24.4	25.0	25.6	26.1
80	26.7	27.2	27.8	28.3	28.9	29.4	30.0	30.6	31.1	31.7
90	32.2	32.8	33.3	33.9	34.4	35.0	35.6	36.1	36.7	37.2
100	37.8	38.3	38.9	39.4	40.0	40.6	41.1	41.7	42.2	42.8
110	43.3	43.9	44.4	45.0	45.6	46.1	46.7	47.2	47.8	48.3
120	48.9	49.4	50.0	50.6	51.1	51.7	52.2	52.8	53.3	53.9
130	54.4	55.0	55.6	56.1	56.7	57.2	57.8	58.3	58.9	59.4
140	60.0	60.6	61.1	61.7	62.2	62.8	63.3	63.9	64.4	65.0

PROPORTIONALITY TABLE.

Fahrenheit.	Centigrade.	Fahrenheit.	Centigrade.	Fahrenheit.	Centigrade.
0.0	0.0	0.4	0.2	0.8	0.4
0.1	0.1	0.5	0.3	0.9	0.5
0.2	0.1	0.6	0.3		
0.3	0.2	0.7	0.4		

Example:

Temperature +42.6° F. Table 9 A gives...... +42.0° F. = +5.6° C.
Proportionality table gives for..................... + 0.6° F. = +0.3° C.
+42.6° F. = +5.9° C.

Table 10 A.—*Sea-temperatures reduced from Fahrenheit to centigrade.*

Degrees Fahrenheit.	0	1	2	3	4	5	6	7	8	9
20									−2.22	−1.67
30	−1.11	−0.56	0.00	0.56	1.11	1.67	2.22	2.78	3.33	3.89
40	4.44	5.00	5.56	6.11	6.67	7.22	7.78	8.33	8.89	9.44
50	10.00	10.56	11.11	11.67	12.22	12.78	13.33	13.89	14.44	15.00
60	15.56	16.11	16.67	17.22	17.78	18.33	18.89	19.44	20.00	20.56
70	21.11	21.67	22.22	22.78	23.33	23.89	24.44	25.00	25.56	26.11
80	26.67	27.22	27.78	28.33	28.89	29.44	30.00	30.56	31.11	31.67
90	32.22	32.78	33.33	33.89	34.44	35.00	35.56	36.11	36.67	37.22

PROPORTIONALITY TABLE.

Fahrenheit.	Centigrade.	Fahrenheit.	Centigrade.	Fahrenheit.	Centigrade.
0.00	0.00	0.40	0.22	0.80	0.44
0.10	0.06	0.50	0.28	0.90	0.50
0.20	0.11	0.60	0.33		
0.30	0.17	0.70	0.39		

Example:

Temperature +54.31° F. Table 10 A gives... +54.00° F. = 12.22° C.
Proportionality table gives for..................... + 0.31° F. = 0.17° C.
54.31° F. = 12.39° C.

Table 11 A. — *Virtual temperature of saturated air in degrees centigrade, the pressure being given in millimeters of mercury.*

Pressure (mm. mercury).	Temperature (°C.). -50	-40	-30	-20	-15	-10	-5	-2	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
150	0.0	0.1	0.2	0.5	0.8	1.3	2.0	2.6	3.1	3.4	3.7	4.0	4.3															
200	0.0	0.0	0.1	0.4	0.6	1.0	1.5	2.0	2.4	2.5	2.8	3.0	3.2	3.4	3.7	4.0	4.3	4.6										
250	0.0	0.0	0.1	0.3	0.5	0.8	1.2	1.6	1.9	2.0	2.2	2.4	2.6	2.7	3.0	3.2	3.4	3.7										
300	0.0	0.0	0.1	0.2	0.4	0.6	1.0	1.3	1.6	1.7	1.8	2.0	2.1	2.3	2.5	2.6	2.8	3.1	3.3	3.5	3.8	4.1	4.3					
350	0.0	0.0	0.1	0.2	0.3	0.6	0.9	1.1	1.3	1.5	1.6	1.7	1.8	2.0	2.1	2.3	2.4	2.6	2.8	3.0	3.2	3.5	3.7					
400	0.0	0.0	0.1	0.2	0.3	0.5	0.8	1.0	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.3	2.5	2.6	2.8	3.0	3.2	3.5	3.7	4.0	4.3	4.5
450	0.0	0.0	0.1	0.2	0.3	0.4	0.7	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.9	2.0	2.2	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.8	4.0
500	0.0	0.0	0.1	0.1	0.2	0.4	0.6	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.3	2.4	2.6	2.8	3.0	3.2	3.4	3.6
550	0.0	0.0	0.0	0.1	0.2	0.4	0.6	0.7	0.9	0.9	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.7	1.8	1.9	2.0	2.2	2.4	2.5	2.7	2.9	3.1	3.3
600	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.7	0.8	0.8	0.9	1.0	1.1	1.1	1.2	1.3	1.4	1.5	1.6	1.8	1.9	2.0	2.2	2.3	2.5	2.6	2.8	3.0
650	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.6	0.7	0.8	0.8	0.9	1.0	1.1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.9	2.0	2.1	2.3	2.4	2.6	2.8
700	0.0	0.0	0.0	0.1	0.2	0.3	0.4	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.3	2.4	2.6
750	0.0	0.0	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.3	2.4
800	0.0	0.0	0.0	0.1	0.2	0.2	0.4	0.5	0.6	0.6	0.7	0.7	0.8	0.9	0.9	1.0	1.1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0	2.1	2.3

Pressure (mm. mercury).	Temperature (°C.). 20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
450	4.3	4.6	4.9	5.2	5.6																				
500	3.9	4.1	4.4	4.7	5.0	5.4	5.7	6.1	6.5	6.9															
550	3.5	3.8	4.0	4.3	4.6	4.9	5.2	5.5	5.9	6.3	6.7	7.1	7.5	8.0	8.5										
600	3.2	3.4	3.7	3.9	4.2	4.5	4.7	5.0	5.4	5.7	6.1	6.5	6.9	7.3	7.8										
650	3.0	3.2	3.4	3.6	3.8	4.1	4.4	4.7	5.0	5.3	5.6	6.0	6.4	6.7	7.2	7.6	8.1	8.6	9.1	9.7	10.2	10.9	11.5	12.2	12.9
700	2.8	2.9	3.1	3.3	3.6	3.8	4.1	4.3	4.6	4.9	5.2	5.5	5.9	6.3	6.7	7.1	7.5	8.0	8.5	9.0	9.5	10.1	10.7	11.3	11.9
750	2.6	2.7	2.9	3.1	3.3	3.6	3.8	4.0	4.3	4.6	4.8	5.2	5.5	5.8	6.2	6.6	7.0	7.4	7.9	8.3	8.8	9.4	9.9	10.5	11.1
800	2.4	2.6	2.7	2.9	3.1	3.3	3.5	3.8	4.0	4.3	4.5	4.8	5.1	5.5	5.8	6.2	6.5	6.9	7.4	7.8	8.3	8.8	9.3	9.8	10.4

Example.

Pressure 631 mm. of mercury, temperature + 2.9° C. Table 11 A gives 0.9.
74 per cent of 0 9 gives.............................. 0.7
Virtual temperature .... 3.6° C. for air of 2.9° C. and 74 per cent relative humidity at the pressure of 631 mm.

Table 12 A.—*Virtual temperature of saturated air in degrees Fahrenheit, the pressure being given in inches of mercury.*

Pressure (inches of Hg).	Temperature (° F.).													
	−50	−30	−10	0	10	20	25	30	32	34	36	38	40	42
2	0.2	0.6	1.9	3.3	5.7	9.5	12.2							
4	0.1	0.3	0.9	1.7	2.8	4.7	6.0							
6	0.1	0.2	0.6	1.1	1.9	3.1	4.0	5.1	5.6	6.1	6.6	7.2		
8	0.0	0.1	0.5	0.8	1.4	2.3	3.0	3.8	4.2	4.6	5.0	5.4	5.9	6.4
10	0.0	0.1	0.4	0.7	1.1	1.9	2.4	3.1	3.3	3.7	4.0	4.3	4.7	5.1
12	0.0	0.1	0.3	0.6	0.9	1.6	2.0	2.5	2.8	3.0	3.3	3.6	3.9	4.2
14	0.0	0.1	0.3	0.5	0.8	1.3	1.7	2.2	2.4	2.6	2.8	3.1	3.3	3.6
16	0.0	0.1	0.2	0.4	0.7	1.2	1.5	1.9	2.1	2.3	2.5	2.7	2.9	3.2
18	0.0	0.1	0.2	0.4	0.6	1.0	1.3	1.7	1.9	2.0	2.2	2.4	2.6	2.8
20	0.0	0.1	0.2	0.3	0.6	0.9	1.2	1.5	1.7	1.8	2.0	2.1	2.3	2.5
22	0.0	0.1	0.2	0.3	0.5	0.9	1.1	1.4	1.5	1.6	1.8	1.9	2.1	2.3
24	0.0	0.0	0.1	0.3	0.5	0.8	1.0	1.3	1.4	1.5	1.6	1.8	1.9	2.1
26	0.0	0.0	0.1	0.3	0.4	0.7	0.9	1.2	1.3	1.4	1.5	1.6	1.8	1.9
28	0.0	0.0	0.1	0.2	0.4	0.7	0.9	1.1	1.2	1.3	1.4	1.5	1.7	1.8
30	0.0	0.0	0.1	0.2	0.4	0.6	0.8	1.0	1.1	1.2	1.3	1.4	1.5	1.7
32	0.0	0.0	0.1	0.2	0.4	0.6	0.7	0.9	1.0	1.1	1.2	1.3	1.4	1.6

Pressure (inches of Hg).	Temperature (° F.).												
	44	46	48	50	52	54	56	58	60	62	64	66	68
2													
4													
6													
8	6.9	7.5	8.1										
10	5.5	6.0	6.5	7.0	7.6	8.2	8.9	9.6					
12	4.6	5.0	5.4	5.8	6.3	6.8	7.4	8.0	8.6	9.3	10.0	10.7	11.5
14	3.9	4.3	4.6	5.0	5.4	5.8	6.3	6.8	7.3	7.9	8.5	9.2	9.9
16	3.4	3.7	4.0	4.4	4.7	5.1	5.5	5.9	6.4	6.9	7.4	8.0	8.6
18	3.0	3.3	3.6	3.9	4.2	4.5	4.9	5.3	5.7	6.1	6.6	7.1	7.6
20	2.7	3.0	3.2	3.5	3.8	4.1	4.4	4.7	5.1	5.5	5.9	6.4	6.9
22	2.5	2.7	2.9	3.2	3.4	3.7	4.0	4.3	4.6	5.0	5.4	5.8	6.2
24	2.3	2.5	2.7	2.9	3.1	3.4	3.6	3.9	4.2	4.6	4.9	5.3	5.7
26	2.1	2.3	2.5	2.7	2.9	3.1	3.4	3.6	3.9	4.2	4.6	4.9	5.3
28	2.0	2.1	2.3	2.5	2.7	2.9	3.1	3.4	3.6	3.9	4.2	4.5	4.9
30	1.8	2.0	2.1	2.3	2.5	2.7	2.9	3.2	3.4	3.7	3.9	4.2	4.5
32	1.7	1.9	2.0	2.2	2.3	2.5	2.7	3.0	3.2	3.4	3.7	4.0	4.3

Pressure (inches of Hg).	Temperature (° F.).																								
	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100	102	104	106	108	110	112	114	116	118
16	9.3	10.0	10.7	11.5	12.4																				
18	8.2	8.8	9.5	10.2	10.9																				
20	7.4	7.9	8.5	9.2	9.9	10.6	11.3	12.1	13.0	13.9															
22	6.7	7.2	7.7	8.3	8.9	9.6	10.3	11.0	11.8	12.6	13.5	14.5	15.5	16.6	17.7										
24	6.2	6.6	7.1	7.6	8.2	8.8	9.4	10.1	10.8	11.6	12.4	13.2	14.2	15.1	16.2	17.3	18.5	19.7	22.0	22.4	24.0	25.5	27.2	28.9	30.8
26	5.7	6.1	6.5	7.0	7.5	8.1	8.7	9.3	10.0	10.7	11.3	12.2	13.1	14.0	14.9	15.9	17.0	18.2	19.4	20.7	22.0	23.5	25.0	26.6	28.3
28	5.3	5.6	6.0	6.5	7.0	7.5	8.0	8.6	9.2	9.9	10.6	11.3	12.1	12.9	13.8	14.8	15.8	16.8	17.9	19.1	20.4	21.7	23.1	24.6	26.2
30	4.9	5.3	5.6	6.1	6.5	7.0	7.5	8.0	8.6	9.2	9.9	10.5	11.3	12.1	12.9	13.8	14.7	15.7	16.7	17.8	19.0	20.2	21.5	22.9	24.4
32	4.6	4.9	5.3	5.7	6.1	6.5	7.0	7.5	8.1	8.6	9.2	9.9	10.6	11.3	12.1	12.9	13.7	14.7	15.6	16.7	17.8	18.9	20.1	21.4	22.8

Example:

Pressure 26.79 inches of mercury, temperature......... 37.5° F. Table 12 A gives 1.6.
81 per cent of 1.6 gives... 1.3° F.
Virtual temperature .... 38.8° F. for air of 37.5° F. and 81 per cent relative humidity at the pressure of 26.79 inches.

Table 13 A.—*Virtual temperature of saturated air in degrees Fahrenheit, the height being given in dynamic meters.*

Height (dynamic meters).	Temperature (°F.).																										
	−50	−30	−10	0	10	20	25	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68
10000	0.0	0.2	0.5	0.9																							
9500	0.0	0.1	0.5	0.8																							
9000	0.0	0.1	0.4	0.8																							
8500	0.0	0.1	0.4	0.7	1.2																						
8000	0.0	0.1	0.4	0.6	1.1																						
7500	0.0	0.1	0.3	0.6	1.0	1.7																					
7000	0.0	0.1	0.3	0.6	1.0	1.6																					
6500	0.0	0.1	0.3	0.5	0.9	1.5	1.9																				
6000	0.0	0.1	0.3	0.5	0.8	1.4	1.8	2.2																			
5500	0.0	0.1	0.3	0.5	0.8	1.3	1.6	2.1	2.3																		
5000	0.0	0.1	0.2	0.4	0.7	1.2	1.5	2.0	2.1	2.4	2.5																
4500	0.0	0.1	0.2	0.4	0.7	1 1	1.4	1.8	2.0	2.2	2.4	2.6	2.8	3.0													
4000	0.0	0.1	0.2	0.4	0.6	1.1	1.3	1.7	1.9	2.1	2.2	2.4	2.6	2.8	3.1	3.3	3.6										
3500	0.0	0.1	0.2	0.3	0.6	1.0	1.3	1.6	1.8	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.4	3.7	4.0	4.3	4.6	5.0					
3000	0.0	0.1	0.2	0.3	0.6	0.9	1.2	1.5	1.6	1.8	1.9	2.1	2.3	2.5	2.7	2.9	3.2	3.4	3.7	4.0	4.3	4.7	5.0	5.4	5.9	6.3	6.7
2500	0.0	0.1	0.2	0.3	0.5	0.9	1.1	1.4	1.5	1.7	1.8	2.0	2.1	2.3	2.5	2.7	3.0	3.2	3.5	3.8	4.1	4.4	4.7	5.1	5.5	5.9	6.3
2000	0.0	0.1	0.2	0.3	0.5	0.8	1.0	1.3	1.4	1.6	1.7	1.9	2.0	2.2	2.4	2.6	2.8	3.0	3.3	3.5	3.8	4 1	4.4	4.8	5.1	5.5	5.9
1500	0.0	0.0	0.2	0.3	0.5	0.8	1.0	1.2	1.4	1.5	1.6	1.7	1.9	2.1	2.2	2.4	2.6	2.8	3.1	3.3	3.6	3.9	4.1	4.5	4.8	5.2	5.5
1000	0.0	0.0	0 1	0.3	0.4	0.7	0.9	1.2	1.3	1.4	1.5	1.6	1.8	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.4	3.6	3.9	4.2	4.5	4.9	5.2
500	0.0	0.0	0.1	0.2	0.4	0.7	0.9	1.1	1.2	1.3	1.4	1.5	1.7	1.8	2.0	2.1	2.3	2.5	2.7	2.9	3.1	3.4	3.7	4.0	4.3	4.6	4.9
0	0.0	0.0	0.1	0.2	0.4	0.6	0.8	1.0	1.1	1.2	1.3	1.4	1.6	1.7	1.8	2.0	2.2	2.3	2.5	2.7	3.0	3.2	3.4	3.7	4.0	4.3	4.6

Height (dynamic meters).	Temperature (°F.).																								
	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100	102	104	106	108	110	112	114	116	118
2500	6.8	7.3	7.9	8.5	9.1																				
2000	6.4	6.9	7.4	7.9	8.5	9.1	9.8	10.5	11.3	12.1															
1500	6 0	6 4	6.9	7.4	8.0	8.6	9.2	9.9	10.6	11.3	12.1	12.9	13.8	14.8	15.8										
1000	5.6	6.1	6.5	7.0	7.5	8.0	8.6	9.3	9.9	10.6	11.4	12.2	13.0	13.9	15.0	15.9	16.9	18.1	19.3	20.6					
500	5.3	5.7	6.1	6.6	7.0	7.6	8.1	8.7	9.3	10.0	10.7	11.4	12.2	13.0	13.9	14.9	15.9	17.0	18.1	19.3	20.6	21.9	23.3	24.8	26.4
0	5.0	5.3	5.7	6.2	6.6	7.1	7.6	8.2	8.7	9.4	10.0	10.7	11.4	12.2	13.1	14.0	14.9	15.9	17.0	18.1	19.3	20.5	21.9	23.3	24.8

Example:

Height, 3191 dynamic meters, temperature......... 40.9° F. Table 13 A glves 2.5.
88 per cent of 2.5 gives....................................... 2.2° F.

Virtual temperature........................... ...... 43.1° F. for air of 41.9° F. and 88 per cent relative humidity at the height of 3191 dynamic meters.

Table 14 A.—*Distances in dynamic meters from the earth's surface to the nearest standard isobaric surfaces, the average virtual temperature of the sheet being 0° C. and the pressure at the station being given in millimeters of mercury.*

Pressure at the station (mm. Hg).	Standard surfaces (m-bars).			
	700	600	500	Δ
450	−1209	−1	1428	17
451	−1192	16	1445	18
452	−1174	34	1463	17
453	−1157	51	1480	17
454	−1140	68	1497	17
455	−1123	85	1514	18
456	−1105	103	1532	17
457	−1088	120	1549	17
458	−1071	137	1566	17
459	−1054	154	1583	17
460	−1037	171	1600	17
461	−1020	188	1617	17
462	−1003	205	1634	17
463	− 986	222	1651	17
464	− 969	239	1668	17
465	− 952	256	1685	17
466	− 935	273	1702	16
467	− 919	289	1718	17
468	− 902	306	1735	17
469	− 885	323	1752	17
470	− 868	340	1769	16
471	− 852	356	1785	17
472	− 835	373	1802	17
473	− 818	390	1819	16
474	− 802	406	1835	17
475	− 785	423	1852	16
476	− 769	439	1868	17
477	− 752	456	1885	16
478	− 738	472	1901	16
479	− 720	488	1917	17
480	− 703	505	1934	16
481	− 687	521	1950	16
482	− 671	537	1966	16
483	− 655	553	1982	17
484	− 638	570	1999	16
485	− 622	586	2015	16
486	− 607	602	2031	16
487	− 590	618	2047	16
488	− 574	634	2063	16
489	− 558	650	2079	16
490	− 542	666	2095	16
491	− 526	682	2111	16
492	− 510	698	2127	16
493	− 494	714	2143	16
494	− 478	730	2159	16
495	− 462	746	2175	16
496	− 446	762	2191	15
497	− 431	777	2206	16
498	− 415	793	2222	16
499	− 399	809	2238	16
500	− 383	825	2254	15
501	− 368	840	2269	16
502	− 352	856	2285	15
503	− 337	871	2300	16
504	− 321	887	2316	16
505	− 305	903	2332	15
506	− 290	918	2347	15
507	− 275	933	2362	16
508	− 259	949	2378	15
509	− 244	964	2393	16

Pressure at the station (mm. Hg).	Standard surfaces (m-bars).				
	800	700	600	500	Δ
510		−228	980	2409	15
511		−213	995	2424	15
512		−198	1010	2439	16
513		−182	1026	2455	15
514		−167	1041	2470	15
515		−152	1056	2485	15
516		−137	1071	2500	16
517		−121	1087	2516	15
518		−106	1102	2531	15
519		− 91	1117	2546	15
520		− 76	1132	2561	15
521		− 61	1147	2576	15
522		− 46	1162	2591	15
523		− 31	1177	2606	15
524		− 16	1192	2621	15
525	−1047	− 1	1207	2636	15
526	−1032	14	1222	2651	15
527	−1017	29	1237	2666	15
528	−1002	44	1252	2681	14
529	− 988	58	1266	2695	15
530	− 973	73	1281	2710	15
531	− 958	88	1296	2725	15
532	− 943	103	1311	2740	14
533	− 929	117	1325	2754	15
534	− 914	132	1340	2769	15
535	− 899	147	1355	2784	15
536	− 884	162	1370		14
537	− 870	176	1384		15
538	− 855	191	1399		14
539	− 841	205	1413		15
540	− 826	220	1428		14
541	− 812	224	1442		15
542	− 797	249	1457		14
543	− 783	263	1471		15
544	− 768	278	1486		14
545	− 754	292	1500		15
546	− 739	307	1515		14
547	− 725	321	1529		14
548	− 711	335	1543		14
549	− 697	349	1557		15
550	− 682	364	1572		14
551	− 668	378	1586		14
552	− 654	392	1600		14
553	− 640	406	1614		14
554	− 626	420	1628		15
555	− 611	435	1643		14
556	− 597	449	1657		14
557	− 583	463	1671		14
558	− 569	477	1685		15
559	− 554	492	1700		13
560	− 541	505	1713		14
561	− 527	519	1727		14
562	− 513	533	1741		14
563	− 499	547	1755		14
564	− 485	561	1769		14
565	− 471	575	1783		13
566	− 458	588	1796		14
567	− 444	602	1810		14
568	− 430	616	1824		14
569	− 416	630	1838		14

Pressure at the station (mm. Hg).	Standard surfaces (m-bars).				
	900	800	700	600	Δ
570		−402	644	1852	13
571		−389	657	1865	14
572		−375	671	1879	14
573		−361	685	1893	13
574		−348	698	1906	14
575		−334	712	1920	14
576		−320	726	1934	13
577		−307	739	1947	14
578		−293	753	1961	13
579		−280	766	1974	14
580		−266	780	1988	13
581		−253	793	2001	14
582		−239	807	2015	13
583		−226	820	2028	14
584		−212	834	2042	13
585		−199	847	2055	14
586		−185	861	2069	13
587		−172	874	2082	13
588		−159	887	2095	14
589		−145	901	2109	13
590		−132	914	2122	13
591		−119	927	2135	13
592		−106	940	2148	14
593		− 92	954	2162	13
594		− 79	967	2175	13
595		− 66	980	2188	13
596		− 53	993	2201	13
597		− 40	1006	2214	13
598		− 27	1019	2227	13
599		− 14	1032	2240	13
600	−924	− 1	1045	2253	14
601	−910	13	1059	2267	13
602	−897	26	1072	2280	13
603	−884	39	1085	2293	13
604	−871	52	1098	2306	13
605	−858	65	1111	2319	13
606	−845	78	1124	2332	12
607	−833	90	1136	2344	13
608	−820	103	1149	2357	13
609	−807	116	1162	2370	13
610	−794	129	1175	2383	13
611	−781	142	1188	2396	13
612	−768	155	1201	2409	13
613	−755	168	1214	2422	12
614	−743	180	1226	2434	13
615	−730	193	1239	2447	13
616	−717	206	1252	2460	13
617	−704	219	1265	2473	12
618	−692	231	1277	2485	13
619	−679	244	1290	2498	12
620	−667	256	1302	2510	13
621	−654	269	1315	2523	13
622	−641	282	1328	2536	12
623	−629	294	1340	2548	13
624	−616	307	1353	2561	12
625	−604	319	1365	2573	13
626	−591	332	1378	2586	12
627	−579	344	1390	2598	13
628	−566	357	1403	2611	12
629	−554	369	1415	2623	13

Table 14 A (continued).—*Distances in dynamic meters from the earth's surface to the nearest standard isobaric surfaces, the average virtual temperature of the sheet being 0° C. and the pressure at the station being given in millimeters of mercury.*

Pressure at the station (mm. Hg).	Standard surfaces (m-bars).				
	1000	900	800	700	Δ
630		−541	382	1428	12
631		−529	394	1440	13
632		−516	407	1453	12
633		−504	419	1465	12
634		−492	431	1477	13
635		−479	444	1490	12
636		−467	456	1502	12
637		−455	468	1514	13
638		−442	481	1527	12
639		−430	493	1539	12
640		−418	505	1551	12
641		−406	517	1563	13
642		−393	530	1576	12
643		−381	542	1588	12
644		−369	554	1600	12
645		−357	566	1612	12
646		−345	578	1624	13
647		−332	591	1637	12
648		−320	603	1649	12
649		−308	615	1661	12
650		−296	627	1673	12
651		−284	639	1685	12
652		−272	651	1697	12
653		−260	663	1709	12
654		−248	675	1721	12
655		−236	687	1733	12
656		−224	699	1745	12
657		−212	711	1757	12
658		−200	723	1769	12
659		−188	735	1781	11
660		−177	746	1792	12
661		−165	758	1804	12
662		−153	770	1816	12
663		−141	782	1828	12
664		−129	794	1840	12
665		−117	806	1852	11
666		−106	817	1863	12
667		− 94	829	1875	12
668		− 82	841	1887	11
669		− 71	852	1898	12
670		− 59	864	1910	12
671		− 47	876	1922	12
672		− 35	888	1934	11
673		− 24	899	1945	12
674		− 12	911	1957	11
675	−827	− 1	922	1968	12
676	−815	11	934	1980	12
677	−803	23	946	1992	11
678	−792	34	957	2003	12
679	−780	46	969	2015	11
680	−769	57	980	2026	12
681	−757	69	992	2038	11
682	−746	80	1003	2049	12
683	−734	92	1015	2061	11
684	−723	103	1026	2072	12
685	−711	115	1038	2084	11
686	−700	126	1049	2095	12
687	−688	138	1061	2107	11
688	−677	149	1072	2118	11
689	−666	160	1083	2129	12
690	−654	172	1095	2141	11
691	−643	183	1106	2152	11
692	−632	194	1117	2163	12
693	−620	206	1129	2175	11
694	−609	217	1140	2186	11
695	−598	228	1151	2197	12
696	−586	240	1163	2209	11
697	−575	251	1174	2220	11
698	−564	262	1185	2231	11
699	−553	273	1196	2242	12
700	−541	285	1208	2254	11
701	−530	296	1219	2265	11
702	−519	307	1230	2276	11
703	−508	318	1241	2287	11
704	−497	329	1252	2298	11
705	−486	340	1263	2309	11
706	−475	351	1274	2320	12
707	−463	363	1286	2332	11
708	−452	374	1297	2343	11
709	−441	385	1308	2354	11
710	−430	396	1319	2365	11
711	−419	407	1330	2376	11
712	−408	418	1341	2387	11
713	−397	429	1352	2398	11
714	−386	440	1363	2409	11
715	−375	451	1374	2420	11
716	−364	462	1385	2431	11
717	−353	473	1396	2442	10
718	−343	483	1406	2452	11
719	−332	494	1417	2463	11
720	−321	505	1428	2474	11
721	−310	516	1439		11
722	−299	527	1450		10
723	−289	537	1460		11
724	−278	548	1471		11
725	−267	559	1482		11
726	−256	570	1493		11
727	−245	581	1504		10
728	−235	591	1514		11
729	−224	602	1525		11
730	−213	613	1536		11
731	−202	624	1547		10
732	−192	634	1557		11
733	−181	645	1568		11
734	−170	656	1579		11
735	−159	667	1590		10
736	−149	677	1600		11
737	−138	688	1611		10
738	−128	698	1621		11
739	−117	709	1632		11
740	−106	720	1643		10
741	− 96	730	1653		11
742	− 85	741	1664		10
743	− 75	751	1674		11
744	− 64	762	1685		10
745	− 54	772	1695		11
746	− 43	783	1706		10
747	− 33	793	1716		11
748	− 22	804	1727		10
749	− 12	814	1737		11
750	−1	825	1748		10
751	9	835	1758		11
752	20	846	1769		10
753	30	856	1779		11
754	41	867	1790		10
755	51	877	1800		10
756	61	887	1810		11
757	72	898	1821		10
758	82	908	1831		10
759	92	918	1841		11
760	103	929	1852		10
761	113	939	1862		10
762	123	949	1872		10
763	133	959	1882		11
764	144	970	1893		10
765	154	980	1903		10
766	164	990	1913		10
767	174	1000	1923		11
768	185	1011	1934		10
769	195	1021	1944		10
770	205	1031	1954		10
771	215	1041	1964		10
772	225	1051	1974		10
773	235	1061	1984		11
774	246	1072	1995		10
775	256	1082	2005		10
776	266	1092	2015		10
777	276	1102	2025		10
778	286	1112	2035		10
779	296	1122	2045		10
780	306	1132	2055		10
781	316	1142	2065		10
782	326	1152	2075		10
783	336	1162	2085		10
784	346	1172	2095		10
785	356	1182	2105		10
786	366	1192	2115		10
787	376	1202	2125		10
788	386	1212	2135		10
789	396	1222	2145		10
790	406	1232	2155		10
791	416	1242	2165		10
792	426	1252	2175		10
793	436	1262	2185		10
794	446	1272	2195		9
795	455	1281	2204		10
796	465	1291	2214		10
797	475	1301	2224		10
798	485	1311	2234		10
799	495	1321	2244		10

Example:

	Dynamic meters.
Height of station above sea-level	39
Table 14 A gives for the 1000 m-bar surface and the pressure at the station of 762.0 mm	123
Virtual-temperature diagram giving for the sheet between the station and the 1000 m-bar surface the average virtual temperature + 25°; table 12 M gives for this temperature and the height 123 the correction	+ 11
Height of standard surface 1000 m-bars above sea-level	173

Table 15 A.—*Distances in dynamic meters from the earth's surface to the nearest standard isobaric surfaces, the average virtual temperature of the sheet being 0° F., and the pressure at the station being given in inches of mercury.*

Pressure at the station (inch Hg).	Standard surfaces (m-bars). 800	700	600	500	Δ
17.0		−1433	−304	1032	43
17.1		−1390	−261	1075	43
17.2		−1347	−218	1118	43
17.3		−1304	−175	1161	42
17.4		−1262	−133	1203	42
17.5		−1220	− 91	1245	41
17.6		−1179	− 50	1286	42
17.7		−1137	− 8	1328	41
17.8		−1096	33	1369	41
17.9		−1055	74	1410	41
18.0		−1014	115	1451	41
18.1		− 973	156	1492	40
18.2		− 933	196	1532	40
18.3		− 893	236	1572	40
18.4		− 853	276	1612	40
18.5		− 813	316	1652	39
18.6		− 774	355	1691	40
18.7		− 734	395	1731	39
18.8		− 695	434	1770	39
18.9		− 656	473	1809	38
19.0		− 618	511	1847	39
19.1		− 579	550	1886	38
19.2		− 541	588	1924	38
19.3		− 503	626	1962	38
19.4		− 465	664	2000	38
19.5		− 427	702	2038	37
19.6		− 390	739	2075	37
19.7		− 353	776	2112	37
19.8		− 316	813	2149	37
19.9		− 279	850	2186	37
20.0	−1220	− 242	887	2223	37
20.1	−1183	− 205	924	2260	36
20.2	−1147	− 169	960	2296	36
20.3	−1111	− 133	996	2332	36
20.4	−1075	− 97	1032	2368	36
20.5	−1039	− 61	1068	2404	35
20.6	−1004	− 26	1103	2439	36
20.7	− 968	10	1139	2475	35
20.8	− 933	45	1174	2510	35
20.9	− 898	80	1209	2545	35
21.0	− 863	115	1244	2580	35
21.1	− 828	150	1279	2615	35
21.2	− 793	185	1314	2650	34
21.3	− 759	219	1348	2684	35
21.4	− 724	254	1383	2719	34
21.5	− 690	288	1417	2753	34
21.6	− 656	322	1451	2787	34
21.7	− 622	356	1485	2821	33
21.8	− 589	389	1518	2854	34
21.9	− 555	423	1552	2888	33

Pressure at the station (inch Hg).	Standard surfaces (m-bars). 1000	900	800	700	600	Δ
22.0			−522	456	1585	34
22.1			−488	490	1619	33
22.2			−455	523	1652	33
22.3			−422	556	1685	33
22.4			−389	589	1718	32
22.5			−357	621	1750	33
22.6			−324	654	1783	32
22.7			−292	686	1815	32
22.8			−260	718	1847	32
22.9			−228	750	1879	32
23.0			−196	782	1911	32
23.1			−164	814	1943	32
23.2			−132	846	1975	31
23.3			−101	877	2006	31
23.4			− 70	908	2037	32
23.5		−901	− 38	940	2069	31
23.6		−870	− 7	971	2100	31
23.7		−839	24	1002	2131	31
23.8		−808	55	1033	2162	30
23.9		−778	85	1063	2192	31
24.0		−747	116	1094	2223	30
24.1		−717	146	1124	2253	31
24.2		−686	177	1155	2284	30
24.3		−656	207	1185	2314	30
24.4		−626	237	1215	2344	30
24.5		−596	267	1245	2374	30
24.6		−566	297	1275	2404	30
24.7		−536	327	1305	2434	29
24.8		−507	356	1334	2463	29
24.9		−477	386	1364	2493	29
25.0		−449	415	1393		29
25.1		−419	444	1422		29
25.2		−390	473	1451		29
25.3		−361	502	1480		29
25.4		−332	531	1509		29
25.5		−303	560	1538		28
25.6		−275	588	1566		29
25.7		−246	617	1595		28
25.8		−218	645	1623		29
25.9		−189	674	1652		28
26.0	−933	−161	702	1680		28
26.1	−905	−133	730	1708		28
26.2	−877	−105	758	1736		28
26.3	−849	− 77	786	1764		28
26.4	−821	− 49	814	1792		28
26.5	−794	− 21	842	1820		27
26.6	−766	6	869	1847		28
26.7	−738	34	897	1875		27
26.8	−711	61	924	1902		27
26.9	−683	88	951	1929		27

Pressure at the station (inch Hg).	Standard surfaces (m-bars). 1000	900	800	700	Δ
27.0	−656	115	978	1956	28
27.1	−629	143	1006	1984	27
27.2	−602	170	1033	2011	27
.27.3	−575	197	1060	2038	26
27.4	−549	223	1086	2064	27
27.5	−522	250	1113	2091	27
27.6	−495	277	1140	2118	26
27.7	−469	303	1166	2144	26
27.8	−442	329	1192	2170	27
27.9	−416	356	1219	2197	26
28.0	−390	382	1245	2223	26
28.1	−364	408	1271	2249	26
28.2	−338	434	1297	2275	26
28.3	−312	460	1323	2301	26
28.4	−286	486	1349	2327	26
28.5	−260	511	1374		26
28.6	−234	537	1400		26
28.7	−209	563	1426		25
28.8	−183	588	1451		26
28.9	−158	614	1477		25
29.0	−133	639	1502		25
29.1	−107	664	1527		25
29.2	− 82	689	1552		25
29.3	− 57	714	1577		25
29.4	− 32	739	1602		25
29.5	− 7	764	1627		25
29.6	17	789	1652		25
29.7	42	814	1677		24
29.8	67	838	1701		25
29.9	91	863	1726		24
30.0	116	887	1750		25
30.1	140	912	1775		24
30.2	165	936	1799		24
30.3	189	960	1823		24
30.4	213	984	1847		24
30.5	237	1008	1871		24
30.6	261	1032	1895		24
30.7	285	1056	1919		24
30.8	309	1080	1943		24
30.9	332	1104	1967		23
31.0	356	1127	1990		24
31.1	380	1151	2014		23
31.2	403	1174	2037		24
31.3	427	1198	2061		23
31.4	450	1221	2084		24
31.5	473	1245	2108		23
31.6	497	1268	2131		23
31.7	520	1291	2154		23
31.8	543	1314	2177		23
31.9	566	1337	2200		23

Table 15 A (continued).—*Distances in dynamic meters from the earth's surface to the nearest standard isobaric surfaces, the average virtual temperature of the sheet being 0° F. and the pressure at the station being given in inches of mercury.*

Pressure at the station (inch Hg).	Standard surfaces (m-bars).			
	1000	900	800	Δ
28.80	−183	588	1451	3
28.81	−180	591	1454	2
28.82	−178	593	1456	3
28.83	−175	596	1459	2
28.84	−173	598	1461	3
28.85	−170	601	1464	2
28.86	−168	603	1466	3
28.87	−165	606	1469	2
28.88	−163	608	1471	3
28.89	−160	611	1474	2
28.90	−158	613	1476	3
28.91	−155	616	1479	3
28.92	−152	619	1482	2
28.93	−150	621	1484	2
28.94	−148	623	1487	2
28.95	−145	626	1489	3
28.96	−143	628	1492	2
28.97	−140	631	1494	3
28.98	−138	633	1497	2
28.99	−135	636	1499	3
29.00	−132	639	1502	2
29.01	−130	641	1504	3
29.02	−127	644	1507	2
29.03	−125	646	1509	3
29.04	−122	649	1512	2
29.05	−120	651	1514	3
29.06	−116	655	1517	2
29.07	−115	656	1519	3
29.08	−112	659	1522	2
29.09	−110	661	1524	3
29.10	−107	664	1527	3
29.11	−104	667	1530	2
29.12	−102	669	1532	3
29.13	− 99	672	1535	2
29.14	− 97	674	1537	3
29.15	− 94	677	1540	2
29.16	− 92	679	1542	3
29.17	− 89	682	1545	2
29.18	− 87	684	1547	3
29.19	− 84	687	1550	2
29.20	− 82	689	1552	3
29.21	− 79	692	1555	2
29.22	− 77	694	1557	3
29.23	− 74	697	1560	2
29.24	− 72	699	1562	3
29.25	− 69	702	1565	2
29.26	− 67	704	1567	3
29.27	− 64	707	1570	3
29.28	− 61	710	1573	2
29.29	− 59	712	1575	2

Pressure at the station (inch Hg).	Standard surfaces (m-bars).			
	1000	900	800	Δ
29.30	−57	714	1577	3
29.31	−54	717	1580	2
29.32	−52	719	1582	3
29.33	−49	722	1585	2
29.34	−47	724	1587	3
29.35	−44	727	1590	2
29.36	−42	729	1592	3
29.37	−39	732	1595	2
29.38	−37	734	1597	3
29.39	−34	737	1600	2
29.40	−32	739	1602	3
29.41	−29	742	1605	2
29.42	−27	744	1607	3
29.43	−24	747	1610	2
29.44	−22	749	1612	3
29.45	−19	752	1615	2
29.46	−17	754	1617	3.
29.47	−14	.757	1620	2
29.48	−12	759	1622	3
29.49	− 9	762	1625	2
29.50	− 7	764	1627	2
29.51	− 5	766	1629	3
29.52	− 2	769	1632	2
29.53	0	771	1634	3
29.54	3	774	1637	2
29.55	5	776	1639	3
29.56	8	779	1642	2
29.57	10	781	1644	3
29.58	13	784	1647	2
29.59	15	786	1649	3
29.60	18	789	1652	2
29.61	20	791	1654	3
29.62	23	794	1657	2
29.63	25	796	1659	3
29.64	28	799	1662	2
29.65	30	801	1664	3
29.66	33	804	1667	2
29.67	35	806	1669	3
29.68	38	809	1672	2
29.69	40	811	1674	3
29.70	43	814	1677	2
29.71	45	816	1679	2
29.72	47	818	1681	3
29.73	50	821	1684	2
29.74	52	823	1686	3
29.75	55	826	1689	2
29.76	57	828	1691	3
29.77	60	831	1694	2
29.78	62	833	1696	3
29.79	65	836	1699	2

Pressure at the station (inch Hg).	Standard surfaces (m-bars).			
	1000	900	800	Δ
29.80	67	838	1701	3
29.81	70	841	1704	2
29.82	72	843	1706	3
29.83	75	846	1709	2
29.84	77	848	1711	2
29.85	79	850	1713	3
29.86	82	853	1716	2
29.87	84	855	1718	3
29.88	87	858	1721	2
29.89	89	860	1723	2
29.90	91	862	1725	3
29.91	94	865	1728	3
29.92	97	968	1731	2
29.93	99	870	1733	2
29.94	101	872	1735	3
29.95	104	875	1738	2
29.96	106	877	1740	3
29.97	109	880	1743	2
29.98	111	882	1745	3
29.99	114	885	1748	2
30.00	116	887	1750	3
30.01	119	890	1753	2
30.02	121	892	1755	2
30.03	123	894	1757	3
30.04	126	897	1760	2
30.05	128	899	1762	3
30.06	131	902	1765	2
30.07	133	904	1767	3
30.08	136	907	1770	2
30.09	138	909	1772	3
30.10	141	912	1775	2
30.11	143	914	1777	2
30.12	145	916	1779	3
30.13	148	919	1782	2
30.14	150	921	1784	3
30.15	153	924	1787	2
30.16	155	926	1789	3
30.17	158	929	1792	2
30.18	160	931	1794	2
30.19	162	933	1796	3
30.20	165	936	1799	2
30.21	167	938	1801	3
30.22	170	941	1804	2
30.23	172	943	1806	3
30.24	175	946	1809	2
30.25	177	948	1811	2
30.26	179	950	1813	3
30.27	182	953	1816	2
30.28	184	955	1818	3
30.29	187	958	1821	2

Example:

	Dynamic meters.
Height of station above sea-level	39
Table 15 A gives for the 1000 m-bar surface and the pressure at the station of 30.00 inches of mercury	116
Virtual-temperature diagram giving for the sheet between the station and the 1000 m-bar surface the average virtual temperature 77° F.; table 16 A gives for this temperature and 116 dynamic meters	+19
Height of standard surface 1000 m-bars above sea-level	174

Table 16 A.—*Corrections to table 15 A for temperature.*

Height (dynamic meters).	Temperature (°F.).																			
	0	10	20	30	40	50	60	70	80	90	100	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	1	1	1	1	2	2	2	2	0	0	0	0	0	0	0	0	0	0
20	0	0	1	1	2	2	3	3	3	4	4	0	0	0	0	0	0	0	0	0	0
30	0	1	1	2	3	3	4	5	5	6	7	0	0	0	0	0	0	0	0	1	1
40	0	1	2	3	3	4	5	6	7	8	9	0	0	0	0	0	0	1	1	1	1
50	0	1	2	3	4	5	7	8	9	10	11	0	0	0	0	0	1	1	1	1	1
60	0	1	3	4	5	7	8	9	10	12	13	0	0	0	0	1	1	1	1	1	1
70	0	2	3	5	6	8	9	11	12	14	15	0	0	0	0	1	1	1	1	1	1
80	0	2	3	5	7	9	10	12	14	16	17	0	0	0	1	1	1	1	1	1	2
90	0	2	4	6	8	10	12	14	16	18	20	0	0	0	1	1	1	1	1	2	2
100	0	2	4	7	9	11	13	15	17	20	22	0	0	0	1	1	1	1	2	2	2
110	0	2	5	7	10	12	14	17	19	22	24	0	0	0	1	1	1	1	2	2	2
120	0	3	5	8	10	13	16	18	21	24	26	0	0	1	1	1	1	2	2	2	2
130	0	3	6	8	11	14	17	20	23	25	28	0	0	1	1	1	1	2	2	2	3
140	0	3	6	9	12	15	18	21	24	27	30	0	0	1	1	1	2	2	2	2	3
150	0	3	7	10	13	16	20	23	26	29	33	0	0	1	1	1	2	2	2	3	3
160	0	3	7	10	14	17	21	24	28	31	35	0	0	1	1	1	2	2	2	3	3
170	0	4	7	11	15	19	22	26	30	33	37	0	0	1	1	1	2	2	3	3	3
180	0	4	8	12	16	20	24	27	31	35	39	0	0	1	1	2	2	2	3	3	4
190	0	4	8	12	17	21	25	29	33	37	41	0	0	1	1	2	2	2	3	3	4
200	0	4	9	13	17	22	26	30	35	39	44	0	0	1	1	2	2	3	3	3	4
210	0	5	9	14	18	23	27	32	37	41	46	0	0	1	1	2	2	3	3	4	4
220	0	5	10	14	19	24	29	34	38	43	48	0	0	1	1	2	2	3	3	4	4
230	0	5	10	15	20	25	30	35	40	45	50	0	1	1	2	2	3	3	4	4	5
240	0	5	10	16	21	26	31	37	42	47	52	0	1	1	2	2	3	3	4	4	5
250	0	5	11	16	22	27	33	38	44	49	54	0	1	1	2	2	3	3	4	4	5
260	0	6	11	17	23	28	34	40	45	51	57	0	1	1	2	2	3	3	4	5	5
270	0	6	12	18	24	29	35	41	47	53	59	0	1	1	2	2	3	4	4	5	5
280	0	6	12	18	24	30	37	43	49	55	61	0	1	1	2	2	3	4	4	5	5
290	0	6	13	19	25	32	38	44	51	57	63	0	1	1	2	3	3	4	4	5	6
300	0	7	13	20	26	33	39	46	52	59	65	0	1	1	2	3	3	4	5	5	6
310	0	7	13	20	27	34	40	47	54	61	67	0	1	1	2	3	3	4	5	5	6
320	0	7	14	21	28	35	42	49	56	63	70	0	1	1	2	3	3	4	5	6	6
330	0	7	14	22	29	36	43	50	57	65	72	0	1	1	2	3	4	4	5	6	6
340	0	7	15	22	30	37	44	52	59	67	74	0	1	1	2	3	4	4	5	6	7
350	0	8	15	23	30	38	46	53	61	69	76	0	1	2	2	3	4	5	5	6	7
360	0	8	16	24	31	39	47	55	63	71	78	0	1	2	2	3	4	5	5	6	7
370	0	8	16	24	32	40	48	56	64	72	81	0	1	2	2	3	4	5	6	6	7
380	0	8	17	25	33	41	50	58	66	74	83	0	1	2	2	3	4	5	6	7	7
390	0	8	17	25	34	42	51	59	68	76	85	0	1	2	3	3	4	5	6	7	8
400	0	9	17	26	35	44	52	61	70	78	87	0	1	2	3	3	4	5	6	7	8
410	0	9	18	27	36	45	54	62	71	80	89	0	1	2	3	4	4	5	6	7	8
420	0	9	18	27	37	46	55	64	73	82	91	0	1	2	3	4	5	5	6	7	8
430	0	9	19	28	37	47	56	66	75	84	94	0	1	2	3	4	5	6	7	7	8
440	0	10	19	29	38	48	57	67	77	86	96	0	1	2	3	4	5	6	7	8	9
450	0	10	20	29	39	49	59	69	78	88	98	0	1	2	3	4	5	6	7	8	9
460	0	10	20	30	40	50	60	70	80	90	100	0	1	2	3	4	5	6	7	8	9
470	0	10	20	31	41	51	61	72	82	92	102	0	1	2	3	4	5	6	7	8	9
480	0	10	21	31	42	52	63	73	84	94	104	0	1	2	3	4	5	6	7	8	9
490	0	11	21	32	43	53	64	75	85	96	107	0	1	2	3	4	5	6	7	9	10
500	0	11	22	33	44	54	65	76	87	98	109	0	1	2	3	4	5	7	8	9	10

Example:

	Dynamic meters.
Given the temperature of 62.3° F. and the height of..........................	330
Table 16 A gives for 330 meters and 60° F. the correction....................	43
Table 16 A gives for 330 meters and 2.3° F. the correction...................	1
∴ 330 meters at 0° F. reduced to 62.3° F. gives the height of.........	374

Table 16 A (continued).—*Corrections to table 15 A for temperature.*

Height (dynamic meters).	Temperature (° F.).																			
	0	10	20	30	40	50	60	70	80	90	100	0	1	2	3	4	5	6	7	8	9
500	0	11	22	33	44	54	65	76	87	98	109	0	1	2	3	4	5	7	8	9	10
510	0	11	22	33	44	56	67	78	89	100	111	0	1	2	3	4	6	7	8	9	10
520	0	11	23	34	45	57	68	79	91	102	113	0	1	2	3	5	6	7	8	9	10
530	0	12	23	35	46	58	69	81	92	104	115	0	1	2	3	5	6	7	8	9	10
540	0	12	24	35	47	59	71	82	94	106	118	0	1	2	4	5	6	7	8	9	11
550	0	12	24	36	48	60	72	84	96	108	120	0	1	2	4	5	6	7	8	10	11
560	0	12	24	37	49	61	73	85	98	110	122	0	1	2	4	5	6	7	9	10	11
570	0	12	25	37	50	62	74	87	99	112	124	0	1	2	4	5	6	7	9	10	11
580	0	13	25	38	51	63	76	88	101	114	126	0	1	3	4	5	6	8	9	10	11
590	0	13	26	39	51	64	77	90	103	116	128	0	1	3	4	5	6	8	9	10	12
600	0	13	26	39	52	65	78	91	104	118	131	0	1	3	4	5	7	8	9	10	12
610	0	13	27	40	53	66	80	93	106	120	133	0	1	3	4	5	7	8	9	11	12
620	0	13	27	40	54	67	81	94	108	121	135	0	1	3	4	5	7	8	9	11	12
630	0	14	27	41	55	69	82	96	110	123	137	0	1	3	4	5	7	8	10	11	12
640	0	14	28	42	56	70	84	98	111	125	139	0	1	3	4	6	7	8	10	11	13
650	0	14	28	42	57	71	85	99	113	127	141	0	1	3	4	6	7	8	10	11	13
660	0	14	29	43	57	72	86	101	115	129	144	0	1	3	4	6	7	9	10	11	13
670	0	15	29	44	58	73	88	102	117	131	146	0	1	3	4	6	7	9	10	12	13
680	0	15	30	44	59	74	89	104	118	133	148	0	1	3	4	6	7	9	10	12	13
690	0	15	30	45	60	75	90	105	120	135	150	0	2	3	5	6	8	9	11	12	14
700	0	15	30	46	61	76	91	107	122	137	152	0	2	3	5	6	8	9	11	12	14
710	0	15	31	46	62	77	93	108	124	139	155	0	2	3	5	6	8	9	11	12	14
720	0	16	31	47	63	78	94	110	125	141	157	0	2	3	5	6	8	9	11	13	14
730	0	16	32	48	64	79	95	111	127	143	159	0	2	3	5	6	8	10	11	13	14
740	0	16	32	48	64	81	97	113	129	145	161	0	2	3	5	6	8	10	11	13	14
750	0	16	33	49	65	82	98	114	131	147	163	0	2	3	5	7	8	10	11	13	15
760	0	17	33	50	66	83	99	116	132	149	165	0	2	3	5	7	8	10	12	13	15
770	0	17	34	50	67	84	101	117	134	151	168	0	2	3	5	7	8	10	12	13	15
780	0	17	34	51	68	85	102	119	136	153	170	0	2	3	5	7	8	10	12	14	15
790	0	17	34	52	69	86	103	120	138	155	172	0	2	3	5	7	9	10	12	14	15
800	0	17	35	52	70	87	104	122	139	157	174	0	2	3	5	7	9	10	12	14	16
810	0	18	35	53	71	88	106	123	141	159	176	0	2	4	5	7	9	11	12	14	16
820	0	18	36	54	71	89	107	125	143	161	178	0	2	4	5	7	9	11	12	14	16
830	0	18	36	54	72	90	108	126	145	163	181	0	2	4	5	7	9	11	13	14	16
840	0	18	37	55	73	91	110	128	146	165	183	0	2	4	5	7	9	11	13	15	16
850	0	19	37	56	74	93	111	130	148	167	185	0	2	4	6	7	9	11	13	15	17
860	0	19	37	56	75	94	112	131	150	168	187	0	2	4	6	7	9	11	13	15	17
870	0	19	38	57	76	95	114	133	152	170	189	0	2	4	6	8	9	11	13	15	17
880	0	19	38	57	77	96	115	134	153	172	192	0	2	4	6	8	10	11	13	15	17
890	0	19	39	58	77	97	116	136	155	174	194	0	2	4	6	8	10	12	14	15	17
900	0	20	39	59	78	98	118	137	157	176	196	0	2	4	6	8	10	12	14	16	18
910	0	20	40	59	79	99	119	139	158	178	198	0	2	4	6	8	10	12	14	16	18
920	0	20	40	60	80	100	120	140	160	180	200	0	2	4	6	8	10	12	14	16	18
930	0	20	40	61	81	101	121	142	162	182	202	0	2	4	6	8	10	12	14	16	18
940	0	20	41	61	82	102	123	143	164	184	205	0	2	4	6	8	10	12	14	16	18
950	0	21	41	62	83	103	124	145	165	186	207	0	2	4	6	8	10	12	14	17	19
960	0	21	42	63	84	104	125	146	167	188	209	0	2	4	6	8	10	13	15	17	19
970	0	21	42	63	84	106	127	148	169	190	211	0	2	4	6	8	[illegible]	13	15	17	19
980	0	21	43	64	85	107	128	149	171	192	213	0	2	4	6	9	[illegible]	13	15	17	19
990	0	22	43	65	86	108	129	151	172	194	215	0	2	4	6	9	[illegible]	13	15	17	19
1000	0	22	44	65	87	109	131	152	174	196	218	0	2	4	7	9	[illegible]	13	15	17	20

Example:

	Dynamic meters.
Given the temperature 44° F. and the height of.	1462
Table 16 A gives for 1000 meters and 40° F............	87
Table 16 A gives for 1000 meters and 4° F......................	9
Table 16 A gives for 462 meters and 40° F......................	40
Table 16 A gives for 462 meters and 4° F........................	4
1462 meters at 0° F. reduced to 44° F. gives.........	1602

Table 17 A.—*Temperature correction to be added to the virtual temperature at the earth's surface in order to give the most probable average virtual temperatures in the sheet between the earth's surface and the nearest standard isobaric surfaces (based upon statistics).*

Height of standard surfaces above station (dynamic meters).	Temperature correction (° F.).											
	Winter.			Spring.			Summer.			Autumn.		
	High.	Mean.	Low.	High.	Mean.	Low.	High.	Mean.	Low.	High.	Mean.	Low.
2400	4.6	1.6	−2.9	−5.8	−10.1	−10.1	−7.3	−11.3	−11.2	−2.0	−8.4	−11.5
2300	4.8	1.9	−2.7	−5.4	− 9.6	− 9.7	−6.7	−10.9	−10.7	−1.7	−8.1	−11.0
2200	4.9	2.1	−2.4	−5.0	− 9.2	− 9.2	−6.6	−10.5	−10.1	−1.4	−7.6	−10 5
2100	5.0	2.3	−2.2	−4.7	− 8.8	− 8.8	−6.2	−10.1	− 9.7	−1.1	−7.3	−10.0
2000	5.1	2.5	−2.0	−4.3	− 8.3	− 8.4	−5.8	− 9.7	− 9.2	−0.9	−6.9	− 9.6
1900	5.1	2.7	−1.8	−4.0	− 7.9	− 7.9	−5.5	− 9.3	− 8.6	−0.6	−6.6	− 9.1
1800	5.1	2.8	−1.6	−3.6	− 7.5	− 7.5	−5.1	− 8.9	− 8.1	−0.4	−6.2	− 8.6
1700	5.1	2.9	−1.4	−3.2	− 7.1	− 7.1	−4.7	− 8.5	− 7.6	−0.1	−5.9	− 8.2
1600	5.0	3.0	−1.2	−2.9	− 6.6	− 6.7	−4.4	− 8.1	− 7.1	+0.1	−5.5	− 7.7
1500	4.9	3.1	−1.1	−2.5	− 6.2	− 6.3	−4.0	− 7.7	− 6.7	+0.3	−5.2	− 7.2
1400	4.7	3.1	−1.0	−2.2	− 5.8	− 5.9	−3.6	− 7.3	− 6.2	+0.5	−4.8	− 6.8
1300	4.5	3.1	−0.8	−1.9	− 5.4	− 5.5	−3.3	− 6.9	− 5.7	+0.7	−4.5	− 6.3
1200	4.2	3.0	−0.7	−1.6	− 5.0	− 5.1	−2.9	− 6.5	− 5.2	+0.8	−4.1	− 5.8
1100	4.0	2.9	−0.6	−1.3	− 4.6	− 4.7	−2.5	− 6.1	− 4.8	+0.9	−3.8	− 5.4
1000	3.8	2.7	−0.5	−1.1	− 4.2	− 4.3	−2.2	− 5.6	− 4.3	+1.0	−3.4	− 4.9
900	3.6	2.5	−0.5	−0.9	− 3.8	− 3.9	−1.9	− 5.2	− 3.9	+1.0	−3.1	− 4.5
800	3.3	2.3	−0.4	−0.7	− 3.4	− 3.4	−1.6	− 4.7	− 3.4	+1.0	−2.7	− 4.0
700	2.9	2.1	−0.4	−0.5	− 3.0	− 3.0	−1.2	− 4.1	− 3.0	+0.9	−2.5	− 3.5
600	2.5	1.8	−0.3	−0.4	− 2.5	− 2.6	−1.0	− 3.6	− 2.5	+0.8	−2.1	− 3.0
500	2.2	1.5	−0.2	−0.3	− 2.2	− 2.2	−0.8	− 3.0	− 2.1	+0.7	−1.9	− 2.5
400	1.7	1.2	−0.1	−0.3	− 1.7	− 1.7	−0.6	− 2.5	− 1.6	+0.6	−1.4	− 2.1
300	1.3	0.9	0	−0.2	− 1.3	− 1.3	−0.4	− 2.1	− 1.2	+0.5	−1.1	− 1.6
200	0.9	0.6	0	−0.1	− 0.9	− 0.9	−0.3	− 1.4	− 0.7	+0.3	−0.7	− 1.1
100	0.5	0.3	0	−0.1	− 0.4	− 0.4	−0.2	− 0.7	− 0.4	+0.2	−0.4	− 0.5
0	0	0	0	0	0	0	0	0	0	0	0	0

Extrapolation below the earth's surface (common for all pressures and seasons).

Dynamic meters.	Temperature correction (° F.).	Dynamic meters.	Temperature correction (° F.).	Dynamic meters.	Temperature correction (° F.).
0	0	−500	2.3	−1000	4.5
−100	0.5	−600	2.7	−1100	5.0
−200	0.9	−700	3.2	−1200	5.4
−300	1.4	−800	3.6		
−400	1.8	−900	4.1		

Example: Low pressure, spring; at station pressure 28.87 inches of mercury; virtual temperature + 50.1° F.

1	2	3	4	5	6
800	1469	−6.2	+43.9	141	1610
900	606	−2.6	+47.5	63	669
1000	−165	+0.7	+50.8	−18	−183

Column 1. Standard surfaces.
2. Approximate height of these surfaces, found from table 15 A for the pressure of 28.87 inches of mercury at the station.
3. Temperature corrections according to table 17 A for low pressure spring and for the heights of column 2.
4. Most probable average virtual temperature of the sheets between the earth and the standard surfaces of column 1, found by addition of the corrections of column 3 to the virtual temperature at the station + 50.1° F.
5. Corrections to the height of column 2, found from table 16 A for the heights of column 2 and the average virtual temperature of column 4.
6. Height of the standard surfaces above the station found by addition of the approximate heights of column 1 and the corrections of column 5.

Table 18 A.—*Change of isobaric charts, given in millimeters of mercury for sea level, into charts of dynamic topography of the 1000 m-bars isobaric surface.*

Height (dyn. meters).	Temperature (° C.) in sea-level.										
	−50	−40	−30	−20	−10	0	10	20	30	40	50
−500	693	696	698	700	702	703	705	707	708	709	711
−450	699	701	703	705	706	708	709	711	712	713	714
−400	704	706	708	710	711	713	714	715	716	717	718
−350	710	712	713	715	716	717	718	720	721	721	722
−300	716	717	718	720	721	722	723	724	725	725	726
−250	721	722	724	725	726	726	727	728	729	730	730
−200	727	728	729	730	730	731	732	733	733	734	734
−150	733	734	734	735	735	736	736	737	737	738	738
−100	738	739	739	740	740	741	741	741	742	742	742
− 50	744	745	745	745	745	745	746	746	746	746	746
0	750	750	750	750	750	750	750	750	750	750	750
50	756	756	755	755	755	755	755	755	754	754	754
100	762	761	761	760	760	760	759	759	759	758	758
150	768	767	766	766	765	765	764	764	763	763	762
200	774	773	772	771	770	769	769	768	767	767	766
250	780	779	777	776	775	774	774	773	772	771	770
300	786	784	783	782	780	779	778	777	776	775	775
350	792	790	789	787	786	784	783	782	781	780	779
400	798	796	794	793	791	789	788	786	785	784	783
450	804	802	800	798	796	794	793	791	790	788	787
500	811	808	806	803	801	799	797	796	794	793	791

Given the isobaric chart in millimeters of mercury and the isothermic chart, centigrade, both for sea-level, the latter reduced to sea-level under the supposition of a fall of temperature of 0.5° C. for every hundred dynamic meters of height. On account of the smallness of the reductions no distinction between true and virtual temperature is required.

(1) Required the level curve of height 0 on the 1000 m-bars surface. The table shows this curve to be identical with the isobaric curve of 750 mm. pressure.

(2) Required the level curve of height 250 dynamic meters on the 1000 m-bars surface. The table shows that it passes closely by the points of intersection of the isothermic curve —50° and the isobaric 780 mm.; the isothermic —40° and the isobaric 779 mm.; the isothermic —30° and the isobaric 777 mm., and so on.

(3) Required the level curve of height —150 dynamic meters on the 1000 m-bars surface. The table shows that it passes closely by the points of intersection of the isothermic curve —50° with the isobaric 733 mm.; of the isothermic curves —40° and —30° with the isobaric 734 mm.; of the isothermic curves —20° and —10° with the isobaric 735 mm., and so on.

The main result is a close accordance of the level lines for the interval of 50 dynamic meters with the isobaric lines for the interval of 5 mm. of mercury.

Table 19 A.—*Change of isobaric charts, given in inches of mercury for sea-level, into charts of dynamic topography of the 1000 m-bars isobaric surface.*

Height (dyn. meters).	Temperature (° F.) at sea-level.															
	−50	−40	−30	−20	−10	0	10	20	30	40	50	60	70	80	90	100
−500	27.34	27.39	27.44	27.49	27.53	27.57	27.61	27.65	27.60	27.73	27.76	27.79	27.83	27.85	27.89	27.91
−450	27.55	27.60	27.64	27.68	27.73	27.76	27.80	27.84	27.87	27.90	27.93	27.96	27.99	28.02	28.05	28.07
−400	27.77	27.81	27.85	27.88	27.92	27.96	27.99	28.02	28.05	28.08	28.11	28.13	28.16	28.18	28.21	28.23
−350	27.98	28.02	28.05	28.09	28.12	28.15	28.18	28.21	28.23	28.26	28.28	28.31	28.33	28.35	28.37	28.39
−300	28.20	28.23	28.26	28.29	28.32	28.34	28.37	28.39	28.42	28.44	28.46	28.48	28.50	28.52	28.53	28.55
−250	28.42	28.44	28.47	28.49	28.52	28.54	28.56	28.58	28.60	28.62	28.63	28.65	28.67	28.68	28.70	28.71
−200	28.64	28.66	28.68	28.70	28.72	28.74	28.75	28.77	28.78	28.80	28.81	28.82	28.84	28.85	28.86	28.88
−150	28.86	28.87	28.89	28.90	28.92	28.93	28.94	28.96	28.97	28.98	28.99	29.00	29.01	29.02	29.03	29.04
−100	29.08	29.09	29.10	29.11	29.12	29.13	29.14	29.15	29.16	29.16	29.17	29.18	29.18	29.19	29.20	29.20
− 50	29.31	29.31	29.32	29.32	29.33	29.33	29.33	29.34	29.34	29.35	29.35	29.35	29.36	29.36	29.36	29.37
0	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53	29.53
50	29.76	29.75	29.75	29.74	29.74	29.73	29.73	29.72	29.72	29.72	29.71	29.71	29.71	29.70	29.70	29.70
100	29.99	29.98	29.97	29.96	29.95	29.94	29.93	29.92	29.91	29.90	29.90	29.89	29.88	29.88	29.87	29.86
150	30.22	30.20	30.18	30.17	30.16	30.14	30.13	30.11	30.10	30.09	30.08	30.07	30.06	30.05	30.04	30.03
200	30.45	30.43	30.41	30.39	30.37	30.35	30.33	30.31	30.30	30.28	30.27	30.25	30.24	30.22	30.21	30.20
250	30.68	30.65	30.63	30.60	30.58	30.55	30.53	30.51	30.49	30.47	30.45	30.43	30.42	30.40	30.38	30.37
300	30.92	30.88	30.85	30.82	30.79	30.76	30.74	30.71	30.69	30.66	30.64	30.62	30.60	30.58	30.56	30.54
350	31.15	31.11	31.08	31.04	31.00	30.97	30.94	30.91	30.88	30.85	30.83	30.80	30.77	30.75	30.73	30.71
400	31.39	31.34	31.30	31.26	31.22	31.18	31.15	31.11	31.08	31.05	31.02	30.99	30.96	30.93	30.90	30.88
450	31.63	31.58	31.53	31.48	31.44	31.39	31.35	31.31	31.28	31.24	31.21	31.17	31.14	31.11	31.08	31.05
500	31.87	31.81	31.76	31.70	31.65	31.61	31.56	31.52	31.48	31.44	31.40	31.36	31.32	31.29	31.26	31.23

Given the isobaric chart in inches of mercury and the isothermic chart, Fahrenheit, both for sea-level, the latter reduced to sea-level under the supposition of a fall of temperature of 1° F. for every 100 dynamic meters of height. On account of the smallness of the reductions no distinction between true and virtual temperature is needed.

(1) Required the level curve of height 0 on the 1000 m-bars surface. The table shows this curve to be identical with the isobaric curve of 29.53 inches pressure.

(2) Required the level curve of height 250 dynamic meters on the 1000 m-bars surface, The table shows it to pass closely by the points of intersection of the isothermic curve —50° F. with the isobaric 30.68 inches; of the isothermic —40° F. with the isobaric 30.65 inches; of the isothermic —30° F. with the isobaric 30.63 inches, and so on.

(3) Required the level curve of height —50 dynamic meters on the 1000 m-bars surface. The table shows it to pass closely by the points of intersection of the isothermic curves —50 and —40° F. with the isobaric 29.31 inches; of the isothermic curves —30 and —20° F. with the isobaric 29.32 inches; of the isothermic curves —10, 0, and +10° F. with the isobaric 29.33 inches, and so on.

DYNAMIC METEOROLOGY AND HYDROGRAPHY

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AND

DIFFERENT COLLABORATORS

PART II.—KINEMATICS

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DYNAMIC METEOROLOGY AND HYDROGRAPHY

PART II. KINEMATICS.

BY

V. BJERKNES, TH. HESSELBERG AND O. DEVIK

CHAPTER I.

GENERAL CONSIDERATIONS ON THE OBJECT AND THE METHODS OF DYNAMIC METEOROLOGY AND HYDROGRAPHY.

87. The General Problem.—Treating statics of atmosphere and of hydrosphere we have considered invariable states of these media. Although passing occasionally the strict limits of statics, we never considered the states from the point of view of their variations, time never entering into our equations. But in entering upon the investigation of these states, not only from the point of view of their distribution in space, but also from that of their variation in time, we have to introduce *time* as a new independent variable. This allows us to view our problem in its generality and it will be useful to do this before returning to investigations of detail.

Considering the problem from a mathematical point of view, we have first to define our independent and our dependent variables.

We consider meteorological and hydrographic phenomena in relation to space and time, *i. e.*, our independent variables are *coordinates* and *time*. The system of coordinates is always rigidly attached to the earth. Two of the coordinates are the geographical ones, serving to define points at the surface of the land or of the sea; while the third has to give the height above or the depth below sea-level. In our static investigations we have found it convenient to measure this third coordinate in dynamic instead of in geometrical measure, and this will generally be convenient during the continued work.

As dependent variables we have to introduce the quantities required for defining the state of the atmosphere and the hydrosphere, or formulating the laws of the changes of these states. We shall designate these dependent variables as meteorological or hydrographic *elements*. The distribution in space of any of these elements is called its *field*. For the description of atmospheric states we have to consider at least five fields, those of pressure, of mass, of temperature, of humidity, and of motion. The first four of these are scalar fields; the fifth, that of motion, is a vector-field. The question may be raised if the full description of atmospheric states and of the laws of their changes will not require the introduction of still more fields. Thus there may be a mutual dependency upon one another of the meteorological processes and the electric or the magnetic fields of the earth. This would require the introduction of vectors describing these fields as further meteorological elements. But the rational plan will be, first, to treat the problem, as far as possible, with the smallest number of variables. We therefore restrict ourselves to the consideration of the five fields already defined for the case of the atmosphere. The five corresponding fields for describing the states of the hydrosphere and for formulating the laws of their changes are the fields of pressure, of mass, of temperature, of salinity, and of motion, precisely the same as in the case of the atmosphere, except that salinity takes the place of humidity.

The fields of pressure, of temperature, of humidity, and of salinity are described by the values of the corresponding elements observed in the different points of space. The fields of mass can be described in either of two ways, by the mass per unit volume or by the volume of unit masses. That is, we can consider either *density* or *specific volume* as the scalar element describing this field. In the same way we can use two different elements of vector-nature for describing the field of motion, either *velocity* or *specific momentum* (Statics, section 3).

Having defined our variables, we can thus concisely state the problem of meteorology and hydrography: *To investigate the five meteorological and the five hydrographic elements as functions of coordinates and time.*

88. Investigation of Phenomena Depending upon More Variables.—The general principle for investigating phenomena depending upon more variables is this: systematically to keep constant a certain variable or group of variables, in order to examine the effect of varying another variable or group of variables.

We have used this principle in statics already. Independent variables were then only the three coordinates. Among them the two geographical ones evidently form a natural group, having other relations to the investigated fields than the third coordinate, height. This difference determined the method. We began by considering the conditions of equilibrium along certain vertical (or quasi-vertical) lines, namely, the lines along which meteorological ascents or hydrographic soundings had taken place (Statics, Chapters VI and VIII); or in mathematical language, we gave to the geographical coordinates the constant values defining the stations and examined the effect of varying the third variable, height.

Using the results thus obtained, we afterwards drew synoptical charts, representing the fields by horizontal sections instead of by vertical soundings (Statics, Chapters VII and IX). This representation involves a modified use of the same general principle; for a chart shows the effect of varying the two geographical coordinates, while the third independent variable keeps constant.

When performing investigations according to this general principle it is occasionally convenient to let a certain dependent and a certain independent variable change parts. In this way we interchanged pressure and height. Retaining height as the third independent variable, to which the constant values were given, we arrived at isobaric charts drawn in level surfaces (section 65). Using pressure as the third independent variable to which the constant values were given, we arrived at topographic charts of isobaric surfaces (section 64). But in both cases the general result was the same, namely, a representation of the field of pressure in its relation to space, *i. e.*, in reference to coordinates as independent variables.

Introducing now a fourth independent variable, time, besides the three old ones, the coordinates, we have to apply the same general principle. The first question will then be that of the grouping of the variables. About this question there can be no doubt; for evidently the three coordinates form a natural group, having other relations to the phenomena than the fourth variable, time. The grouping of the variables being agreed upon, we can proceed along two ways: (1) Giving constant values to the coordinates, we can examine the effect of letting time vary; or (2) giving a constant value to time, we can examine the effect of letting coordinates vary. These two different ways lead to two essentially different branches of meteorological and of hydrographic science.

89. **Climatological Method.**—First let us give constant values to the coordinates, and examine the effect of letting time vary. We can imagine the investigation performed in the following way: Self-recording instruments are set up at a number of fixed points (stations) in atmosphere or hydrosphere. The different records of the meteorological or hydrographic elements then show directly the effect of letting time vary, while the coordinates have the constant values defining a certain station.

When we examine the records we find great irregular changes, the explanation of which can not be found by a direct examination of the curves; but conspicuous signs of *regular* changes are also discovered. Forming averages in different ways, the irregular phenomena will more or less disappear. The regular ones will then, for the most part, present a periodical character, having the periods of the solar day, of the solar year, of the sunspots, and perhaps of still other cosmic phenomena. Besides the decidedly periodic phenomena, slow secular changes may also be discovered.

The different kinds of averages thus formed of the meteorological or hydrographic elements may be called the *climatological* elements for atmosphere or hydrosphere. Inasmuch as time enters into the definition of these elements, it is the local time of each station, not universal simultaneous time. The elements found at the different stations may be compared to each other. This leads to the drawing of climatological maps, showing the average influence of geographical data, just as the single curves showed that of astronomical events; but no way leads to the investigation of the nature or the causes of what we called irregular phenomena. These were eliminated, and to investigate them we must follow another way.

90. **Dynamic Method.**—In order to examine the other method, we can start with the records obtained from the same set of self-recording instruments, but shall make a modified use of them. Giving time a certain constant value, we read off from all records the values of meteorological or hydrographic elements at this epoch, and draw continuous synoptical representations of the field of each element. Having thus got a complete picture of the state of the atmosphere or the hydrosphere at this epoch, we give time a new constant value, read off the new values of the elements, and produce new synoptical representations of the fields, which give a complete picture of the state of atmosphere or hydrosphere at this second epoch, and so on.

A series of such pictures being produced, the next step will be to make them the subject of a comparative investigation. This comparative investigation of the successive states must lead to the solution of the ultimate problem of meteorological or hydrographic science, viz, that of discovering the laws according to which an atmospheric or hydrospheric state develops out of the preceding one.

We shall call this the *dynamic* method; for in virtue of the laws of hydrodynamics and thermodynamics which govern atmospheric or hydrospheric phenomena, preceding states are in relation of causality to subsequent states. Inasmuch as we know the laws of hydrodymanics and thermodynamics, we know the intrinsic laws according to which the subsequent states develop out of the preceding ones. We are therefore entitled to consider the ultimate problem of meteorological and hydrographic science, that of the precalculation of future states, as one of which we already possess the *implicit* solution, and we have full reason to believe that we shall succeed in making this solution an *explicit* one according as we succeed in finding the methods of making full practical use of the laws of hydrodynamics and thermodynamics.

91. Three Partial Problems.—Evidently general investigations according to the dynamic plan must lead to occupation with three special problems. The first is the question of the *organization of observations* serving these investigations. The observations being given, the next problem will be to work out from them synoptical representations of the fields serving to define actual states of atmosphere or hydrosphere. Introducing a terminology taken from medical science, we shall call this the problem of *diagnosis* of atmospheric or hydrospheric states. The result of a diagnosis being given, the final problem will be that of precalculating future states. Making continued use of the same terminology, we shall call this the problem of *prognosis* of future states. Before returning to details, we shall make some general remarks on each of these three problems, taking as the leading idea that the condition for real progress is to arrange so that full use can be made of the knowledge contained in the laws of hydrodynamics and thermodynamics.

92. Principles for the Organization of Observations.—It is of course not possible to know how observations will be organized later, when the problems of diagnosis and of prognosis are completely solved in explicit form. But the question interesting the present generation of investigators is to get that organization which would facilitate as much as possible the work with the solution of these problems.

From what we have evolved already it will be clear that the dynamic method requires simultaneous observations. The *principle of simultaneity* being therefore agreed upon as the fundamental one, the next questions will be those of the *distribution in space* of each set of simultaneous observations and the *distribution in time* of the successive epochs of observations.

In order to answer these questions, we have to remark that the fundamental laws of hydrodynamics and thermodynamics have the form of partial differential equations giving relations between the continuous space-variations and time-variations of the different elements. To make it as easy as possible to bring them into application, we must try to organize observations so as to realize an approximation toward *continuity* in space and time. In other words, the distances in space between the points of observation and the distances in time between the epochs of observation must be small enough to be used, with a certain degree of approximation, as line-differentials and time-differentials.

The test that the distribution in space of the points of observation fulfil this condition will be, that it turns out to be possible to draw synoptical maps, by use of the observations; for such maps give *continuous* representations of the fields of the observed elements. The distances to be allowed in the net of observations will therefore depend upon the space-variations of the elements. The network must be satisfactory for the element having the strongest space-variations. But nothing hinders elements which have less irregular distribution in space from being observed at a smaller number of points in the network.

A suitable time-differential must be determined by a comparison of synoptic charts representing the field of the same element at successive epochs. The changes which the element has undergone from epoch to epoch must be small enough to allow us to form satisfactory approximate values of the time-derivative of the element. The time-differential must therefore be chosen so as to suit the element which has the most rapid time-variation. But nothing hinders elements having slower time-variations from being observed, only, for instance, at every second or every third of the epochs of observation, which have thus been chosen.

93. **Special Remarks on Meteorological Observations.**—In passing to concrete meteorological observations, we shall first make some remarks regarding the *principle of simultaneity.*

The ideal is of course the use of self-recording instruments having sufficiently large time-scale. But whichever instruments or methods of observation be used, it will be neither possible nor required to realize simultaneity in the mathematical sense of the word. Most meteorological elements will under ordinary circumstances change very little during as small an interval of time as, for instance, half an hour. Departures of this magnitude from the precise epoch of observation will therefore not usually produce errors of greater importance, though exceptions are not excluded,

The general slowness of the variations makes it possible to use averages registered during suitable intervals of time instead of true instantaneous values. For one element, wind, the use of averages, as we shall see, will be unavoidable, and it will have certain advantages also in connection with other elements, especially inasmuch as time-integrations should be performed afterwards. But if averages be used, they should be used at all the cooperating stations, and taken according to the same rules at all. These conditions have been excellently fulfilled by hourly averages which we have obtained from the U. S. Weather Bureau.

Observations obtained from the higher strata by meteorological ascents will cause certain difficulties inasmuch as the records taken by the same instrument at different levels are not taken simultaneously. But the departures will be reduced according as we increase the velocity of the ascent. A registering balloon can be made to mount from the ground to the lower limits of the isothermal layer in less than an hour. Departures up to half an hour from the true epochs of observation being considered allowable, we are entitled to consider the observations obtained by such a balloon in different levels as simultaneous with observations taken near the ground half an hour after its launching.

Thus a tolerably satisfactory simultaneity *can* be realized even for the observations from the higher strata. But still the principle of simultaneity is not carried through universally, not even for the observations at the ground, where its realization should not cause any real difficulty. Thus departures by far exceeding the half-hour limit exist still in the European net of daily observations. Fortunately in the United States the principle of simultaneity is completely carried through for the whole net of stations. This circumstance, in connection with the complete homogeneity of the observations, all being obtained from self-recording instruments of the same construction and treated according to the same rules, make these observations the best which we have had at our disposal for the study of the conditions of the atmosphere near the ground.

Passing to the *distribution in space* of the points of observation, we must distinguish the points of observation near the ground from those in the free atmosphere. As to the investigation of the lowest atmospheric sheet, the greater nets of observation, as that of Europe, of the United States, or of India, may be said to be satisfactory, exceptions being made for certain specially difficult regions, for instance the western mountainous parts of the United States. For practical reasons the net of stations is here less close, while the space-variations of meteorological elements are stronger than in the flat land. For the most variable element, wind, this has caused us great difficulties.

In the free space fixed points of observation can not be maintained, and would not, unless they could be kept up in great number, be of appreciable use; for the lengths which can be used as line-differentials in vertical direction are much smaller than those which can be used in horizontal direction. But on account of the relative slowness of the variations in time and the rapidity with which meteorological ascents can be performed, we can get continuous records along vertical lines, representing approximately the instantaneous state of things along these lines.

As the variation of meteorological elements in horizontal direction is necessarily much smaller in the free atmosphere than near the ground, where the local influences of topography come in, it will not be necessary to provide all stations at the ground with the implements for meteorological ascents. But only experience can show how close the net of aerological stations should be. Further, it will not be required to give all aerological stations equally complete equipment, for the scalar elements have much less pronounced space-variations than the vector-element, velocity. As air-velocity is also much easier to observe, thanks to the method of pilot-balloons, it will be rational and economical to erect two classes of aerological stations, complete aerological stations and pilot-balloon stations. How close the net of each kind should be, will be evident by and by from the synoptical maps drawn by use of the ascents. The erection of aerological stations, including pilot-balloon stations in great numbers, will be of special importance in mountainous regions, where the effectivity of the common stations is so limited on account of the local irregularities.

The last and most delicate question is that of the determination of a suitable *time-differential* separating the epochs of observation. Inasmuch as continuity in time is realized in as great extent as possible by providing the stations at the ground with self-recording instruments, the question will be reduced to that of a suitable interval between the successive aerological soundings. As time-variations of the meteorological elements have the same rapidity near the ground as in the free air, this question can be answered by examination of charts for the ground concerning the element which has the most rapid time-variations, namely, velocity. According to our preliminary experience regarding these charts (see Chapters XII and XIII) it seems reasonable to try time-differentials of three hours for this element, while differentials of double the length may be used for the other elements.

Observations of the completeness thus required can not be kept up continuously. It will be necessary to organize special periods of investigation extended for each time over a series of days. An effective organization of such a period would be this:

During the whole period continuous observations or observations for every hour of Greenwich time are kept up at all stations at the ground.

For every third hour of Greenwich time ascents are made from the pilot-balloon stations.

For every sixth hour of Greenwich time ascents are made from the complete aerological stations.

94. **Remarks on Hydrographic Observations.**—Oceanographic observations are not yet organized systematically. But the general principles for their organization will be the same as for the meteorological observations. Hydrographic expeditions going out occasionally can only contribute to the knowledge of the average state, *i. e.*, to the climatology of the sea. But the final aim must be that of investigating the actual states and their variations. The organization must then be governed by the principle of simultaneity. The investigations will have to be performed not by one luxuriously fitted ship, passing months or years at sea, but by the cooperation of small ships going out simultaneously.

The demands regarding the degree of simultaneity and the intervals between the epochs of observation will depend upon the rapidity of the changes. There are indications both for rapid changes (among which the tidal phenomena in the deeper strata will play an important part) as well as for slow seasonal changes and changes from year to year. The problem will be to organize observations so as to separate from each other the changes of different rapidity and to investigate them as much as possible independently of each other. But a serious discussion on the suitable method of organization will only be possible by and by, as our knowledge of the oceanic phenomena advances.

95. **The Problem of Diagnosis.**—The observations being given, the diagnosis will consist in working out continuous synoptical representations of the field of each element. This involves first the choice of proper methods of representing each field synoptically. This choice being made, methods for passing from the observations to the synoptic representation must be worked out. These diagnostic methods must take into consideration not only the observations themselves, but also all intrinsic relations existing between observed quantities and quantities to be represented. It is due to these intrinsic relations that we are able to work out relatively complete

representations in spite of the extreme incompleteness of the observations. According as we introduce the different relations of dynamics and thermodynamics, we shall have to examine carefully their possible diagnostic use.

In statics our work was exclusively of this diagnostic nature. We chose our methods for representing two fields, those of pressure and of mass, and we developed the methods of arriving at these representations, making diagnostic use of two relations, viz, the equation of hydrostatics and the gas-equation, respectively the relation existing between temperature, salinity, pressure, and specific volume of the sea-water. Passing now to kinematics, we shall have to occupy ourselves with the diagnosis of the field of motion. We shall choose methods for representing this field, and try to make complete diagnostic use of all relations of kinematic origin.

96. The Problem of Prognosis.—The present state being diagnosticated, the final problem is that of the precalculation of future states. The solution of this problem will involve the simultaneous use of all intrinsic relations of hydrodynamic and thermodynamic origin, to be used in connection with the initial conditions, the surface conditions, and data regarding exterior effects of terrestrial or cosmic origin. Evidently the problem is of enormous complexity. But in order to try to prepare its solution, we shall solve one by one a series of partial problems belonging to it. For every equation introduced we shall examine its prognostic as well as its diagnostic value. In kinematics we shall meet with the first partial problem of prognosis, for the definition of the fundamental kinematic vectors involves the idea of time. When we know the instantaneous velocity of a moving particle, we shall know the place of this particle a differential of time later. The changes of place of the moving particles can therefore be determined in the first approximation by purely kinematic principles. The solution of this problem of kinematic prognosis is the first step in the solution of the general problem.

During the work with the problem of prognosis, it will be apparent that while we are probably in possession of all the intrinsic relations to be used for its solution, certain empirical data required for bringing them into application must be sought. The missing data can in many cases be found by reversing the problem of prognosis. The state being known at two epochs, we calculate the missing data, which, used in the intrinsic relations, should allow us to calculate the second state when the first is given. Having this method in view, we shall treat the different partial problems of prognosis both in direct and in inverse form.

Reversing the problem of kinematic prognosis, we shall thus arrive at the purely kinematic determination of accelerations. When we determine afterwards the same accelerations by dynamic principles, we get the opportunity of finding the value of a term in the dynamic equation, of which we have not *a priori* a sufficient knowledge, namely, of that representing frictional resistance.

CHAPTER II.

THE OBSERVATIONS OF AIR AND SEA MOTIONS.

97. The Common Wind-Observations.—Taking up the subject of the kinematics of atmosphere and hydrosphere, we have first to discuss the observations to be used as the basis of the kinematic diagnosis. We shall begin by considering the observations of wind.

Even a rough examination shows the wind to be very irregular, its direction and intensity changing rapidly in varying limits. By using finer methods of observation smaller irregular air-movements will be discovered which would otherwise escape our attention. Directions and intensities of wind noted at meterological stations are therefore always averages, the smaller irregularities not being discovered and the greater ones being smoothed out by the personal estimate of the observer or by a regular treatment of the records of the self-recording instruments.

It is therefore only certain average air-motions which can be submitted to a kinematic analysis. Neglecting the small irregularities in the large-scale meteorology, we make a similar approximation as when in laboratory experiments on fluid motion we neglect the irregular molecular motions existing according to the kinetic theory. But in both cases indirect effects of the small motion arise in the form of an apparent increase of frictional resistance. The question of this resistance will be taken up in the dynamic part of this book.

For our kinematic investigations we have to mention these irregularities only on account of the uncertainty which they cause in the noted average direction and intensity of the wind. When quantitative use is to be made of the wind-observations, it will be important to use rational methods both for taking the observations and for smoothing out the irregularities. Especially it will be important that the *same* method should be used for these purposes at all cooperating stations. The best results will be obtained by self-recording instruments, the averages being taken from the values registered during an interval of time extended equally long before and after the epoch of observation. The average should be formed by *vector-addition* and registering instruments should allow an easy determination of this average. The vector formed by taking the separate averages of the recorded directions and of the recorded intensities will not be the true vector-average; but it may be used approximately instead of the true vector-average if the variations of direction and intensity have not been too strong during the interval for which the average is formed. As meteorological wind-observations have not been organized in view of our quantitative applications, they are very imperfect from our point of view. In Europe, besides the fundamental imperfection that the principle of simultaneity is not carried through, all sorts of wind-observations are used, from personal estimates to averages obtained by the best self-recording instruments. Greater homogeneity is highly desirable. The best wind-observations which we have had at our disposal are from the United States. We have obtained them partly from the published weather maps, and partly from unpublished material, thanks to the kindness of the United States Weather Bureau. They give the registered average wind-velocities from hour to hour, and eight corresponding average wind-directions. We have considered these averages as defining the vector-average of the wind for the half-hour epoch, though for periods of rapid changes they may differ considerably from the true vector-average. Quite independently of the method of averaging, it would be a great improvement to increase the number of wind-directions noted from eight to at least sixteen.

98. Preliminary Synoptic Representation of the Wind-Observations.—A set of simultaneous observations of the wind being given, the first step in the subsequent diagnostic work will be to introduce these observations in a convenient form on the map.

The most direct way will be to draw on the map a set of arrows representing the observed wind-directions, and to add numbers representing the observed wind-velocities in meters per second. These numbers, giving the result in quantitative form, should always be introduced instead of the different qualitative signs used to represent the strength of wind according to the different "wind-scales." Plates XXXI, XXXVI, and LIII give examples of charts containing in this way a representation of wind-observations.

Besides representing the directions by arrows, we shall use a method of representing them by numbers. This will be useful not only for purposes of registration, but also for quantitative work. The correspondence between directions and numbers which we shall use is illustrated by fig. 32.

The numbers defined by this figure may be used not only as names of the directions, but also as measure of the angles which the different directions form with the initial direction, the direction toward E. We get in this manner a measure of the angles by dividing the circle into 64 instead of 360 degrees. We have chosen this measure of angles for our purposes by two reasons; first, 64 is the highest two-figure number which is a power of 2; and then its tenth part, 6.4, differs only by 1.9 per cent from 2π or 6.28. This difference will as a rule be insignificant for us. We can therefore consider the numbers 1 to 64, after division by 10, as representing the angles in absolute measure. The choice of the direction E. N. W. S., *i. e.*, the direction against the motion of the hands of a watch, as the positive, and the direction toward E. as the initial direction, is made for reasons which will be apparent later, when we shall choose our systems of coordinates and give the corresponding rule of signs.

When this correspondence between numbers and directions is used, it will be found convenient to have the diagram of fig. 32 engraved on a transparent sheet of glass or of celluloid. By use of this divided plate we can then easily pass from an arrow to the corresponding number, or vice versa.

These numbers can now be introduced on the charts instead of the arrows. The chart will then contain a representation of the wind-observations by use of two sets of numbers, one set representing the wind-directions and another representing the wind-intensities. It will be found convenient to use ciphers of different type or of different color for the two different sets. The heavy numbers on plates XXXVI and LIII represent wind-directions.

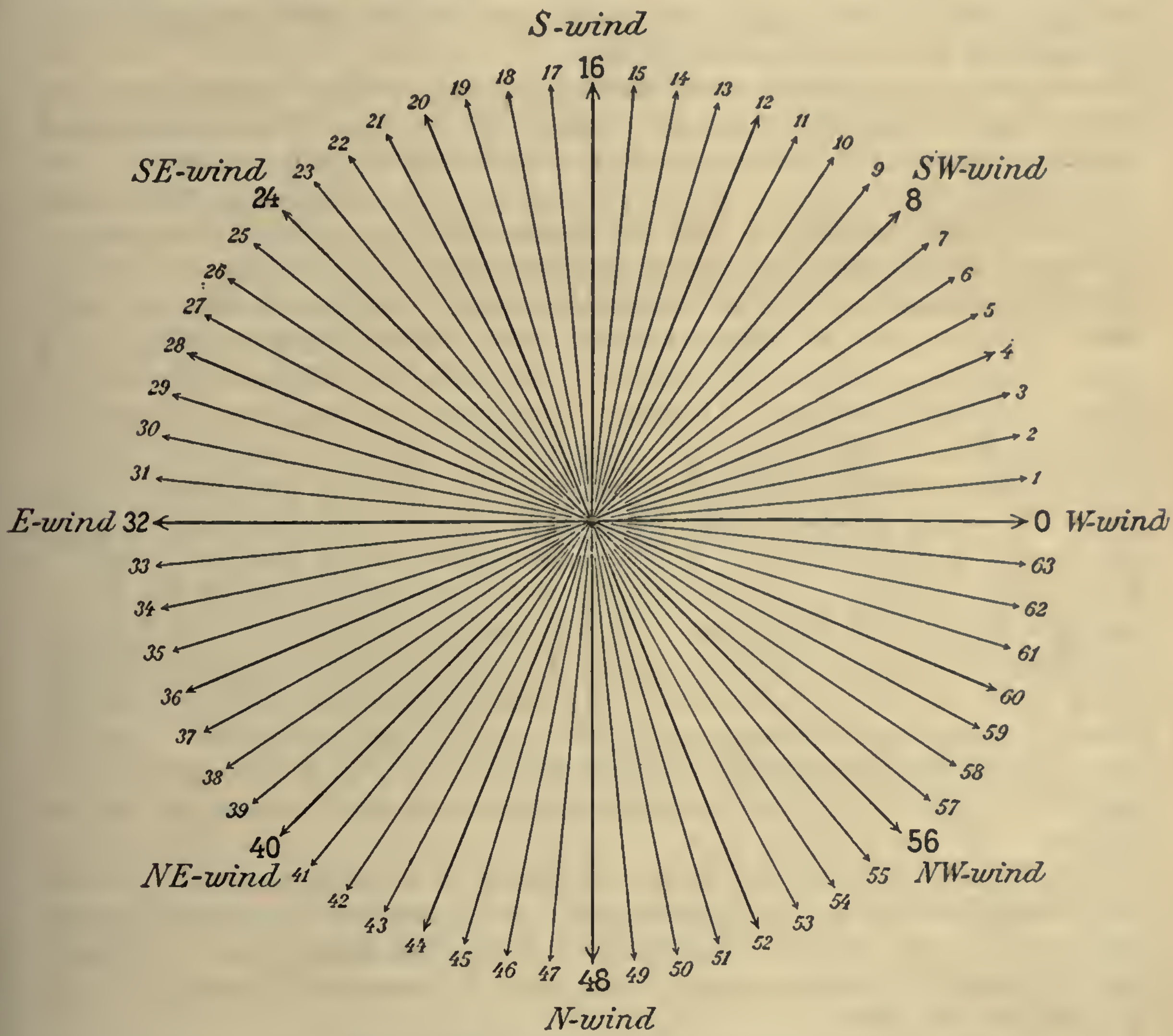

FIG. 32.—Representation of directions by numbers.

Charts of this description, which contain a representation of the observed motions either by arrows and numbers or by two sets of numbers, will form the starting-point for the whole subsequent work of kinematic diagnosis.

99. Observations of Air-Motion in the Free Atmosphere.—Until lately the drift of the clouds gave the only information on air-motion in the higher strata. Qualitative results were obtained by observing the cloud-form and the direction of the drift, and quantitative results in the first approximation by also measuring the angular velocity of the cloud by a nephoscope. For as the height of the cloud can be estimated approximately by its form, the velocity can be calculated. But besides the smaller errors caused by the use of the estimated heights, great errors may arise on account of the difficulty of recognizing with perfect certainty the cloud-forms. Even if finer methods be used, based upon the measurement of the parallax, the diffuseness of the objects observed causes difficulties, and the process of formation and dilution of clouds going on simultaneously with their motion makes the interpretation of the observations difficult.

But the great drawback of the cloud-observations is that they only give sporadic information, depending upon where clouds happen to be. What is wanted are *continuous records* of the air-motion taken along vertical or quasi-vertical lines, corresponding to the continuous records of pressure, temperature, and humidity, which we have considered in Statics. Continuous records of air-motion may be obtained by the same ascents which give records of the scalar meteorologic elements. Besides the other instruments, a kite can lift an anemometer registering the wind-intensity, while the direction can be estimated roughly by the direction of the kite-line. But the best results are obtained by observing the motion of free balloons by theodolites. If the height of the balloon can be found from other data, only one theodolite is required. Thus if the balloon carries a registering barograph, the direction and intensity of the wind may be found as function of the registered pressure or as function of the height calculated from this pressure. But an important simplification has recently been introduced. A closed caoutchouc-balloon has been found to mount with a practically constant velocity, which can be calculated by the dimensions and buoyancy of the balloon. Thus the height is known simply by the time elapsed since the moment of its launching. The air-motion can therefore be determined much more easily than all other meteorological elements in the free air, the instruments required being simply a theodolite and a small pilot-balloon. According to Hergesell* this method gives better determinations of the air-motion in the higher strata than our ordinary station-instruments can give for the layer near the ground.

Just as other instruments, the pilot-balloons give the air-motion with the small irregularities to some degree smoothed out. Small oscillations of the balloon are seen as long as the distance is not too great, but as the observations are taken at intervals which are long compared to the period of these small irregularities, only averages are obtained.

It should be observed that these averages in reality are of a complex nature, being averages simultaneously as regards intervals of time and of height, and further, that the air-motions found by the same pilot-balloon in different heights are not strictly simultaneous. We have mentioned already this general imperfection of observations obtained by aerological ascents (section 93) and its relatively small importance when the ascents are arranged so as to be made with sufficient vertical velocity.

*H. Hergesell: Die Bedeutung der Pilotballonaufstiege für die praktische Aerologie. Sixième Réunion de la Commission Internationale pour l'Aérostation scientifique à Monaco, 1909. Strassbourg, 1910.

100. Horizontal Motion and Vertical Motion.—The considered observations, those from the earth's surface as well as those from the higher strata, do not give full information on the direction of the motion. They only give the azimuth of the direction, not its inclination relatively to the horizon. Observations of the vertical components of the motion are difficult. It has been proved possible lately to derive the vertical velocity of the air from the motion of pilot-balloons, the observations being taken in a more complete way by two theodolites and base.* This or other methods of making the observations more complete are very much to be recommended, especially also on account of the more correct values thus obtained for the horizontal velocity. But even if it be possible thus to obtain valuable results on the local ascending or descending currents, it may turn out difficult to arrange a sufficient number of observations for the purpose of getting a complete picture of the general vertical motion. As long as a sufficient system of observation of this nature has not been organized, we shall be obliged to derive the vertical motion indirectly. This can be done by proper diagnostic methods which will be developed later, provided that we know sufficiently well the horizontal motion. We shall therefore first examine this part of the motion as completely as possible.

101. Direct Result of a Pilot-Balloon Ascent.—Directing our attention to the horizontal motion only, we shall consider the result of the ascent of a viséed balloon. Table A, columns 1, 3, and 4, shows the result of an ascent as given in the publications of the International Committee for Aeronautical Meteorology.†

A table like this gives more detailed information on the air-motion than we can use in the subsequent work, when the result of a great number of simultaneous ascents are to be worked out. The contents of the table must therefore be condensed, and evidently by forming vector-averages of the air-motion for thicker sheets than those appearing in table A.

102. Vector-Averages of Horizontal Motion Formed with Height as Independent Variable.—As the required averages have to be found by vector-addition, a graphical method will be best. From table A we derive a curve giving a geometrical representation of the distribution of velocity in the different heights. We form the numbers noted in column 5, obtained as products of the velocities, column 4, into the thicknesses Δz of the corresponding sheets, column 2; drawing then in succession segments of line having the lengths represented by the numbers in column 5 and the directions given in column 3, we get a polygonal curve which is seen in each of the diagrams, figs. 33 and 34 (pages 15 and 17). The numbers added in the corners represent the heights.

Now let us mark on the curve two points, representing two heights, and let us draw the chord joining them. This chord then represents the vector-sum (or the vector-integral) of velocities within the sheet defined by the two points, formed with height as independent variable; and dividing by the thickness of the sheet we shall get the average velocity within this sheet. In each of the figures 33 and 34 a

*See note, p. 12.
†Publications de la Commission Internationale pour l'Aérostation scientifique, 1907, p. 358. Strassbourg, 1909.

set of such chords are seen, drawn to determine the average velocities in the corresponding sheets.

If we wish to have the air-motion represented by specific momenta instead of by velocities, the direct way of proceeding will be to change the velocities contained in column 4 of table A into specific momenta, multiplying them by the corresponding densities of the air. Afterwards the construction is performed exactly as in the case of the other vector, velocity. We multiply the numerical values of the specific momenta by the corresponding thicknesses of sheet Δz, and draw in succession segments of line having the directions given in column 3 and the lengths represented by these products. By use of the curve thus obtained we form the vector-average for any sheet precisely as in the case of velocities.

TABLE A.—*Horizontal velocity of viséed balloon in different sheets. Pavia (lat. 45° 11′, long. 9° 10′ E.), July 25, 1907, 7^{h}33^m—7^{h}48^m a. m., Greenwich.*

1 Height z (meters).	2 Thickness of sheet Δz (meters).	3 Direction of motion within the sheet.	4 Velocity v within the sheet (m/sec.)	5 $v.\Delta z$.
77				
	603	S 50° E	3.4	2050
680				
	280	S 57° E	4.0	1120
960				
	280	S 36° E	5.3	1484
1240				
	290	S 28° W	1.5	435
1530				
	280	S 2° W	1.8	504
1810				
	280	S 2° W	2.0	560
2090				
	340	S 35° W	1.5	510
2430				
	300	S 53° W	1.8	540
2730				
	310	S 69° W	1.8	558
3040				
	360	S 55° W	3.0	1080
3400				
	310	S 53° W	2.8	868
3710				
	320	S 58° W	4.4	1408
4030				
	370	S 37° W	10.2	3774
4400				

103. The Choice of Suitable Atmospheric Sheets.—The method of forming the vector-averages being given, we have to settle the choice of sheets for which the averages should be formed in our practical work. Here different ways may be thought of.

One possibility will be to retain the principle used in table A, viz, to use a division into sheets characterized by the motion itself, only trying to reduce the number of sheets. This method will be very natural if corresponding changes of wind-direction and wind-intensity are found by the simultaneous ascents from other stations, and especially if these changes of motion are connected with changes of temperature or humidity.

We emphasize this as a method which may be seriously thought of, especially later, when more complete observations can be obtained. But for the first attempts we shall prefer a more summary method, using the same sheets which we have already introduced in statics for the representation of the fields of pressure and of mass. This will be convenient for several reasons. First, the kinematic diagnosis is not complete as long as we know only the velocities. We must also know the amounts of mass which have these velocities. Using the sheets introduced in

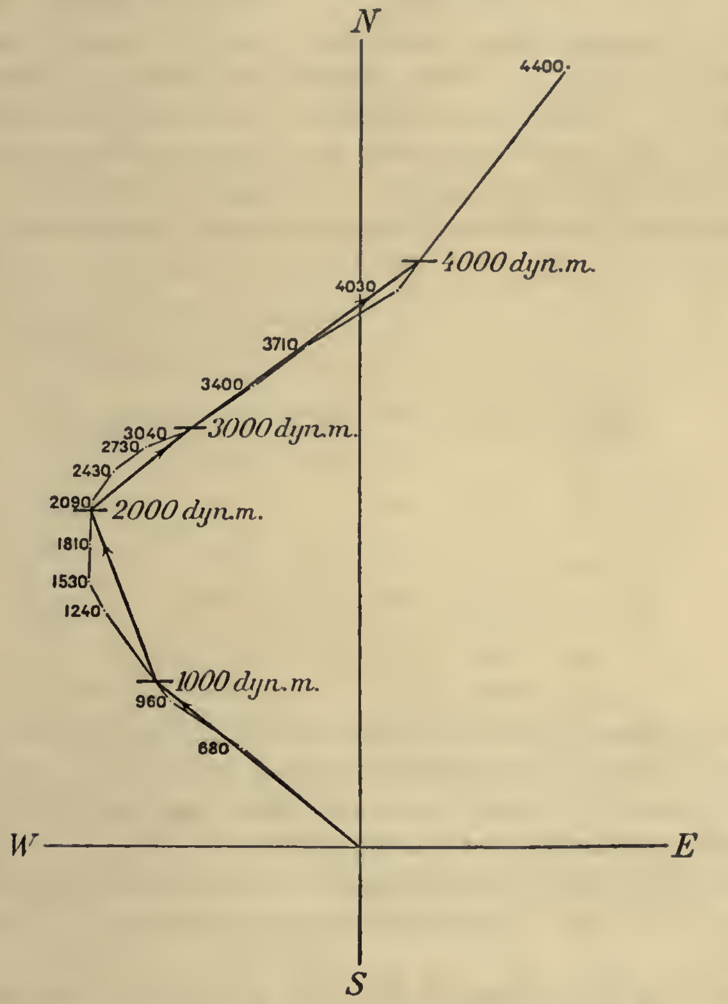

Fig. 33.—Construction of average velocities for standard level sheets.

statics, we get a coherent representation of velocities and of masses. Further, our final aim being the performance of dynamic investigations, we shall arrange everything convenient for future purposes by choosing our representation of the field of motion in as close connection as possible with that of the field of pressure. We shall therefore use either level sheets of the thickness of 1,000 dynamic meters, or isobaric sheets corresponding to the difference of pressure of 100 m-bars. In so doing, we shall evidently often get sheets which are too thick for a detailed representation of the motions. But the way of refining the representation by the choice of thinner sheets is evident. Thus level sheets of 500 or of 100 dynamic meters, or isobaric sheets of 50 or of 10 m-bars may be used, especially in the lowest strata near the ground, where the greatest irregularities occur.

104. **Use of Standard Level Sheets.**—Fig. 33 shows the construction leading from the observations given in table A to the average velocities in standard level sheets. The curve having been constructed as described in section 102, points are marked on it corresponding to the heights of 1020, 2040, 3060, 4080 . . . meters, *i. e.*, to 1000, 2000, 3000, 4000 . . . dynamic meters. Then the chords are drawn, and the numbers representing their directions found by use of the divided sheet described in section 98. These numbers are given in column 4 of table B. Further, the length of the chords is measured and the numbers representing these lengths divided by 1020, which represents the common thickness of the level sheets. The velocities found in this way are given in column 5 of table B.

As the balloon carried self-recording instruments, the pressures in the standard level surfaces have been calculated (Statics, secs. 53–54) and are given in column 2

TABLE B.—*Average horizontal motion in standard level sheets. Pavia (lat. 45° 11′, long. 9°10′ E.), July 25, 1907, $7^h33^m-7^h48^m$ a. m., Greenwich.*

1 Height dyn. meters.	2 Pressure m-bars.	3 Density ton/m^3.	4 Direction of motion.	5 Velocity m/sec.	6 Sp. momentum 10^{-3}ton/m^{2}sec.
4000	622				
		0.00083	6	3.8	3.2
3000	705				
		0.00092	7	1.6	1.5
2000	797				
		0.00102	20	2.4	2.5
1000	899				
		0.00112	25	3.7	4.1
75	1003				

of table B. The difference between these pressures multiplied by 10^{-5} gives the average density of the air in the different level sheets (Statics, sections 38 and 54). These densities appear in column 3 and give full information on the masses moving with the velocities specified by the columns 4 and 5.

To get the corresponding specific momenta, we can simply multiply the average velocities, column 5, by the corresponding densities, column 3. The result is noted in column 6.

Regarding the specific momenta obtained in this way, it should, however, be emphasized, that they are not identical in direction and intensity with those average momenta which would be found if we multiplied each velocity given in table A by the corresponding density and repeated the construction for forming the vector-average. But on account of the relatively slow decrease of density with the height, the difference will generally be insignificant in comparison with the unavoidable uncertainty of the observations of velocity.

105. **Use of Standard Isobaric Sheets.**—Fig. 34 shows the construction leading from the observations given in table A to the average velocities in standard isobaric sheets. The curve is precisely the same as that of fig. 33. But now we

have to mark on it the points which represent the heights of the standard isobaric surfaces. From the records in the case before us these heights are found equal to 99, 989, 1970, 3057, 4274 dynamic meters, as noted in column 2 of table C. The corresponding thicknesses of sheet, also expressed in dynamic meters, are given in column 3. These two columns then represent, as we have seen (Statics, secs. 35, 54), the distribution of pressure and mass along the vertical, the thickness of the standard isobaric sheets giving the average specific volume of the air within the sheet.

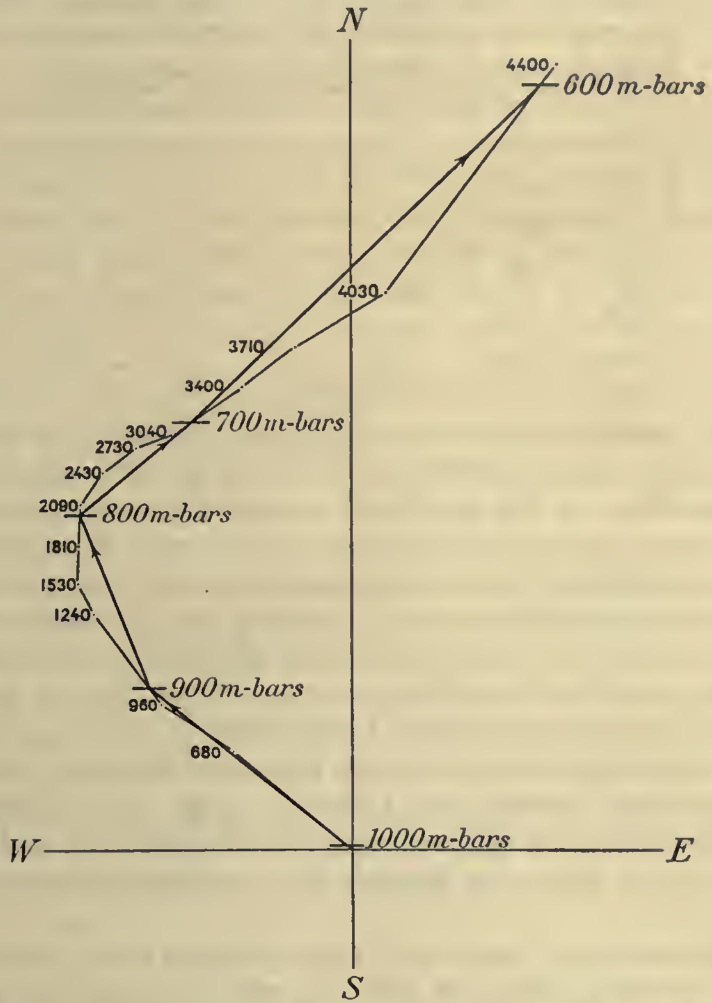

Fig. 34.—Construction of average velocities for standard isobaric sheets.

On the curve we now mark points representing the heights noted in column 2, *i.e.*, the heights of 101, 1009, 2009, 3121, 4362 common meters, and draw the corresponding chords. The directions of these chords determined by the transparent sheet (fig. 32) are noted in column 4. Then the lengths of these chords are measured and divided by the thicknesses of sheet, viz, 908, 1000, 1112, 1241 common meters, respectively. The average velocities found in this way are noted in column 5.

As the numbers in column 3 represent the average specific volume of the air in the sheets, we get the specific momenta simply by dividing the velocities (column 5) by the specific volumes (column 3). The result is given in column 6. But as in the preceding case, we have to remark that the values of specific momentum found in this way are not the exact height-averages of specific momentum for these sheets; yet they give as a rule sufficient approximation toward these averages. If the exact values are required, we have to change the velocities given in table A into specific momenta before performing the construction leading to the averages.

TABLE C.—*Average horizontal motion in standard isobaric sheets. Pavia (lat. 45°11′, long. 9°10′E.), July 25, 1907, $7^h33^m-7^h48^m$ Greenwich.*

1 Pressure m-bars.	2 Height dyn. meters.	3 Sp. volume m³/ton.	4 Direction of motion.	5 Velocity m/sec.	6 Sp. momentum 10^{-3}ton/m² sec.
600	4274				
		1217	8	5.2	4.3
700	3057				
		1087	7	1.7	1.6
800	1970				
		981	20	2.4	2.4
900	989				
		890	25	3.7	4.2
1000	99				
		....	25	3.4	4.0
1002.6	76				

106. Special Remarks.—As we see, the result contained in columns 4 and 5 of table B, *i.e.*, the average velocities of the air in the level sheets, can be found without any knowledge of the registered values of pressure, temperature, and humidity. For everything regarding specific momenta or motions in isobaric sheets, a certain knowledge of the fields of pressure and of mass is required. But on account of the very limited accuracy of the observations of velocities we shall never, for the purely kinematic purposes, need to know these fields of pressure and of mass with the accuracy used in statics for the purpose of drawing their synoptical representations. Pressure and temperature being observed near the ground, and the values of the temperature being estimated for greater heights, it will be easy, by using our meteorological tables (11 M, 15 M, 12 M, 10 M), to find the heights of the standard isobaric surfaces or the pressures in standard levels and the correlated data regarding specific volume or density, with sufficient accuracy for kinematic purposes.

It is furtherworth while mentioning that the result of an ascent (as contained, for instance, in columns 4 and 5 of table C), may be condensed into a telegram which could be sent to a central office. (Compare Statics, section 57.) While two men are taking the observations, a third can perform the constructions and the calculations, and it will be possible to send off the telegram a few minutes after the last observation of the pilot-balloon is taken. Thus there will be no technical difficulty in bringing observations of this kind into application for the daily weather service.

107. Main Example: Europe, July 25, 1907.—The most complete observations which we have had at our disposal for working out a diagnosis of atmospheric motions are those obtained on July 25, 1907. This day belonged to a period of six days, extending from July 22 to July 27, during which the North Pole was surrounded by a circle of aerological stations. During this period in all 89 registering balloons, 20 manned balloons, 100 kites and captive balloons, and 41 pilot-balloons were sent up.

From our point of view these observations were spread out over too great an area as well as over too long a period of time. The number of pilot-balloons launched was also far too small compared with that of registering balloons (compare sections 92, 93). In order to try a diagnosis of atmospheric motions we can only think of using the observations from a more limited area, where the network of stations was closest, namely, central Europe. And we shall choose the epoch when the greatest number of fairly simultaneous ascents took place, namely, July 25, about the time of the daily meteorological observations, 7 a. m. Greenwich. From one hour before to two hours after this epoch 13 balloons were followed by theodolites. This number is not sufficient for working out a real diagnosis of atmospheric motions over Central Europe, but we shall at least be able to illustrate the formal methods.

When working out the example we shall choose the method of dividing the atmosphere into isobaric sheets; the corresponding use of level sheets will be understood without difficulty. Using the methods developed in Statics, as well as those given in section 105, we get the result of the ascents condensed in table D. In each of the 13 subdivisions of this table, the first column gives the standard pressures and the pressure at the station; the next gives the dynamic height of these pressures, and the three following the thickness of sheets, direction, and velocity of air-motion in the sheets. It may be remarked regarding the observations that those of the wind in subdivision 8 are obtained from the ascent of a kite, those in subdivisions 11 and 12 from the course of manned balloons. The registering balloon, subdivision 13, could not be viséed for cloudiness. In subdivision 7, heights of standard surfaces and thickness of standard sheets are estimated from the ascents at the other stations.

From the numbers registered in table D we shall now work out the corresponding synoptical representations. Using the numbers representing the heights of the standard isobaric surfaces and the thickness of the sheets contained between them, we shall first work out representations of these sheets. For the sake of brevity we shall denote these sheets, counted from below, by the Roman numbers X, IX, VIII, . . ., X being the sheet limited below by the 1000 m-bar surface, IX that limited below by the 900 m-bar surface, and so on. The always incomplete sheet contained between the 1000 m-bar surface and the ground may be denoted by XI.

To distinguish the curves for absolute and those for relative topography we shall draw the first as single and the second as double lines. The double lines consist of a thick and a thin line, the thin being drawn on that side where the isobaric sheet, whose thickness is represented, is thinner. Fig. A of plate LVII represents the isobaric sheet X, the single lines giving the dynamic height of the 1000 m-bar surface above sea-level and the double lines giving the height of the 900 m-bar surface above the 1000 m-bar surface. Or, as we express it: the single lines give the absolute topography of the 1000 m-bar surface and the double lines the relative topography of the 900 m-bar surface. In the same manner fig. A of plate LVIII

TABLE D.—*Aerological observations* (*Europe, July 25, 1907*).

1. Uccle. Lat. 50° 48′ Long. 4° 22′ E Dyn. height 98 6^h52^m–7^h45^m

100	16374			
		4427	0	4.7
200	11947			
		2627	10	3.2
300	9320			
		2019	18	3.4
400	7301			
		1653	19	3.3
500	5648			
		1408	8	2.6
600	4240			
		1220	5	2.5
700	3020			
		1082	4	2.5
800	1938			
		963	4	1.4
900	975			
		867	36	4.5
1000	107			
		—	35	4.5
1001.2	98			

2. Crinan. Lat. 56° 6′ Long. 5° 32′ W Dyn. height 5 9^h20^m–10^h12^m

400	7187			
		1654		
500	5533			
		1374		
600	4159			
		1184		
700	2975			
		1052	9	3.4
800	1923			
		945	4	2.1
900	978			
		863	11	2.7
1000	115			
		—	4	4.0
1013.2	5			

3. Guadalajara. Lat. 40° 39′ Long. 3° 10′ W Dyn. height 622 8^h29^m–9^h13^m

300	9578			
		2145	53	17.5
400	7433			
		1725	54	13.6
500	5708			
		1432	55	11.4
600	4276			
		1226	53	7.3
700	3150			
		1077	52	6.8
800	1973			
		966	52	4.1
900	1007			
		—	48	1.2
942.1	622			

4. Pavia. Lat. 45° 11′ Long. 9° 10′ E Dyn. height 76 7^h33^m–8^h10^m

200	11889			
		2601		
300	9288			
		1993		
400	7295			
		1631		
500	5664			
		1390		
600	4274			
		1217	8	5.2
700	3057			
		1087	7	1.7
800	1970			
		981	20	2.4
900	989			
		890	25	3.7
1000	99			
		—	25	3.4
1002.6	76			

5. Zürich. Lat. 47° 23′ Long. 8° 33′ E Dyn. height 471 7^h17^m–8^h3^m

100	16238			
		4421	3	10.0
200	11817			
		2577	6	6.5
300	9240			
		1992	7	7.6
400	7248			
		1622	2	10.2
500	5626			
		1382	3	6.7
600	4244			
		1206	2	6.8
700	3038			
		1083	62	5.3
800	1955			
		978	4	0.6
900	977			
		—	30	2.1
955.9	471			

6. Strassburg. Lat. 48° 35′ Long. 7° 45′ E Dyn. height 138 8^h8^m–8^h42^m

300	9235			
		1982		
400	7253			
		1630		
500	5623			
		1385		
600	4238			
		1203	0	7.0
700	3035			
		1078	2	6.7
800	1957			
		974	8	2.0
900	983			
		—	37	2.1
995.9	138			

7. Hamburg. Lat. 53° 33′ Long. 9° 59′ E Dyn. height 17 7^h48^m–ca. 8^h40^m

200	11890			
		2649	59	9.2
300	9241			
		2001	57	10.5
400	7240			
		1597	58	8.8
500	5643			
		1447	55	8.0
600	4196			
		1205	49	2.9
700	2991			
		1064	41	2.9
800	1927			
		946	38	1.9
900	981			
		863	56	4.3
1000	118			
		—	55	3.4
	17			

8. Lindenberg. Lat. 52° 36′ Long. 13° 37′ E Dyn. height 116 6^h40^m–8^h58^m

200	11862			
		2624		
300	9238			
		1999		
400	7239			
		1644		
500	5595			
		1395		
600	4200			
		1210		
700	2990			
		1062		
800	1928			
		948	56	9.0
900	979			
		864	59	6.8
999.9	116			

9. München. Lat. 48° 9′ Long. 11° 37′ E Dyn. height 506 7^h4^m–7^h34^m

200	11874			
		2579		
300	9295			
		2002		
400	7293			
		1641		
500	5652			
		1396		
600	4256			
		1215	62	10.0
700	3041			
		1082	62	10.0
800	1959			
		974		
900	985			
		—		
953.2	506			

10. Wien. Lat. 48° 15′ Long. 16° 21′ E Dyn. height 157 7^h0^m–9^h40^m

500	5631			
		1394	60	17.3
600	4237			
		1213	61	14.3
700	3024			
		1072	62	15.3
800	1952			
		959	62	9.7
900	993			
		—	51	3.6
995.9	157			

11. Novogeorgievsk. Lat. 52° 26 Long. 20° 44′ E Dyn. height 108 5^h37^m–9^h12^m

700	2947			
		1054	53	12.0
800	1893			
		947	54	8.4
900	946			
		—	57	3.0
997.2	108			

12. Koutchino. Lat. 55° 45′ Long. 37° 59′ E Dyn. height 138 6^h38^m–7^h15^m

200	11566			
		2588		
300	8978			
		1939		
400	7039			
		1584		
500	5455			
		1345		
600	4110			
		1176		
700	2934			
		1043	60	7.5
800	1891			
		942	62	7.0
900	949			
		—	62	7.0
993.2	138			

13. Pawlowsk. Lat. 59° 41′ Long. 30° 29′ E Dyn. height 29 5^h9^m–6^h8^m

300	8442			
		1920		
400	7022			
		1578		
500	5444			
		1346		
600	4098			
		1174		
700	2924			
		1046		
800	1878			
		946		
900	932			
		859		
1000	73			
		—		
1005.2	29			

represents the isobaric sheet IX, the single lines giving the absolute topography of the 900 m-bar surface and the double lines the relative topography of the 800 m-bar surface, and so on.

On the charts representing thus the different isobaric sheets, we now introduce the arrows and corresponding numbers representing the air-velocities given in table D. These data regarding the air-motions in the higher strata, in connection with the corresponding data for the ground which are given on plate LIII, will now form the basis for the further diagnostic work regarding the air-motion above central Europe, July 25, 1907, about 7 a. m. Greenwich.

108. On the Observations of the Sea-Motions.—If the observations of the air-motions are too scarce, this is still more the case with those of oceanic motions. Quantitative measurements are only to be had exceptionally. The motions of the sea's surface is in many cases known qualitatively from the drift of floating objects or of bottles thrown out for the purpose of investigation. Qualitative conclusions as to the motions in the deeper sheets can be drawn from the measurements of the salinity, this giving information as to the origin of the waters. Similar conclusions can be made also on the basis of the examination of the organisms contained in the water. But none of these observations are of the quantitative nature which can give rise to a closer kinematic analysis.

For this reason we can work out no example of a kinematic diagnosis of the sea-motions. But the principle of the methods to be employed in the case of the sea, as soon as serviceable material of observations is produced, will be sufficiently illustrated by the example worked out for the case of the atmosphere. We shall therefore only make occasional references to the sea.

The most important point to emphasize is the necessity of producing sufficient data of direct observations of the sea-motions from the surface as well as from all depths. Suitable instruments for doing it have already been invented.* It remains only to bring them into application on a sufficiently large scale and according to rational principles.

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CHAPTER III.

ELEMENTARY PRINCIPLES OF KINEMATICS OF CONTINUOUS MEDIA.

109. **Kinematics of a Continuous Medium.**—We have considered the observations from which we shall derive our diagnosis of atmospheric or hydrospheric states of motion. We shall then proceed to develop the general principles of kinematics which shall govern the diagnostic work.

In order to arrive at these general principles, we shall consider atmosphere, hydrosphere, and solid earth as a material system which fills space continuously. We shall neglect phenomena related to the molecular structure of these bodies, such as the diffusion of water-vapor through air or of salt through water. In the same manner we shall neglect every transfer of mass from one of these bodies to any other of them. Thus we shall set out of consideration the transfer of mass from the sea or from the moist ground to the air by the evaporation of water, and the return of these masses to the sea or to the porous ground in the form of rain. These processes will be of high importance in connection with the thermodynamics of atmosphere and hydrosphere. But from the pure kinematic point of view they will be insignificant, as they will give mass-transports which are small compared with those connected with the great air-motions or sea-motions.

It will therefore be sufficient for our present purpose to consider a material medium which fills space continuously. Density or specific volume may vary from particle to particle of the medium, even in discontinuous manner, as at the surface of separation between air and sea. The dynamic properties are not taken into consideration. The only condition to be observed is that of the material nature of the medium, involving the principle *that every moving particle shall have an invariable mass*, together with the supplementary condition *that the medium shall fill space continuously*.

To describe the instantaneous state of motion of this medium we shall use two vectors, velocity and specific momentum. The conditions of the material nature of the medium, and of its continuity in space, do not restrict the generality of the fields of these vectors. The methods of representing them geometrically will therefore be the methods of representing geometrically a vector-field of unlimited generality. From a formal point of view this chapter will therefore deal with the subject of the geometrical representation of vector-fields, and will thus contain results which we shall use later in connection with other vectors.

While the conditions of the material nature of the medium and of its continuity in space do not restrict the geometrical properties of the field of motion, they will lead to a fundamental relation connecting these fields with that of mass. For as motion consists in the displacement of invariable masses having to fill space continuously, the knowledge of the present field of motion involves a certain knowledge regarding the future field of mass. Thus the two fundamental suppositions regarding the medium lead to an intrinsic relation of *prognostic* nature, in its mathematical form called the *equation of continuity*. In special cases time drops out, and the equation is reduced to a *diagnostic* one, submitting the fields of motion to certain restrictive conditions. In connection with the geometrical principles for representing the fields of motion, we shall therefore develop this prognostic equation and pay special attention to the cases when it is reduced to a diagnostic equation.

110. Vector-Lines, Vector-Surfaces, and Tensor-Surfaces.—In Statics we have considered the methods of representing geometrically certain special vectors, the ascendants or the gradients of a scalar quantity (sections 16, 17). The field of the scalar quantity gave a complete representation also of the vector derived from it. But in the general case a vector will, for its geometrical as well as for its analytical representation, require the use of three instead of only one scalar quantity.

In order to represent first the *direction* of a vector at every point of the field, we can draw a set of curves running tangentially to the direction of the vector. These lines are called *vector-lines*, or for a field of motion *lines of flow*. A set of curves in space is obtained by the intersection of two sets of surfaces. Each set of surfaces being the equiscalar surfaces of a certain scalar field, we see that the representation of the direction of a vector by vector-lines involves the use of two scalar fields.

The surfaces used to represent the vector lines may be chosen in an infinite number of ways; but they have the common property of being surfaces generated by vector-lines. Any surface generated in this way will be called a *vector-surface*, or for the field of motion a *surface of flow*.

The direction of the vector being thus given by two scalar fields, we can use a third for representing its numerical value or its *tensor*. An equiscalar surface of this third field will pass through all points where the vector has a certain constant numerical value. These surfaces may be called *tensor-surfaces*, or *surfaces of equal intensity*.

The vectors considered by us will have a uniquely determined direction at every point where it is different from zero. As intersections of vector-lines under finite angles would give two or more different directions for the vector in the point of intersection we conclude:

Vector-lines can intersect each other only at zero-points of the field.

Nothing prevents vector-lines from touching each other; for, having a common tangent, both lines indicate the same direction at the point of tangency.

111. Vector-Tubes and Surfaces of Equal Transport.—The two sets of vector-surfaces cutting each other along the vector lines will divide the field into a set of elementary tubes which have parallelogrammatic cross-sections. These may be called *vector-tubes*, or, for a field of motion, *tubes of flow*.

Cutting a vector-tube by any surface σ let $d\sigma$ denote the area of the section. A being the vector and A_n its component normal to the section, let us consider the product $A_n\, d\sigma$. This product does not depend upon the angle contained between the normal to the section and the axis of the tube; for as this angle varies, the area $d\sigma$ of the section and the vector-component A_n normal to it will vary in inverse proportion to each other, always giving a product equal to that of the area of the normal section into the tensor of the vector. We will call this product the *transport* through the section. The name is derived from the case to be examined more fully below, when the field represents motion. The bundle of tubes cutting through a finite surface σ divides this surface into elements $d\sigma$ and determines a certain transport $A_n\, d\sigma$ through each of them. Forming their sum, we get

$$\text{(a)} \qquad \textit{Transport through surface } \sigma = \int A_n d\sigma$$

The excess of transport leading out of a closed surface over that leading into it may be called the *outflow*, and the same quantity with the sign changed the *inflow*. The outflow is obtained by taking the integral (a) over the closed surface, counting the normal directed outward as positive. The inflow is obtained in the same way, counting the normal directed inward as positive.

Returning to an elementary vector-tube, let the section be moved from place to place along it. The transport will then, as a rule, be found to vary. Measuring its value from section to section in all tubes, we get numbers representing the *field of transport*. This field can be represented in the common way by drawing *surfaces of equal transport*.

The tubes of flow in connection with the surfaces of equal transport will give a representation of the vector field as complete as that given by the lines of flow in connection with the surfaces of equal intensity; for, being sufficiently narrow, the tubes will represent the direction of the vector equally well as the lines; and from the value of the transport we can come back to the numerical value of the vector dividing by the area of the cross-section of the tube.

Though the field of transport thus performs a similar service as the field of intensity for representing the numerical value of the vector, one important difference should be observed. The intensity-field is uniquely determined, while the field of transport has a definite sense only in connection with a given system of tubes. Choosing new surfaces for defining the tubes, we shall as a rule get tubes which have other cross-sections, and therefore lead to a new field of transport.

112. Solenoidal Vector.—A field may have the property that the outflow is zero out of every closed surface. The transport will then be the same through every section of one and the same tube. The surfaces of equal transport may then be left out as superfluous. It will be sufficient to know the constant of transport for each tube. It will in this case be found convenient to undertake the division of the field into tubes in such a way that each tube gets the same transport, in the simplest case unit transport. Choosing a unit of suitable magnitude, we can still get tubes sufficiently narrow for the purpose of representation. These narrow, in the limiting case infinitely narrow, tubes are called *solenoids*, and every vector which can be represented completely by such tubes is called a *solenoidal vector*.

The solenoidal vector is simpler than the general vector inasmuch as it can be represented completely by two sets of surfaces, *i. e.*, by two scalar fields, while the general vector requires three. In other words, there is a dependency between the three components of the solenoidal vector. Using the *solenoidal condition*, *i. e.*, the condition expressing the fact that the outflow from every closed surface is zero,

$$\int A_n d\sigma = 0 \qquad (a)$$

we can determine the third component of the vector, if we know the value of the two others at all points of the field.

113. Volume-Transport and Mass-Transport.—Passing to concrete fields of motion, we shall consider a tube of flow and a section of it having the area $d\sigma$. The particles situated at a certain time t on this section and having the velocity v, will an element of time dt later be situated on another section which is displaced the distance vdt along the tube. The normal distance between the sections will be $v_n\, dt$. The two sections and the walls of the tube determine an elementary parallelepipedon of volume $v_n\, dt\, d\sigma$, giving the elementary volume of the medium which during the time dt has passed the section $d\sigma$. Multiplying by the density ρ of the medium, we get the mass contained in this volume, *i. e.*, the mass which during the time dt has passed the section $d\sigma$. When we remember that the product of density into velocity gives the specific momentum V of the medium, we get as expression of this elementary mass $V_n\, dt\, d\sigma$. Dividing by dt we get the expressions $v_n\, d\sigma$ and $V_n\, d\sigma$ representing, according to our definition, the transport respectively in the field of velocity and in the field of specific momentum. We thus arrive at this result:

(A) *In the field of velocity the transport through a surface*

$$\int v_n d\sigma \qquad (a)$$

gives the volume of the medium passing the surface per unit time.

(B) *In the field of specific momentum the transport through a surface*

$$\int V_n d\sigma \qquad (b)$$

gives the mass of the medium passing the surface per unit time.

Taken over a closed surface the integral (a) will represent the volume and (b) the mass of the medium conveyed per unit time out through the closed surface.

Considering the transport as given in our m. t. s. units, and returning to the vectors, we arrive at these methods of measuring velocity and specific momentum, which may be useful to bear in mind in the subsequent practical work:

(C) *Velocity is measured by the number of cubic meters and specific momentum by the number of tons passing per second a square meter normal to the direction of the motion.*

114. Equation of Continuity.—The physical significance of the integral expressing transport in a field of motion being thus known, it will be easy to give in quantitative form the dependency of the future fields of mass upon the present field of motion.

Measuring the elementary volume conveyed out of a closed surface in an element of time dt, we evidently get the elementary increase of volume during the time dt of that mass which is momentarily contained in the closed surface. Reducing to unit time we get the velocity of expansion of this mass. Thus:

(A) *The integral of the normal component of velocity taken over a closed surface*

(*a*) $$\int v_n d\sigma$$

is equal to the increase of volume per unit time of the mass momentarily contained in the surface.

Measuring on the other hand the elementary mass conveyed out of a closed surface in the element of time dt, we get the elementary decrease during this time of the mass stored within the surface. Reducing to unit time, we get:

(B) *The integral of the normal component of the specific momentum taken over a closed surface*

(*b*) $$\int V_n d\sigma$$

is equal to the diminution per unit time of the mass contained in the surface.

The dependency of the future field of mass upon the present field of motion is expressed by these two theorems in two different ways, in the first case by the change of volume of moving masses, in the second by the change of mass within stationary volumes. We shall later write in explicit form the "equation of continuity," expressing in mathematical symbols the contents of any of these theorems. Provisionally it will be found more convenient to work directly with the physical facts as contained in the theorems (A) and (B).

115. Conditions Leading to Solenoidal Fields of Motion.—Every reference to variations in time of the field of mass will disappear from the theorem 114 (A) if the mass momentarily contained in the closed surface does not change its volume. In this case the field of velocity will fulfil the solenoidal condition

(*a*) $$\int v_n d\sigma = 0$$

In the same manner, the reference to future fields of mass will drop out of the theorem 114 (B) when the content of mass of every stationary volume is constant. Specific momentum will then fulfil the solenoidal condition

(*b*) $$\int V_n d\sigma = 0$$

We thus get the important results:

(A) *Velocity is a solenoidal vector if the moving medium is incompressible.*

(B) *Specific momentum is a solenoidal vector if the field of mass is stationary in space.*

If the medium be both incompressible and homogeneous, the moving masses will not change volume, and the mass-contents of every stationary volume will be invariable. We thus get the special case:

(C) *Both velocity and specific momentum will be solenoidal vectors if the moving medium be both homogeneous and incompressible.*

Without restricting the physical properties of the medium, we can apply theorem 114 (A) to the infinitely small volume contained between two parallel surfaces running at infinitely small distance from each other. Finite difference between the normal velocity-components at adjacent points on the two surfaces would in this case lead to finite expansion of an infinitely small volume. Thus the continuity would be broken. Therefore a finite difference between the normal components can not exist. This leads to the solenoidal surface-condition:

(D) *The normal component of velocity must have the same value on both sides of any surface in a material system filling space continuously.*

This solenoidal surface-condition must be fulfilled, for instance, at the surface of separation between atmosphere and hydrosphere. It applies only to velocity, not in general to specific momentum. Taking the case of mercury and water in contact with each other, the normal component of velocity will be the same on both sides of the surface; but that of specific momentum will be 13.6 times greater on the side of the mercury than on that of the water.

If the system is at rest on the one side of the surface, there will be no velocity-component normal to it on the other side; consequently the normal component of specific momentum will also be zero. Thus:

(E) *Velocity and specific momentum are directed tangentially to every resting boundary.*

This condition is to be applied to the motion of the air along the ground and of the water along the bottom of the sea.

116. Examples of Volume-Transport and Mass-Transport.—It will be useful here to take a few examples illustrating the difference between the conditions of solenoidal velocity-field and solenoidal field of specific momentum.

Let a tube be filled partly with water and partly with mercury, both fluids being considered incompressible. If the fluid column moves along the tube there will be equal volume-transport through a section in the water and through one in the mercury, say one cubic meter per second through each. The volume-outflow out of the closed surface formed by the walls of the tube and the two cross-sections will be zero, and the field of velocity will be solenoidal. But measuring the transport in tons, we find a transport of one ton per second through the upper and a transport of 13.6 tons per second through the lower section. The difference, 12.6 tons per second, gives the outflow of mass through the walls of the volume, and thus the decrease per unit time of the mass contained in the volume. We shall have outflow or inflow of mass according to the direction of the motion. For there will be a decrease of mass in the volume when water expels mercury, and an increase when mercury expels water. The specific momentum will be solenoidal within each homogeneous part of the fluid column, but non-solenoidal at the surface of of discontinuity separating water and mercury. Instead of a discontinuous system like this, we could also have considered a fluid system with continuously varying density, for instance, a column of water with continuously varying salinity. Even in this case we would have a solenoidal velocity-field and non-solenoidal field of specific momentum, the solenoidal condition being violated by this vector not only at a certain surface of discontinuity, but at every point where density showed variations in space.

Let us now, on the other hand, consider motions in a compressible medium, atmospheric air. Setting aside the insignificant influence of humidity, we know that the density of the air depends upon temperature and pressure. Therefore, if the fields of temperature and of pressure are maintained stationary in space, the field of mass will also be stationary, and the specific momentum will be a solenoidal vector. Let us then consider a tube having its lower end near sea-level and its upper end in the region of cirrus. If one ton of air enters the tube per second at its lower end, one ton per second must leave it at its upper end. But measuring by volumes, we find that one ton of air has at sea-level a volume of about 1000 cubic meters, and at the height of cirrus a volume of about 3000 cubic meters. There is a volume-outflow from the closed volume limited by the walls and the cross-sections of the tube equal to 2000 cubic meters per second. This volume-outflow is equal to the velocity of expansion of the column of air which is contained in the tube. This expansion is due to the motion up toward lower pressures. Reversing the direction of the motion, we get a corresponding inflow, equal to the contraction per second which the column of air will have in virtue of its descending motion.

117. The Fields of Motion in Atmosphere and Hydrosphere.—We can now take up the discussion of the chances of arriving at a satisfactory diagnosis of atmospheric or hydrospheric motions. The great incompleteness of the observations of air-motions is that they give only the horizontal components, and no information on the vertical components. The same has also hitherto been the case with all observations of sea-motions. The conditions for a satisfactory diagnosis will then be that we should be able to derive the unknown vertical components from the observed horizontal components. This will be possible if the motions can be considered solenoidal, and the question will be if we can suppose this to be the case with sufficient approximation for the purpose of the kinematic diagnosis.

In the case of the hydrosphere there is no doubt. We can put out of consideration both the slight compressibility of the sea-water and the slight changes in the field of mass following local changes of temperature, salinity, and pressure. Doing so, we find that both the field of velocity and the field of specific momentum will fulfil the solenoidal condition. Using this condition for deriving the not-observed vertical components from the observed horizontal ones, we shall obtain an accuracy depending entirely upon that of the observations; for the errors introduced by neglecting compressibility and heterogeneity will be insignificant.

In the case of the atmosphere we have seen already that the changes of volume of the moving masses of air are too great to allow us to consider the field of velocity solenoidal. But the field of mass is not very far from being stationary, the changes in this field being caused exclusively by the gradual changes in the fields of temperature and of pressure; we may therefore try to derive the vertical motions, supposing the field of specific momentum to be solenoidal in the first approximation.

In order to see the errors which can then arise, we can consider a cylinder going from the ground up to a certain height in the atmosphere. Calculating the vertical motion through a horizontal section at the top of the cylinder, we set the transport of mass *up* through this section equal to the transport of mass *in* through the walls of the cylinder. The vertical motion thus found will be erroneous, inasmuch as the temperature or pressure within the cylinder is changing. To find the error we shall estimate the additional vertical motion produced by the local changes of temperature and pressure.

First let there be an increase of temperature within the cylinder of 1° C. per hour, *i. e.*, of $\frac{1}{3600}$° C. per second. This will give a cubic meter of air the velocity of expansion of $\frac{1}{273} \cdot \frac{1}{3600}$, or less than one-millionth of a cubic meter per second. The corresponding linear velocity of expansion of the air in the cylinder will be less than one micron per meter in the second. There will thus arise a vertical velocity not exceeding 1 mm. per second at the height of 1000 meters, and not exceeding 1 cm. per second at the height of 10,000 meters. We can only as an exception expect to get changes of temperature greater in average than a few degrees centigrade per hour for columns of air of this height. Therefore, neglecting the local change of temperature, we shall get errors in the vertical velocities not exceeding a few millimeters per second at the height of 1000 meters, and a few centimeters per second at the height of 10,000 meters.

For the corresponding influence of local change of pressure, we can suppose temperature to be constant. For the column of air contained in the cylinder we have then $pK = \text{const}$, p being the average pressure in the cylinder and K its volume. As only the height z of the cylinder is variable, we can write $pz = \text{const}$. Differentiating with respect to time and solving with respect to $\frac{dz}{dt}$, we get

$$\frac{dz}{dt} = -\frac{z}{p}\frac{dp}{dt}$$

Supposing the change of pressure $\frac{dp}{dt}$ to be one m-bar per hour, *i. e.*, $\frac{1}{3600}$ m-bar per second, setting the height of the cylinder equal to 1000 meters, and the average pressure between sea-level and this height equal to 900 m-bars, we get the vertical velocity $\frac{dz}{dt}$ smaller than a third of a millimeter per second. Setting the height of the cylinder equal to 10,000 meters and the average pressure between this level and sea-level equal to 600 m-bars, we get the vertical velocity due to the variation of the pressure smaller than half a centimeter per second. Thus in both cases the change of pressure of one m-bar per hour will have smaller effect than the change of temperature of one degree centigrade per hour. Now the change of pressure of a few m-bars per hour for columns of air of this length will have about the same degree of probability as the change of temperature of a corresponding number of degrees. Thus we have an equal right to neglect the influence of local pressure-changes as of local temperature-changes.

When we determine vertical velocities in the atmosphere by the condition of the solenoidal nature of specific momentum, we may thus get errors amounting to a few millimeters per second at the height of 1000 meters and of a few centimeters per second at the height of cirrus. As the errors due to the uncertainty and the incompleteness of the observations of the horizontal velocities will be much greater, these errors must be considered as insignificant.

We can therefore set down as fundamental principles to be used in the diagnostic work regarding the fields of motion in atmosphere and hydrosphere:

(A) *In hydrosphere both velocity and specific momentum fulfil the solenoidal condition.*

(B) *In atmosphere specific momentum fulfils the solenoidal condition.*

Finally we have, independent of every approximation:

(C) *At every surface velocity fulfils the solenoidal surface-condition.*

As a special case of this condition we have

(D) *Both velocity and specific momentum are tangential to every resting boundary.*

CHAPTER IV.

EXAMPLES OF SOLENOIDAL FIELDS AND THEIR REPRESENTATION BY PLANE DRAWINGS.

118. Two-Dimensional Representations of Three-Dimensional Vector-Fields.—Before passing to practical applications, it will be useful to consider a few simple examples of solenoidal fields and to illustrate different methods of representing them by plane drawings.

In order to see the character of two-dimensional drawings representing any three-dimensional field, let us consider a surface cutting through the field in space. At every point of the surface the vector has a certain direction and intensity. For the representation it will be convenient to consider separately two projections of the vector, that on the normal to the surface, and that on the plane tangential to the surface. *The normal component* can be represented simply by curves for equal numerical values. No representation of the direction is required. The field of this component has lost the character of a vector-field and has completely obtained that of a two-dimensional scalar field.

The tangential component, on the other hand, will represent a true *two-dimensional vector-field.* The methods of representing it geometrically will be special cases of the methods for representing vectors in space (section 110). Precisely as in space, the direction can be represented by *vector-lines.* But instead of surfaces we shall get *curves of equal intensity.* It should be observed that these curves of equal intensity will not, as a rule, be the intersections of the given surface with the surfaces of equal intensity in space. This will be the case only if the given surface happens to be a vector-surface; for then the normal vector will disappear and the vector of the two-dimensional field will be identical with that of the three-dimensional field in space.

A set of two-dimensional drawings representing a three-dimensional vector-field in space can therefore be obtained in the following way: We choose a set of surfaces cutting through the field. The vector defines at each of them a two-dimensional vector-field and a two-dimensional scalar field. The first can be represented by two sets of curves, viz, the vector-lines and curves of equal intensity for the tangential component; and the second by one set of curves, viz, curves for equal values of the normal component.

We shall then have to direct our attention to the two-dimensional vector-fields contained in a surface and to the correlated scalar fields representing a vector-component normal to the surface.

119. General Remarks on the Two-Dimensional Vector-Field.—For the same reasons which we have for vector-lines in space, we get:

> *Vector-lines of the two-dimensional field can intersect each other under finite angles only at points where the two-dimensional vector is zero, i. e., at points where the corresponding vector in space is either zero or normal to the surface containing the two-dimensional field.*

Instead of vector-tubes we shall in the two-dimensional field get *vector-bands* bordered by vector-lines. Transport must be referred to lines instead of to surfaces. A_n being the component of the vector normal to the curve s, we get

(a) $$\textit{transport through curve } s = \int A_n ds$$

Instead of surfaces we get *curves* of equal transport. The solenoidal condition is expressed by

(b) $$\int A_n ds = 0$$

the integral being extended over a closed curve. When condition (b) is fulfilled, the curves of equal transport can be left out, and the field be represented by bands of equal transport, most conveniently of unit transport. If unit bands be used, the numerical value of the vector is given by the reciprocal of the number expressing the breadth of the band.

If a unit band gets infinitely narrow, the solenoidal vector will be infinite. Excluding infinite values, we get this important result:

In the two-dimensional solenoidal field the lines of flow can not touch each other.

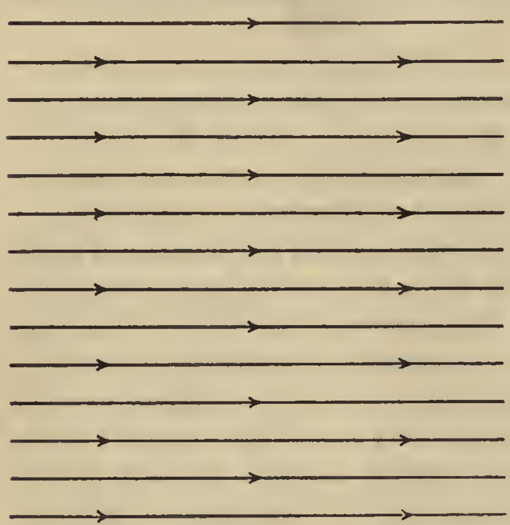

FIG. 35.—Translation-field.

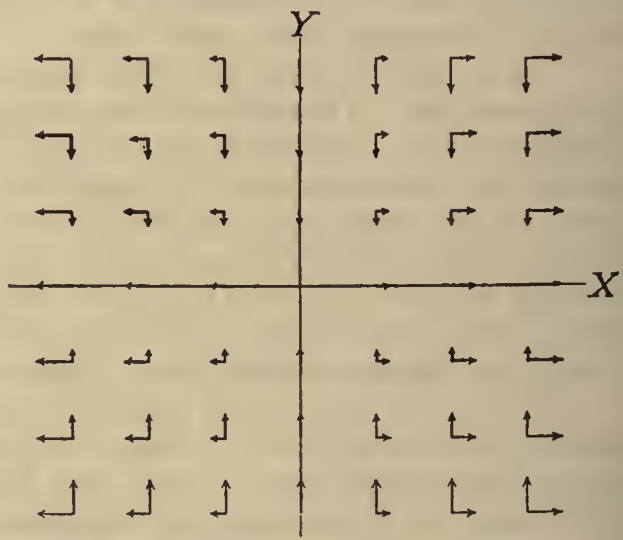

FIG. 36.—Vector-components of a plane deformation-field.

120. Examples of Two-Dimensional Solenoidal Fields.—We shall consider first the case that the two-dimensional field is solenoidal. Let the surface containing the field be plane. The simplest field will be that of a vector having the same direction and the same intensity at all points of the plane. If the vector is velocity the field will represent simple motion of translation. The field evidently fulfils the solenoidal condition. It can be represented geometrically by a set of parallel and equidistant vector-lines (fig. 35).

Let us next consider a field where the component A_x parallel to the axis x is proportional to x, and the component A_y parallel to the axis y is proportional to y:

(a) $$A_x = ax \qquad A_y = by$$

The line-integral of the normal component of the vector is easily found for any closed curve having the form of a rectangle with sides parallel to the axes of coordinates. If two of the sides are the coordinate axes, and the two others the lines $x=x$ and $y=y$, the line-integral taken over the closed curve will be $A_x y+A_y x$. Substituting the values (*a*) of the components, we get the line-integral equal to $axy+bxy$, and the solenoidal condition is seen to be fulfilled if $b = -a$. Thus the formulæ will be

(*b*) $$A_x = ax \qquad A_y = -ay$$

Fig. 36 represents the components of this solenoidal vector. If the vector represents velocity, the motion given by fig. 36 will be the simplest typical fluid motion producing a *deformation* of the fluid masses without change of volume.

A vector-line is determined by the condition that the projections dx and dy of its line-element are proportional to the vector-components A_x and A_y. It is therefore given by the differential equation

(*c*) $$\frac{dx}{A_x} = \frac{dy}{A_y}$$

Substituting the values of A_x and A_y according to (*b*), and integrating, we find

(*d*) $$xy = \text{const.}$$

i. e., the vector-lines are equilateral hyperbolæ, having the axes of coordinates as asymptotes (fig. 37). These axes themselves belong to the system of lines of flow,

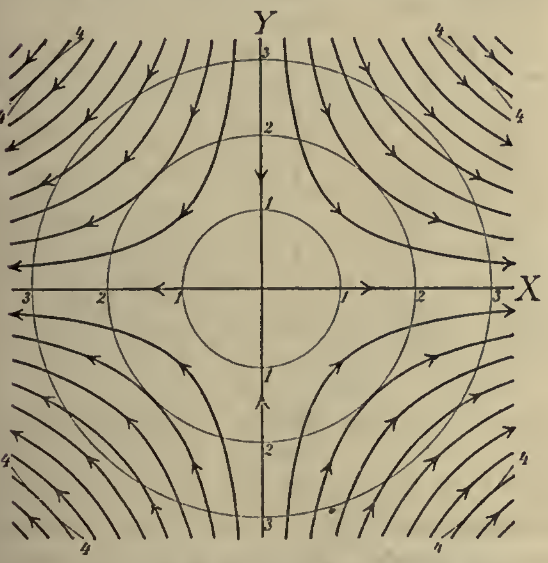

FIG. 37.—Hyperbolic lines of flow and circular curves of equal intensity 1, 2, 3, . . . of a plane deformation-field.

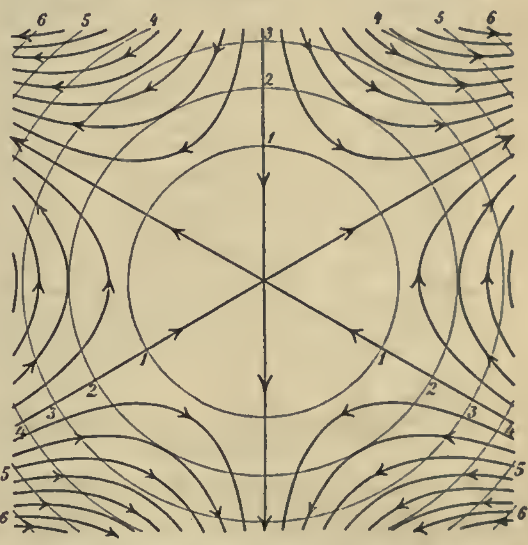

FIG. 38.—Neutral point of higher order.

and cut each other at the *neutral* point of the field where $A_x = A_y = 0$. The intensity A of the vector is

$$A = \sqrt{A_x^2+A_y^2} = a\sqrt{x^2+y^2} = ar$$

Thus the curves for equal intensity are circles $r=$const. around the neutral point. Through equal lengths of a line parallel to one of the axes there will go equal transport. Drawing the hyperbolæ through equidistant points on such a line as made in fig. 37, we get bands of equal transport and can leave out the curves of equal intensity. If this field represents a field of motion, it gives the picture of two currents flowing against each other, bending off against each other, and canceling at the neutral point.

Neutral points of a more complex nature, where three or more currents cancel simultaneously, may also be conceived (fig. 38).

121. Graphical Addition of Two-Dimensional Solenoidal Fields.—The investigation of the two-dimensional solenoidal vectors is much assisted by a construction allowing us to pass from the representations of the fields of two such vectors to that of their vector-sum.

Let the two given fields be represented by the two sets of thin lines of fig. 39. These lines divide the plane into a set of parallelograms. Every diagonal in any one of the parallelograms represents a section simultaneously of two unit bands, viz, of one belonging to the first and of one belonging to the second of the given fields. It is further seen that through one diagonal in a parallelogram goes the sum of the transports in two unit bands, *i. e.*, the transport 2, while through the other goes the difference, *i. e.*, the transport zero. Drawing the diagonal curves formed by the

FIG. 39.—Graphical addition of two-dimensional solenoidal fields.

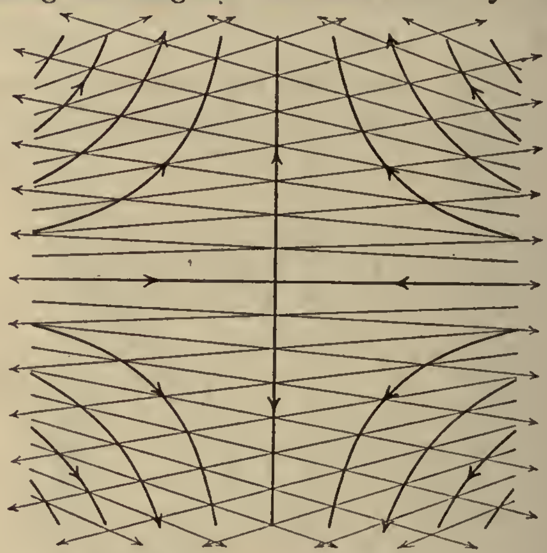

FIG. 40.—Addition of oppositely directed divergent fields.

latter set of diagonals, we evidently get lines of flow of the field due to the coexistence of the two given fields. These lines are drawn heavy in fig. 39. Further, it is seen that the bands separating these lines are unit bands. For two of them correspond to each diagonal through which we found the transport to be equal to 2.

As an application of the construction, fig. 40 shows how a deformation-field with neutral point and hyperbolic vector-lines is produced by the coexistence of two oppositely directed fields with straight, slightly divergent vector-lines.

Figure 41 shows the effect of adding the field with parallel and equidistant straight vector-lines to that with the hyperbolic vector-lines. As is seen, the result is simply a displacement of the latter field, the neutral point turning up where the two fields cancel.

122. Solenoidal Field in Space with Neutral Point.—It will be useful to show the simplest case of a solenoidal field in space having a neutral point. Corresponding to the two-dimensional field of section 120, we shall then consider a field with the rectangular components

(*a*) $$A_x = ax \qquad A_y = by \qquad A_z = cz$$

The integral of the normal component of the vector is easily formed for a surface of parallelepipedic form having sides parallel to the coordinate planes. The solenoidal condition is seen to be fulfilled if

$$a+b+c = 0$$

In order to simplify we shall further set $b = a$, which gives $c = -2a$. We then have the field

(*b*) $$A_x = ax \qquad A_y = ay \qquad A_z = -2az$$

Composing the components A_x and A_y we get a resultant contained in a plane passing through the axis of z. Calling r the distance of any point in this plane from the axis of z and R the resultant of A_x and A_y, we get instead of the two first equations $R = ar$. The field will then be completely given by the two components

(*c*) $$R = ar \qquad A_z = -2az$$

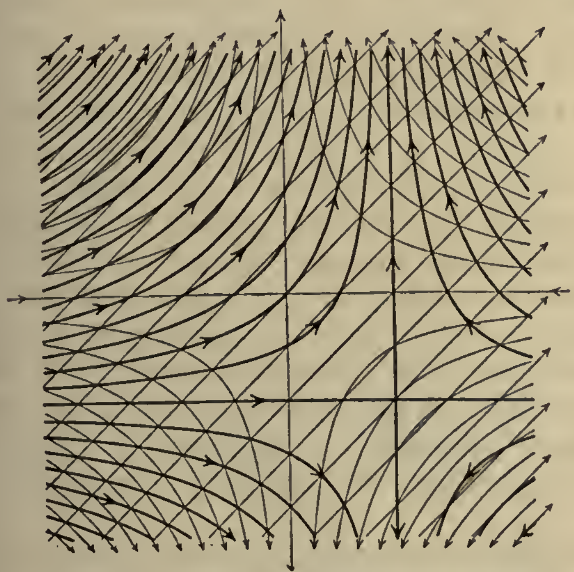

FIG. 41.—Addition of translation-field and deformation-field.

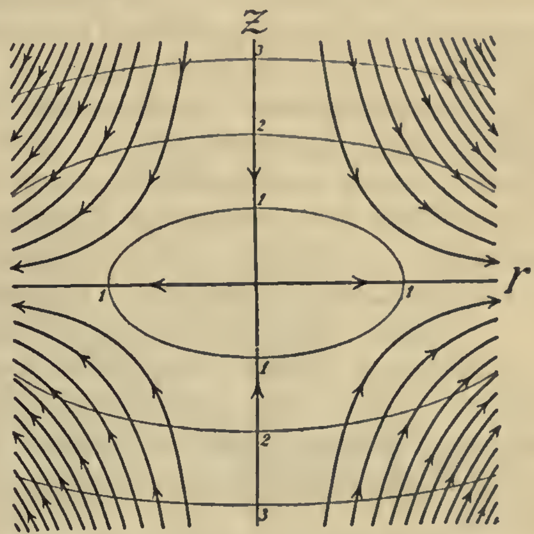

FIG. 42.—Lines of flow and curves of equal intensity, 1, 2, 3, of a symmetrical deformation-field in space.

The field is thus symmetrical around the axis of z, and the vector is contained in the meridian planes passing through this line. Substituting the values of R and z in the differential equation

$$\frac{dr}{R} = \frac{dz}{A_z}$$

and integrating, we get the equation of the vector-lines

(*d*) $$r^2 z = \text{const.}$$

They are a kind of asymmetric hyperbolæ having the axes of r and z for asymptotes, but converging more rapidly toward the first of these axes than toward the second (fig. 42). The axes are themselves vector-lines cutting each other at the neutral point of the field.

The vector is seen to have the constant numerical value A on the curve

$$A^2 = R^2 + A_z^2 = a^2r^2 + 4a^2z^2$$

which is an ellipse of half-axes $\frac{A}{a}$ and $\frac{1}{2}\frac{A}{a}$. These ellipses are drawn in fig. 42 for the values 1, 2, 3, of A.

We can now get a complete picture of the field. The meridian planes passing through the axis of z form one set of surfaces of flow. The other set is generated by the lines of flow of fig. 42, when this figure rotates around the axis of z. Simultaneously the other curves of this figure will generate the surfaces of equal intensity. We get thus the complete representation of the field by three sets of surfaces: two sets of surfaces of flow cutting each other along the lines of flow in space, and one set of surfaces representing equal scalar values of the vector.

As the field is solenoidal, a representation can also be obtained where the last set of surfaces is left out. A_z is constant in a plane $z = \text{const.}$ Thus there goes equal transport through equal areas of this plane. A division of this plane into equal areas is obtained if the radial lines defining the meridian planes are drawn with equal angular intervals and the circles defining the other surfaces of flow are drawn with radii proportional to the numbers $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, ... These intervals have been chosen already for the meridian curves of fig. 42, which represent these surfaces of flow. Thus the intersection of these surfaces with meridian planes which have constant angular distance from each other will produce tubes of equal transport representing the field completely. The surfaces of equal intensity may then be left out.

As atmosphere and hydrosphere have a limited extent in vertical direction but an enormous extent in horizontal direction, the best representations of fields of motion in these media will be obtained by charts in horizontal projection. It will be useful to consider different types of charts representing a simple field of motion, as that which we have just examined. Fig. 43 gives four different types of such charts.

(A) In fig. 43 A, the full-drawn concentric circles are contour-lines representing the topography of one of the surfaces of flow, namely, that of which a profile-curve is drawn at the top of the figure. The radial lines represent the lines of flow on this surface. Their vertical course is given directly by the topography of the surface. Finally the stippled circles are curves for the equal intensity of the vector. Evidently a set of charts of this kind each containing three sets of curves, contour-lines, lines of flow, and curves of equal intensity, will give a complete representation of the field.

(B) A varied method of representation, derived from the solenoidal property, is given in fig. 43 B. The contour-lines giving the topography of a surface of flow are retained and the lines of flow on it are drawn as before. But these lines are supposed to represent the projections of vertical walls separating from each other a set of tubes of flow. A third set of lines is then drawn, representing the height of these tubes. The curves for the equal height of the tubes will be a new set of con-

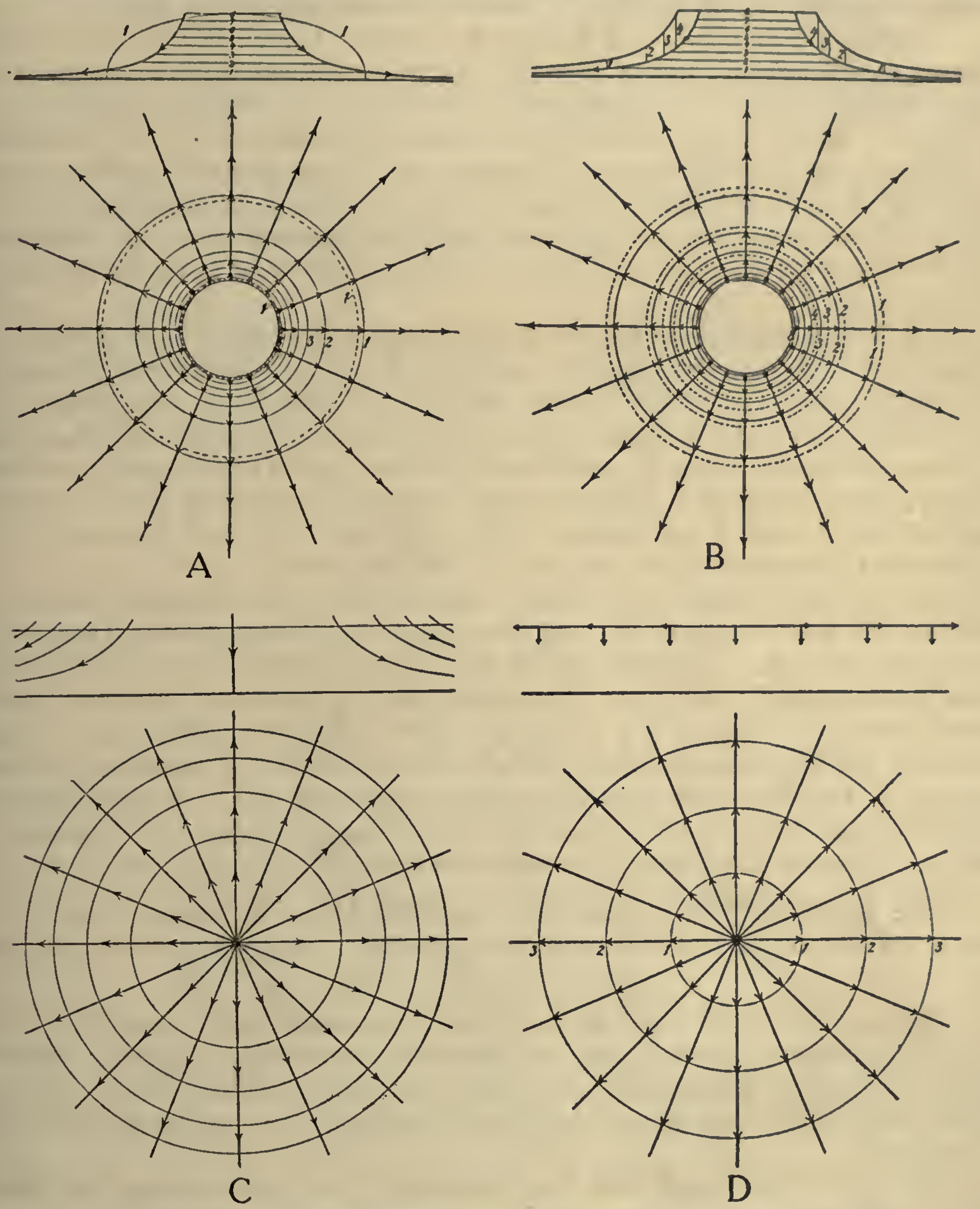

FIG. 43.—Field with singular point in space represented by different charts in horizontal projection.

A. Surface of flow represented by contour-lines (circles) and containing lines of flow (radii) and curves of equal intensity (stippled circles).
B. Tubes of flow represented by contour-lines for absolute and for relative topography (full and stippled circles) and lines of flow (radii).
C. Horizontal section through the system of tubes of flow.
D. Two-dimensional field in a horizontal plane represented by lines of flow (radii) and curves of equal intensity (circles). Normal component constant and not represented.

tour-lines, giving the topography of a second surface of flow relatively to the first. The stippled circles of fig. 43 B are these contour-lines. A set of charts of this kind, each containing three sets of curves, lines of flow, curves of absolute and curves of relative topography, will also give a complete representation of the field.

Instead of using surfaces of flow, as in the cases (A) and (B), we can use arbitrary surfaces cutting through the field. We can then simplify by choosing surfaces of simple configuration, instead of the surfaces of flow, which as a rule will not be simple. But in return we must give special representations of the component fields tangential to and normal to the surface. In the case before us it will be easiest to cut the field by horizontal planes z = const. As above, we shall then get two different representations according as we make explicit use or not of the solenoidal property of the field. We shall then arrive at the following two representations, (C) and (D):

(C) Let us imagine the field in space to be given by tubes of equal transport *i. e.*, by the meridian planes and the surfaces of revolution which form the walls of these tubes. The two sets of surfaces will cut the horizontal plane in two sets of curves, the radii and the circles of fig. 43 C. These curves divide the plane into areas which are sections of the unit tubes, and thus areas of equal transport normal to the plane. While these areas represent the normal component-field, the radial lines of flow represent the tangential field. Evidently the field in space can be represented completely by a set of charts of this description.

(D) Instead of using the solenoidal property of the field, we can draw the vector-lines and the curves of equal intensity which represent the tangential field contained in the plane z = const, and the curves of equal intensity which represent the normal field, as developed in section 118. In the case before us the vector tangential to any of the planes z = const. is $R = ar$. It has radial lines of flow and curves of equal intensity which are concentric circles with radii increasing in arithmetical series (fig. 43D). As in the case before us the normal component $A_z = -2az$ is independent of the coordinates x and y, no curves for representing the normal field are required. Only the constant value of the component will have to be noted for each plane.

123. Solenoidal Field in Space with Asymptotic Line.—As another example of a solenoidal field in space, we shall consider that defined by the rectangular components

(*a*) $$A_x = ax \qquad A_y = b \qquad A_z = -az$$

It consists of two partial fields which we have examined already (section 120), the field of the constant vector A_y and the field of the linear vectors A_x and A_z defining a two-dimensional deformation-field in planes parallel to the xz-plane. Each of these partial fields being solenoidal, that produced by their co-existence will also be solenoidal.

The vector-lines of the field thus produced will be represented by the differential equations

(*b*) $$\frac{dx}{A_x} = \frac{dy}{A_y} = \frac{dz}{A_z}$$

or, substituting the values of the components,

(c) $$\frac{dx}{ax} = \frac{dy}{b} = -\frac{dz}{az}$$

Integrating each of the three equations contained in this system, we get

(d) $$x = e^{\frac{a}{b}(y+c_1)} \qquad xz = c_2 \qquad z = e^{-\frac{a}{b}(y+c_3)}$$

The surfaces for equal scalar values A of the vector are given by the equation

(e) $$A^2 = A_x^2 + A_y^2 + A_z^2 = b^2 + a^2(x^2+z^2)$$

representing for every constant value of A a circular cylinder around the axis of y.

The second equation (d) shows that the lines of flow in space project themselves as equilateral hyperbolæ on the plane of xz. As the cylindrical surfaces of equal intensity cut this same plane along concentric circles, we get in this plane a figure precisely similar to that of fig. 37. The third equation (d) shows that the lines of flow in space project themselves on the yz-plane as exponential curves converging asymptotically toward positive y. The first equation (d) shows in the same manner that the lines of flow in space project themselves on the xy-plane as exponential curves diverging out asymptotically from negative y (fig. 44).

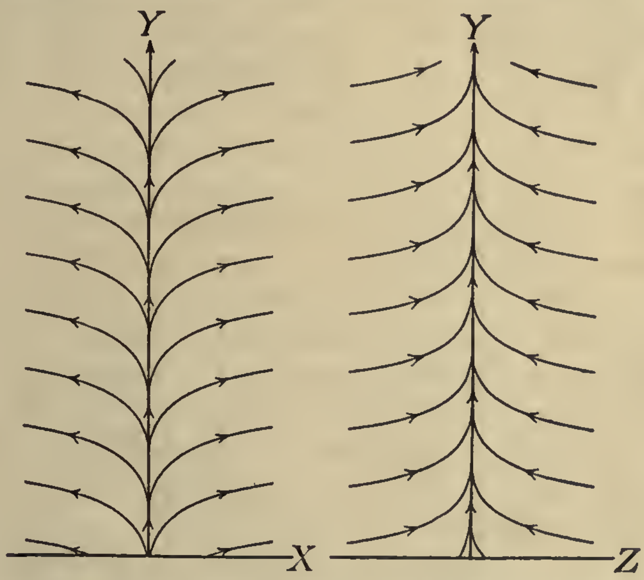

FIG. 44.—Lines of flow in the xy-plane diverging from, and in the yz-plane converging to the axis of y, which is a singular line of flow.

As the planes of xy and yz are themselves surfaces of flow, fig. 44 represents directly the lines of flow contained in these planes. The axis of y is itself a singular line of flow, and toward this line an infinity of lines of flow converge in asymptotically in the vertical plane and diverge out asymptotically in the horizontal plane.

In order to get a more complete view of the field, we can use the different representations by charts in horizontal projection.

(A) Fig. 45 A gives the topographical representation of two surfaces of flow which cut the xz-plane along two equilateral hyperbolæ. The course in space of the lines of flow contained in these hyperbolic surfaces and projecting themselves on the horizontal plane as exponential curves is thus easily conceived. Adding the lines of equal intensity (stippled straight lines), we get a complete representation of the field contained in these hyperbolic surfaces.

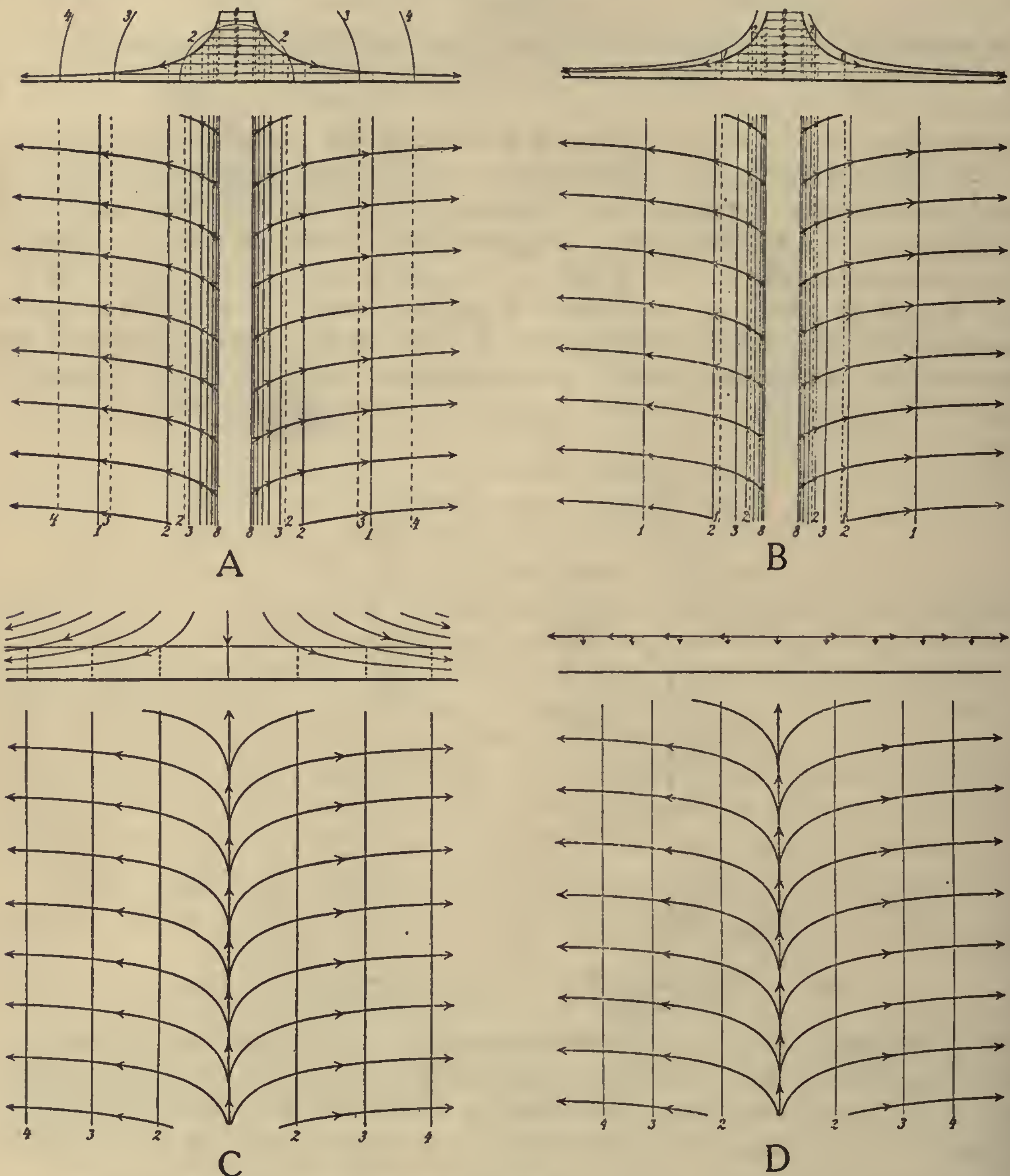

FIG. 45.—Field with asymptotic line in space, represented by different charts in horizontal projection.

A. Surface of flow represented by contour-lines (straight) and containing lines of flow (exponential curves) and curves of equal intensity (stippled straight lines).
B. Tubes of flow represented by contour-lines for absolute and for relative topography (full-drawn and stippled straight lines), and lines of flow (exponential curves).
C. Horizontal section through the system of tubes of flow.
D. Two-dimensional field in horizontal plane represented by lines of flow (exponential curves) and curves of equal intensity (straight lines).

(*B*) Leaving out the lines of equal intensity, and introducing in their place contour-lines giving the relative topography of a second surface of flow over the first, we get the solenoidal representation of the field contained between the two surfaces of flow (fig. 45 B).

(C) Fig. 45 C gives the horizontal section through the system of unit-tubes. The diagram shows the horizontal projection of the tubes and represents the vertical motion by a division of the horizontal plane into areas of equal transport normal to this plane.

(D) Fig. 45 D gives the lines of flow and the curves of equal intensity for the two-dimensional field contained in a horizontal plane. As in the example in section 122 (*D*), the vertical component $A_z = -az$ is independent of x and y and does not therefore require any special representation. But the principle of representing a variable normal component by drawing equiscalar curves is evident at once.

124. Charts Representing Fields of Motion in Atmosphere and Hydrosphere.— Referring to simple examples, we have given four different types of charts for representing fields of motion in space. Each type can be used practically in the case of atmospheric or hydrospheric motions, and we shall later indicate the methods of arriving at each of them. For the purpose of representation each type will have special advantages and special disadvantages. But it would lead too far to develop and exemplify them all in full detail. We shall therefore choose one of the methods as the principal one, namely the method D, *i.e.*, we shall choose arbitrary surfaces cutting through the field, and consider separately the two-dimensional vector-field contained in the surface and the scalar field representing the normal component of the vector.

As surfaces cutting through the field, we shall use level surfaces, isobaric surfaces, or for more limited purposes surfaces running parallel to the ground. In order to reduce as much as possible the number of drawings, we shall compose the two-dimensional vector-fields for a series of surfaces. In this manner we shall get two-dimensional vector-fields representing the *average tangential motion* within sheets of a certain thickness, level sheets, isobaric sheets, or sheets parallel to the ground. We have already made the introductory steps for the determination of such two-dimensional vector-fields from the observations (Chapter II).

These two-dimensional vector-fields being found as the direct result of the observations, we shall afterwards use the solenoidal condition for deriving the corresponding scalar fields representing the normal component of motion. It will be most convenient to determine them for the surfaces separating from each other the sheets for which the two-dimensional vector-fields have been drawn.

The methods for deriving the two-dimensional vector-fields from the observations will be considered in Chapters V–VII. Then Chapters VIII and IX will give from general points of view the graphical methods of performing mathematical operations to be used in the subsequent work. These methods being developed, we shall apply them in Chapters X and XI to complete the kinematic diagnosis by deriving the scalar fields which represent the normal component of the motion.

CHAPTER V.

DIRECT DRAWING OF THE LINES OF FLOW AND THE CURVES OF EQUAL INTENSITY FOR THE TWO-DIMENSIONAL VECTOR-FIELDS.

125. Continuous Representation of the Two-Dimensional Vector-Fields.—Passing to the practical diagnostic work, our first problem will be this: From the observations of motion (local values or averages for certain sheets) to draw the lines of flow and the curves of equal intensity for the corresponding two-dimensional field. Drawing these curves we shall get a continuous representation of this field instead of the discontinuous representation given by the observations themselves.*

Our solutions of concrete problems of this kind are given on plates XXXII, XXXVIII, LV, and LVII B to LX B. The lines of flow are represented by heavy curves provided with arrow-heads, the curves of equal intensity by thinner curves.

As such continuous representations of the two-dimensional fields are to form the basis for every further step in kinematic diagnosis or prognosis, we can not discuss too carefully the methods for drawing them as correctly as possible. Referring to the mentioned plates as examples, we shall take up this discussion, which will occupy us in this as well as in the two following chapters.

126. Equiscalar Curves in the Field of Single-Valued Scalar Quantities.—The numbers representing the numerical value of the vectors velocity or specific momentum define a scalar field having the same geometrical features as the well-known fields of other scalars, like pressure or temperature. The method of drawing the curves of equal intensity of a vector is therefore precisely the same as that of drawing isothermal or isobaric curves; but as the curves in the case before us will have an irregular course, the drawing will require a good deal of care.

Equiscalar curves are never drawn exclusively by the use of the numbers representing the observations. Otherwise an infinite number of observations would be required for the determination of their course. The intrinsic properties of the scalar quantity are also taken into consideration. The main property used in drawing the common synoptical charts is this, that the scalar is *single-valued.* As it can never have two different values in one point, *two different curves, representing different values of the scalar, can never intersect each other.* This property gives to the field of the single-valued scalar features which are totally different from those of the multiple-valued scalar, which we shall have to consider later.

*That charts of this character have not yet been used in practical meteorology, must be on account of their apparent complexity. The only charts containing lines of flow of atmospheric motions which we have been able to find in literature have been drawn by René de Saussure (Archives des Sciences Physiques et Naturelles, Quatrième Période, T. 5, p. 497, Genève, 1898) and by Jean Bertrand (Bulletin de la Societé belge d'Astronomie et de Meteorologie, 1905, No. 7 and 8; see also Physikalische Zeitschrift, 1905, p. 853).

The property of never intersecting each other very much limits the course of the curves, and makes it possible to draw them as soon as the values of the scalar are known in a relatively small number of points. But the course is never completely determined by a limited number of observations. There will always be a certain limited freedom in the way of drawing each curve. But as the number of observations is increased this freedom will be reduced, and finally the course of the curve will be perfectly determined from a practical point of view, *i. e.*, with a certain finite degree of precision.

The curves will obtain their characteristic features by the situation of the points where the scalar has its extreme values. At the maximum points and the minimum points the equiscalar curve will be reduced to a point. These points are surrounded by closed equiscalar curves. Between the maximum and minimum points there will be maximum-minimum points. In each a certain singular equiscalar curve cuts itself. The two branches of this singular curve divide the field in the neighbor-

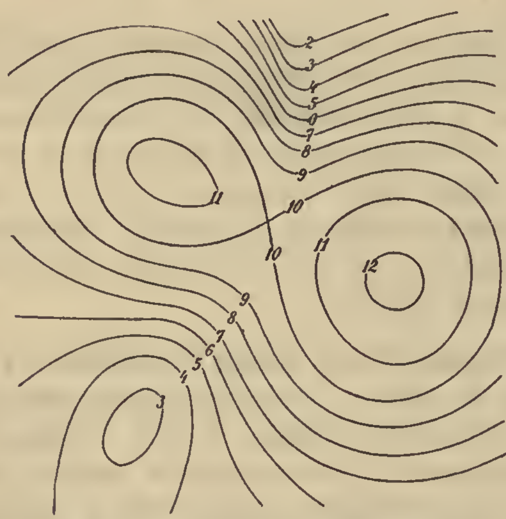

Fig. 46.—Maximum points, minimum points, and a maximum-minimum point of a scalar field.

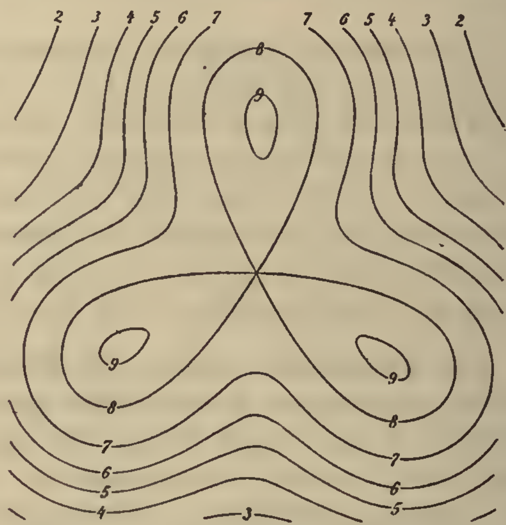

Fig. 47.—Maximum-minimum point of higher order.

hood of the maximum-minimum point into four angular areas. In two of them the scalar has greater and in two of them smaller values than in the point of intersection (fig. 46). More complex maximum-minimum points may also be mentioned, though they will rarely be met with in practice. Thus three branches of the singular equiscalar curves may cut each other in this point, dividing the surrounding field into six angular areas of alternately higher and lower values of the scalar (fig. 47), and so on.

The said features of the field give the practical rules for drawing the curves. Examining the numbers we first look for the points where the scalar has its extreme values. Around these points we then draw closed curves, proceeding subsequently to the curves representing intermediate values of the scalar and having the more complicated course between the areas of greater and those of smaller values. Among these curves the ones intersecting themselves should not be avoided. They give more information regarding the field than any other single curve.

It is important to observe the remarkable completeness of the graphical representation of a scalar function. When we draw the equiscalar curves for unit intervals, these curves will represent not only the scalar itself, but also its ascendant or its gradient (section 17). Any one of these vectors gives complete information regarding the result of any differentiation of the first order performed upon the scalar. The drawing of the equiscalar curves involves therefore a differentiation of the scalar function. The representation gives not only the function itself, but also its differentials. We shall derive great advantage from this later, when we have to perform differential operations in graphical form.

127. The Drawing of Vector-Lines.—The drawing of vector-lines by the use of arrows representing observed directions of a vector and the drawing of equiscalar curves by the use of observed values of a scalar are analogous operations, inasmuch as interpolations have to be performed by eye-measure. But in one case the interpolations are of scalar nature, in the other of vector-nature, interpolations of direction.

This difference regarding the nature of the interpolations is intimately related to a difference of principle between the two operations: The drawing of equiscalar curves involves a differentiation in graphical form of a scalar function; the drawing of the vector-lines involves an integration in graphical form of a differential equation, namely, the differential equation for the vector-curves. We have performed the corresponding analytical integrations in special cases above (sections 120, 122, 123). This graphical integration would not contain any difficulty if arrows of absolutely correct direction completely covered the plane of the drawing. But the curves have to be drawn by the use of the minimum of data given by the observations, and with attention paid to the limited accuracy, or to the direct errors of the observations. Under these circumstances, in order to get the lines drawn as correctly as possible, it will be important to make as complete use as possible of the general properties of the field. We must derive from them qualitative rules which allow us to make the correct use of the data contained in the observations.

For this we shall have to pay special attention to the *singularities* of the field, *i. e.*, to the mutual intersections and touchings of the lines of flow; for as soon as the places are determined where intersections or touchings take place, and as soon as the manner is known in which the lines of flow pass through these places, the general feature of the field will to a great extent be given; for everywhere else in the field the lines will be limited in their course by the condition of not cutting or touching each other.

128. Simplest Singularities in the Field of the Lines of Flow.—We have chosen our examples in the preceding chapter so as to illustrate the simplest singularities which can arise in the three-dimensional solenoidal field; and forming the horizontal sections through these fields we have seen the character of the corresponding singularities in the two-dimensional vector-fields which we shall use to represent the three-dimensional one. In the simple cases treated analytically, the fields had simple properties of symmetry. Drawing correspondingly crooked and asymmetric figures, we get pictures of the singularities and of the field surrounding them as they will appear in the case of concrete motions. In this manner we get the schemes of singularities presented by the different diagrams of fig. 48. The following remarks regarding each of them will easily be understood by a comparison with the results obtained analytically in sections 120, 122, 123 of the preceding chapter.

I. *Neutral Points.*—Points of this description appear when opposite currents meet each other and bend off against each other without producing any motion normal to the sheet (section 120). In the singular point two lines of flow will intersect each other. Points of higher order, in which a greater but still finite number of lines of flow intersect each other under finite angles, are also theoretically possible (fig. 38), though they will occur rarely.

II. *Points of divergence and of convergence.*—Let a field in space as that of fig. 42 (p. 37) be given. The corresponding two-dimensional field contained in a horizontal plane is represented by fig. 43 D. It contains a point in which an infinite number of lines of flow intersect each other. A tangential motion of this kind in a sheet always depends upon the existence of a motion normal to the sheet, leading masses into it or taking masses away from it. In the atmospheric sheet near the ground a point of divergence will appear where there is a descending current (centre of anticyclone) and a point of convergence where there is an ascending current (center of cyclone). The lines of flow are drawn in diagrams B–E of fig. 48, with the common spiral-formed curvature due to the earth's rotation, which is so well known from the air-motions near the centers of cyclones or anticyclones. In the sheet of water near the sea's surface a point of divergence will depend upon an ascending motion and a point of convergence upon a descending motion of the water masses below. When the sheet is situated at a greater distance from the bounding surfaces, divergence in the tangential motion shows that the normal motion brings greater masses into the sheet on one side than it brings out on the other, and vice versa for convergence in the tangential motion. But no definite conclusion can be drawn regarding the general direction of this normal motion, which may even have opposite directions on the two sides of the sheet.

III. *Lines of divergence and of convergence.*—Let a field in space, as that described in section 123, be given. Fig. 45 D shows that the two-dimensional field in a horizontal plane will contain a singular line of flow from which an infinite number of other lines of flow diverge out asymptotically (fig. 48 F). Reversing the direction of the motion, we get a similar line toward which an infinite number of lines of flow converge asymptotically (fig. 48 G). Evidently the lines of divergence and convergence are in precisely the same relation to the normal motion as the points of convergence and of divergence. In the case of rapid convergence, the designer can make no difference between common and asymptotical touching. When the singular line is represented by a stroke of finite breadth, it will completely absorb the lines converging toward it. The case of an infinitely rapid convergence arises when the lines go normally into the singular line, the case $A_y = 0$ or $b = 0$ in the example of section 123. In this case the asymptotic line ceases to be a line of flow and is reduced to be a line for zero numerical value of the vector.

The singularities presented by the lines of flow are in a definite relation to the field of intensity. As we have remarked already, wherever vector-lines intersect each other under finite angles, the vector must have the numerical value zero. In the same

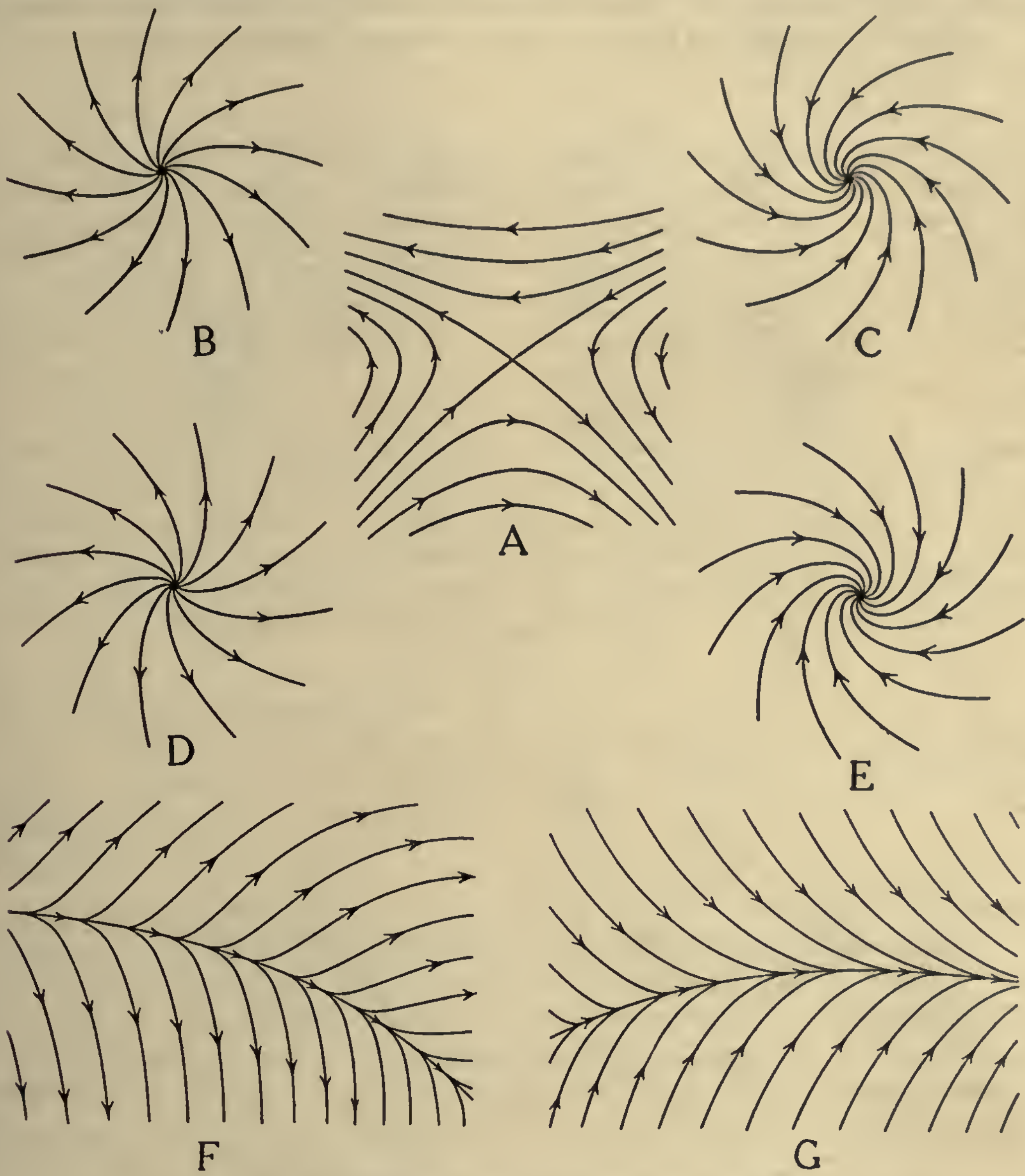

FIG. 48.—Simplest singularities in two-dimensional vector-field.

A. Neutral point.
B. Point of divergence, northern hemisphere.
C. Point of convergence, northern hemisphere.
D. Point of divergence, southern hemisphere.
E. Point of convergence, southern hemisphere.
F. Line of divergence.
G. Line of convergence.

manner the vector must have smaller numerical values in the asymptotic lines than on both sides of it, because the components normal to the line disappear in the line. Thus:

The numerical value of the vector is zero in the singular points, and has a relative minimum in the singular lines.

The curves of equal intensity must therefore be closed around the neutral points and around the points of convergence and divergence, and make a bend as they pass lines of divergence or of convergence. This bend may be very slight and impossible to discover by the observations when the lines of flow have a slow convergence toward the singular line. But in the case of rapid convergence the bend should come out strongly.

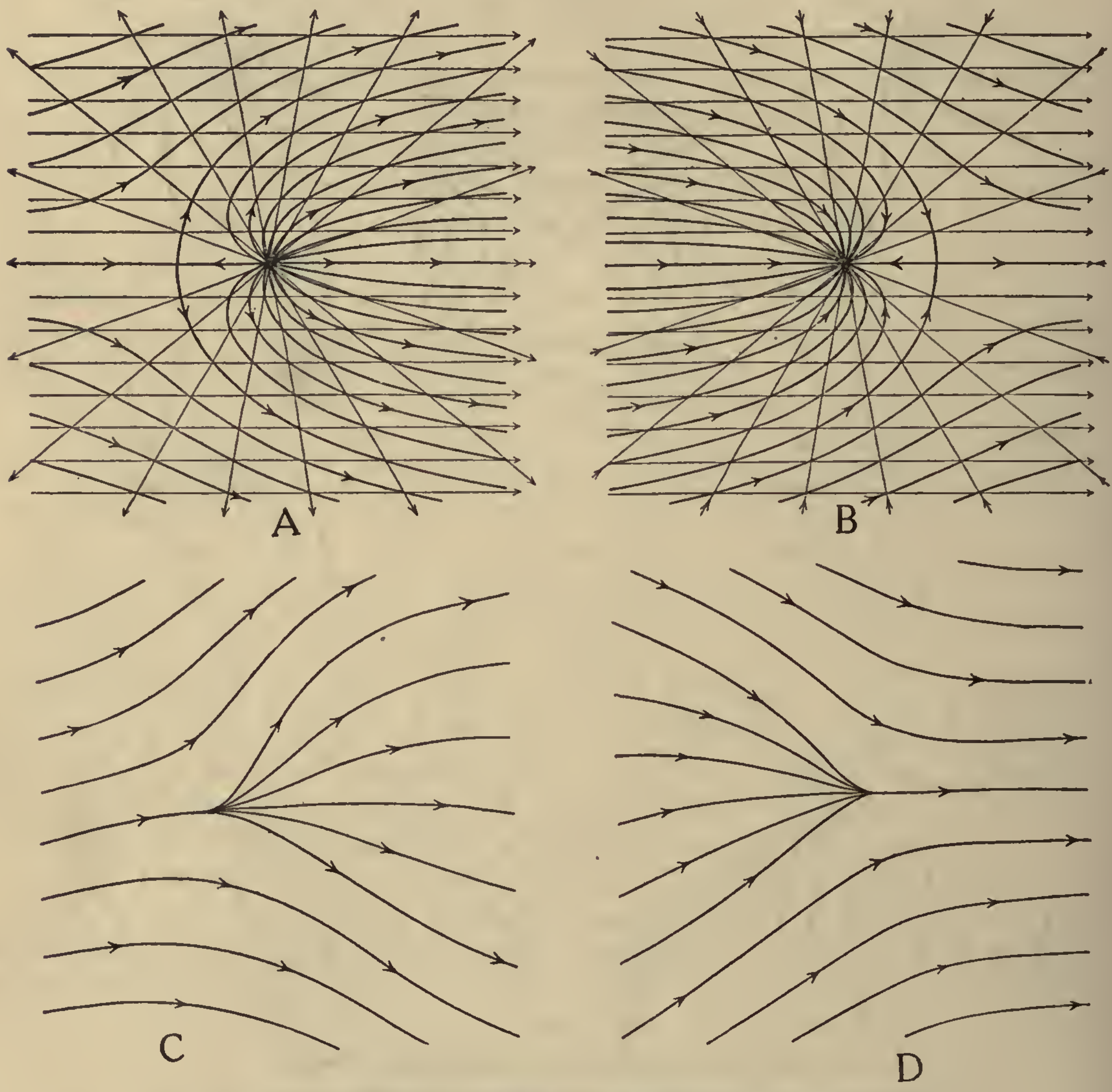

FIG. 49.—Complexes of singular points.

A. Neutral point and point of divergence.
B. Neutral point and point of convergence.
C. Line of flow branching out into several lines.
D. Lines of flow joining into one.

129. Complexes of Singular Points.—When good observations are at hand, it will generally cause no greater difficulty to discriminate the nature of the singular points as long as they are separated from each other by sufficiently large spaces; but it may be more difficult when singular points of different nature appear close together. It will therefore be important to consider the conditions for the formation of some such complexes of singularities.

Let us for this purpose consider two coexistent fields, a simple field of translation represented by the parallel straight lines of flow, and a field containing a point of divergence, having the straight radial lines of flow of fig. 49 A. For the sake of simplicity we may consider also the last field as solenoidal except at the center itself, the normal supply being localized to this point instead of being spread over a finite area. Under these conditions we can add the solenoidal fields graphically (section 121). We then get the resultant field represented by the heavy lines of fig. 49 A, containing the constellation of two singular points, a point of divergence and a neutral point. Fig. 49 B shows the result of the same construction when the field of translation is retained, while the second field is changed into one containing a center of convergence. The field has the same character as the preceding one, only reversed.

This constellation of a point of convergence or divergence and a hyperbolic point will often occur on the charts of air-motion along the earth's surface. It appears as the result of a main horizontal wind and a vertical descending, respectively ascending, current. The discrimination of this constellation will cause no difficulty when the phenomenon is on a sufficiently large scale, and the two singular points are thus at sufficiently great distances from each other; but they may also get so near to each other that no observations of the air-motion is obtained between them. The direct drawing of the lines of flow from the observations will then give points or places where a line of flow branches out into several branches (fig. 49 C), or several lines of flow join into one (fig. 49 D). At the point of ramification the different branches may touch each other or cut each other under finite angles. The first case presumes a minimum and the second zero numerical value of the vector at this point.

130. Complex Phenomena in Connection with Lines of Convergence and of Divergence.—The theoretical possibility of certain complex singularities is seen at once. A line of convergence or of divergence can contain a neutral point in which the direction of the motion tangential to the line changes its sign (fig. 50 A, B). A line of divergence can come out from a point of divergence, and a line of convergence can end in a point of convergence (fig. 50 C, D). The latter seems to be no rare phenomenon in well-developed cyclones. Several lines of convergence are also often seen to join into one (fig. 50 E).

A specially interesting feature is the closed line of convergence containing within the inclosed area a point of divergence (fig. 50 F). This gives the kinematic aspect of the phenomenon called *eye of cyclone*, which seems to be common in strong cyclones. Corresponding eyes of anticyclone are also kinematically possible, though for dynamic reasons less probable.

A remarkable feature sometimes found on synoptical maps representing the air-motion along the ground is lines alternately of convergence and of divergence running more or less parallel to each other.

FIG. 50.—Complex singularities.

A. Line of divergence with neutral point.
B. Line of convergence with neutral point.
C. Line of divergence issuing from point of divergence.
D. Line of convergence ending in point of convergence.
E. Two lines of convergence joining into one.
F. Eye of cyclone.
G. Rolling mass of air bordered by a line of convergence and a line of divergence.

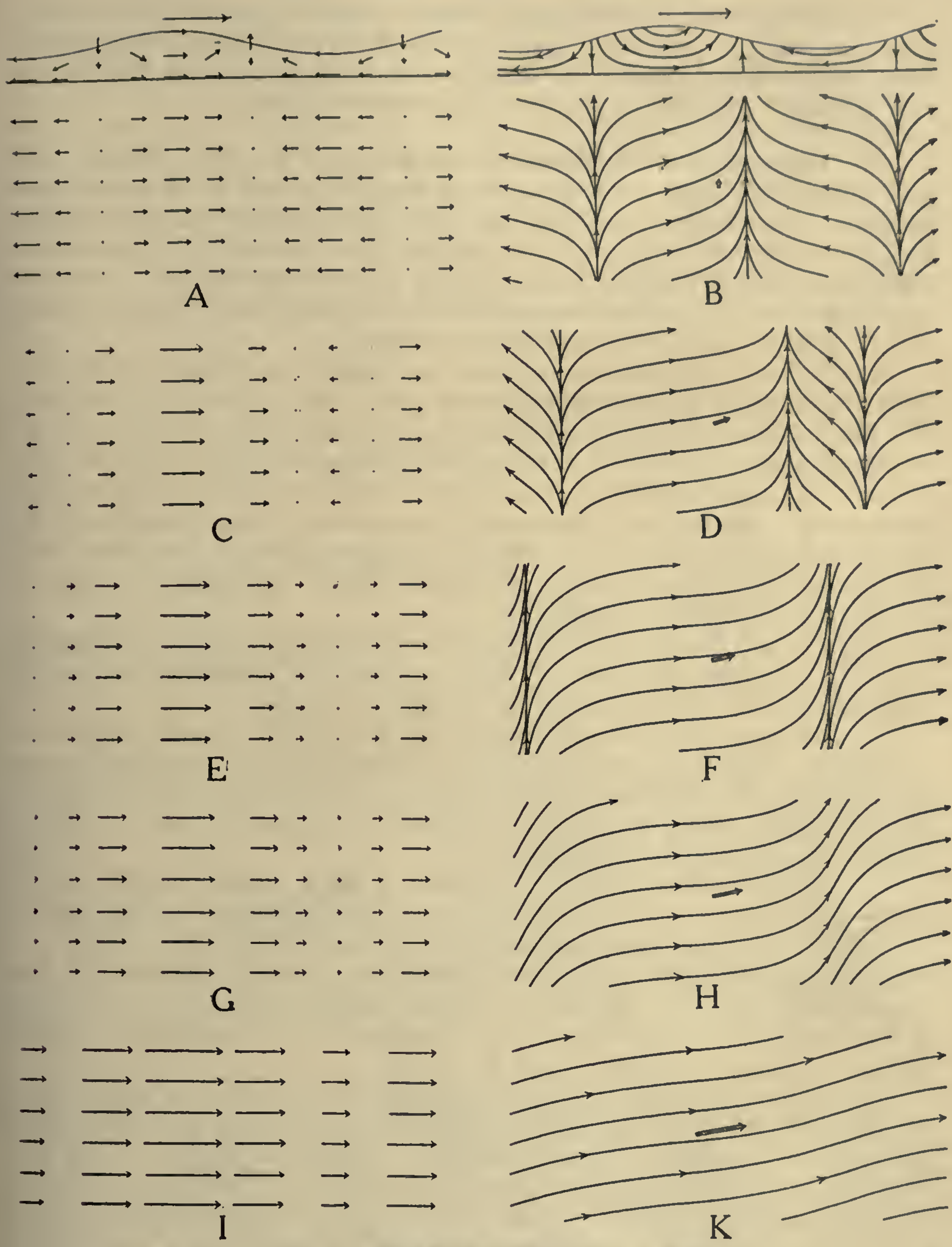

FIG. 51.—Effect of combined wave-motion and motion of translation.

A. Pure wave-motion.
B. Translation parallel to wave-ridges.
C. Translation normal to wave-ridges.
D. Translation oblique to wave-ridges.
E. Stronger translation normal to wave-ridges.
F. Stronger translation oblique to wave-ridges.
G. Still stronger translation normal to wave-ridges.
H. Still stronger translation oblique to wave-ridges.
I. Still stronger translation normal to wave-ridges.
K. Still stronger translation oblique to wave-ridges.

The corresponding motions in space may be of different kinds. Thus a rolling mass of air (fig. 50 G) will be bounded by a line of convergence and a line of divergence parallel to each other. But the most common origin of such lines may be wave-motions.* We shall therefore examine this case separately.

131. Influence of Wave-Motions on the Aspect of the Lines of Flow.—The large-scale waves which can arise in the atmosphere will be of the same nature as long waves in shallow water. During the propagation of the waves the different particles will describe elliptic orbits in vertical planes normal to the wave-ridges. Every ellipse has its long axis horizontal and its short axis vertical. The latter axis will decrease as we go downward, and be zero at the ground. Thus the motion near the ground will consist in rectilinear oscillations.

Remembering the difference of phase from particle to particle, we can draw arrows representing the *simultaneous* motion of a set of particles at a given epoch. This distribution of arrows in a vertical plane is shown at the top of fig. 51 A, and the corresponding lines of flow at the top of fig. 51 B. As will be seen, the propagation of the waves depends upon a conflux of masses below the front-slope and a corresponding afflux below the back-slope of the waves. In the horizontal projection we shall therefore always get a line of convergence below the front slope and a line of divergence below the back-slope of every wave. These lines will follow the waves in their motion of propagation.

Fig. 51 A will thus give the instantaneous distribution of motion at the ground in the case of a pure wave-motion. With the system of velocities thus given we shall compose the constant velocity due to a pure translation.

(1) First let us add a constant velocity which is parallel to the direction of the wave-ridges. Performing the parallelogram-constructions and afterwards drawing the lines of flow, we get the picture of fig. 51 B. The picture shows lines of flow running between a system of parallel and equidistant asymptotic lines, alternately lines of convergence and of divergence.

(2) To the velocities of fig. 51 A we shall now add a constant velocity which is normal to the direction of the wave-ridges and of smaller intensity than the greatest velocity due to the pure wave-motion. We shall then get the picture of fig. 51 C. When we afterwards add the same constant velocity parallel to the direction of the wave-ridges as above, perform the parallelogram-constructions, and draw the lines of flow, we get the picture of fig. 51 D. The picture shows parallel, but no more equidistant, lines of convergence and divergence.

(3) To the velocities of fig. 51 A we shall again add a constant velocity of direction normal to the wave-ridges, but now of intensity equal to the greatest occurring in the pure wave-motion. We shall then get the velocities presented by fig. 51 E. When we add in this case the same constant velocity parallel to the wave-ridges as above, perform the parallelogram-constructions, and draw the lines of flow, we shall get the picture of fig. 51 F. Here we have a set of wave-formed lines of flow, touching

*Cf. J. W. Sandström: Ueber die Beziehung zwischen Luftdruck und Wind. K. Svenska Vetenskapsakademiens Handlingar, T. 45, No. 10. 1910.

each other along a set of singular lines, each produced by the coincidence of a line of convergence and a line of divergence.

(4) To the velocities of fig. 51 A we shall finally add a constant velocity of direction normal to the wave-ridges and now of greater intensity than the greatest velocity due to pure wave-motion. We then get velocities which are periodically increasing and decreasing, but without any change of direction (fig. 51 G). If we add to these velocities the same constant velocity parallel to the direction of the wave-ridges as above, we shall get the system of asymmetric wave-formed lines of fig. 51 H, containing no singularity.

(5) If we increase still further the velocity normal to the wave-ridges, and then add the same velocity parallel to the wave-ridges as before, we shall get fig. 51 I and K respectively. The lines of flow of the latter figure are very nearly sinus-lines, but of very small amplitude.

In all of the figures B, D, F, H, K, the velocity parallel to the wave-ridges has the same value, and a very small value. If we increase this velocity, the lines of flow of the figures B, D, F will be stretched out in the direction of the singular lines, *i.e.*, in the direction of the wave-ridges, and the lines of flow of the figures H and K will get higher waves.

132. Practical Rules for the Direct Drawing of the Lines of Flow and the Curves of Equal Intensity.—When a chart is given containing arrows and numbers representing the observations of the motion, the first thing to do in order to pass on to the continuous representation of the motion will be this: by examination of the distribution of arrows and of the corresponding intensities to find out the nature and the approximate situation of the singularities.

This being done, it will generally be best first to draw certain of the lines of flow issuing from the singularities. Some lines of flow will generally be found whose course can be drawn with great certainty. A set of such lines being drawn, the general character of the whole field will practically be determined, for they will divide the chart into areas within which the other lines must have their course, as intersections are excluded except in the singularities.

The lines of flow and those of equal intensity should be drawn with continuous attention to each other. The closed intensity-curves surrounding the singular points are first drawn, then other closed curves surrounding other places of maximum or of minimum values of the vector, and then by and by the curves which have a more complicated course.

In this way, it will generally not be found too difficult to draw the lines of flow and curves of equal intensity, representing the air-motions along the ground over the areas where we have a satisfactory network of meteorological stations. Cases of doubt as to the character of the singularities as well as to the detailed course of the curves may arise. But making the experiment of letting different workers draw the curves of flow from the same observations independently, we have always found that the result has been very nearly the same as soon as the observations have the completeness of those from Europe or from the United States.

CHAPTER VI.

SUPPLEMENTARY RULES TO ASSIST IN THE DRAWING OF THE LINES OF FLOW AND OF THE CURVES OF EQUAL INTENSITY.

133. **Remarks on the Digression.**—We have emphasized the fact that the drawing of the lines of flow and of the curves of equal intensity would cause no difficulty, if we had at our disposal a sufficient number of really good observations; but as a matter of fact the observations are often so scarce and so heterogeneous that great doubts arise as to the course of the lines. In such cases we must look for other diagnostic methods than the pure kinematic ones.

This leads us to give here, in anticipation, diagnostic rules depending upon dynamic, partly also upon thermodynamic and other principles. The foundation of these rules will be considered more fully in later parts of this work. Deviating thus for practical reasons from the strictly theoretical plan, it will be important to make certain reservations in connection with this digression.

If the aim be simply this, to find the most probable motion of atmosphere or hydrosphere on a certain occasion, it is perfectly legitimate to bring into application all diagnostic methods which may serve the purpose. But if further conclusions should be drawn from the picture of motions thus obtained, we must take care to avoid the *circulus vitiosus*. If rules derived from dynamic or thermodynamic principles have been used to produce the picture of atmospheric motions, this picture can not be used legitimately afterwards to verify these same rules.

It can not therefore be too strongly recommended to develop the system of direct observations of atmospheric and hydrospheric motions, in order to make it possible to arrive at the synoptical representations of the motions by methods of a purely kinematic nature. Representations obtained in this way will be the only ones which can be legitimately used for subsequent investigations regarding the dynamic and thermodynamic phenomena which are the causes of the motions.

134. **Relation of the Kinematic Singularities to Dynamic and Thermodynamic Phenomena.**—Motion has a general tendency to go from higher toward lower pressures. From this we easily derive the following special rule:

Within a barometric depression there is a probability for existence of points or lines of convergence; within areas of high pressure there is a probability for the existence of points or lines of divergence. Long ridges of high pressure will as a rule contain a line of divergence; long ridges of low pressure a line of convergence. In the neighborhood of a maximum-minimum point of pressure situated between two high and two low areas there will be a probability for the existence of a neutral point with hyperbolic lines of flow.

Where the given observations of the wind do not give sufficient evidence for the nature and placement of the singularities, the required supplementary evidence may be obtained by examining the chart of pressure. But in doing so we should remember that there is no necessity for the motion to go always, and under all conditions, toward lower pressure. There will seldom be an absolute coincidence between the points of convergence or of divergence with the points of minimum or of maximum pressure, or between the neutral point and the saddle point on the isobaric surfaces. The draftsman will often find that the observations of the wind give full evidence for the existence of kinematic singularities, especially of neutral points and of lines of convergence and of divergence at places where the chart of pressure does *not* show the expected peculiarities. Examples where the pressure for theoretical reasons shows other peculiarities will be considered below.

For thermodynamic reasons the kinematic singularities are in similar relation to the distribution of precipitation, cloudiness, and blue sky, as to that of pressure. Within an area of precipitation or of cloudiness there is, as a rule, ascending motion and therefore a probability of the existence of a point or of a line of convergence. In the same manner within areas of blue sky there will usually be descending motion and therefore a probability for the existence of a point or a line of divergence. The neutral point, which has no relation to vertical motion, will be indifferent in its relation to precipitation and to blue sky.

The charts of precipitation, of cloudiness, and of blue sky may therefore be used precisely as those of pressure, to get additional evidence in cases where the observations of the wind are not sufficient. But as in the case of pressure, the conclusion can not be reversed. Especially there will often be found lines of convergence causing no precipitation. Examining the relation of the kinematic singularities to pressure and precipitation, cloudiness and blue sky, it will probably be possible to decide whether the singularity is a local one, concerning only the lowest strata, or whether it has any connection with the motion also at greater heights.

135. Consequences of the Stability of Atmospheric or Hydrospheric Equilibrium.—The different layers of the air or the sea as a rule rest upon each other in stable equilibrium. A mass of air or of water will not leave its level except it be forced to do so. The currents will therefore always prefer to some extent to go round instead of going over obstacles. In other words, the lines of flow will have a certain tendency to follow the level curves representing the topography of the bounding surfaces. Many striking examples of this are seen on the accompanying maps of the air-motion. This dependency of the wind-direction upon topography is so strong that it can be recommended to draw the lines of flow on outline-maps containing a simplified representation of the topography of the land. In many cases the apparent irregularity in the distribution of arrows representing the observed wind-directions will be understood at once, by a comparison with the level curves of this map.

Sea-motions will depend upon the configuration of the bottom still more than air-motions on the configuration of the ground. The remarkable correspondence of lines of equal salinity, or equal temperature,* even at the surface of the sea, with

*In his paper "Some oceanographic results of the expedition with the 'Michael Sars', 1900" (Nyt Magasin for Naturvidenskab, T. 39, Christiania, 1901), Professor Nansen says, p. 153: "If we consider the chart (Plate I) of the surface-salinity and temperature it must strike one how almost exactly the most saline surface-water follows the deepest channel of the Norwegian sea, and how the isotherms especially of 10° C. and 9° C. seem to be deflected in a way similar to the isobaths." Further observations on this and allied subjects are found in the same author's "Oceanography of the North Polar Basin," pp. 260 et seq. (The Norwegian North Polar Expedition, 1893–96, Scientific Results, Vol. III, Christiania, 1902), and in Helland-Hansen and Nansen: The Norwegian Sea, Chapter X, p. 311 (Christiania, 1909).

the course of bathymetric curves several thousand meters below is a very striking sign of this dependency.

Sudden disturbances of the equilibrium will give rise to wave-motions. There seems to be good evidence for the existence of large-scale waves in the bounding surfaces between different layers in the sea.* Motions of the same kind are equally possible in the atmosphere, and lines of flow of the character described in section 131 seem to show that they actually occur. When the motion has this character, we have no right to expect a minimum of pressure along the lines of convergence and a maximum of pressure along lines of divergence. In case of pure wave-motions, the maxima of pressure should be under the summits and the minima under the troughs of waves, while the line of convergence is under the front-slope and the line of divergence under the back-slope of the advancing wave. If a progressive motion is added, displacements of the lines of convergence and divergence take place, and their relation to the pressure will not be easy to see on the chart.

136. Consequences of Kinetic Instability, Discontinuous Motions, and Eddies.— A kinetic phenomenon which is equally well known, though not so well understood

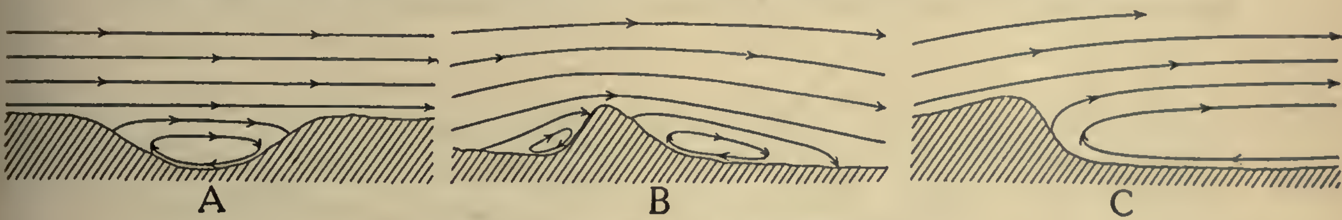

FIG. 52.—Motions due to kinetic instability.

A. Eddy in a valley. B. Eddies on windward and leeward side of a mountain. C. Eddy joining the great atmospheric motions.

as that of the formation of waves, is that of the formation of eddies. They are very often formed in the neighborhood of obstacles, both on the windward side of the obstacles, and still more frequently behind them. A motion going on without eddies is in such cases kinematically possible, but dynamically unstable, and has therefore no chance of persisting even if it be produced for a moment.

An eddy due to the instability of the motion may fill a valley across which there passes a main wind (fig. 52 A). It may be produced both on the windward and on the leeward side of a mountain (fig. 52 B), the latter case being the most frequent. The observations of the wind at the ground will then give pictures like that of fig. 50 G, with a parallel line of divergence and of convergence, the latter being as a rule the one which appears most distinctly. The line of divergence may also disappear completely when the eddy enters as a part of great atmospheric motions (fig. 52 C). In such cases only a line of convergence will be discovered following the ridge of a chain of mountains or the edge of a plateau-land. Eddies having a vertical axis may be formed in the same way. This latter kind of eddy will be very frequent in the atmosphere and perhaps still more so in the sea.† The eddies can exist on

*Regarding this question on submarine waves, cf. Helland-Hansen and Nansen's work just quoted, Chapter VI. See also V. W. Ekman's paper, "On Dead Water" (The Norwegian North Polar Expedition, 1893–96, Scientific Results, Vol. V, Christiania, 1906).

†Concerning eddies of large scale in the sea, cf. figs. 2, 37, 39, 105–107 of Helland-Hansen and Nansen's work just quoted; and especially pp. 311–312.

every scale, down to the smallest, which must be considered as local disturbances. These local eddies in connection with the sheltering effect of mountains and the deviating effect of valleys make the use of wind-observations from mountainous regions difficult. For such regions it would be good to have special information as to the peculiarities of each station, *i. e.*, to know the relation of the observed local wind to the general wind to be found higher up, where the influence of the obstacles is reduced or has disappeared. Signs representing these peculiarities could be introduced on the outline maps. The best method of investigating these peculiarities would be by sending up simultaneously from all stations pilot-balloons, giving the motions in the free air with which the local motions at the ground should be compared.

137. Cold Wave, Warm Wave.—Let us suppose a certain mass of air has been cooled down below the temperature of other masses in the same level. Equilibrium will then be disturbed, and in order to restore it the cool and heavy air will tend to spread out along the ground, driving away the warmer and lighter masses previously covering it. These will on the other hand go up, in order to fill the space from which the heavy masses of air sink down. In this case there will appear at the ground a line of convergence a little before the front of the advancing cold wave (fig. 53).*

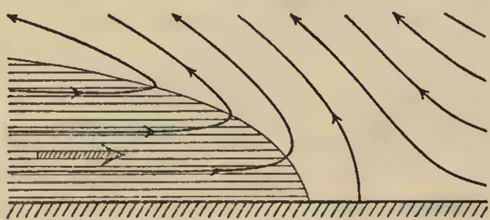

Fig. 53.—Cold wave.

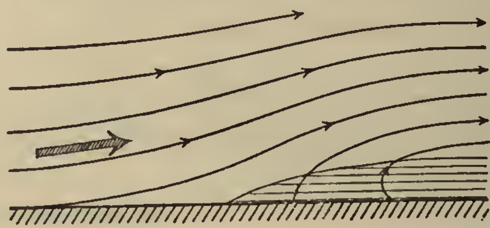

Fig. 54.—Warm wave.

Let us consider, on the other hand, a warm mass of air resting originally in hydrostatic equilibrium upon a thin sheet of cooler air. This arrangement will be stable as long as there is no motion or only a feeble motion. But if the upper layer has a sufficiently strong motion, the arrangement will be kinetically unstable. The warm air will then roll up and sweep away the thin layer of cool air. In this case there will arise at the ground a line of convergence a little before the front of an advancing warm wave (fig. 54).

In such cases there is no reason to expect a minimum of pressure along the line of convergence. There may come a sudden change of pressure as the line passes, but the most striking effect will be the sudden change of temperature along the line, and such a discontinuity of temperatures may give additional evidence for the existence of a line of convergence when the wind-observations themselves are insufficient.†

138. Lines of Convergence at the Sea's Surface.—While the observations of the motions themselves are difficult at sea, the situation of a line of convergence will under favorable circumstances be strikingly visible, for the reason that all sorts

*Cf. Sandström's paper, quoted p. 54.

†Cf. R. G. K. Lempfert and Richard Corless: Line squalls and associated phenomena. Quarterly Journal of the Royal Meteorological Society. London, April, 1910.

of floating objects, such as foam, seaweed, wood, etc., are collected in this line. Such lines are seen on a small scale near the shores when the wind is directed against the land. They then run parallel to the shore, often only like an oily band, marking the limit between the somewhat brackish water near the shore and the more salt water outside. Mr. Sandström has investigated directly the motion in the neighborhood of this line and found horizontal and vertical motion to be that represented by fig. 55.* Under the same condition of wind against the coast these lines exist on greater scale several kilometers from the coast, separating the coast-water from the salter sea-water. They are very well known by the fishermen, especially on account of the danger to the nets when they are set out across the line. These lines may also be seen under favorable circumstances on the open ocean, separating sea-currents of opposite directions.†

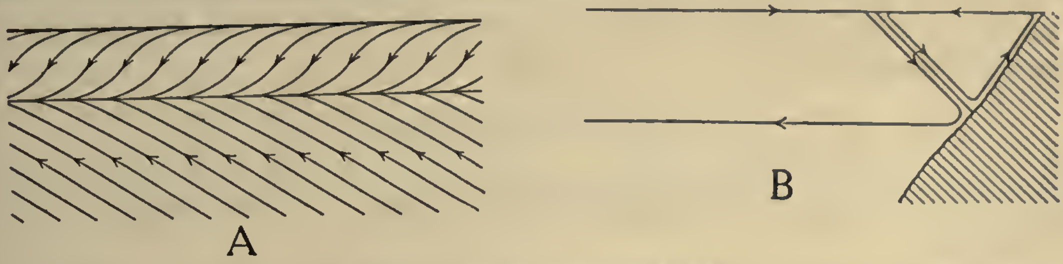

Fig. 55.—Line of convergence at sea.

A. Motion at sea's surface. B. Motion in a vertical section.

The investigation of these lines, their course, the degree of their constancy, etc., may be of great use for the kinematic investigation of the oceans.

139. Dynamic Diagnosis of Motion in the Free Space.—The observations of the air-motion in the higher strata are still too scarce to form the basis of a satisfactory construction of the motions, if only direct kinematic methods should be used. The arrows on the charts are far too few to determine the course of the lines of flow, and the numbers added to them are too few to determine the course of the lines of equal wind-intensity. We can not therefore avoid relying upon dynamic principles, if in such a case as this we should be able to give a fairly probable reconstruction of the air-motion on this occasion.

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*J. W. Sandström: Windströme in Gullmarfjord. Svenska Hydrografisk-biologiska Kommissionens Skrifter II.

†Although the phenomenon must often have been observed, not only near the coasts, but also in the open sea, I have not been able to find any reference to it in literature in the latter case. I am indebted to Professor Fridtjof Nansen for the following communication concerning this case:

"Lines of convergence, as you mention, are frequently met with in the open sea, wherever a surface-current formed by light surface-water meets with another current formed by heavier water. Such conditions are quite common along the margin of the East-Greenland Polar Current. I remember especially to have observed such a remarkably distinct line of convergence in the Denmark Strait, northwest of Iceland, in about 66° 42′ N. Lat. and 26° 40′ W. Long. where we were with the 'Michael Sars' on August 3, 1900. The cold but light surface-water of the Polar Current met here with the warmer but more saline and consequently heavier water of the Irminger Current, coming from the south. One could distinctly see how the latter water flowed in under the surface-layer of polar water, and everything floating on its surface was, as it were, skimmed off by the polar water, especially of course all kinds of foam, and the line of convergence between the two currents was consequently marked with quantities of this foam which had been skimmed off, and we could thus easily trace the line across the sea surface, as far as the eye could reach toward the horizon, both northeastward and southwestward."

Setting aside on the one hand frictional resistance, and on the other hand the acceleration of the particles of air, we get a motion determined dynamically by the equilibrium between pressure-gradient and deviating force of the earth's rotation. Recent observations have shown that the true motion in the higher strata is usually not very different from that determined by this equilibrium condition.* The ideal motion existing when this condition is fulfilled is directed along the level curves on the isobaric surfaces and goes on with a velocity represented by the formula

$$v = \frac{1}{2\,\omega\,a\sin\varphi}$$

ω is the angular velocity of the earth, measured in radians per second ($\omega = 0.000073$); φ is the latitude, and a the distance in meters between level lines corresponding to unit difference of level (one dynamic decimeter). The difference of level between the successive curves being on some of our charts 10, on others 50 dynamic meters, we can use the formula

$$v = \frac{100}{1.46\,a\sin\varphi}, \text{ or, for the greater interval, } v = \frac{500}{1.46\,a\sin\varphi}$$

measuring the distance a between the curves in millimeters on our chart in the scale 1 : 10 000 000.

To use this principle to complete the observations on the charts, we have first constructed the level curves for the isobaric surfaces representing a pressure equal to the arithmetical mean of the pressures at the upper and the lower limits of the sheet. These curves are easily found by the principle of graphic addition, by drawing the diagonal curves through the parallelograms formed by the curves of absolute topography of the lower and the relative one of the upper bounding surface of the sheet, after having left out every second of the last curves.

The accordance of these curves with the direction of the arrows is never complete, and should be complete only in exceptional cases. Drawing the lines of flow (fig. B of the plates LVII–LX) we have made them cut the level lines under angles similar to those under which the arrows cut them (fig. A of the same plates). Further, the numbers representing the observed wind-intensities are never in full accordance with the formula. We have drawn the curves of equal wind-intensity (fig. B of the mentioned plates) so as to get departures from the theoretical value similar to those presented by the observations as seen by fig. A of the same plates.

Of course, many different drawings of the lines of flow and curves of intensity can be produced which are in accordance with these elastic rules. To what degree we have succeeded in reconstructing by plates LVII–LX, the true horizontal motion within each sheet will therefore remain an open question. We can not therefore too strongly recommend further work to produce satisfactory direct observations of atmospheric motions. Provisionally, the synoptic representations in higher strata which we have obtained will serve our nearest aim, viz, that of illustrating formally the further steps in the work of kinematic diagnosis.

*E. Gold: Barometric gradient and windforce. London, 1908.

CHAPTER VII.

ISOGONAL CURVES.

140. Isogonal Curves.—The drawing of vector-lines from the observed directions of a vector is an operation of the nature of an integration (section 127). On account of the incompleteness of the observations this integration is combined with interpolations. But it will be possible to separate from each other these two heterogeneous operations of interpolation and of integration. This is obtained by the method of isogonal curves devised by Mr. Sandström.*

We have agreed to represent observed directions by numbers (section 98). Instead of inscribing the arrows we can inscribe these numbers on a chart. Then we can draw curves joining the points where these numbers are equal. In all points of such a curve the vector will have the same direction, *i. e.*, form the same angle with the north-south line. These curves may therefore be called *isogonal* curves or *isogons*.

A chart containing these curves may be considered a completely interpolated representation of the differential equation determining the vector-curves. This representation being obtained, the integration will cause no difficulty. Across each isogonal curve we can draw short lines of the direction represented by the curve. These will be line-elements of the vector-lines. In this manner we can get the whole plane filled with such line-elements, and joining them to continuous curves we get the vector-lines.

141. Singular Points in the Field of a Multiple-Valued Scalar.—The isogonal curves represent the field of a multiple-valued scalar, the angle. The angle has no true greatest and no true smallest value. From the highest number, 64, used in our representation, we interpolate to the lowest, 1; for 1 represents the same angle as 65 would do.

In order to see the consequences which this peculiarity of the scalar has on the appearance of the field, let us suppose observations to have been taken at the points of a closed curve and to have given in succession the numbers from 1 to 64; in this case isogonal curves representing all angles must run in through the closed curve, in order to cut each other somewhere in the area contained within it. The point of intersection will be a *singular point.*

In the diagrams of figs. 56 and 57 the isogonal curves passing through the singular point are for the sake of simplicity drawn as straight radii. The numbers belonging to these radii may be arranged in two different ways: they can increase in the

*J. W. Sandström: Ueber die Bewegung der Flüssigkeiten. Annalen der Hydrographie und der maritimen Meteorologie. Berlin, 1909.

same direction as the numbers on the dial of fig. 32, the singular point will then be called *positive;* or in the opposite direction, the singular point will then be called *negative.* The eight diagrams of fig. 56 represent positive, the two of fig. 57 negative singular points, the successive diagrams are differing from each other by the situation of the initial isogon, that represented by 0 or 64. The change from

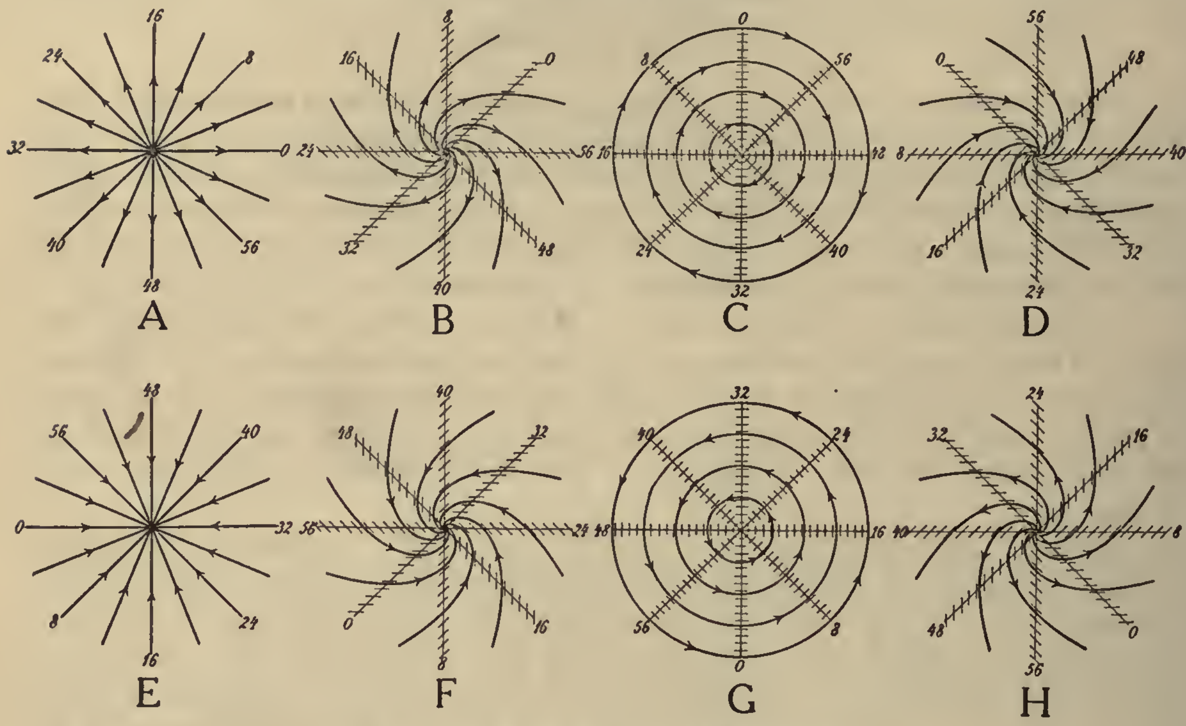


Fig. 56.—Positive singular points of isogonal curves.

A. Pure divergence.
B. Anticyclonic spirals of northern hemisphere.
C. Anticyclonic circles of northern hemisphere, cyclonic of southern hemisphere.
D. Cyclonic spirals of southern hemisphere.
E. Pure convergence.
F. Cyclonic spirals of northern hemisphere.
G. Cyclonic circles of northern hemisphere, anticyclonic of southern hemisphere.
H. Anticyclonic spirals of southern hemisphere.

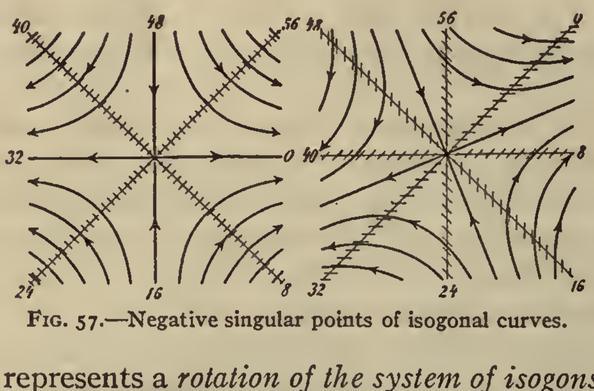


Fig. 57.—Negative singular points of isogonal curves.

diagram to diagram represents a *rotation of the system of isogons* of 45°. In all cases we can draw the short lines across the isogonal curves, and then the vector-curves. The diagrams will then show the features of the vector-field, which in the different cases corresponds to the singular point in the field of isogonal curves.

The examination of the figures leads to the following results:

(1) *The positive singular point of isogons corresponds to a point of divergence or convergence, the negative singular point to a neutral point of the vector-field.*

(2) The rotation of the system of isogons of a positive point has as a consequence that the vector-lines take the form of spiral curves of all types, including the limiting cases of straight radial lines and of circles.

(3) The rotation of the system of isogons of a negative point has as a consequence a rotation of the system of hyperbolic vector-lines without any change in their form; the angle of rotation of the vector-lines is half as great as that of the isogonal curves.

When the isogonal curves are no longer straight radii with constant angular intervals, but curves with more irregular intervals, the vector-lines of the corresponding vector-field will no longer be true logarithmic spirals or true hyperbolæ; but otherwise the character of the field will remain unchanged. If the numbers 1 to 64 are repeated twice or a greater number of times on a contour surrounding the singular point, always increasing in the same direction, the singular point will be of higher order. Only the negative singular points will be physically possible; but even they will occur rarely and be of small practical interest. (Cf. fig. 38.)

142. Further Remarks on the Field of Isogonal Curves and their Relation to the Vector-Field.—When the isogonal curves are to be drawn, the first thing will be to discover the situation of the singular points. For this we have to examine whether closed contours can be found on which the numbers representing the observations always increase in the same direction. If this be the case we are sure that there must be a singular point within the contour. As these singular points will always coincide with the singular points of the vector-field, we can also find these points by the use of rules which we have developed in the preceding chapters.

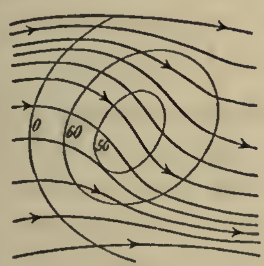


FIG. 58.—Closed isogonal curves. Inflexions of vector-lines.

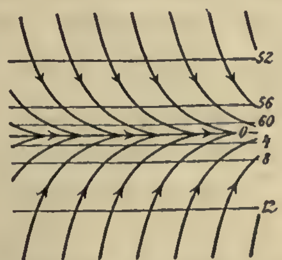


FIG. 59.—Parallel isogonal curves.

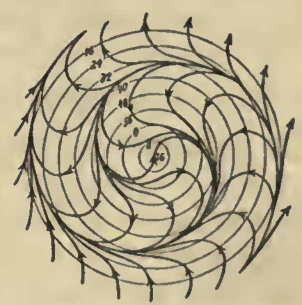


FIG. 60.—Concentric circles as isogonal curves.

The situation of the singular points being found, in which the curves intersect each other, the drawing of the curves will involve no other difficulties than those connected with the drawing of the equiscalar curves of the single-valued scalars; for isogonal curves representing different angles can never intersect each other in other points. Besides curves issuing from or entering into the singular points there will be found closed curves surrounding places of what may be called maxima or minima. Within these regions the lines of flow will have points of inflexion (fig. 58). As in the fields of the single-valued scalar, there may appear complexes of such maxima and minima, containing between them a maximum-minimum point where a certain singular isogonal curve cuts itself (fig. 46).

It is remarkable that no special singularity of the isogonal curves corresponds to lines of convergence or of divergence in the field of motion. Fig. 59 shows a case where such lines appear in the case of rectilinear and parallel isogonal curves, fig. 60 a case where they appear in the case of circular concentric isogonal curves. The feature of the isogonal curves in the case of the wave-motions described in section 131 is remarkably simple. Let the numbers on the rectilinear and parallel isogonal curves oscillate between two extreme values for instance, 52 and 12. If the isogonal curves run parallel to the average wind-direction, we get the parallel and equidistant lines of convergence and divergence of fig. 61 A. As the angle between the average

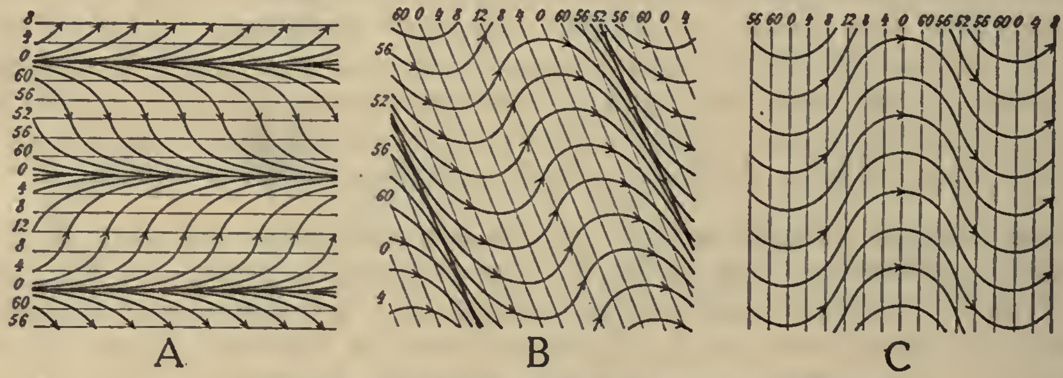


FIG. 61.—Isogonal curves for combined wave-motion and motion of translation.

A. Isogonal curves parallel to the main wind-direction.
B. Isogonal curves oblique to the main wind-direction.
C. Isogonal curves normal to the main wind-direction.

wind-direction and the isogonal curves increases, the singular lines are displaced relatively to each other, until finally two and two join into one, as in fig. 61 B. For still smaller angles we get sinusoidal lines of flow, the case of symmetry (fig. 61 C) arising when the isogonal curves are normal to the main wind-direction.

143. Sandström's Integration-Machines.—Mr. Sandström has based a method for graphical integration of differential-equations upon the representation of these equations by isogonal curves.* These curves being drawn, the tracing of the curves representing the integral, *i. e.*, the vector-curves, will cause no difficulty. Still, the draftsman will find it time-wasting to measure out the precise angles which these curves will have as they pass the different isogonal curves. But the work of drawing the vector-lines is very much facilitated by special machines constructed by Mr. Sandström, which trace automatically line-elements of the required direction across the isogonal curves. The construction of these machines will depend upon the system of coordinates to which the angles are referred. If the angles are referred to the meridians of a chart drawn in conical projection, very simple devices may be used. Fig. 62 shows a simple instrument serving the purpose in this case. A rule

*See note, p. 63.

R can slide through a guide which can turn around the pivot *P*. This pivot is fixed at the point of convergence of the meridians of the chart. At its other end the rule carries a toothed wheel *W*, which may be fixed in a position, where the edge of the teeth (*i. e.*, of the axis of the wheel) forms any given angle with the meridians. This angle is measured at the dial *D*. If the wheel is colored and carried along the isogonal curves, it will mark lines of the required direction across them.

During the motion the wheel partly slides and partly rolls. As the resistance against these two motions is not equal, it requires some care to follow precisely the

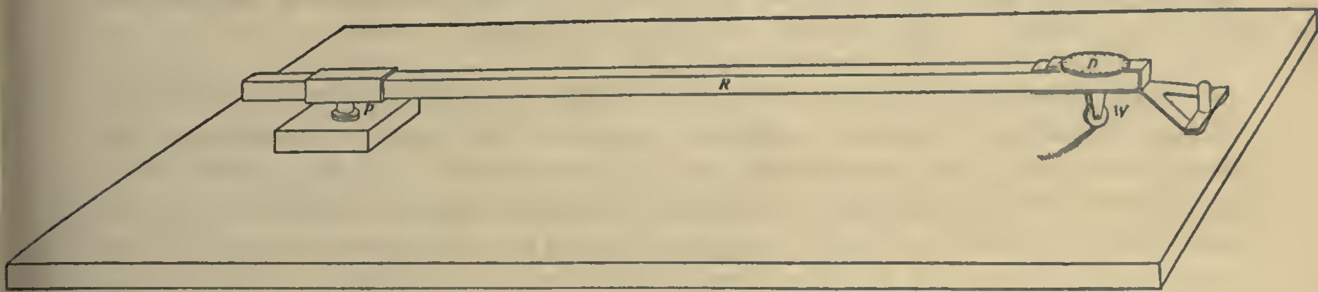


Fig. 62.—Machine for tracing line-elements across isogonal curves.

given curves. It will for this reason be advantageous to have an adjustable friction at the pivots of the toothed wheel. Fig. 63 shows another instrument by which this difficulty is avoided. Instead of a toothed wheel, the rule *R* carries a drum *D* with a caoutchouc membrane. This membrane carries a metal plate with a chisel *C*, which writes a line-element when it touches the paper. By an alternating air-current the membrane is set in motion, making the edge go up and down. When the chisel has this motion and is guided along the curve, it will mark the required

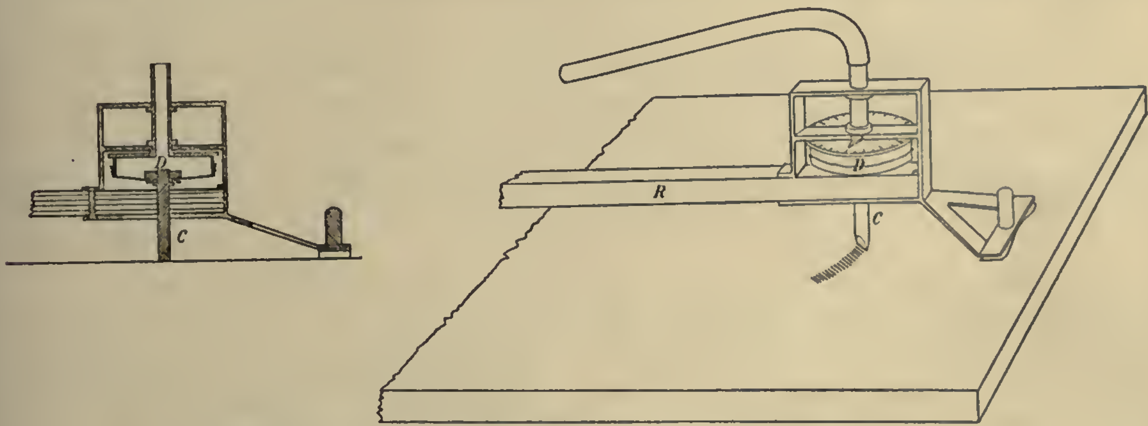


Fig. 63.—Other machine for tracing line-elements across isogonal curves.

line-elements across it. The desired angle with the meridian can be obtained by turning the drum, which on its upper face carries a dial with the required divisions. The alternating air-current for driving the membrane is obtained from another drum, joined with [the crank of a rotating wheel, which is driven by a little electromotor.

When the charts are drawn on semi-transparent paper, no special device is required to color the tooth-wheel or the chisel. A coloring paper can be placed under the transparent sheet upon which the isogonal curves are drawn. The line-elements will then come on the under-side of the sheet, but will be seen through it.

When an instrument like one of these is at hand, it will be found very convenient to draw the lines of flow in the indirect way, using the isogons as auxiliary curves. Of course the indirect method will always require more time than the direct one. The latter will therefore be preferable for rapid work. But the indirect method gives a much higher degree of precision, and should therefore be preferred when the purpose is quantitative scientific investigations.

144. Equivalence of Isogons and Vector-Lines for the Representation of Vector-Fields.—We have introduced the isogons as auxiliary curves for tracing the vector-lines; but in reality they are perfectly equivalent to these lines for the representation of the field. We shall therefore have henceforth to reckon with two different representations of the vector-fields; by intensity-curves in connection with vector-lines, and by intensity-curves in connection with isogons. We have used the first consistently hitherto because it gives the most conspicuous picture. But our work will consist henceforth in the performance of mathematical operations upon the field, and these operations are in many cases performed more easily when the direction of the vector is given by the isogons. Therefore, in the following chapters, when we are going to study methods for performing elementary algebraic or infinitesimal operations upon the fields, we shall have to take into consideration the one method of representing the vector as well as the other, trying to utilize the special advantages of each of them.

CHAPTER VIII.

GRAPHICAL ALGEBRA.

145. **Graphical Mathematics.**—When the synoptical charts are found which can be derived directly from the observations, the further work for the diagnosis of present or for the prognosis of future states will consist in the performance of mathematical operations with the data given by these charts. The development of proper graphical methods for performing these operations directly upon the charts will be of the same importance for the progress of dynamic meteorology and hydrography as the methods of graphical statics and of graphical dynamics have been for the progress of technical sciences. The first serious problems of these graphical mathematics will present themselves as soon as we shall accomplish kinematic diagnosis by determining the vertical motions. Afterwards we shall meet with such problems continuously. This will therefore be the moment for taking a general view of the character of these problems and of methods to be used for solving them.

The problems will present themselves in this form: a chart or a set of charts is given, representing the fields of certain scalars or vectors. Another chart or set of charts is to be derived from them, representing the field of other scalars or vectors, which are defined as functions of the first by relations in finite or in infinitesimal form.

One way for the solution of such problems will always be open. We perform discontinuously, for a certain number of points, the operations defined by the relations. This gives the values of the required scalars or vectors in a certain number of points. By use of these values we draw the charts representing the new scalars or vectors, just as we draw such charts by use of the observations taken at a finite number of points. By following this method we give up the idea of continuous fields during the performance of the mathematical operations, in order to return to the fields as soon as the operations have been performed. We shall call this the *discontinuous* method.

But on the other hand it will be possible to find methods by which the idea of the field is never given up. The method will then consist in the continuous tracing of curves guided by the data contained on the given charts, and by the relations containing the implicit definition of the new charts. Every operation leads to a chart representing a field, and it will, as a rule, be necessary to pass through several auxiliary fields in order to arrive at the required fields. We shall call these methods *continuous*, and the development of them will be our main object.

146. **Drawing-Board.**—Certain practical arrangements should be mentioned at once. It will be impossible to draw all the different curves on one sheet of paper. They must be distributed on several sheets. But at the same time we must be able

to make different systems of curves simultaneously visible in their true mutual position, as if they had been drawn upon the same sheet of paper. Certain measures must be taken to attain this.

We have found it most convenient to draw the different charts upon sheets of semi-transparent paper, and to have at hand a special drawing-board. This board consists of a sheet of glass with a wooden frame and has a contrivance for producing illumination from below. This illumination is obtained most easily by an incandescent electric lamp. The sheets of paper should cover the glass completely. They can be fixed to the wooden frame by drawing-pins. The paper should be sufficiently transparent, or the illumination sufficiently strong, to allow us to have at least three sheets simultaneously upon the board, two containing given systems of curves and a third upon which the derived curves are drawn. The plates accompanying this book have been printed upon paper which we have found convenient for this kind of work.

147. Graphical Algebra with One Variable.—Let a be a scalar function represented by a chart of equiscalar curves. These curves are to be drawn for what we shall call "integer values" of the scalar, using the expression in a widened sense as a shortened expression for "integer values multiplied by a positive or negative power of 10." By a suitable change of units they will get integer values in the common sense of the word. It is required to find the equiscalar curves which represent in the same way the field of another scalar

$$\varphi = f(a) \tag{a}$$

In this case a curve $a =$ const. will also be a curve $\varphi =$ const. But the curves which represent integer values of a will as a rule not coincide with those which represent integer values of φ.

The discontinuous method of finding the curves for integer values of φ will be this: by direct calculation to find the values of φ in a certain number of points, and then to interpolate between them the points where φ has integer values. These points will give the placing of the curves for integer values of φ between those for integer values of a.

But we can give a continuous method of solving the same problem: We then solve equation (a) with respect to the *known* variable a,

$$a = F(\varphi) \tag{b}$$

and construct an auxiliary table in which the values of a are tabulated for integer values of the argument φ. Thus

TABLE E.—*Table-scheme for graphical algebra with one variable.*

φ	0	1	2	3	4	5	6	7	8	9
0	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
10	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}
20	a_{20}	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}

Table E shows at once for which values of α we shall get integer values of φ. We can then at once draw the equiscalar curves for integer values of φ in their proper places between the given equiscalar curves for integer values of α.

As an example we can consider the square of a given field, $f(\alpha) = \alpha^2$. Thus

(c) $$\varphi = \alpha^2$$

Solving with respect to α we get

(d) $$\alpha = \sqrt{\varphi}$$

α is tabulated for integer values of φ in table F.

TABLE F.—*Square-root table for passing from the field of a scalar to the field of its square.*

φ	0	10	20	30	40	50	60	70	80	90
0	0	3.2	4.5	5.5	6.3	7.1	7.7	8.4	8.9	9.5
100	10.0	10.5	11.0	11.4	11.8	12.2	12.6	13.0	13.4	13.8
200	14.1	14.5	14.8	15.2	15.5	15.8	16.1	16.4	16.7	17.0

This table shows that the curve $\varphi = 50$ coincides with the curve $\alpha = 7.1$, curve $\varphi = 60$ with curve $\alpha = 7.7$, and so on. Fig. 64 shows how by use of this information

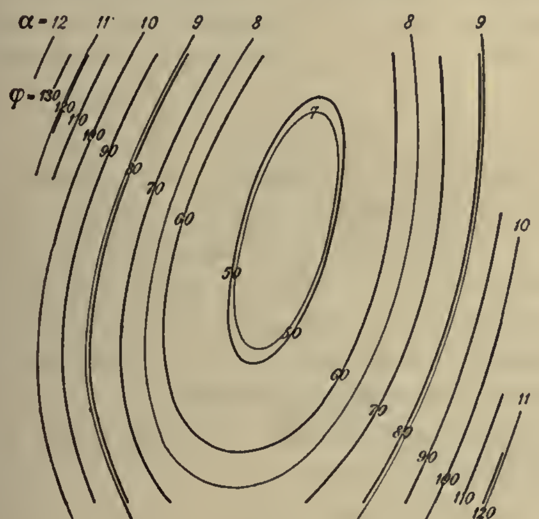


FIG. 64.—Field of a given scalar (fine lines $\alpha = 7, 8, 9, \ldots$) and field of its square (thick lines $\varphi = 50, 60, 70, \ldots$)

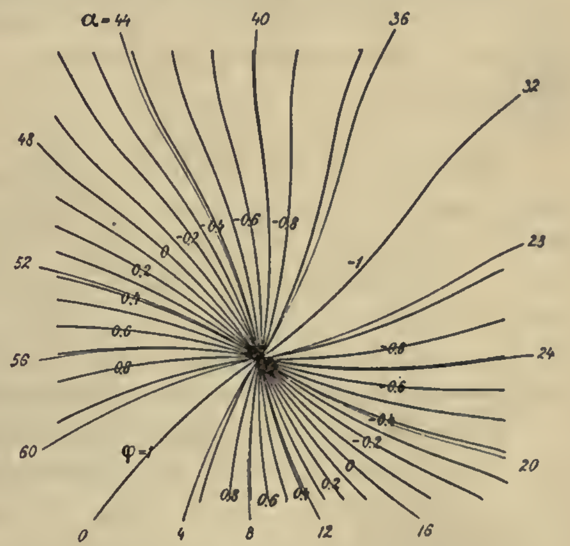


FIG. 65.—Field of an angle (fine lines $\alpha = 0, 4, 8, 12, \ldots$) and field of its cosine (thick lines $\varphi = 1.0, 0.9, 0.8 \ldots$)

the curves for integer values of φ are drawn in their proper places between the curves for integer values of α. For evident reasons we have drawn the curves φ = const. for ten times greater intervals than the curves α = const.

To use another example, let the field of a multiple-valued scalar, the angle α, be given, expressed by the numbers 0–63. It is required to find the field of the scalar

(e) $$\varphi = \cos \alpha$$

We then construct a table (table F′) according to the equation

$$\alpha = \operatorname{arc} \cos \varphi \qquad (f)$$

By use of this table we can easily draw the curves for "integer" values of cos α between those for integer values of α (fig. 65).

TABLE F′.—*Arcus-cosine table for passing from the field of an angle to the field of its cosine.*

φ	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
	16	15.0	13.9	12.9	11.8	10.7	9.4	8.1	6.6	4.6	0
	48	49.0	50.1	51.1	52.2	53.3	54.6	55.9	57.4	59.4	0

φ	−0.0	−0.1	−0.2	−0.3	−0.4	−0.5	−0.6	−0.7	−0.8	−0.9	−1.0
	48	47.0	45.9	44.9	43.8	42.7	41.4	40.1	38.6	36.6	32
	16	17.0	18.1	19.1	20.2	21.3	22.6	23.9	25.4	27.4	32

148. Graphical Algebra with Two Variables.—Let α and β be two scalar functions, each represented by a chart of equiscalar curves. The problem is to draw the equiscalar curves representing the field of a third scalar φ, which is determined by the relation

$$\varphi = f(\alpha, \beta) \qquad (a)$$

The discontinuous method of solving this problem will be this: we choose a point, take out from the charts the values of α and β and calculate by equation (a), the corresponding values of $f(\alpha, \beta)$. This is repeated for a sufficient number of points. The values thus found for φ are inscribed upon a sheet of paper, and then the equiscalar curves $\varphi = 1, 2, 3, \ldots$ are drawn by leading of the values thus found. Evidently the work can be facilitated by the construction of an auxiliary table containing the values of φ tabulated with α and β as arguments.

But a corresponding continuous method can also be given. To see it we solve equation (a) with respect to *one of the known* quantities α or β,

$$\beta = F(\alpha, \varphi) \quad \text{or} \quad \alpha = F'(\beta, \varphi) \qquad (b)$$

According to these equations we construct the auxiliary tables G.

Let us first follow one of the vertical columns in the table and the corresponding curve α = const. on the chart. We shall then see that the curve $\alpha = 0$ will be cut by the curve $\varphi = 0$ at the point where β has the value β_{00}, by the curve $\varphi = 1$ at the point where β has the value β_{10}, by the curve $\varphi = 2$ at the point where β has the value β_{20}, and so on. The situation of the points $\beta_{00}, \beta_{10}, \beta_{20}, \ldots$ is seen at once, as the intersection of the curve $\alpha = 0$ with the curves $\beta = 0, \beta = 1, \beta = 2 \ldots$ shows where on the curve $\alpha = 0$ we have the integer values of β. Interpolating by eye-measure we can mark the points where the curves α = const. are cut by the curves for integer values of φ. These points being marked, we can draw at once the curves φ = const.

Instead of following the vertical columns we can also follow the horizontal lines of the table, and then draw directly one by one the curves φ = const., performing successively the interpolations by eye-measure which give the points of intersection with the different curves α = const. This method will usually be the most convenient.

TABLES G.—*Table-schemes for graphical algebra with two variables.*

φ	α					
	0	1	2	3	4	5
0	β_{00}	β_{01}	β_{02}	β_{03}	β_{04}	β_{05}
1	β_{10}	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}
2	β_{20}	β_{21}	β_{22}	β_{23}	β_{24}	β_{25}
3	β_{30}	β_{31}	β_{32}	β_{33}	β_{34}	β_{35}

φ	β					
	0	1	2	3	4	5
0	α_{00}	α_{01}	α_{02}	α_{03}	α_{04}	α_{05}
1	α_{10}	α_{11}	α_{12}	α_{13}	α_{14}	α_{15}
2	α_{20}	α_{21}	α_{22}	α_{23}	α_{24}	α_{25}
3	α_{30}	α_{31}	α_{32}	α_{33}	α_{34}	α_{35}

The second table G can be used in precisely the same way to find the points of intersection of the required curves φ = const. with the given curves β = const. When φ is a symmetric function of α and β, the two tables will be identical with each other. Then one table will be sufficient, which may be provided with two sets of arguments, one set above and on the left side, the other below and on the right side. (Cf. tables H and I below).

We shall now make a few special applications of this general principle, taking the simplest algebraical operations, and giving the schemes for the construction of the most important auxiliary tables. More extensive tables will be given later in our collections of tables for practical use.

149. Addition of Scalar Fields.—Let the function be $f(\alpha, \beta) = \alpha+\beta$. That is, we shall determine the field of the scalar φ which is the sum of the scalars α and β

(*a*) $$\varphi = \alpha+\beta$$

The discontinuous method will consist in forming directly the sum $\alpha+\beta$ in a certain number of points, and to draw the equiscalar curves of φ by leading of these values.

In order to use the continuous method we write equation (*a*) in the form

(*b*) $$\beta = \varphi-\alpha \text{ or } \alpha = \varphi-\beta$$

Both equations lead to the same table, table H, where on account of the symmetry we have an equal right to interpret α as argument and β as the tabulated quantity or β as argument and α as tabulated quantity.

The table shows that the curves representing the sum of the scalars α and β pass through the points for simultaneously integer values both of α and β as a set of *diagonal curves* (figs. 66 and 67), *i. e.*, we return to the simple process of graphical addition, of which we have made so frequent use. In this simple case the auxiliary table is superfluous. We have introduced it only to show the connection with the more complicated corresponding problems.

It will be seen at once that while the sum $\alpha+\beta$ is represented by the one set of diagonal curves, the difference $\beta-\alpha$ or $\alpha-\beta$ will be represented by the other set of diagonal curves.

TABLE H.—*Graphical addition.*

One addend tabulated as function of the sum and the other addend.

Sum. φ	First addend α.												
	1	2	3	4	5	6	7	8	9	10	11	12	
9	8	7	6	5	4	3	2	1	0	−1	−2	−3	9
10	9	8	7	6	5	4	3	2	1	0	−1	−2	10
11	10	9	8	7	6	5	4	3	2	1	0	−1	11
12	11	10	9	8	7	6	5	4	3	2	1	0	12
13	12	11	10	9	8	7	6	5	4	3	2	1	13
14	13	12	11	10	9	8	7	6	5	4	3	2	14
15	14	13	12	11	10	9	8	7	6	5	4	3	15
16	15	14	13	12	11	10	9	8	7	6	5	4	16
	1	2	3	4	5	6	7	8	9	10	11	12	φ Sum.
	Second addend β.												

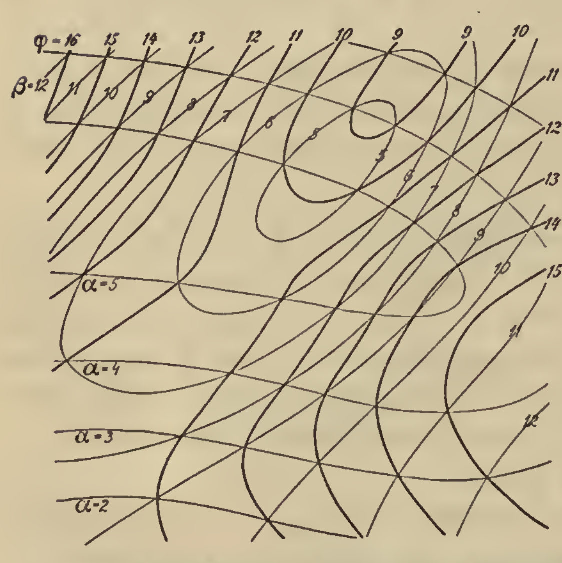


FIG. 66.—Graphical addition of single-valued scalar fields. (The fine lines represent the given fields, the thick lines their sum.)

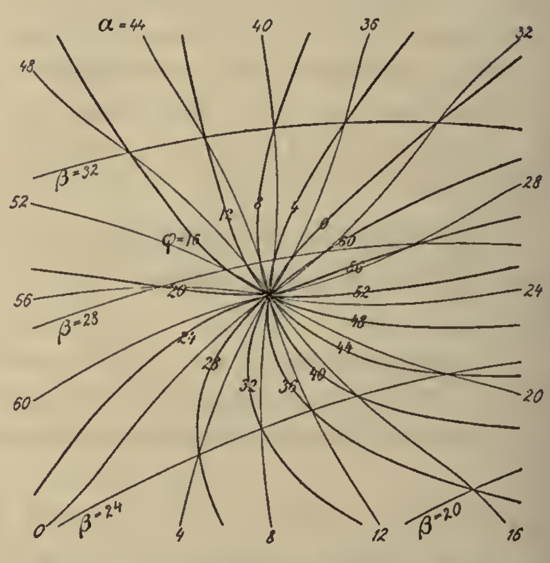


FIG. 67.—Graphical addition of multiple-valued scalar fields (fields of angles). The fine lines represent the given fields, the thick lines their sum. Observe the consequences of the multiple-values: 32+32 =64=0, 48+32=80=16, etc.

150. Multiplication of Scalar Fields.—Let the function $\varphi = f\ (\alpha, \beta)$ be

(*a*) $$\varphi = \alpha\,\beta$$

In order to use the continuous method we solve with respect to β or α

(*b*) $$\beta = \frac{\varphi}{\alpha} \text{ or } \alpha = \frac{\varphi}{\beta}$$

TABLE I.—*Graphical multiplication.*

One factor tabulated as function of the product and the other factor.

Product. φ	First factor α.												
	1	2	3	4	5	6	7	8	9	10	11	12	
15	15.0	7.5	5.0	3.8	3.0	2.5	2.1	1.9	1.7	1.5	1.4	1.2	15
20	20.0	10.0	6.7	5.0	4.0	3.3	2.9	2.5	2.2	2.0	1.8	1.7	20
25	25.0	12.5	8.3	6.3	5.0	4.2	3.6	3.1	2.8	2.5	2.3	2.1	25
30	30.0	15.0	10.0	7.5	6.0	5.0	4.3	3.7	3.3	3.0	2.7	2.5	30
35	35.0	17.5	11.7	8.8	7.0	5.8	5.0	4.4	3.9	3.5	3.2	2.9	35
40	40.0	20.0	13.3	10.0	8.0	6.7	5.7	5.0	4.4	4.0	3.6	3.3	40
45	45.0	22.5	15.0	11.3	9.0	7.5	6.4	5.6	5.0	4.5	4.1	3.8	45
50	50.0	25.0	16.7	12.5	10.0	8.3	7.1	6.2	5.6	5.0	4.5	4.2	50
	1	2	3	4	5	6	7	8	9	10	11	12	φ Product.
	Second factor β.												

These two equations lead to the same table, table I, in which we are equally right in interpreting α as argument and β as tabulated quantity or β as argument and α as tabulated quantity.

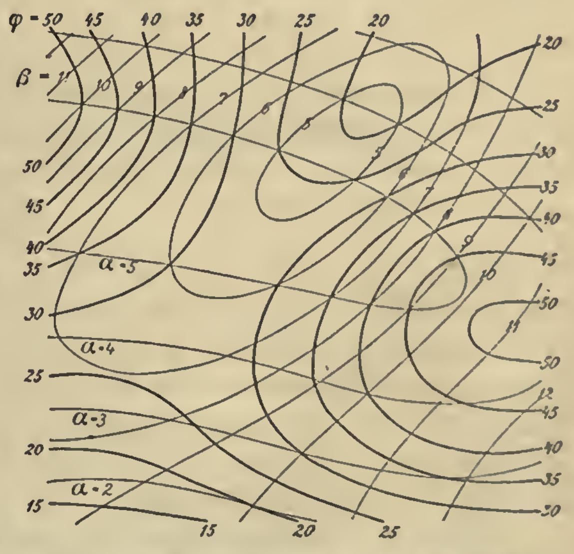


FIG. 68.—Graphical multiplication. The fine lines $\alpha = 2, 3, 4, \ldots$ and $\beta = 11, 10, 9, \ldots$ represent the factors, the thick lines $\varphi = 50, 45, 40, \ldots$ their product.

Fig. 68 exemplifies the use of the table. In drawing, for instance, the curve for the constant value $\varphi = 30$ of the product, we use the line in the table which has the argument $\varphi = 30$. When we consider α as argument and β as the tabulated quantity, this line of the table tells us that the curve $\varphi = 30$ is to be drawn through that point of the curve $\alpha = 3$ where $\beta = 10$, through that point of the curve $\alpha = 4$ where $\beta = 7.5$, through that point of the curve $\alpha = 5$ where $\beta = 6$, and so on. If we consider β as the argument and α as the tabulated quantity, we see that the curve $\varphi = 30$ is to be drawn through that point of the curve $\beta = 12$ where $\alpha = 2.5$, through that point of the curve $\beta = 11$ where $\alpha = 2.7$, and so on.

TABLES J.—*Graphical division.*

I. *Divisor tabulated as function of quotient and dividend.*

Quotient. φ	Dividend α.			
	2	3	4	5
0.2	10.0	15.0	20.0	25.0
0.3	6.7	10.0	13.3	16.7
0.4	5.0	7.5	10.0	12.5
0.5	4.0	6.0	8.0	10.0
0.6	3.3	5.0	6.7	8.3
0.7	2.9	4.3	5.7	7.1
0.8	2.5	3.8	5.0	6.3
0.9	2.2	3.3	4.4	5.6
1.0	2.0	3.0	4.0	5.0
1.1	1.8	2.7	3.6	4.5

II. *Dividend tabulated as function of quotient and divisor.*

Quotient. φ	Divisor β.							
	5	6	7	8	9	10	11	12
0.2	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4
0.3	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6
0.4	2.0	2.4	2.8	3.2	3.6	4.0	4.4	4.8
0.5	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
0.6	3.0	3.6	4.2	4.8	5.4	6.0	6.6	7.2
0.7	3.5	4.2	4.9	5.6	6.3	7.0	7.7	8.4
0.8	4.0	4.8	5.6	6.4	7.2	8.0	8.8	9.6
0.9	4.5	5.4	6.3	7.2	8.1	9.0	9.9	10.8
1.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0
1.1	5.5	6.6	7.7	8.8	9.9	11.0	12.1	13.2

151. Division of Scalar Fields.—Now let $f(\alpha, \beta) = \frac{\alpha}{\beta}$. We shall then have to construct the field of the scalar φ, which is the ratio of the two scalars α and β,

$$(a) \qquad \varphi = \frac{\alpha}{\beta}$$

We here meet with the case that the function φ is asymmetric with respect to the two variables. Solving with respect to each of them we get

$$(b) \quad \beta = \frac{\alpha}{\varphi} \qquad \text{or} \qquad \alpha = \beta\varphi$$

These equations lead to the two tables J. The first of them is the same as that serving graphical multiplication (table I), though other values of the arguments appear to suit the example of fig. 69. The second is an ordinary multiplication-table.

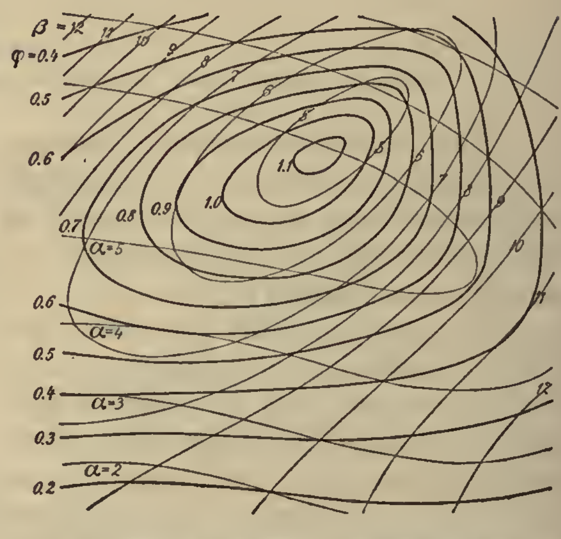


FIG. 69.—Graphical division. The fine lines $\alpha = 2, 3, 4, \ldots$ represent the dividend; the fine line $\beta = 12, 11, 10, 9, \ldots$ the divisor; the thick lines $\varphi = 1.1, 1.0, 0.9, \ldots$ the quotient.

The first of tables J shows for instance that the curve $\varphi = 0.6$ is to be drawn through that point of the curve $\alpha = 5$ where $\beta = 8.3$, through that point of the curve $\alpha = 4$ where $\beta = 6.7$, through that point of the curve $\alpha = 3$, where $\beta = 5.0$, and so on. The second table J shows in the same manner that the curve $\varphi = 0.6$ is to be drawn through that point of the curve $\beta = 9$ where $\alpha = 5.4$, through that point of the curve $\beta = 8$ where $\alpha = 4.8$, and so on. Observing thus the tabulated numbers, we can draw continuously one by one the curves $\varphi =$ const., which represent the required field.

152. Case of Three or More Variables.—Now let the scalar φ be a function of any number of variables

$$\varphi = f(\alpha, \beta, \gamma \ \ . \ . \ . \ )$$

In this case the discontinuous method, which consists of calculating the values of φ in any sufficient number of points and subsequent tracing of the equiscalar curves $\varphi =$ const., may be used precisely as in the case of two variables. But if we solve with respect to one of the given scalars, for instance α, in order to bring the continuous method into application, we meet with the practical difficulty connected with the tabulation of functions of more than two variables; for numerical tables can not easily be provided with more than two arguments.

In special cases it may be possible to decompose the complex operation into a series of partial operations each depending upon two variables only. Then all difficulties connected with the greater number of variables will drop out, and we can bring into application the methods which we have developed already, depending upon the construction of numerical tables with two arguments.

In the general case this decomposition of the problem will not, however, be possible. We must then look for other auxiliaries than numerical tables, and it will always turn out to be possible to produce special graphical or mechanical auxiliaries which will serve the same purpose as tables with more than two arguments would have done. These auxiliaries will, however, as a rule be more laborious to use than the tables with two variables. If, therefore, a reduction to problems with two variables is possible, it should generally be performed even if the number of single operations be thereby considerably increased.

We shall give the general method for constructing graphical tables which serve the purpose in the case when the number of variables is limited to three. Then let α, β, γ be three given scalar quantities. The field of each of them is represented by equiscalar curves. The problem is to find the equiscalar curves $\varphi =$ const., which represent the field

$$(a) \qquad \varphi = f(\alpha, \beta, \gamma)$$

In order to find the points of the curve $\alpha = \alpha_1$ in which φ has integer values, we have to examine the values of

$$(b) \qquad \varphi = f(\alpha_1, \beta, \gamma)$$

Here only β and γ are variables, and when we follow the curve $\alpha = \alpha_1$ (fig. 70B), we see that to any value of β will correspond a definite value of γ, and vice versa.

In order to find those values of one of them for which φ has integer values, we construct a graphical table. We set off β and γ as abscissa and as ordinate of a rectangular system of coordinates (fig. 70 A) and draw in this system of coordinates the curves $\varphi = 1, 2, 3, \ . \ . \ .$ according to equation (b). We observe on the given chart (fig. 70 B) the values which β and γ have along the curve $\alpha = \alpha_1$. These values will define a certain curve in the system of coordinates β, γ. We draw this curve on a transparent sheet of paper, laid upon the graphical table fig. 70 A. This curve will cut the curves $\varphi = 1, 2, 3, \ldots$ of the graphical table, and we can read off those values of β or of γ for which φ has integer values. Then we can set off these points along the curve $\alpha = \alpha_1$ on the given chart (fig. 70 B).

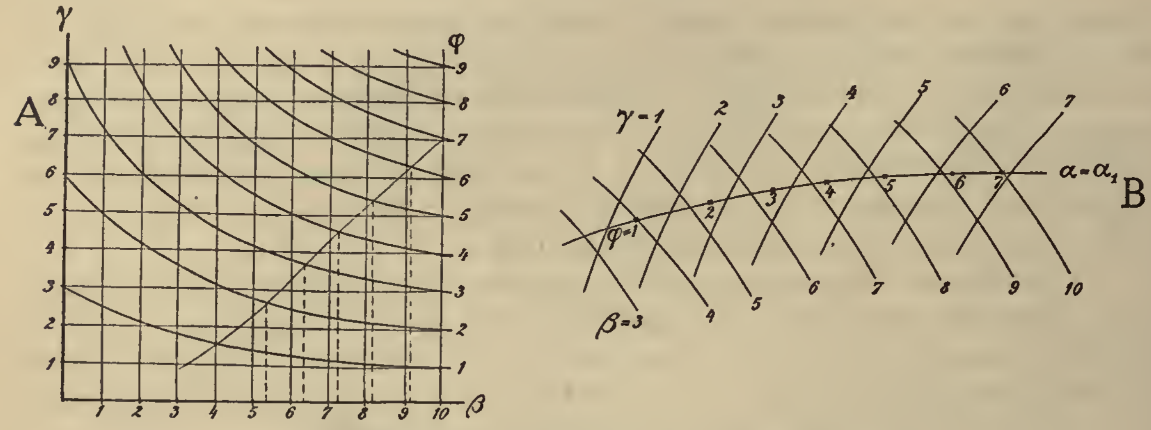


FIG. 70.—Example of graphical operations with three variables.

A. Scheme of graphical table.
B. α, β, γ, given fields. Construction of $\varphi = f(\alpha_1, \beta, \gamma)$.

If we construct a graphical table as that of fig. 70 A for each of the curves $\alpha =$ const., we can thus find a complete system of points determining the course of the curves $\varphi =$ const.

153. **Vector-Algebra.**—It will be of special importance for us to bring graphical methods into application for mathematical operations concerning vector-fields. It will be useful and save circumlocution when at the same time we introduce a few simple notations of modern vector-analysis.*

A vector considered as a quantity which has both magnitude and direction will be denoted by a letter in heavy print. The corresponding letter in common print will denote its scalar value or tensor (intensity). The same letter in common print and with the suffix s will denote the projection of the vector on the direction s. In the same manner we shall by the suffixes x, y, z denote the projections on the three rectangular axes x, y, and z. Thus

Vector.	Tensor.	Projection on direction s.	Projections on rectangular axes.
A	A	A_s	A_x, A_y, A_z
B	B	B_s	B_x, B_y, B_z
. . .	. . .	. . . .	. . . .
F	F	F_s	F_x, F_y, F_z

*Compare: Gibbs-Wilson, Vector-Analysis, New York, 1901.

The fact that the vector **F** is the *vector-sum* according to the parallelogram-law of the two vectors **A** and **B** will be denoted by the vector-equation.

(*a*) $$\mathbf{F} = \mathbf{A}+\mathbf{B}$$

This equation can be considered as equivalent to the three scalar equations

(*a'*) $$F_x = A_x+B_x \qquad F_y = A_y+B_y \qquad F_z = A_z+B_z$$

which express the projections of **F** as the scalar sum of the projections of **A** and of **B** (fig. 71). The scalar-sum of the tensors $A+B$ must be carefully distinguished from the scalar value or tensor $|\mathbf{A}+\mathbf{B}|$ of the vector-sum. There will be identity between the scalar sum of the tensors and the tensor of vector-sum when the two given vectors have the same direction, and between the scalar differences of the tensors and the tensor of the vector-sum when the two given vectors have opposite directions.

A scalar quantity which is equal to the product of the tensors of two given vectors and the cosine of the included angle will be called the *scalar product* of the two given vectors. When the given vectors are **A** and **B**, their scalar product shall be denoted by **A.B**, thus

(*b*) $$\mathbf{A}.\mathbf{B} = AB\cos\theta$$

By the fundamental formulæ of analytical geometry it is easily verified that the scalar product is equal to the sum of the products of the rectangular components of the given vectors,

(*b'*) $$\mathbf{A}.\mathbf{B} = A_xB_x+A_yB_y+A_zB_z$$

The vector-operations defined by the preceding formulæ are symmetrical with respect to the two given vectors **A** and **B**. In the vector-formulæ the symbols for the vectors can therefore be commutated

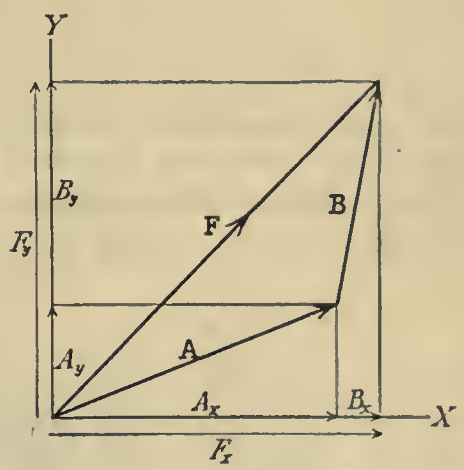


FIG. 71.—Vector-addition.

(*c*) $$\mathbf{A}+\mathbf{B} = \mathbf{B}+\mathbf{A} \qquad \mathbf{A}.\mathbf{B} = \mathbf{B}.\mathbf{A}$$

We shall define finally an important *unsymmetric* vector-operation, in which this commutation of the symbols will no more be allowed. The succession of the symbols will be used to serve an important purpose, namely, to distinguish between opposite directions in space. In order to give the definition of this operation, we must first make an important remark concerning the geometry of translations and rotations.

Let an axis in space be given. Two opposite translations will be possible along it, and two opposite rotations will be possible around it. We must agree upon a definite connection by which we can define the positive direction of rotation as soon as the positive direction of translation is chosen, and vice versa. We shall attain this by the rule of the *positive or right-handed screw*. When this screw moves in its nut, it can not advance along its axis in a definite direction unless it performs a rotation around this axis in a corresponding definite direction; and vice versa it can not turn around its axis in a definite direction unless it advances along this axis in a corresponding definite direction (see fig. 72). Thus this screw connects a definite direction of translation with a corresponding definite direction of rotation, and vice versa. We shall agree to give the same sign to directions of translation and of rotation which are connected to each other in this way.

Two vectors in space, **A** and **B**, define two rotations which are smaller than two right angles, that from **A** to **B** and that from **B** to **A**. Both rotations take place around an axis which is normal both to **A** and to **B**, and can be represented symbolically by arrows pointing along the axis of rotation, in that direction which by the screw-rule is positive in reference to the direction of rotation.

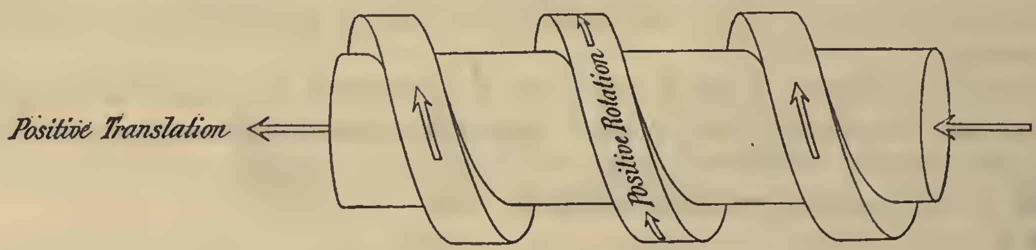


Fig. 72.—Positive-screw rule.

Now let us consider a vector **F** which is normal to the two given vectors **A** and **B**, which by its direction represents the rotation from **A** to **B**, and which has a tensor equal to the product of the tensors of the given vectors and the sine of the included

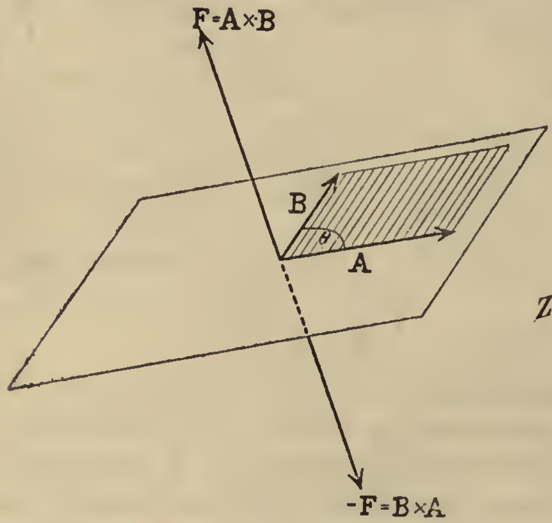


Fig. 73.—Vector-product.

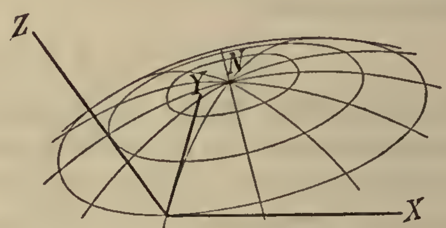


Fig. 74.—Positive system of rectangular coordinates.

angle. The fact that the vector **F** has this relation to the two vectors **A** and **B** will be expressed by the formula

$$\text{(d)} \qquad \mathbf{F} = \mathbf{A} \times \mathbf{B}$$

and **F** will be called the *vector-product* of the vectors **A** and **B**.

The relation of the vector-product **F** to the vector-factors **A** and **B** is illustrated by fig. 73: **F** is directed along the normal to the plane which contains **A** and **B**; the positive rotation around **F** transfers the first vector-factor **A** into the second **B**; and **F** has the scalar value F, which is given by the formula

$$(d') \qquad F = AB \sin\theta$$

or which is represented geometrically by the area of the parallelogram which has sides representing the vector-factors.

It follows immediately from the definitions that when we commutate the vectors **A** and **B**, we get the vector $-\mathbf{F}$, which is directed oppositely to **F**, thus

$$(e) \qquad \mathbf{B}\times\mathbf{A} = -\mathbf{A}\times\mathbf{B}$$

When we bring coordinates into application we shall agree to use consistently what we shall call a *positive system of coordinates*. Let the positive direction along each of the rectangular axes be chosen. The corresponding positive rotation around an axis will then either be a rotation *in* or *against* that direction which is defined by the succession of letters X, Y, Z, X, . . . In the first case the system will be called positive, in the second case negative. Thus when the system is positive, the positive rotation around Z will go from X to Y, the positive rotation around X will go from Y to Z, the positive rotation around Y will go from Z to X. A positive system of coordinates, of which we shall make a frequent use, is one which has its axis of X directed toward the east, its axis of Y directed toward the north, and its axis of Z directed upward. (See fig. 74.)

When we use a positive system of coordinates, it is easily verified by the fundamental formulæ of analytical geometry that the rectangular components F_x, F_y, F_z of the vector-product **F** are

$$(f) \qquad F_x = A_yB_z - A_zB_y \qquad F_y = A_zB_x - A_xB_z \qquad F_z = A_xB_y - A_yB_x$$

The vector-equation (d) may be considered as a shortened symbolic expression for the three equations (f). Equations (f) also at once lead to the result expressed by equation (e), that the vector-product changes its sign when the succession of the vector-factors is interchanged; for we get $-F_x, -F_y$, and $-F_z$ when in equations (f) we change A_x with B_x, A_y with B_y, and A_z with B_z.

154. Consistent Use of Rectangular Components in Graphical Vector-Algebra.— As drawings are two-dimensional, our methods can deal directly only with two-dimensional fields. Vector-fields in space must be treated indirectly. We have introduced for this the method of solving the three-dimensional field into fields tangential to and normal to a set of surfaces (section 118). The normal field may be treated as a two-dimensional scalar, while the tangential field represents a true two-dimensional vector. Our subject will therefore be that of developing graphical methods for performing mathematical operations upon these two-dimensional vectors.

One general method presents itself at once. We can introduce two sets of curves cutting each other under right angles, and use them as coordinate-curves. In the simplest case the two sets of curves will be two sets of parallel lines, which are mutually perpendicular to each other. A vector is represented in every point of the field by its components along each of the two coordinate-curves passing through the point. The coordinate-curves are the vector-lines of the two vector-components. But as these vector-lines are given invariable curves which are common to the components of all vectors, no operations will have to be performed upon them. Although these components $\mathbf{A}_x$, $\mathbf{A}_y$, $\mathbf{B}_x$, $\mathbf{B}_y$, . . . are primarily vectors, we never need take into account their vector-nature. They will be represented completely by the fields of their scalar values A_x, A_y, B_x, B_y, ... The sign of the scalar value will give the direction of the component along the coordinate-curves. The graphical methods for scalar fields which we have developed will then come directly into application to all problems of vector-algebra.

When we follow this method, the problems of graphical vector-algebra are solved already.

Thus the *vector-sum* **F** of two vectors **A** and **B** will be represented by the two scalar components F_x and F_y, and each of them is found by graphical addition of the fields of the scalar components A_x and B_x, respectively A_y and B_y, in accordance with the equations

(*a*) $$F_x = A_x + B_x \qquad F_y = A_y + B_y$$

The *scalar product* of the two vectors **A** and **B** will be found by two graphical multiplications and one graphical addition in accordance with the formula

(*b*) $$A_xB_x + A_yB_y$$

In the case of the two-dimensional fields, the *vector-product* of two vectors will be normal to the surface which contains the field. From the point of view of two-dimensional geometry it therefore loses its character of a vector. We have to deal simply with a scalar

(*c*) $$A_xB_y - A_yB_x$$

and the field of this scalar is derived from those of four given scalars A_x, B_y, A_y, B_x by two graphical multiplications and one graphical subtraction.

The advantages gained by the consistent use of vector-components are great enough to make it a serious question whether it should not be favorable from the beginning to work exclusively with components, and not with the vectors themselves. From the point of view of the observations there will be no objection against this. It would be a good plan to observe separately the N.–S. and the E.–W. component of the wind or of the sea-motion. If the observations were taken with self-recording instruments, the vector-averages required (section 97) would be obtained by taking the ordinary average of each component separately. Neither would there be any objection from the point of view of the meteorological telegraphic service. Whichever system be used, two numbers will have to be telegraphed. In the one case the two numbers will have to represent the two rectangular components. In the other case one number must be used to represent the wind-intensity, and another to represent the wind-direction.

But as long as the observations are not very good and complete it may be a question if it be advisable to *draw* the charts for each component separately, without compounding them to a vector. The formal process of drawing equiscalar curves would be simple enough. But the difficulty would consist in smoothing out the irregularities and filling up gaps in the observations. This must be done with full understanding of the kinematical situation of which the true vector-chart gives a conspicuous picture, but the two separate component-charts present only a very imperfect picture. This full understanding of the situation will also be of use for the control when mathematical operations are to be performed on the charts. We shall therefore as a rule avoid the artificial representation of the vectors by two component-fields, and use as much as possible the direct representations.

155. Use of Angles to Represent the Directions of Vectors.—We have introduced two direct representations of the two-dimensional vector, by intensity-curves and vector-lines, and by intensity-curves and isogons. We shall as a rule prefer the latter when mathematical operations are to be performed. The angles which

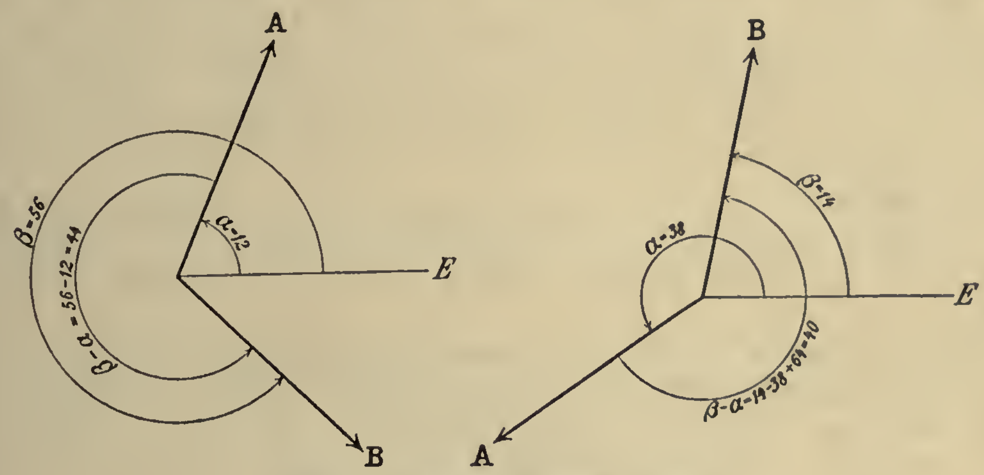


FIG. 75.—Angles and differences of angle.

represent the directions of the vectors **A**, **B**, . . . **F** will be represented by Greek letters α, β, . . . φ. We shall find it convenient occasionally in two-dimensional vector-algebra to use the symbols (A, α), (B, β), . . . (F, φ) as symbols for the vectors instead of **A**, **B**, . . . **F**. Thus we shall have identically

$$(a) \qquad \mathbf{A} = (A, \alpha), \ \mathbf{B} = (B, \beta), \ . \ . \ . \ \mathbf{F} = (F, \varphi)$$

In order to define completely the angles α, β, . . . φ, we must agree upon the choice of *an initial direction* from which they should be counted, and *the direction of that rotation by which they should be produced.* The initial direction must be agreed upon by an arbitrary choice. On our charts in horizontal projection we will choose the direction toward E as this initial direction. The *positive rotation around a point*, or, what comes to the same, the *positive circulation around a closed curve*, is always to be defined in accordance with the positive-screw rule. Most of our charts will represent fields which are contained in horizontal or quasi-horizontal surfaces.

As we count the normal to these surfaces positive upward, the positive rotation around the normal will be a rotation against the motion of the hands of a watch and the positive circulation will be the cyclonic circulation E.–N.–W.–S. of the northern hemisphere. (Compare the dial of fig. 32.)

We shall agree to consider all given angles α, β, . . . *which are used to represent the direction of given vectors as produced by positive rotation from the chosen initial direction.* Thus all initially given angles will be represented by positive numbers which are smaller than the number used to represent four right angles, *i. e.*, in our measure positive numbers smaller than 64 (see fig. 75).

When we form sums or differences of the numbers which represent the given angles we may come both to positive numbers which are greater than 64, and to negative numbers. In such cases we shall always by subtraction or addition of 64 (or a multiple of 64) reduce to a positive number smaller than 64. This will always be allowed by the general reason that there is no difference between the direction represented by α and that represented by $\alpha \pm$ four right angles. This remark is of special importance in connection with *the difference of angle* $\beta-\alpha$, which represents the direction of the vector **B** *relatively to that of* **A**. When we agree always to represent this difference of angle by a positive number, it implies that we agree to count it as *produced by a rotation in positive direction from the vector* **A**, of which the angle α a ppears as subtractor *to the vector* **B**, of which the angle appears as minuend (see fig. 75).

These agreements must be remembered for the understanding of our charts, where the isogons, whether they represent absolute angles α, β . . . or differences of angle $\beta-\alpha$, are always numbered with positive numbers contained between 0 and 64.

Two vectors which cut each other under constant angle will have the same system of isogons, only with different numbers appearing on the isogons. The difference will be zero, if the two vectors have the same direction, 32 if they have the opposite direction, and 16 or 48 if they cut each other under right angle. Evidently two opposite directions will have equal right to be called normal to a given direction. We shall therefore agree to distinguish between these two directions by a rule of signs, namely this:

> *From a given direction we pass to that of its positive normal by a rotation of one right angle and to that of its negative normal by a rotation of three right angles in positive direction.*

It follows from this rule that when the vector **B** is directed along the positive normal to the vector **A**, the vector **A** will be directed along the negative normal to the vector **B**. Or in the notations (*a*): The vector

$$(B, \beta) = (B, \alpha+16)$$

is directed along the positive normal to the vector (A, α). But then

$$(A, \alpha) = (A, \beta-16) = (A, \beta+48)$$

will be directed along the negative normal to the vector (B, β).

156. Projections of a Vector; Scalar Product and Vector-Product.—Let a direction represented by the angle α be given everywhere in the field. We shall form the projection A_1 of a given vector (F, φ) on this direction. This projection will have the positive or the negative sign according as it points *in* or *against* the direction represented by the given angle α.

The projection is given by

(*a*) $$A_1 = F \cos(\varphi - \alpha)$$

We solve with respect to F, and to $\varphi - \alpha$

$$F = \frac{A_1}{\cos(\varphi - \alpha)} \qquad \varphi - \alpha = \text{arc}\cos\frac{A_1}{F}$$

We tabulate F as function of the variables A_1 and $\varphi - \alpha$ (first of tables K). In the same manner we should have tabulated $\varphi - \alpha$ as function of F and A_1. But as we deal here only with the general principles, and not with the tables for practical use, we shall give here and in several cases below only one table. The field of the projection A_1 can then be found in two operations. By graphical subtraction we form the field of the angle $\varphi - \alpha$. This field is placed upon that which represents the scalar value F of the given vector. Using the first table K, we derive from the curves $\varphi - \alpha$ = const. and F = const., the field of the scalar A_1, proceeding as we have exemplified several times already for graphical operations with two variables. In this, as well as in several of the following tables, each tabulated number corresponds to different sets of arguments. The arguments on the left side and above belong together, and so do the arguments on the right side and below. In order to avoid mistakes it may be favorable for practical use to have two tables containing the same tabulated numbers, but each only with one set of arguments.

We can now form the projection of (F, φ) on the positive normal to that direction which is given by the angle α. For this projection we have

(*b*) $$A_2 = F \sin(\varphi - \alpha)$$

We solve this equation with respect to F

(*b'*) $$F = \frac{A_2}{\sin(\varphi - \alpha)}$$

and tabulate F as function of A and $\varphi - \alpha$ (second table K). Thus, in order to find the field of this projection A_2, we first form the same auxiliary field $\varphi - \alpha$ as in the preceding case, place this field upon that which represents the intensity F of the given vector, and draw the curves A_2 = const. by use of the second table K.

By the two tables K we can thus solve a vector F into orthogonal components A_1 and A_2. We thus have the way open to bring coordinate-methods into application when this should be desirable.

From expressions of the form (*a*) and (*b*) there is only one step to expressions of the form

$$AB \cos(\beta - \alpha) \text{ and } AB \sin(\beta - \alpha)$$

i. e., to the formation of the complete scalar product or the complete vector-product of two-dimensional vectors. Tables K in connection with a table for graphical multiplication will thus give the complete solution of the formation of these two products.

It is easier to explain the graphical procedures by formulæ and text than to illustrate them by text-figures, for the text-figures can not be placed upon each other on the illuminated drawing-board in order to make any two systems of curves visible at once as if they were drawn on the same sheet. This should be remembered when studying the example given in fig. 76, which illustrates the formation of the projection of the vector (F, φ) on the direction defined by the angle α. The chart A

TABLES K.—*Projections of a vector* (F, φ).

I. *Table for drawing the field of the projection* $A_1 = F \cos(\varphi - \alpha)$ *(F tabulated).*

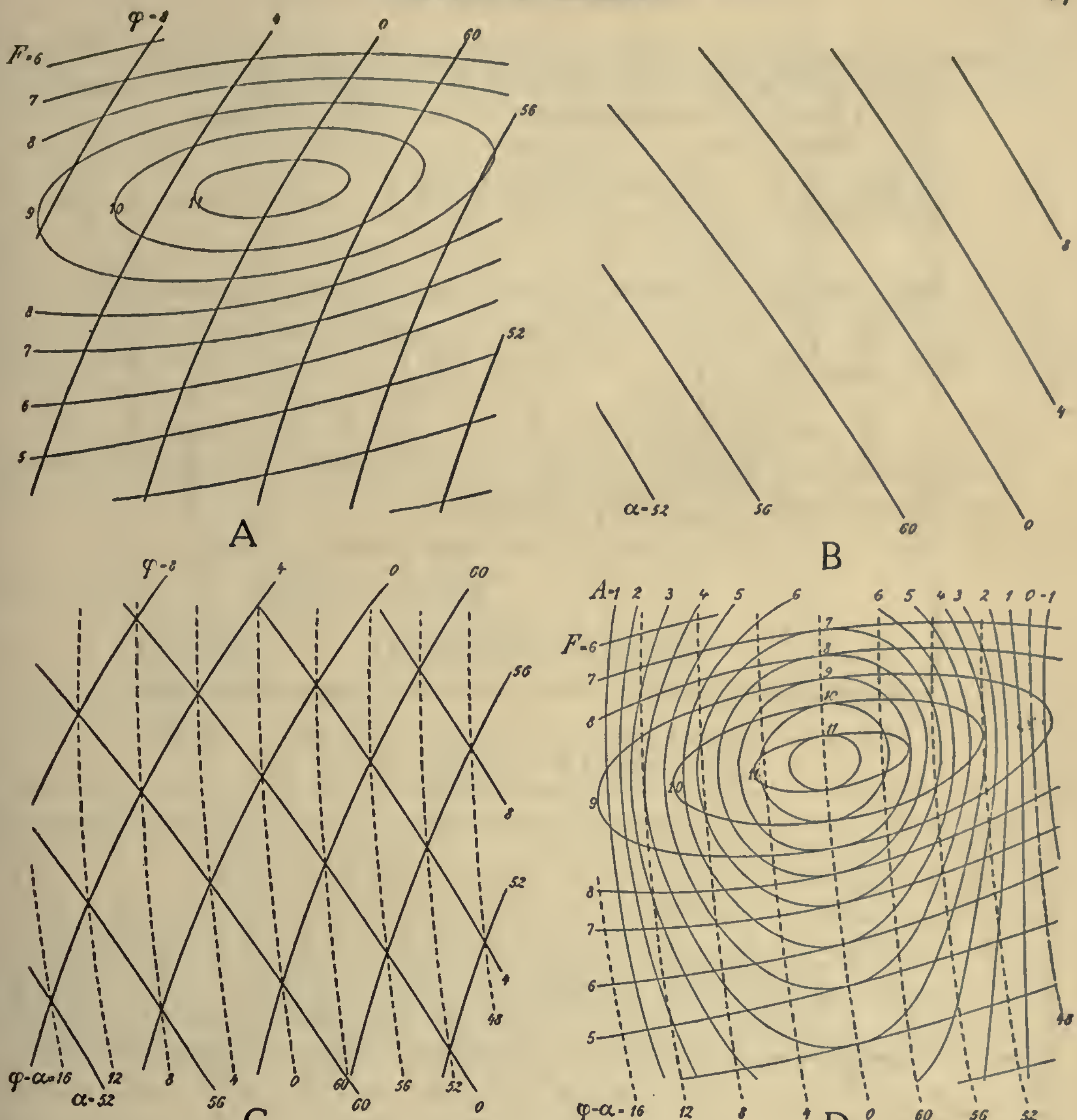
Projection A_1	Angle $(\varphi-\alpha)$.					
	0	4	8	12	16	
	64	60	56	52	48	
0	0	0	0	0	$\frac{0}{0}$	0
1	1	1.1	1.4	2.6	∞	− 1
2	2	2.2	2.8	5.2	∞	− 2
3	3	3.2	4.2	7.8	∞	− 3
4	4	4.3	5.7	10.5	∞	− 4
5	5	5.4	7.1	13.1	∞	− 5
6	6	6.5	8.5	15.7	∞	− 6
7	7	7.6	9.9	18.3	∞	− 7
8	8	8.7	11.3	20.9	∞	− 8
9	9	9.7	12.7	23.5	∞	− 9
10	10	10.8	14.1	26.1	∞	−10
	32	28	24	20	16	A_1 Projection.
	32	36	40	44	48	
	Angle $(\varphi-\alpha)$					

II. *Table for drawing the field of the projection* $A_2 = F \sin(\varphi - \alpha)$ *(F tabulated).*

Projection A_2	Angle $(\varphi-\alpha)$.					
	0	4	8	12	16	
	32	28	24	20	16	
0	$\frac{0}{0}$	0	0	0	0	0
1	∞	2.6	1.4	1.1	1	− 1
2	∞	5.2	2.8	2.2	2	− 2
3	∞	7.8	4.2	3.2	3	− 3
4	∞	10.5	5.7	4.3	4	− 4
5	∞	13.1	7.1	5.4	5	− 5
6	∞	15.7	8.5	6.5	6	− 6
7	∞	18.3	9.9	7.6	7	− 7
8	∞	20.9	11.3	8.7	8	− 8
9	∞	23.5	12.7	9.7	9	− 9
10	∞	26.1	14.1	10.8	10	−10
	0	60	56	52	48	A_2 Projection.
	32	36	40	44	48	
	Angle $(\varphi-\alpha)$					

represents the field of the given vector (F, φ), the chart B that of the given angle α. The sheets containing these two charts are placed upon each other, and on a third sheet we draw the curves for the difference of angle $\varphi - \alpha$ as illustrated by chart C. We then place the three sheets upon each other in reversed order, taking that which contains the field of α uppermost and draw upon it the curves $A =$ const. illustrated by chart D. On this sheet we have then obtained the chart E which contains the complete result. Thus, while fig. 76 contains five charts, only three sheets of paper have been used.

157. Addition of Vectors which are Normal to Each Other.—In two-dimensional vector-algebra the vector-product has lost its character of a proper vector. It can be treated as a scalar, and is therefore easier to form than the vector-sum, which always remains a true vector. In two-dimensional vector-algebra the vector-addition is therefore the only typical vector-operation, and as a rule an operation of more complicated nature than the formation of the products.



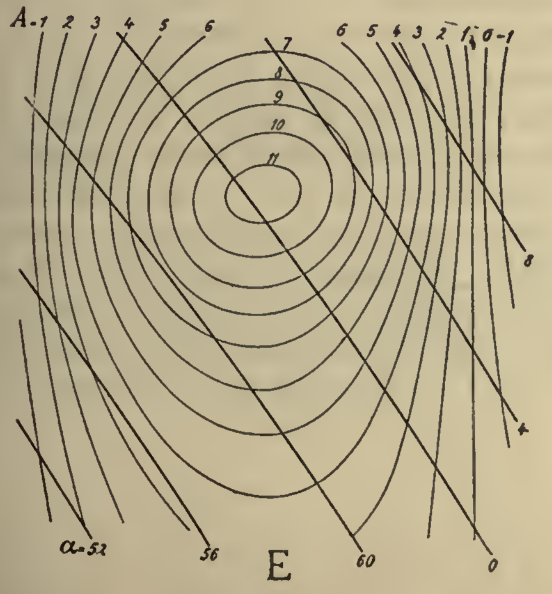


FIG. 76.—Projection of a vector.

A. Field of given vector (F, φ) represented by intensity-curves $F = 6, 7, 8, \ldots$ (fine) and isogons $\varphi = 8, 4, 0, 60 \ldots$ (heavy).

B. Isogons of angle $\alpha = 52, 56, 60, \ldots$

C. Isogons of angle α of the angle φ and of the difference $\varphi - \alpha = 16, 12, 8, \ldots$ (stippled).

D. Isogons $\varphi - \alpha$, intensity-curves F, and intensity-curves $A = 1, 2, 3, \ldots$ of projection derived by first of tables K.

E. Field of projection (A, α) represented by intensity-curves A and isogons α.

TABLES L.—*Graphical addition of mutually normal vectors.*

I. *Table for drawing the intensity field* $F = \sqrt{A^2+B^2}$ *(B or A tabulated).*

Intensity of vector-sum. F	Intensity of first vector-addend *A*.											
	0	1	2	3	4	5	6	7	8	9	10	
0	0											0
1	1	0										1
2	2	1.7	0									2
3	3	2.8	2.2	0								3
4	4	3.9	3.5	2.6	0							4
5	5	4.9	4.6	4.0	3.0	0						5
6	6	5.9	5.7	5.2	4.5	3.3	0					6
7	7	6.9	6.7	6.3	5.7	4.9	3.6	0				7
8	8	7.9	7.7	7.4	6.9	6.2	5.3	3.9	0			8
9	9	8.9	8.8	8.5	8.1	7.5	6.7	5.7	4.1	0		9
10	10	9.9	9.8	9.5	9.2	8.7	8.0	7.1	6.0	4.4	0	10
	0	1	2	3	4	5	6	7	8	9	10	F
	Intensity of second vector-addend *B*.											Intensity of vector-sum.

II. *Table for drawing the field of the angle* $\varphi-\alpha$ *between the vector-sum F and the vector-addend A (B or A tabulated).*

Angle ($\varphi-\alpha$)		Intensity of first vector-addend *A*.												
$\beta-\alpha=16$	$\beta-\alpha=48$	0	1	2	3	4	5	6	7	8	9	10		
0	64 or 0	0	0	0	0	0	0	0	0	0	0	0	16	48
4	60	0	0.4	0.8	1.2	1.7	2.1	2.5	2.9	3.3	3.7	4.1	12	52
8	56	0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	8	56
12	52	0	2.4	4.8	7.2	9.7	12.1	14.5	16.9	19.3	21.7	24.1	4	60
16	48	$\frac{0}{0}$	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	0	64 or 0
		0	1	2	3	4	5	6	7	8	9	10	$\beta-\alpha=16$	$\beta-\alpha=48$
		Intensity of second vector-addend *B*.											Angle ($\varphi-\alpha$)	

We shall first treat the case in which the two given vectors **A** and **B** are normal to each other. The problem of their addition may then be considered as inverse to the problem of solution into rectangular components treated in the preceding section. It will be precisely inverse if the two vectors **A** and **B** are given in the form used for components along orthogonal coordinate-curves, *i. e.*, by equiscalar curves for positive and negative values of the intensity. On the other hand, it will take a slightly changed form if the vector is given in the ordinary way by isogons and intensity curves for an always positive intensity. We shall treat this case only. It will easily be seen that in the other case we can use the same tables, only with a somewhat changed arrangement as to the sign of the tabulated numbers and the arguments. For the scalar value of the vector (F, φ) which is the resultant of the vectors (A, α) and (B, β) we have

(a) $$F^2 = A^2 + B^2$$

We solve this equation with respect to one of the given quantities A or B,

(a′) $$B = \sqrt{F^2 - A^2}, \text{ or } A = \sqrt{F^2 - B^2}$$

Both formulæ lead to the same table, the first of tables L, where, according to circumstances we can consider F and A or F and B as arguments. By this table we can thus derive the intensity-curves for the vector-sum from the intensity-curves of the two orthogonal vector-addends.

In order to form the isogons of (F, φ) we have to remember that the vector (B, β) in some regions of the field may be directed along the positive and in others along the negative normal to (A, α). In the two cases we shall have respectively

$$\beta - \alpha = 16 \text{ and } \beta - \alpha = 48$$

with corresponding values of the angle $\varphi - \alpha$

$$\varphi - \alpha < 16 \text{ and } \varphi - \alpha > 48$$

The rectangular triangle will give for the determination of this angle in the two cases respectively

(b) $$tg\ (\varphi - \alpha) = \frac{B}{A} \text{ and } tg\ (\varphi - \alpha) = -\frac{B}{A}$$

We solve these equations with respect to one of the given quantities A or B, thus

(b′) $$B = A \text{ tg } (\varphi - \alpha) \text{ and } B = -A \text{ tg } (\varphi - \alpha)$$
$$A = B \text{ cotg } (\varphi - \alpha) \qquad A = -B \text{ cotg } (\varphi - \alpha)$$

By suitable change of arguments all formulæ can be represented by one table, the second of tables L. This table allows us to derive the field of the angle $\varphi - \alpha$ from the fields of the two tensors A and B.

When the field of the angle $\varphi - \alpha$ is found, we find by the graphical addition

(c) $$\varphi = (\varphi - \alpha) + \alpha$$

the field of the angle φ which represents the direction of the vector-sum.

The illustration of the procedure by text-figures would seem complicated, but when the illuminated drawing-board is used, only four sheets of paper are required: two contain the fields of the given vectors; a third is used for the field of the auxiliary quantity $\varphi - \alpha$; on the fourth we draw directly the final curves giving the fields of F and of φ.

158. Addition of Any Vectors.—When the two given vectors (A, α) and (B, β) cut each other under a variable angle, the operation of determining their vector-sum (F, φ) will depend upon four variables, A, α, B, β. But the complex operation can be decomposed into the following series of operations, each involving the use of two variables.

(1) By graphical subtraction we form the auxiliary field of the scalar $\beta-\alpha$ which represents the angle between the two given vectors.

(2) By graphical division (section 151) we form the auxiliary field representing the ratio $\frac{B}{A}$ of the numerical values of the two given vectors.

(3) By the elementary properties of the triangle with the sides A, B, and F we get the following relation connecting the angle $\varphi-\alpha$ with the known angle $\beta-\alpha$ and the known ratio $\frac{B}{A}$

$$(a) \qquad \left(\frac{B}{A}-1\right) \operatorname{tg} \frac{\beta-\alpha}{2} = \left(\frac{B}{A}+1\right) \operatorname{tg} \left(\varphi-\alpha-\frac{\beta-\alpha}{2}\right)$$

We solve this equation with respect to $\frac{B}{A}$ and tabulate this quantity as function of the two angles $\beta-\alpha$ and $\varphi-\alpha$. Using this table, the first of tables M, we can derive the field of the angle $\varphi-\alpha$ from the fields of the two auxiliary quantities $\beta-\alpha$ and $\frac{B}{A}$.

(4) By the properties of the same triangle we find the following relation which connects the ratio $\frac{F}{A}$ with the ratio $\frac{B}{A}$ and the angle $\beta-\alpha$,

$$(b) \qquad \left(\frac{F}{A}\right)^2 = 1+\left(\frac{B}{A}\right)^2+2\frac{B}{A}\cos(\beta-\alpha)$$

We solve this equation with respect to $\frac{B}{A}$ and tabulate this quantity with the ratio $\frac{F}{A}$ and the angle $\beta-\alpha$ as arguments. This gives the second of tables M. Using this table we can derive the field of the ratio $\frac{F}{A}$ from the fields of the two auxiliary quantities $\beta-\alpha$ and $\frac{B}{A}$. When two numbers are given in the same place in the table, the curve $\frac{F}{A}$ = const. has two points of intersection with the curve $\beta-\alpha$ = const. Both will have to be used.

(5) By graphical multiplication (section 150) we derive the field of the intensity F of the required vector from the fields of the ratio $\frac{F}{A}$ and of the intensity A of the given vector.

(6) By graphical addition we derive the field of the angle φ of the required vector from the fields of the angles $\varphi-\alpha$ and α.

It should be emphasized that as soon as we have drawn the first two systems of auxiliary curves (1) and (2), we know the situation of all zero-points of the field.

Every singular point of a vector is a zero-point for its absolute value. The resultant can be zero only in points where the two given vectors have equal magnitude and opposite direction. Now the two given vectors have equal magnitude in the points of the curve $\frac{B}{A}=1$, and opposite direction in the points of the curve $\beta-\alpha=32$.

Hence it follows:

The singular points in the field of the vector-sum are the points of intersection of the curves

$$(c) \qquad \beta - \alpha = 32 \text{ and } \frac{B}{A} = 1$$

On account of the ample information which the singular points give regarding the field, it will be important to draw with as great care as possible these two curves. In return much labor can be saved in other parts of the field, where few additional data are required besides those involved in the knowledge of the situation of the singular points.

TABLES M.—*Graphical addition of any vectors.*

I. *Table for drawing the field of the angle between the vector-sum and the vector-addend A.*

Angle $\varphi-\alpha$	Angle $(\beta-\alpha)$.									
	0	4	8	12	16	20	24	28	32	
0	0/0	0	0	0	0	0	0	0	<1	0
4		∞	1.00	0.54	0.41	0.38	0.41	0.54	1.00	60
8			∞	1.85	1.00	0.76	0.71	0.76	1.00	56
12				∞	2.4	1.30	1.00	0.92	1.00	52
16					∞	2.6	1.41	1.08	1.00	48
20						∞	2.4	1.30	1.00	44
24							∞	1.85	1.00	40
28								∞	1.00	36
32									>1	32
	64	60	56	52	48	44	40	36	32	$\varphi-\alpha$ Angle
	Angle $(\beta-\alpha)$									

II. *Table for drawing field of ratio of intensity of the vector-sum to that of the vector-addend A.*

Ratio $\frac{F}{A}$	Angle $(\beta-\alpha)$								
	0	4	8	12	16	20	24	28	32
	64 or 0	60	56	52	48	44	40	36	32
0									1.00
0.5								0.60	0.50
0.5								1.25	1.50
1.0						0.00	0.00	0.00	0.00
1.0					0	0.77	1.42	1.85	2.00
1.5	0.50	0.53	0.62	0.80	1.12	1.57	2.04	2.38	2.50
2.0	1.00	1.04	1.16	1.39	1.73	2.16	2.58	2.89	3.00
3.0	2.0	2.1	2.2	2.4	2.8	3.2	3.6	3.9	4.0
4.0	3.0	3.1	3.2	3.5	3.9	4.3	4.6	4.9	5.0
5.0	4.0	4.1	4.2	4.5	4.9	5.3	5.6	5.9	6.0
6.0	5.0	5.1	5.2	5.6	5.9	6.3	6.7	6.9	7.0
7.0	6.0	6.1	6.3	6.6	6.9	7.3	7.7	7.9	8.0
8.0	7.0	7.1	7.3	7.6	8.0	8.3	8.7	8.9	9.0
9.0	8.0	8.1	8.3	8.6	9.0	9.3	9.7	9.9	10.0
10.0	9.0	9.1	9.3	9.6	10.0	10.3	10.7	10.9	11.0

159. Easily Accessible Data Regarding the Field of the Vector-Sum.—Just as we can find the singular points, we can easily find a series of further data regarding the field of the vector-sum. It will save much labor to use these data as completely as possible.

The vector-addition will be performed according to the simplest law, that of scalar addition or subtraction, in all points where the two given vectors have either the same or opposite directions, *i.e.*, in the points of the curves

(*a*) $\beta - \alpha = 0$

(*b*) $\beta - \alpha = 32$

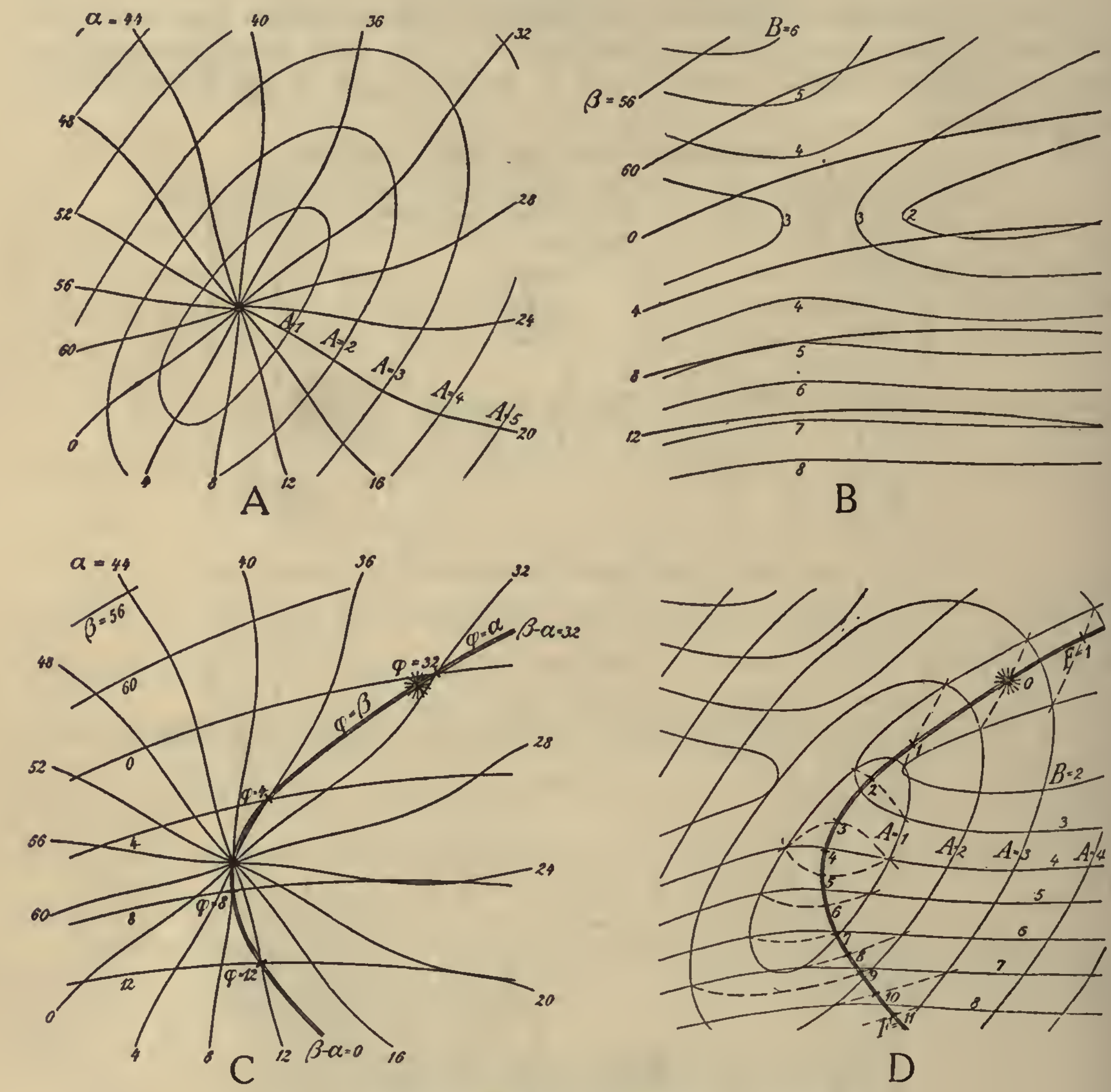


FIG. 77.—Easily accessible data on the field of the vector-sum.

A. First given vector (A, α).
B. Second given vector (B, β).
C. Curves $\beta - \alpha = 0$ and $\beta - \alpha = 32$, and their points of intersection with the required curves $\varphi = 12, 8, 4$.
D. Curves $\beta - \alpha = 0$ and $\beta - \alpha = 32$ and their points of intersection with the required curves $F = 11, 10, 9, \ldots$

In all points of the first curve the vector-sum **F** will have the same direction as both **A** and **B**, and a numerical value equal to their scalar sum. In all points of the second curve the vector-sum **F** will have the same direction as the greater of the vectors **A** and **B**, and a numerical value equal to their scalar difference. We can therefore with the greatest ease find all data regarding direction and intensity of the vector-sum in all points of the two curves (*a*) and (*b*).

The curves (*a*) and (*b*) belong to the first set of auxiliary curves drawn for the determination of the vector-sum, section 158 (1). While we draw the curve (*a*) we can mark on it the points where it will be cut by the required curves $\varphi = 0, 1, 2, \ldots$ for these will be the same points as those which serve for the determination of the curve $\beta - \alpha = 0$ itself, namely, the points of intersection of the curve $\alpha = 0$ with $\beta = 0$, of the curve $\alpha = 1$ with $\beta = 1$, of the curve $\alpha = 2$ with $\beta = 2$, and so on (fig. 77 C). In order to find the points where the same curve is cut by the required intensity-curves $F = 0, 1, 2, \ldots$, we have simply to draw the short parts of the curves $A + B = 0, 1, 2, \ldots$ which cut the curve (*a*) (see fig. 77 D).

In the same manner, while we draw the curve (*b*), we can mark on it the points where it will be cut by the curves $\varphi = 0, 1, 2, \ldots$; for these points will again be the same as those points of intersection of the given curves $\alpha =$ const. and $\beta =$ const. which serve to determine the curve (*b*). We have to remark that the integer values of φ which should be noted at these points will be those of α when $A > B$ and those of β when $B > A$. In order to find the points where the same curve is cut by the intensity-curves $F = 0, 1, 2, \ldots$, we have to draw the parts of the curves $|A - B| = 0, 1, 2, \ldots$, which cut the curve (*b*) (see fig. 77 D). Evidently the intersection of the curve $A - B = 0$ with the curve (*b*) gives the singular points of the field of the vector-sum. It should be observed that the curve $B - A = 0$ is identical with the curve $\frac{B}{A} = 1$, which we have used already in the preceding section for the determination of the singular points. These points will divide the curve (*b*) into distinct branches. As we pass a singular point the value of the angle φ will change suddenly from $\varphi = \alpha$ to $\varphi = \beta$, or vice versa.

Thus the investigation of the two curves (*a*) and (*b*) gives with great ease both the situation of the singular points in the field of the vector-sum and in addition a great number of points through which different curves representing the field of the vector-sum shall pass. These data can be utilized in different ways, according to the method otherwise used for finding the field of the vector-sum. If the method given in the preceding section be retained, it will be important to remark that the curves (*a*) and (*b*) will turn up again as the curves

(*c*) $$\varphi - \alpha = 0$$
(*d*) $$\varphi - \alpha = 32$$

in the auxiliary field of the angle $\varphi - \alpha$, which is found by the operation (3) of the preceding section. The curve (*c*) will correspond to the curve (*a*) and certain parts of the curve (*b*), the change in the correspondence taking place at the singular points.

160. Graphical Tables for Vector-Addition.—Our first solution of the problem of graphical vector-addition depended upon the decomposition of the general problem with four variables into six partial problems each with two variables. But if we use the method which we have developed in section 152 for three variables, we can reduce to a smaller number of partial problems. The operations with three variables which we shall have then to perform will join themselves directly to those of the preceding section.

After we have drawn the curves

$$(a) \qquad \beta - \alpha = \text{const.}$$

we can pass directly to the determination of the angle $\varphi - \alpha$ and of the intensity F of the resultant by the formulæ

$$(b) \qquad \text{tg}\,(\varphi - \alpha) = \frac{B \sin(\beta - \alpha)}{A + B\cos(\beta - \alpha)}$$

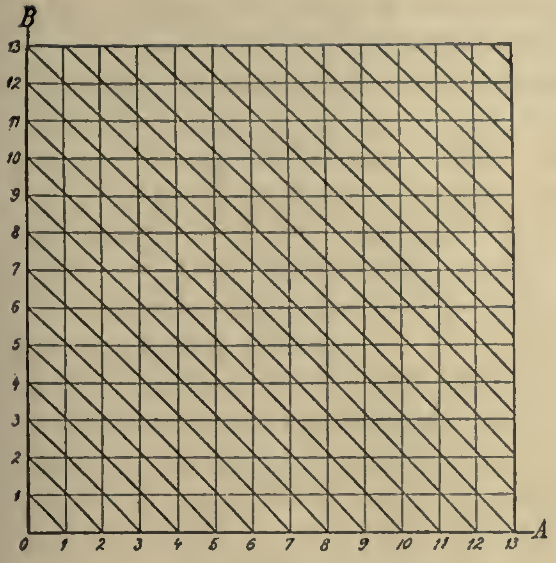
$$(c) \qquad F^2 = A^2 + B^2 + 2AB\cos(\beta - \alpha)$$

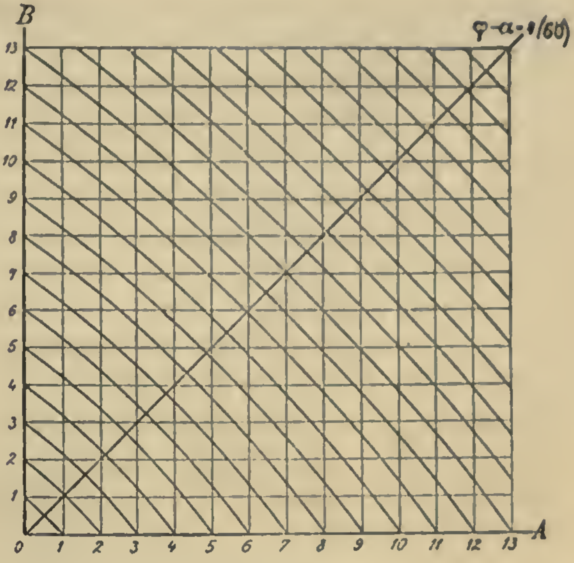
In each of these formulæ we can give $\beta - \alpha$ a certain constant value and by the principles of section 152 construct a graphical table by which we can find the points in which this particular curve $\beta - \alpha = \text{const.}$ is cut by the curves for integer values of $\varphi - \alpha$ and of F. We then set off A and B as abscissa and ordinate of a rectangular system of coordinates, and draw in the one case the curves $\varphi - \alpha = \text{const.}$, in the second the curves $F = \text{const.}$ in this system of coordinates. It will be seen at once that the first curves are simply straight lines through the origin of the coordinates, the second ellipsæ with the origin of the coordinates as center and with the axes forming the angle 8 (45°) with the axes of coordinates. It will be convenient to draw both systems of curves on the same diagram. Then we can read off simultaneously the situation of the required points for integer values both of $\varphi - \alpha$ and of F.

In fig. 78 we have drawn these diagrams for the values $\beta - \alpha = 4, 8, 12, 16, 20, 24$. The radial lines $\varphi - \alpha = \text{const.}$ are drawn in these diagrams for the interval 4. Thus on the first diagram $\beta - \alpha = 4$ we have only two lines $\varphi - \alpha = \text{const.}$, namely, the two axes of coordinates. On the following we have 3, 4, 5, 6, and 7 of them respectively. The ellipsæ are drawn for unit intervals of the intensity F. The ratio of the axes changes as we pass from the one diagram to the other. In the case of $\beta - \alpha = 16$, *i.e.*, at the curve where the vectors cut each other under right angle, the two axes are equal to each other and the curves are circles. It will easily be seen that the same diagrams may be used for the values 60, 56, 52, 48, 44, 40 of $\beta - \alpha$, taken in connection with the values of $\varphi - \alpha$, which are written in brackets on the diagrams.

By use of these diagrams, including the first of figs. 101, p. 127, we can then find the points in which the curves for integer values of $\varphi - \alpha$ and of F cut 14 isogons $\beta - \alpha = \text{const.}$ The points of intersection with the fifteenth and the sixteenth, viz, $\beta - \alpha = 0$ and $\beta - \alpha = 32$, have been found already by the simpler method of the preceding section.

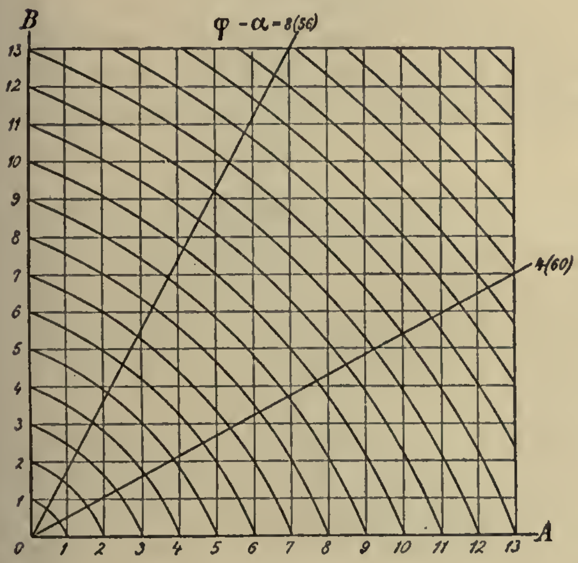
A great advantage of this method is that two draftsmen can cooperate. One has before him a chart containing the three sets of curves $A = \text{const.}$, $B = \text{const.}$, and $\beta - \alpha = \text{const.}$ They may be copied on one paper, or they may be drawn on three different papers which are placed upon each other on the illuminated drawing-board. The other has the graphical table fig. 78 and a transparent paper placed upon it.

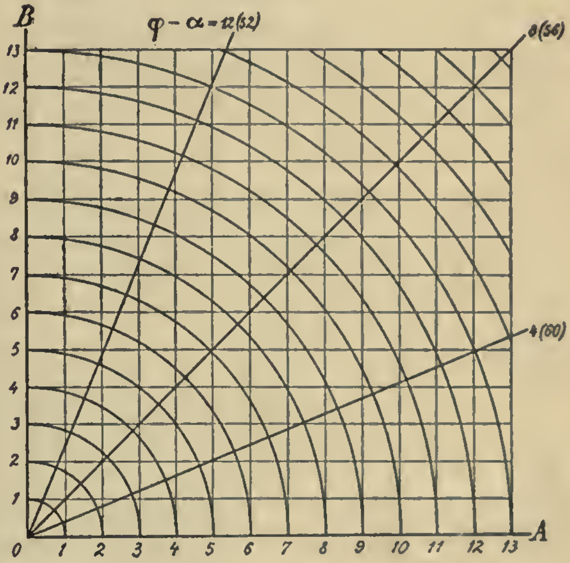




β-α = 4(60)

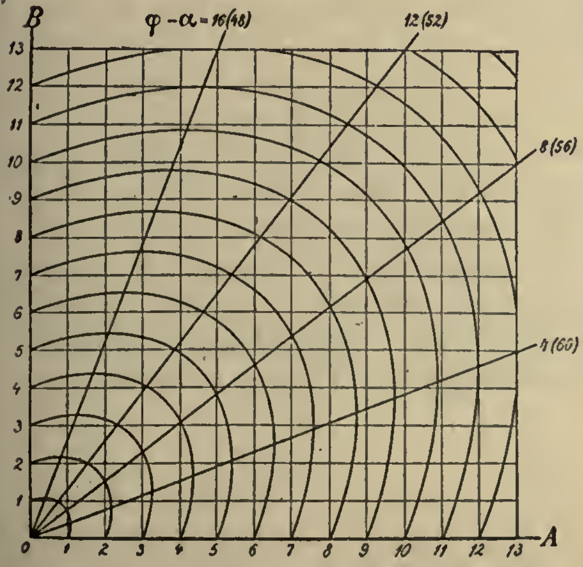
β-α = 8(56)

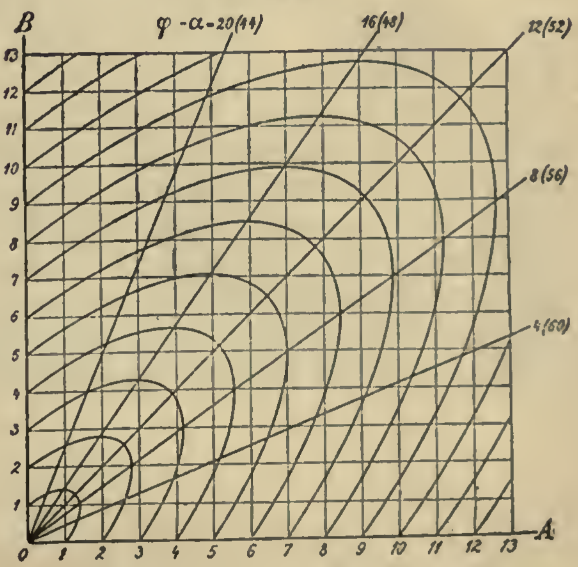




β-α = 12(52)

β-α = 16(48)





Let it be required, for instance, to determine the points in which the curve $\beta-\alpha = 20$ is cut by the curves $\varphi-\alpha = 0, 4, 8, 12, \ldots$ and by the curves $F = 1, 2, 3, 4, \ldots$. The first draftsman then observes the connected values of A and of B along the curve $\beta-\alpha = 20$, and dictates that it cuts the curve $A = 1$ in the point where $B = B_1$, the curve $A = 2$ in the point where $B = B_2$, etc. The second draftsman then draws point by point the corresponding curve on the transparent sheet placed upon fig. 78. Then the second draftsman follows the course of the curve which he has drawn, and dictates to the first that it cuts the curve $F = 1$ at the point where $A = A_1$, the curve $F = 2$ at the point where $A = A_2 \ldots$, the curve $\varphi-\alpha = 0$ at the point where $A = A'_0$. the curve $\varphi-\alpha = 4$ at the point where $A = A'_4, \ldots$. The first draftsman then marks these points on the curve $\beta-\alpha = 20$ on his chart, using different kinds of marks for the curves $F =$ const. and $\varphi-\alpha =$ const., and adding the numerical values of F and $\varphi-\alpha$. When this is repeated for a sufficient number of curves $\beta-\alpha =$ const. we shall get a complete set of points determining the course of the curves $F =$ const. and $\varphi-\alpha =$ const.

From the set of curves $\varphi-\alpha =$ const. we finally pass, by the graphical addition $(\varphi-\alpha) + \alpha = \varphi$, to the curves representing the required angle φ.

When we compare with the method of section 158, we see that the use of the graphical tables replaces the performance of the separate graphical operations (2), (3), (4), (5). Only the simple graphical subtraction (1) and the graphical addition (6) are retained.

161. Complete Resultantometer.—While the method of section 158 required the drawing of four auxiliary systems of curves, besides the fifth and sixth, which represent the result, we succeed by using the graphical tables in arriving at the result by drawing only two auxiliary systems of curves. By introducing a still more complete auxiliary, a complete machine for vector-addition, we can completely avoid the drawing of auxiliary systems of curves.

Instruments for adding vectors can be constructed in various ways, each having an advantage according to the special form in which the problem presents itself. Fig. 79 shows a convenient construction for our purposes. We draw parallel and equidistant lines on two circular transparent sheets and concentric circles on one of them. The sheets are laid upon each other, so that the upper is free to slide inside the divided brass-ring C, while the lower is mounted in a brass-ring which can slide outside the ring C. This ring contains the divisions 0 to 63, which represent the angles. When the instrument is to be used on our charts in conical projection, the ring C is attached to the rule R, which passes through the point of convergence of the meridians. (Compare the integration-machine of fig. 62.) The divisions of the circle C will then always show the true directions relatively to the meridians on the chart. For the practical use of the instrument it will finally be useful to have two screws by which we can attach either of the divided sheets rigidly to the ring C and thus give the lines of the fixed sheet an invariable direction relatively to the meridians of the chart. The two sheets are perforated at the center, in order to make it possible to set marks on the paper underneath by use of a pin or a sharp pencil. All lines are engraved on the upper side of the lower sheet and on the under side of the upper sheet. When using the instrument it will be best to have illumination from below. Otherwise the pictures seen will be blurred by the shadows which the lines will throw on the paper.

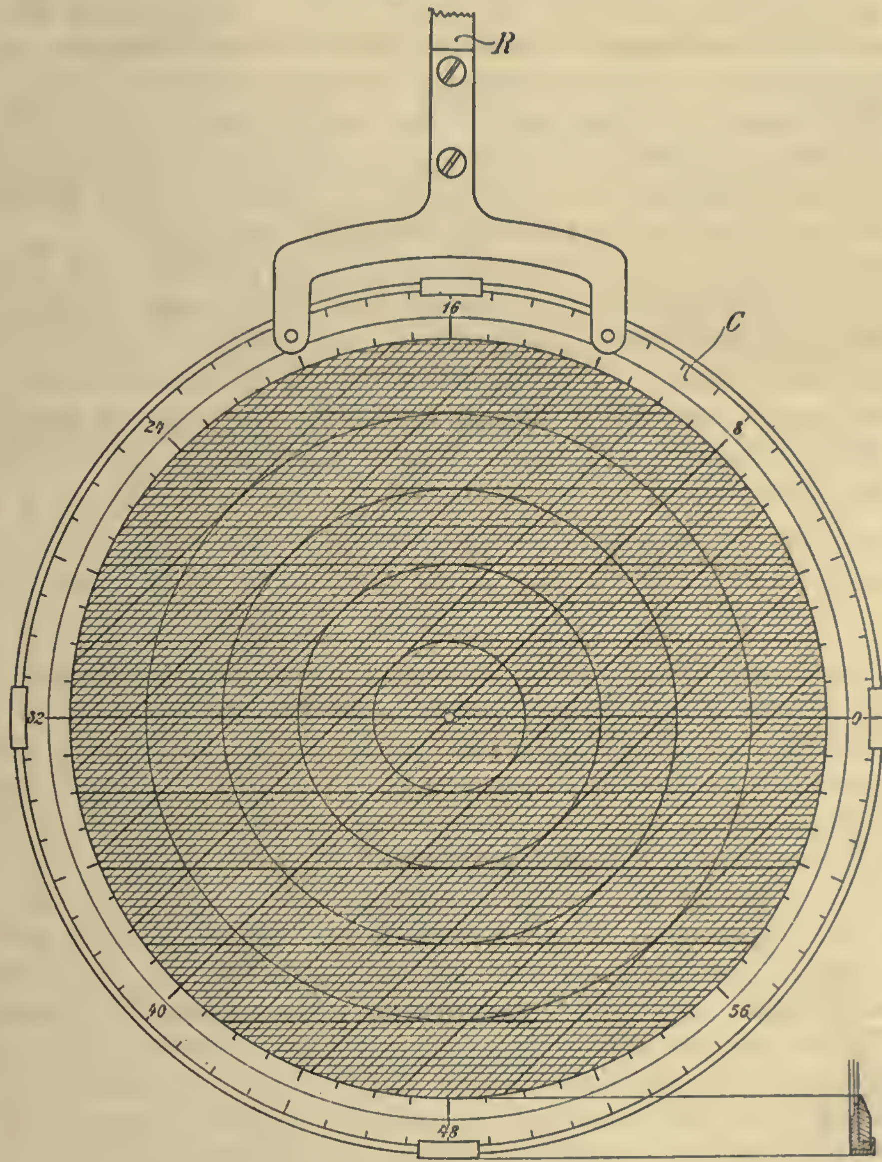


Fig. 79.—Resultantometer.

When one of the divided sheets is made to rotate relatively to the other, the two sets of parallel lines produce parallelograms of all possible shapes. The intersection of the lines gives at the same time equidistant divisions on each line, which can be used for measuring the lengths which shall represent the scalar values of the given vectors. In this manner we use a variable unit length, which varies with the angle between the lines. This is a great advantage, for we get automatically the construction performed on a large scale in the difficult case in which the given vectors are nearly equal and oppositely directed. The direction of the resultant is read off on the divided circle *C*. In order to read off the intensity of the resultant on the same scale as that used for the components, we follow the circles from the point at the end of the resultant to one of the central lines of the one divided sheet.

The discontinuous use of the instrument will be understood at once. If the directions of the given vectors are represented by vector-lines, the two divided sheets are adjusted so as to be tangential to one line of each set. If the directions are given by isogons, the adjustment of the sheets is made by use of the divided circle C. In this case it will not be necessary to place the instrument on the chart. Two workers can cooperate. One manages the instrument, while the other reads off from the chart the given data and introduces the results on it.

Continuous use of the instrument will also be possible. We can then go along an isogonal curve, having the one disk fixed in the angle represented by the isogon, while the other is turned according to the value of the angle represented by the other set of isogons. The intensities of the two vectors are observed, and thus by short steps we can follow the variations of the angle and the intensity of the resultant and mark the points where integer values occur. But this work will require keen attention.

CHAPTER IX.

GRAPHICAL DIFFERENTIATION AND INTEGRATION.

162. Different Forms of the Problems.—We shall meet with problems of graphical differentiation in a variety of forms, each requiring the development of special methods and auxiliaries. The problems will take different forms according as *space* or *time* derivations should be performed. The pure space-derivations will depend upon measurements performed upon a chart which represents the given field at a given epoch. The time derivations will involve a comparative investigation of *two* charts which represent the fields of the same quantity at two different epochs. We shall consider first the space-derivations and afterwards the time-derivations.

The space-derivations will present themselves in different forms, requiring different methods and auxiliaries according as they depend upon the measurement of lengths or of angles. We shall consider first the angular or directional and then the linear differentiations. To each problem of the differentiation will correspond a problem of integration which in the elementary cases will cause no difficulty as soon as the problem of differentiation is solved.

163. Directional Differentiation and Integration.—Let a system of curves s be given; by their tangents they define a system of directions. It is required to find the angle φ which gives the direction of the tangents, *i. e.*, we shall draw the isogonal curves which represent the field of this angle. Evidently this is a problem of differentiation which is inverse to the problem of integration, consisting in the drawing of the vector-lines to a given system of isogons. This drawing of the isogons to a given system of curves can be performed with a certain degree of precision by eye-measure, but a simple auxiliary instrument will be of great use. A transparent circular sheet (fig. 80) can slide in a ring, which has the divisions 0 to 63 or a certain number of these divisions. On the sheet is drawn a diameter and a set of lines parallel to it. Millimetric distance between them will in most cases be convenient. The ring is guided so that it has invariable orientation relatively to the system of coordinates. Thus if cartesian coordinates are used, the ring is guided so that it can perform any motion of translation without rotation. In the case of our charts in conical projection the ring is attached to the rule R, which always passes through the point of convergence of the meridians. (Compare fig. 62.) The sheet has a small perforation at the center, which allows us to mark the points where the desired values of the angle are found.

The sheet is guided in such a way that its center (the hole) follows one of the given curves. During the displacements it is turned so that the diameter remains tangent to the curve. The adjustment to tangency will be very much assisted by the lines which are parallel to the diameter. Whenever the diameter points to one of the integer divisions on the ring we make a mark on the curve through the hole. In this manner the disk is guided along the given curves, and marks are made where the required isogonal curves should intersect them. Afterwards these isogonal curves can be drawn continuously. If they are made to pass precisely through the points marked they will always show oscillations in their course, due to the unavoidable errors accompanying the drawing of the given curves and the use of the differentiating instrument. But these irregularities are easily smoothed out on the final drawing of the curves.

It will be important to remember that the curves which we obtain by this instrument can be numbered so as to be the isogons of the curves s themselves, or so as to be the isogons of the curves which are normal to the curves s. We pass

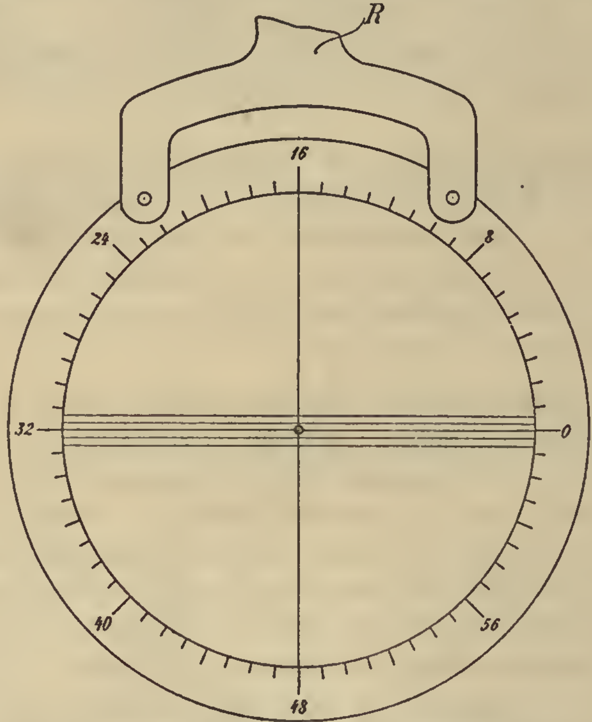


FIG. 80.—Divided sheet for directional differentiation.

from the isogons of the curves s to those of their positive normal curves by an addition of 16, to those of their negative normal curves by an addition of 48 to the numbers which the isogons have when they represent the curves s.

We have treated already (Chapter VII) the problem of integration which is inverse to the directional differentiation. Evidently the sheet of fig. 80, by which we perform the differentiations, may also be used to assist the integrations; and the integration-machine of fig. 62 or 63 can be considered as intrinsically the same instrument as that of fig. 80, only provided with special devices for facilitating the practical work connected with the integration.

164. **Linear Differentiation and Integration.**—Let the scalar a have a definite value at every point of a line s; *i. e.*, let a be a function of the length of arc s

$$a = a(s) \tag{a}$$

We represent this function by marking the points where it has certain integer values, $a_0, a_1, a_2, \ldots a_n, a_{n+1}, \ldots$ The expression "integer" must be taken in the same generalized sense of the word as before (section 147). The differences between the values of a in consecutive points will then also be expressed by "integer" numbers, and they must be small enough to be considered as differentials, $da = a_{n+1} - a_n$. The distance between the points will be the corresponding differentials of line ds, and the problem of differentiation will consist in forming the values of the quotient

(b) $$\varphi(s) = \frac{da}{ds}$$

at the different points of the line s.

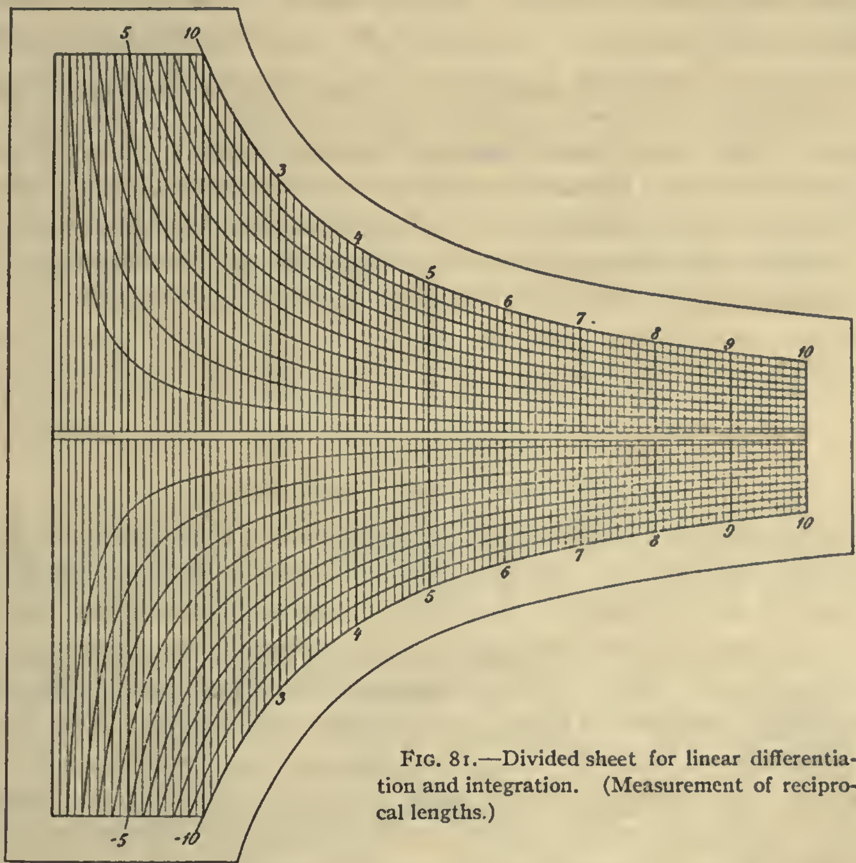


FIG. 81.—Divided sheet for linear differentiation and integration. (Measurement of reciprocal lengths.)

In order to construct a convenient auxiliary for the formation of the value of φ in one operation, we solve equation (b) with respect to da

(c) $$da = \varphi ds$$

We measure off $\varphi = x$ along the axis of abscissæ and $ds = y$ along the axis of ordinates of a rectangular system of coordinates. To each positive or negative integer value of da, viz, $da = \ldots -2, -1, 0, 1, 2, \ldots$ will then correspond an equilateral hyperbola $xy = \text{const.}$ The diagram of fig. 81 contains these curves together with a number of ordinates, one for each millimeter. Now let a value of the differential da be given, say $da = 4$. The abscissæ of the hyperbola $da = 4$ then gives the values of φ corresponding to the line-element ds measured off as ordinates. If the length of this element is given, we can read off on the axis of abscissæ the corresponding value of the ratio $\varphi = \frac{d\alpha}{ds}$. Instead of measuring the length ds between the hyperbola 4 and the axis of abscissæ, we can also measure it between the two symmetric hyperbolæ $d\alpha = +2$ and $d\alpha = -2$. This will as a rule be preferable.

For practical use we engrave the diagram on the under side of a transparent sheet of celluloid, and cut a narrow slit in this sheet along the axis of abscissæ. The slit should just be broad enough to make it possible to make marks with a sharp pencil on the paper below the sheet. The sheet is placed so that the line-element ds is parallel to the ordinates of the sheet. In the case $d\alpha = 4$ it will have one end-point on each of the two hyperbolæ $d\alpha = +2$ and $d\alpha = -2$. The reading on the axis of the abscissa gives the value $\varphi = \frac{d\alpha}{ds}$, which the derivative has in the central point of the line-element ds. This point can be marked through the slit. It will be clear how different hyperbolæ should be used according to the occurring values of the differential $d\alpha$. The procedure is illustrated by the upper line of fig. 82, where the points for integer values of the function α are marked on the upper side of the line, while the values determined for the derivative are noted on the under side.

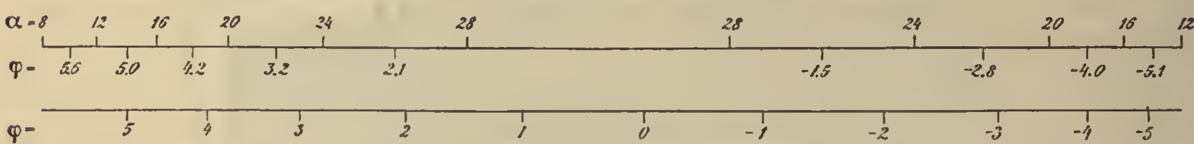


FIG. 82.—Linear differentiation or integration.

When the line-elements ds are short, a small error in the placement of the points where the given function has integer values will cause great errors in the values of the derivative. It will then be an excellent method of reducing these errors to measure two or more elements simultaneously. Thus if the points for all integer values $\alpha = 1, 2, 3, \ldots$ are given, we measure the corresponding elements two by two between the hyperbolæ $+1$ and -1; or we can measure them four by four between the hyperbolæ $+2$ and -2, and so on.

As it is seen, the direct use of the sheet gives the derivative at points where it has all sorts of fractional values. But it will be easy afterwards by interpolation to find the points where the derivative has certain integer values. In the example of fig. 82 these points are marked on the lower line.

We can now treat the inverse problem, the linear integration. Then let the function $\varphi(s)$ be given. The problem is to determine any function $\alpha(s)$, which is in the relation to the given function φ which is defined by equation (b) or (c). Evidently this can be done by the same divided sheet. For the sheet at once gives those lengths ds to which integer increases $d\alpha$ will correspond. The process of integration must begin at a certain initial point $s = s_o$ and we presume that at this the required function has a given value $\alpha = \alpha_o$.

Now let the value of the given function in the region of this initial point be $\varphi = \varphi_o$. In order to find the point where α has the value $\alpha_o + 4$ we set off from the initial point a length ds equal to the ordinate which the hyperbola $d\alpha = 4$ has for the value φ_0 of the abscissa. Using the value $\varphi = \varphi_1$ which φ has in the region of this new point, we measure off in the same manner the length ds, which leads to the point where α has the value $\alpha_0 + 8$, and so on. Inasmuch as α_0 is integer, we find in this manner points for integer values of the function α. If we wish to proceed by other steps $d\alpha$, we use other hyperbolæ.

The marking off of the successive points can be made without removing the sheet from the paper. Thus in the case of $d\alpha = 4$ the sheet is placed with the hyperbola 4 on the point from which the length ds is to be measured. The new point can then be marked through the slit in the sheet.

We have spoken above of the value which the given function φ has in the "region" of the point from which the length ds is to be measured. This "region" will have a maximal extent equal to the length of the line-element ds. The use of one value or another of φ from this region will give no appreciable difference in the lengths ds obtained, if these lengths are sufficiently short; but the greater the lengths ds used, the more important it will be not to choose an arbitrary value of φ in this region, *but the average value of φ along the element ds.* As the approximate length of ds is seen at once, it will cause no difficulty to find a sufficiently approximate value of this average value of φ, and to use it for the final determination of ds.

Evidently the function $\alpha(s)$ which we determine by the process described will be that which is expressed analytically by the integral

(*d*) $$\alpha = \alpha_0 + \int_{s_0}^{s} \varphi(s)\, ds$$

Fig. 82 can be used to examplify this graphical integration. We then consider the divisions $\varphi = 5, 4, 3, \ldots$ on the lower line as given, and find by use of the divided sheet the divisions $\alpha = 12, 16, 20, \ldots$ on the upper line.

165. Application to Two-Dimensional Scalar Fields.—The application of the described process of linear differentiation to scalar fields in two dimensions will be the most important graphical differential operation. It will return in most of the more complex differentiation-problems.

Let the two-dimensional scalar field be represented by a system of equiscalar curves

(*a*) $$\alpha = \alpha_0 \qquad \alpha = \alpha_1 \qquad \alpha = \alpha_2 \quad \ldots$$

where $\alpha_0, \alpha_1, \alpha_2, \ldots$ are integer values in the widened sense of the word as defined above. Let further a system of curves s be given which cut through the field. (Compare fig. 83.) The scalar α will then have a definite value at each point of a curve s. On each of these curves the scalar α will appear as a function of the length of arc s. We can therefore perform a linear differentiation along each curve s, using the divided sheet as described in the preceding section. In this manner we find the value of the derivative

(*b*) $$\varphi = \frac{d\alpha}{ds}$$

at a great number of points. Afterwards we can draw curves for integer values of φ, and thus arrive at the common representation of the field of the scalar φ, which is the derivative of the scalar α.

In the way described we arrive at the field of φ by a discontinuous process. But it can be changed at once into a *continuous* one. Instead of moving the differentiating sheet along the curves s, we move it along the curves α = const., and measure the line-elements which are contained between two curves $\alpha = \alpha_0$ and $\alpha = \alpha_1$. When we come to places where the element ds is seen to give one of the required integer values of φ we make a mark through the slit of the sheet. In this manner we mark points through which the required curves for integer values of φ are to go. Afterwards these curves can be drawn continuously through the points determined.

Vice versa the problem of determining the field of α when that of φ is given, *i. e.*, the problem of integration, will be determinate when an initial value of α is given at

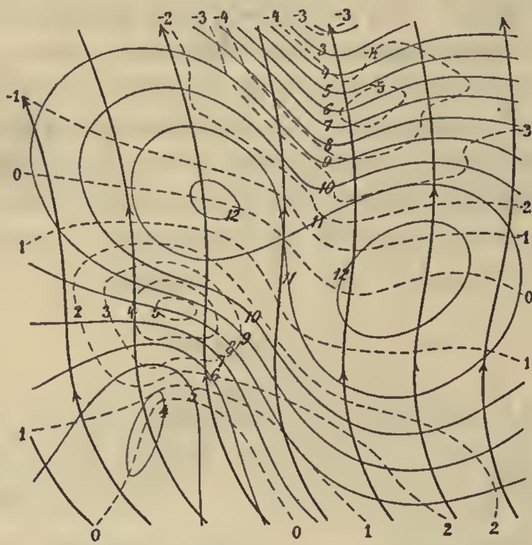


FIG. 83.—The curves s are represented by thick lines with arrow-heads; the curves $\alpha = 12, 11, 10, \ldots$ by fine continuous lines; and the curves $\varphi = \frac{d\alpha}{ds} = \ldots 2, 1, 0, -1, -2, \ldots$ by stippled lines.

one point of each curve s, for instance when an initial curve $\alpha = \alpha_0$ is given. The measurements which are to give the values of α at other points can be performed along one after another of the curves s as described in the preceding section. Or they can be performed first along the initial curve $\alpha = \alpha_0$ in order to determine the points of the next curve $\alpha = \alpha_1$; then along this second curve in order to determine the next curve $\alpha = \alpha_2$, and so on. Both methods are continuous.

It will much facilitate the drawing of the field of the derivative to observe that the curve $\varphi = 0$ can be drawn at once, without any use of the differentiating sheet; for evidently this curve will pass through all the points of tangency of the curves s with the curves α = const., including the points of maximum, minimum, or maximum-minimum, at which the curve α = const. is reduced to a point or cuts itself. Vice versa we conclude that when the field of φ is given, and that of α shall be determined by integration, the curves α = const. must have tangency with the curves s at the points where these curves are cut by the curve $\varphi = 0$.

As we shall make an extensive use of the process of differentiation described, it will be important to direct the attention to the character of the errors which will enter, and the methods of diminishing their influence. Let us for this purpose consider the derivatives of the two fields which are given by fig. 84 A and B. In both cases the lines α = const. have the same general course and the same *average* distance from each other; but on the first figure the distance varies in a regular way, and in the second it shows small irregularities in its variations. The curves which represent the field of the differential quotient are then seen to be very different in the two cases. In the first case they have a regular course, while in the second they show great sinuosities.

Now a free off-hand drawing which should represent a field as that of the first figure will in consequence of the unavoidable errors get more or less the character of the second figure. *Thus the irregularities in the drawing of the given field will cause oscillations in the course of the curves representing the field of the derivative.* But as the oscillations will go equally to both sides, they will be easy to reduce afterwards.

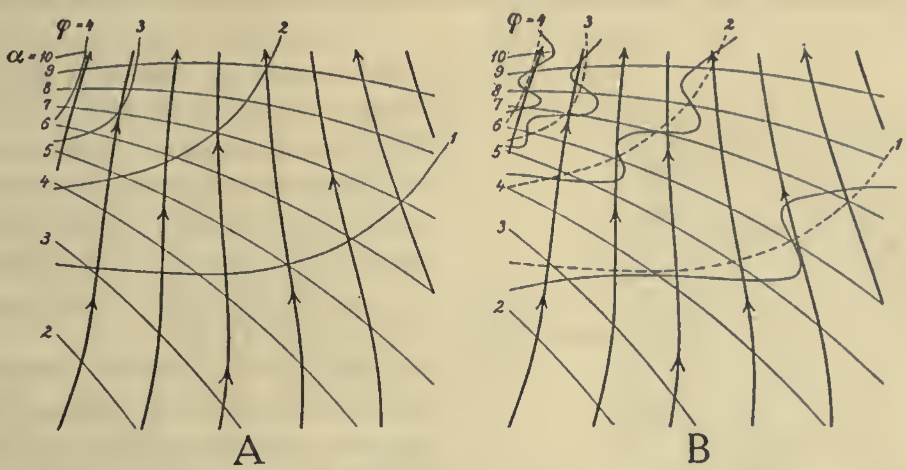


FIG. 84.—Regular course (A) and oscillating course (B) of the curves representing the differential-quotient $\varphi = \frac{d\alpha}{ds}$

A good method of diminishing them from the beginning will be to measure the line-elements not one by one, but two by two or even more of them at a time. On the divided sheet we can always find the proper hyperbolæ for doing this. But the final correction will always consist in reducing those sinuosities which are seen to arise from errors in the drawing and not from the true nature of the given field. By this correction *à posteriori* of the field of the derivative, we get a determination of this field which by far exceeds the accuracy of the single measurements upon which the process of differentiation depends.

For the process of integration, the irregularities in the drawing of the given field will cause no errors of greater importance. The process of integration itself involves a formation of averages, by which the consequences of the irregularities in the drawing are reduced.

166. Other Forms of the Problem of Linear Differentiation and Integration.—Instead of constructing an auxiliary sheet for the determination of the differential quotient itself, $\frac{da}{ds}$, of a given function a, we can construct a sheet for the determination of any function of this differential quotient

$$(a) \qquad \varphi = f\left(\frac{da}{ds}\right)$$

The sheet which allows us to derive this function $\varphi(s)$ from the given function $a(s)$ will also allow us to solve the corresponding problem of integration, viz, when $\varphi(s)$ is given to determine the function $a(s)$ which is defined as function of φ by the differential equation (a).

In order to construct this auxiliary sheet we solve equation (a) with respect to $\frac{da}{ds}$ and obtain $\frac{da}{ds} = F(\varphi)$ or

$$(b) \qquad da = F(\varphi)ds$$

As in the preceding case, we consider $\varphi = x$ as the abscissa and $ds = y$ as the ordinate of a point, and construct the curves $F(x)y = \ldots -2, -1, 0, 1, 2, \ldots$ to positive or negative integer values of da (fig. 85). When a value of da and a value of ds are given, we have a certain curve given on the sheet and a certain ordinate belonging to this curve. The corresponding abscissa then gives the value of the function $\varphi = f\left(\frac{da}{ds}\right)$. This gives the solution of the problem of differentiation. If on the other hand $\varphi(s)$ is given the corresponding ordinate up to a certain curve, da gives the length ds, for which we have a certain integer increase in the value of the required function a. This leads to a method of determining step by step a series of points at which the function

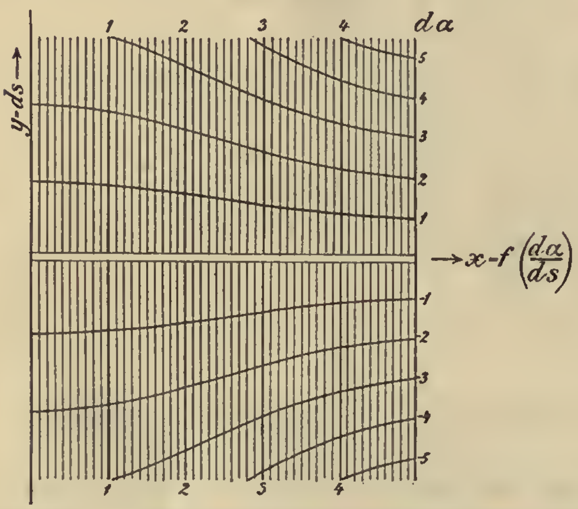


FIG. 85.—Divided sheet for determination of the field of a function $f\left(\frac{da}{ds}\right)$

$$(c) \qquad a = a_0 + \int_{s_0}^{s} F(\varphi(s))ds$$

has given integer values. The procedure is precisely the same as in the preceding case.

We shall consider only one simple example. Let $f\left(\frac{da}{ds}\right) = \frac{ds}{da}$. We shall then determine

$$(d) \qquad \varphi = \frac{ds}{da}$$

that is, we shall determine simply the lengths ds between the points where the function a has the integer values $1, 2, 3, \ldots$ Corresponding to equation (b) we then get

$$(e) \qquad da = \frac{ds}{\varphi}$$

If we consider $\varphi = x$ as abscissa and $ds = y$ as ordinate, the constant values of da give straight lines $\frac{y}{x}$ = const. which pass through the origin of the coordinates. Fig. 86 contains these lines for "integer" values $da = -2.10^{-1}, -1.10^{-1}, 0, 1.10^{-1}, 2.10^{-1} \ldots$ We get the well-known sheet for direct measurements of lengths. When the line-elements are short, it will be convenient to measure them two by two between the lines $da = +1.10^{-1}$ and $da = -1.10^{-1}$. The abscissa will then give the length in ten-fold enlargement, with a corresponding increased accuracy of the reading. Longer elements can be measured without any enlargement by use of the lines $da = \pm 1$.

167. Curves of Equal Intensity and Curves of Equal Transport.—In order to give an example of the use of this sheet, we shall treat the following problem: to draw curves for equal transport when the vector-lines and the intensity-curves of a vector are given. The curves of equal transport will be determinate

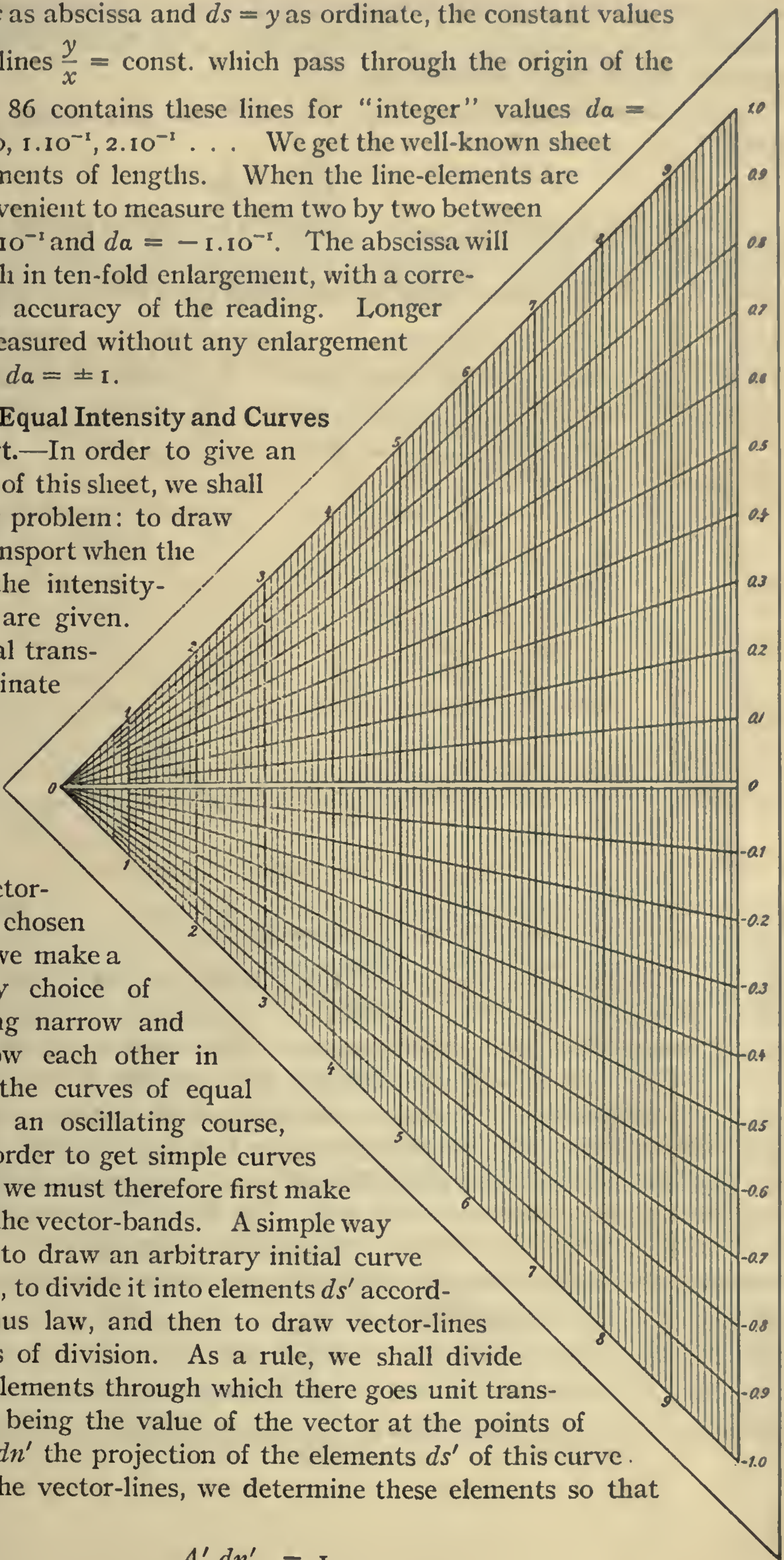


FIG. 86.—Divided sheet for direct length-measurements.

only when the vector-bands have been chosen (section 119). If we make a perfectly arbitrary choice of these bands, letting narrow and broad bands follow each other in an irregular way, the curves of equal transport will get an oscillating course, see fig. 84 B. In order to get simple curves of equal transport, we must therefore first make a careful choice of the vector-bands. A simple way of doing it will be to draw an arbitrary initial curve C' of regular shape, to divide it into elements ds' according to a continuous law, and then to draw vector-lines through the points of division. As a rule, we shall divide the curve C' into elements through which there goes unit transport; that is, A' being the value of the vector at the points of the curve C', and dn' the projection of the elements ds' of this curve on the normal to the vector-lines, we determine these elements so that for each of them

(a) $$A' \, dn' = 1$$

(see fig. 87). This principle for dividing the curve C' into elements has the advantage that it at once leads to the determination of the bands of unit transport in the cases where the vector is solenoidal.

The vector-bands being chosen, we know that the transport T is given by the product

$$(b) \qquad T = A \; dn$$

A being the intensity of the vector and dn the breadth of the band. In order to find the field of the scalar T, we have first to form the field of the line-element dn. This is done by making continuous use of the divided sheet for direct length-measurements

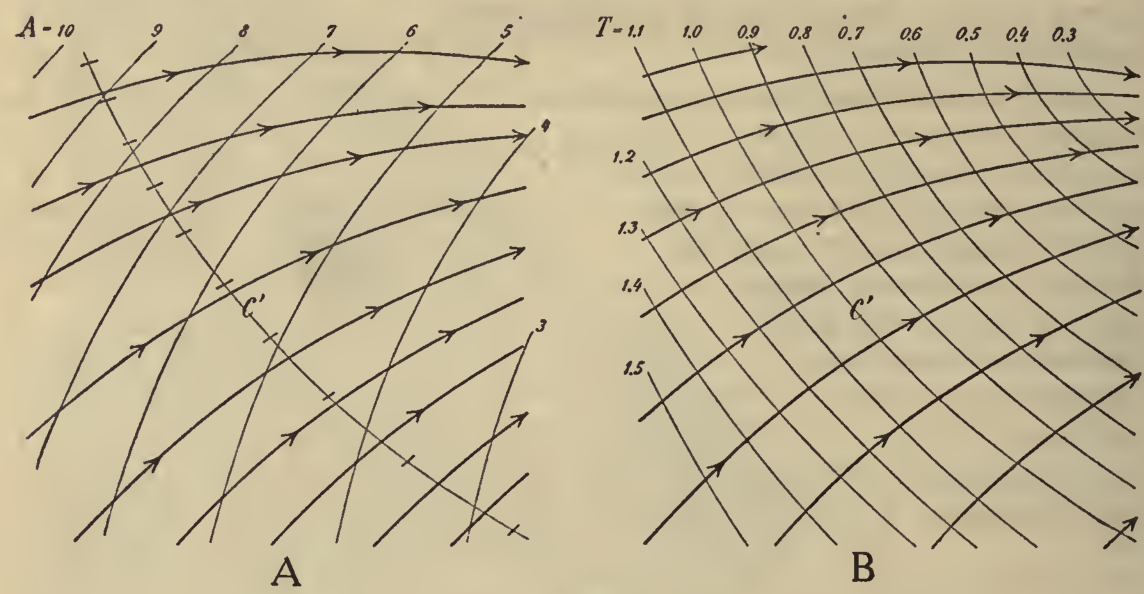


FIG. 87.

A. Vector curves s (with arrow-heads) and intensity curves A = 10, 9, 8, 7, . . . (fine continuous lines).
B. Vector curves s (with arrow-heads) and curves of equal transport T = 1.1, 1.0, 0.9, . . . (fine continuous lines).

(fig. 86). The curves n along which the line-elements dn should be measured need not be drawn; for the sheet can with the same ease be placed with its ordinates normal to as parallel to the given curves s. Afterwards the graphical multiplication of the field of dn by that of the intensity A gives the field of transport T, corresponding to the vector-bands. When the elements of the initial curve C' fulfil the condition (a) this curve will appear as the curve $T = 1$ in the field of transport.

That the use described of the divided sheet is a process of differentiation from the analytical point of view is thus seen: The choice of vector-lines by the division of the initial curve C' into elements corresponds to the choice of a continuous scalar function α which has these vector-lines for equiscalar curves $\alpha = 1, 2, 3, \ldots$ T will then be expressed by the equation:

$$(b') \qquad T = A \frac{dn}{d\alpha}$$

where $d\alpha = 1$ for the chosen interval between the successive curves.

If we wish to return from the field of transport to that of intensity of the vector A, we have to use the formula

(c) $$A = T\frac{1}{dn}$$

or corresponding to (b′)

(c′) $$A = T\frac{da}{dn}$$

We then use the common differentiating sheet for forming the field of $\frac{1}{dn}$ or $\frac{da}{dn}$ and afterwards perform the graphical multiplication of this derivative with the scalar T.

168. Differentiations of Higher Order. Curvature and Divergence of a System of Curves.—The processes described of directional or of linear differentiations can be repeated any number of times. By use of the auxiliaries which we have described

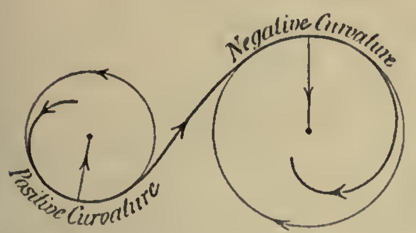


FIG. 88.—Positive and negative curvature of a curve.

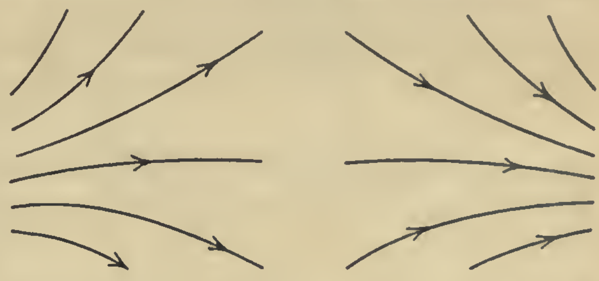


FIG. 89.—Positive and negative divergence of a system of curves.

we can thus form a derivative of any order. In precisely the same manner the process of integration can be repeated, and will then lead back to the primary function from a derivative of any order.

A case of special importance is that in which a directional differentiation is succeeded by a linear one.

In order to consider this case let us suppose that a system of curves s is given. By directional differentiation we can derive the angle a which represents the direction of the tangent to these curves and represent the field of this angle by the isogonal curves

(a) $$a = \text{const.}$$

Upon the field of the scalar a we can perform a linear differentiation, which will then show the variation from place to place of the angle a. Let this linear differentiation be performed along the direction of the originally given curves s themselves. This derivative

(b) $$\gamma = \frac{da}{ds}$$

represents the change of direction of the tangent per unit length along the curve, *i. e.*, the *curvature* of the curves s. The differentiation can be performed as described in section 165 by use of the divided sheet of fig. 81, and will give the *field of curvature* of the given system of curves s.

It must be remarked that (*b*) defines curvature as a quantity which has a definite sign. This sign depends on the direction for the positive increase of the angle (sec. 155), and the positive direction along the curve *s*. It is seen at once that the rule of signs can be given in this form:

An element ds of a curve has positive or negative curvature according as it determines positive or negative circulation on the osculating circle (fig. 88).

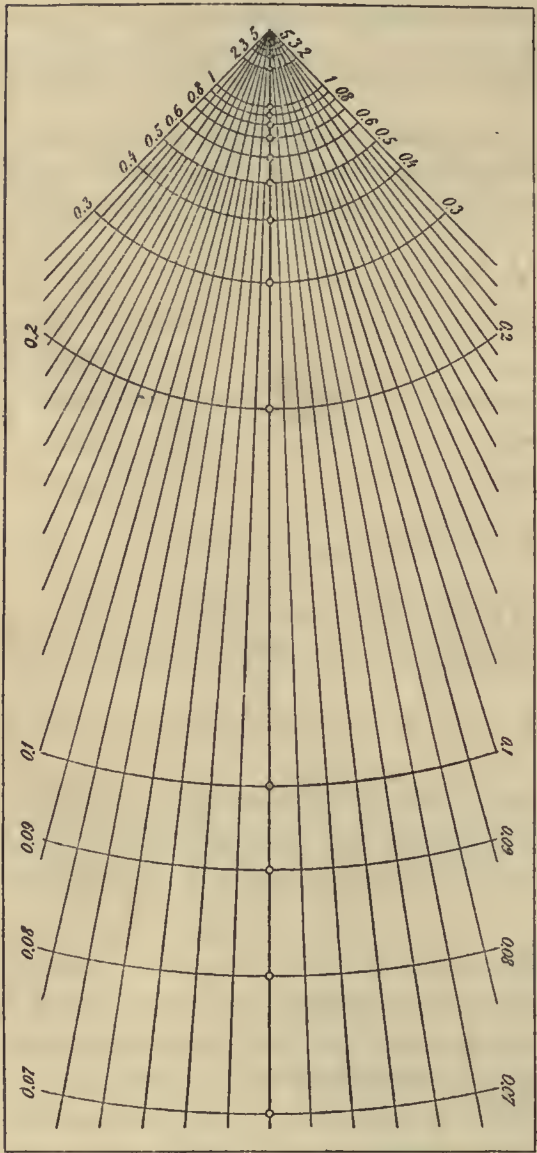


Fig. 90.—Divided sheet for the determination of curvature and divergence of curves.

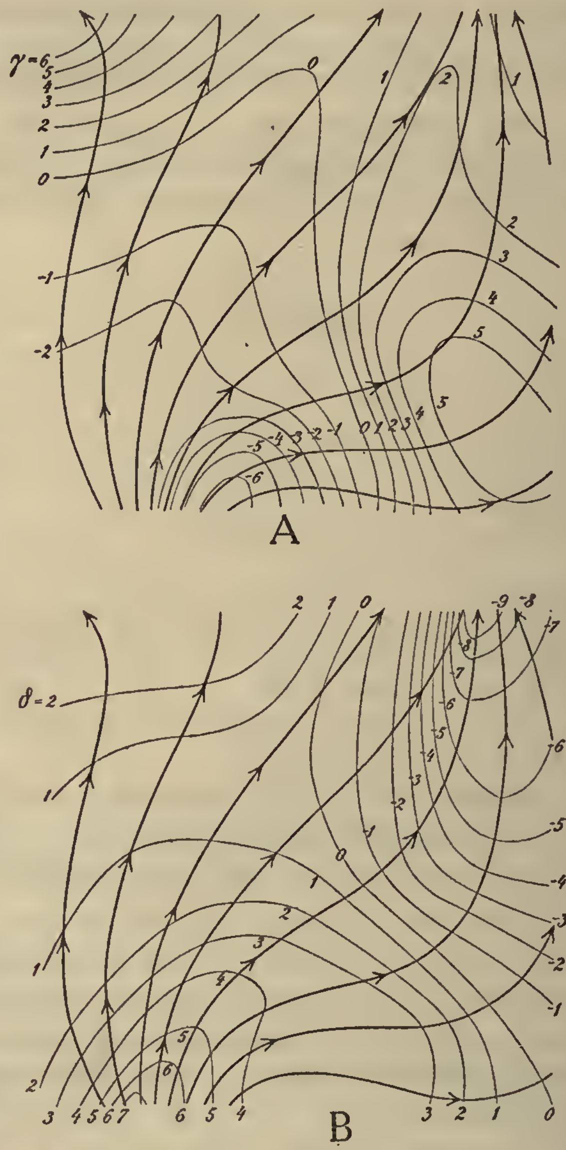


Fig. 91.—Field of curvature (A) and field of divergence (B) of a system of curves.

For our charts in horizontal projection the same rule can be stated thus:

Curvature will be positive or negative according as an observer who looks in the positive direction of the curve has the center of curvature to the left or to the right.

Or, let the linear differentiation be performed along the direction of the normal n to the curves s. This derivative

$$(c) \qquad \delta = \frac{d\alpha}{dn}$$

will represent the change of direction per unit length when we proceed *normally from curve to curve* instead of tangentially along one and the same curve s. It therefore shows how the different curves s diverge from each other. Equation (c) gives the field of divergence of the given system of curves s. This field can also be found by use of the divided sheet of fig. 81, and it will not be necessary to draw the normal curves n, as the sheet can be placed with the same ease both with its ordinates normal to and parallel to the given curves s.

When we remember our definition of the positive normal n to a given direction s (section 155) we see that formula (c) contains the following rule for the sign of the divergence δ:

> *Divergence of a system of curves will be positive or negative according as they appear to an observer looking in the positive direction of the curves to diverge or to converge* (fig. 89).

As will be seen at once, there is a close relationship between the fields of divergence and of curvature. The field of divergence of a system of curves is the field of curvature to the normal curves, and vice versa the field of curvature is the field of divergence to the normal curves.

The derivatives (b) and (c) are derivatives of the second order in reference to the originally given system of curves. The two successive operations, consisting in a directional and a subsequent linear differentiation, can be combined into one which represents a differentiation of the second order and which can be performed by the divided sheet of fig. 90. This sheet contains a set of concentric circles with integer values (multiplied by a power of 10) of the curvature, *i.e.*, integer reciprocal values of the length of the radii and a set of divergent radii with equal and small angular intervals. For continuous use the sheet is perforated at the points of intersection of the circles with the central radius.

This sheet can be placed directly upon the field of the system of curves s originally given. In order to find the field of curvature (fig. 91A) we place it with the circles tangential to and the radii normal to the curves s. One after another of the curves s is followed, and the points are marked where these curves give complete osculation with one of the circles of the sheet. In order to find the field of divergence (fig. 91B) we place the sheet with the circles normal to and the radii tangential to the curves s. One after another of the curves s is followed, and the points are marked where the circles osculate the normal curves, *i. e.*, the points where one of the circles is normal to the curves next to the considered curve s. As a supplementary condition we have that the radii shall be tangential to the curves at the points where these radii are cut by the circles.

169. Partial Derivatives; Ascendant and Gradient.—The two-dimensional scalar α is a function of two coordinates which figure as independent variables.

Now let us consider the curves s as the one set of coordinate-curves. The derivative of a scalar α with respect to s will then be the one partial derivative of the dependent variable α. It is a special case that the curves s are parallel and equidistant straight lines. If we use two such systems of lines which are normal to each other, and call the length of arc along the one set x, and along the other set y, the two partial derivatives will be

$$(a) \qquad F_x = \frac{\partial \alpha}{\partial x} \qquad F_y = \frac{\partial \alpha}{\partial y}$$

The fields of these partial derivatives can be determined by use of the divided sheet of fig. 81.

The two partial derivatives are the rectangular components of the *ascendant* **F** of the scalar. As we have shown already (Statics, section 17), this vector is directed along the normal n to the equiscalar curves α = const., and is numerically equal to the derivative of α with respect to the length of arc n measured along the normal curves

$$(b) \qquad F = \frac{d\alpha}{dn}$$

In order to abbreviate we shall introduce here a useful notation. The fact that the vector **F** is in the defined relation to the scalar α will be expressed by the single vector-equation

$$(c) \qquad \mathbf{F} = \nabla \alpha$$

This equation is by definition equivalent to the two scalar equations (a), and in the case of the three-dimensional field it will be equivalent to three such equations. A vector **G** of the opposite direction

$$(d) \qquad \mathbf{G} = -\nabla \alpha$$

represents the gradient of the scalar α.

The field of the ascendant or of the gradient can be found by algebraic methods (section 157) from the fields of the two rectangular components; but it can also be derived directly from the field of the given scalar α. This direct method will involve separate determinations of the direction and of the magnitude of the vector.

The vector-lines can be drawn at once as normal curves to the equiscalar curves α = const. If we wish to have the direction represented by isogons, we have to use the directional differentiation described in section 163, and to give the isogons such numbers that they represent the direction of the normal curves n, not of the equiscalar curves α = const.

The intensity-field of ascendant or gradient are found by use of the differentiating sheet of fig. 81 in accordance with formula (b). The field will contain no zero-curve. It will only have zero-points at the points of maximum, minimum, and maximum-minimum of the scalar α. These zero-points will be singular points of intersection of the vector-lines as well as of the isogons. As points for absolute minimum of the scalar value of the vector they will be surrounded by closed curves of equal intensity. The drawing of the field is therefore very much facilitated by the circumstance that these zero points are given beforehand.

Fig. 92 represents the ascendant of the same field of which fig. 83 represents a partial derivative.

From the field of the ascendant (a) we can derive that of any other derivative

(e) $$F_s = \frac{\partial a}{\partial s}$$

as we have

(f) $$F_s = F \cos \theta$$

where θ is the angle between the directions n and s. This algebraic method of finding the partial derivative F_s will be convenient if the direction of the ascendant **F** is represented by isogonal curves φ = const., and the direction of s by isogonal curves σ = const. We shall then pass from the field of **F** to that of F_s by the following operations (compare sections 149, 156).

(1) By graphical subtraction we form the field of the angle $\theta = \varphi - \sigma$. In the drawing of these auxiliary curves special attention should be attached to the drawing of the curves $\theta = 16$, and $\theta = 48$, which will be curves $F_s = 0$ in the resultant field.

(2) By use of these auxiliary curves and the curves F = const., we derive the field of the scalar value of F_s according to equation (c) by use of the first of tables K.

By this process we can thus derive the partial derivative of fig. 83 from the ascendant-field of fig. 92.

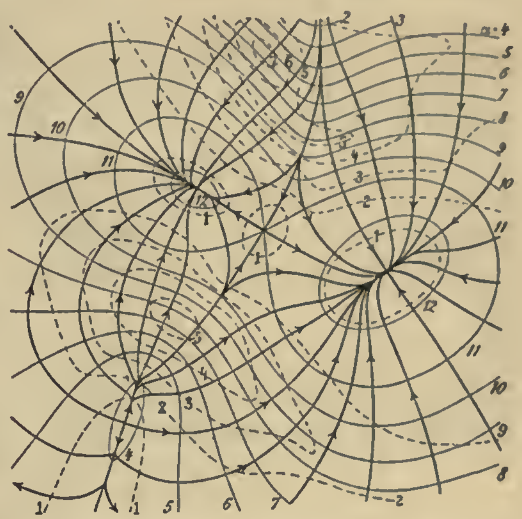


FIG. 92.—Scalar field $a = 12, 11, 10, \ldots$ (fine continuous lines), vector-lines of the ascendant (thick lines with arrow-heads), and intensity-curves of the ascendant (stippled curves).

If we know the field of the ascendant or the gradient and the value of the scalar a at a single point, we can reconstruct the field of the scalar a. The simplest method will be this: We first perform a linear integration along the particular curve n which passes through the point where we have the known value of a. By this integration we find a series of points through which equiscalar curves representing the required integer values of a shall pass. Through these points we may then by directional integration draw the curves a = const., normal to the vector-curves of the ascendant or the gradient.

170. Divergence of a Two-Dimensional Vector.—We have considered already the "transport" in the two-dimensional field (section 119), *i. e.*, the integral of the normal component A_n of a vector taken along a curve.

(a) $$\int A_n \, ds$$

In the special case of a closed curve the transport directed outward was called the "outflow" out from the area limited by the curve.

This outflow has a simple additive property. Let the considered area be divided by a line into two parts (fig. 93). The transport through the dividing line will then appear in the expression for the outflow out of each part. But in the sum of these two outflows this transport will drop out, as it represents the transport *out of* the one and *into* the other area. The sum of the outflows out of the two parts will therefore be equal to the outflow out of the total area. As each part can be divided again, and so on, we get the general result that the outflow out of all the parts into which an area can be divided will be equal to the outflow out of the total area. We symbolize this result by the equation

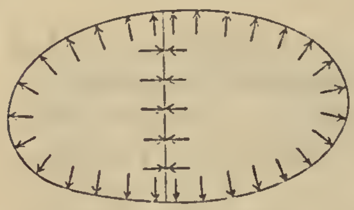


FIG. 93.

(*b*) $$\int A_n\,ds = \Sigma \int A_n\,ds$$

the first member being extended to the contour of the total area, and the integrals in the second member being extended to the contours of all the parts into which the total area has been divided.

The division may be continued indefinitely. The areas of which the contours appear in the second member of equation (*b*) may therefore be considered as elementary areas $d\sigma$. As they can have any form let them be limited by the two elements ds and ds' of two vector-lines, and by the two elements dn and dn' which are normal to these lines (fig. 94). The outflow will be the difference between the transport $A'dn'$ and Adn through the latter elements.

(*c*) $$A'dn' - Adn$$

Here A' will vary as we proceed along a vector-line s, and the same will be the case with the normal distance dn' between the two vector-lines. We may then consider these quantities as functions of s and use the developments

$$A' = A + \frac{\partial A}{\partial s}ds \qquad dn' = dn + \frac{\partial dn}{\partial s}ds$$

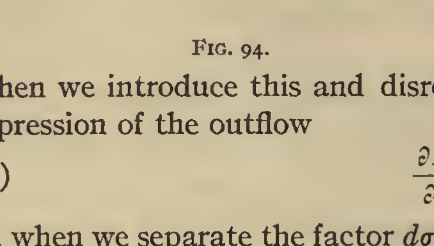


FIG. 94.

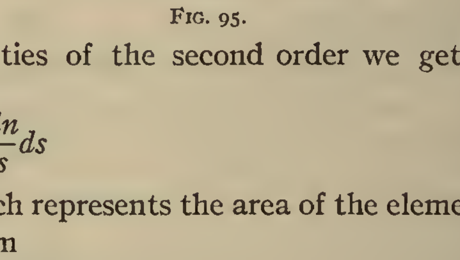


FIG. 95.

When we introduce this and disregard quantities of the second order we get as expression of the outflow

(*c'*) $$\frac{\partial A}{\partial s}dsdn + A\frac{\partial dn}{\partial s}ds$$

or, when we separate the factor $d\sigma = dnds$ which represents the area of the element, we get the expression of the outflow in the form

(*c''*) $$\left(\frac{\partial A}{\partial s} + A\frac{1}{dn}\frac{\partial dn}{\partial s}\right)d\sigma$$

We have thus expressed the outflow through the contour of an elementary area as the product of the area of the elements and a factor which must then represent the *outflow per unit area*. We shall call this outflow per unit area the *two-dimensional divergence* of the vector **A** and introduce the notation

(*d*) $$\text{div}_2\mathbf{A} = \frac{\partial A}{\partial s} + A \cdot \frac{1}{dn}\frac{\partial dn}{\partial s}$$

We can now write every term in the sum which forms the second member of equation (*b*) in the form $\text{div}_2\mathbf{A}\, d\sigma$. The sum then takes the form of an integral extended to all the elements of area $d\sigma$; that is, we get the formula

(*e*) $$\int A_n ds = \int \text{div}_2\mathbf{A}\, d\sigma$$

or expressed in words:

The integral of the normal component of a two-dimensional vector taken around a closed curve is equal to the integral of the two-dimensional divergence of this vector taken over the area bordered by the closed curve.

The two-dimensional divergence, or the outflow per unit area, can be found by a process of differentiation given by equation (*d*). The last term has a simple geometrical sense. As dn represents the elementary distance between two curves s, the derivative $\frac{\partial dn}{\partial s}$ will evidently represent the elementary angle $d\alpha$ between the tangents of two curves s which have the distance dn from each other (see fig. 95). Thus we get

(*f*) $$\frac{1}{dn}\frac{\partial dn}{\partial s} = \frac{d\alpha}{dn} = \delta$$

where δ is the divergence of the system of curves s as defined in section 168. Thus the two-dimensional divergence of the vector **A** can be written in the form

(*g*) $$\text{div}_2\mathbf{A} = \frac{\partial A}{\partial s} + A\delta$$

where δ is the divergence of the vector-lines. When in this formula we give the vector **A** the constant scalar value 1, we get $\text{div}_2\,\mathbf{A} = \delta$, which shows that the divergence of a system of curves is equal to the divergence of a unit vector which has these curves as vector-lines.

By formula (*g*) we have reduced the construction of the field of divergence of a two-dimensional vector to graphical differentiations which we have performed already. We shall find it by the following series of operations:

(1) We perform the graphical differentiation of the intensity-field of the given vector with respect to its vector-lines. (See fig. 83, where we can interpret the curves s as the vector-lines and the given scalar field as the intensity-field of the given vector.)

(2) We form the field of divergence of the vector-lines, using either of the two developed methods according as the isogons of the vector or the vector-lines themselves are given. (See section 168.)

(3) We perform the graphical multiplication of the intensity-field of the vector and the divergence-field of its vector-lines.

(4) We perform the graphical addition of the two fields obtained by the operations (1) and (3).

The construction described will be of great importance for the kinematic diagnosis of air- and sea-motions.

Other expressions of the divergence will also be useful. If the vector-lines happen to run at an invariable distance dn from each other, we shall have the divergence of the vector-lines equal to zero, and the divergence of the vector will be given by one term only, $\frac{\partial A}{\partial s}$. Now, when we express the field of the vector **A** by the fields of its two cartesian components A_x and A_y, the component-fields have straight and parallel lines of flow. The divergence of the two component-fields will be respectively $\frac{\partial A_x}{\partial x}$ and $\frac{\partial A_y}{\partial y}$ and their sum will give the divergence of the resultant field.

$$(h) \qquad \mathrm{div}_2\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y}$$

When the fields of the rectangular components A_x and A_y are given, this expression gives a simple construction of the fields of divergence. By graphical differentiation we form separately the fields of $\frac{\partial A_x}{\partial x}$ and of $\frac{\partial A_y}{\partial y}$, and then by graphical addition that of $\mathrm{div}_2\mathbf{A}$.

Inasmuch as coordinate-methods should be used on our charts, it must be remembered that the meridians are not equidistant coordinate-curves. The divergence of the meridians must be taken into account when the divergence of a velocity-field should be formed separately from charts of the south–north component and of the west–east component of the wind.

171. Divergence of a Vector in Space.—The two-dimensional divergence which we can represent on our charts will have its importance as part of the three-dimensional divergence of that vector in space of which the two-dimensional vector is a component. We shall therefore also consider the divergence of a vector in space.

Transport in the vector-field in space is represented by the surface-integral of the normal component of the vector

$$(a) \qquad \int A_n d\sigma$$

In the case when the surface σ is closed the transport will represent the outflow of the volume bounded by the closed surface (see section 111).

If we divide a given volume into any number of parts and form the sum of the outflows out of each part, the transport through the dividing surfaces will cancel, and we find that the outflow in three dimensions has the same additive property as it has in two dimensions. This property can be expressed by the formula

$$(b) \qquad \int A_n d\sigma = \Sigma \int A_n d\sigma$$

where the integral appearing as the first member is extended to the limiting surface of the total volume, and the integrals appearing in the second member are extended to the limiting surfaces of the different parts into which the total volume is divided.

Now let the total volume be divided into elementary volumes, consisting of infinitely short trunks of infinitely narrow vector-tubes. There will be a transport only through the surface-elements $d\sigma$ and $d\sigma'$ which form sections of the tube (fig. 96). These sections being normal, we get the transport through them equal respectively to $Ad\sigma$ and $A'd\sigma'$, and the outflow equal to their difference

(c) $$A'd\sigma' - Ad\sigma$$

Here we can develop A' and $d\sigma'$ as functions of the length of arc s along the axis of the tube

$$A' = A + \frac{\partial A}{\partial s} ds \qquad d\sigma' = d\sigma + \frac{\partial d\sigma}{\partial s} ds$$

When we introduce this and leave the term of the second order out of consideration, we get the expression of the elementary outflow (c) in the form

(c') $$\frac{\partial A}{\partial s} ds d\sigma + A \frac{\partial d\sigma}{\partial s} ds$$

Introducing the volume of the element $d\tau = d\sigma ds$, this expression may be written

(c'') $$\left(\frac{\partial A}{\partial s} + A \cdot \frac{1}{d\sigma} \frac{\partial d\sigma}{\partial s}\right) d\tau$$

Thus for elementary volumes the outflow is proportional to the volume of the element. The factor of proportionality represents the outflow per unit volume, and

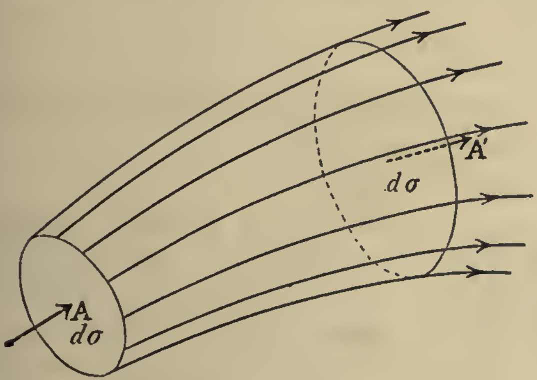


FIG. 96.

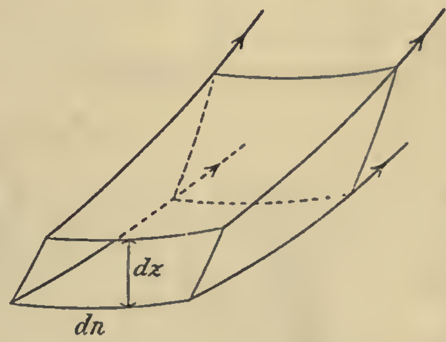


FIG. 97.

is called the *three-dimensional divergence* or simply the *divergence* of the vector **A**

(d) $$\text{div}\,\mathbf{A} = \frac{\partial A}{\partial s} + A \cdot \frac{1}{d\sigma} \frac{\partial d\sigma}{\partial s}$$

As we can now write each term in the second member of equation (b) in the form div **A** $d\tau$, this second member takes the form of a sum extended to all the elements of volume $d\tau$, *i. e.*, the form of a volume-integral. We thus get the important formula

(e) $$\int A_n d\sigma = \int \text{div}\,\mathbf{A}\, d\tau$$

or in words:

The integral of the normal component of a vector taken over a closed surface is equal to the volume-integral of the divergence of the vector taken in the volume limited by the closed surface (Gauss's theorem).

This theorem allows us to bring the solenoidal condition—section 112 (*a*)—into a new form; for when the surface-integral in equation (*e*) is zero for every closed surface in the field, the volume-integral must also be identically zero, and this involves

$$\text{div } \mathbf{A} = 0 \tag{f}$$

This is the differential form of the solenoidal condition.

The expression $\frac{1}{d\sigma}\frac{\partial d\sigma}{\partial s}$ which appears in the equation of definition (*d*) has a similar significance in space as $\frac{1}{dn}\frac{\partial dn}{\partial s}$ in two dimensions. When the area $d\sigma$ of the cross-section of the tube is constant, the considered trunk of the tube may be compared to a cylinder. When $d\sigma$ varies, the trunk of the tube may be compared to a cone, and the derivative $\frac{\partial d\sigma}{\partial s}$ will represent its solid angle. Then $\frac{1}{d\sigma}\frac{\partial d\sigma}{\partial s}$ will represent the ratio of this solid angle to the cross-section of the tube and thus be a measure of what we may call the divergence of the curves *s* in space.

In order to express this divergence by the corresponding divergences in two dimensions we will consider vector-tubes which are produced in the usual way by the intersection of two sets of surfaces of flow (fig. 97). Each tube will then have the well-known parallelogrammatic cross-section. If dn is one side in the parallelogram, and dz the corresponding height, we have $d\sigma = dndz$, and get

$$\frac{1}{d\sigma}\frac{\partial d\sigma}{\partial s} = \frac{1}{dn}\frac{\partial dn}{\partial s} + \frac{1}{dz}\frac{\partial dz}{\partial s}$$

Introducing this in equation (*d*), we get this more developed form of the divergence

$$\text{div } \mathbf{A} = \frac{\partial A}{\partial s} + A \cdot \frac{1}{dn}\frac{\partial dn}{\partial s} + \mathrm{A} \cdot \frac{1}{dz}\frac{\partial dz}{\partial s} \tag{g}$$

The divergence is here given by a trinomial expression, the first two terms of which are seen to express the two-dimensional divergence—equation (*d*) of the preceding section—of the vector **A** in the surface which contains the curves *s* and *n*.

If we resolve the given vector-field into three component-fields, each with vector-lines coinciding with one set of coordinate-curves of a system of curvilinear orthogonal coordinates, we can write the divergence of each component-field in either of the forms (*d*) or (*g*). In the special case of a cartesian system the vector-lines of each component-field are straight and parallel. Each vector-tube will have a constant cross-section $d\sigma$, or constant base dn and height dz, and only the first term in the second member of formulæ (*d*) or (*g*) will be different from zero. Therefore, if we call the vectors of the three component-fields $\mathbf{A}_x$, $\mathbf{A}_y$, $\mathbf{A}_z$, and the lengths of arc measured along the vector-lines *x*, *y*, and *z*, we get for the divergence in each component-field

$$\text{div } \mathbf{A}_x = \frac{\partial A_x}{\partial x} \qquad \text{div } \mathbf{A}_y = \frac{\partial A_y}{\partial y} \qquad \text{div } \mathbf{A}_z = \frac{\partial A_z}{\partial z}$$

When we form the sum, we get the divergence of the resultant-field

$$\text{div } \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \tag{h}$$

This is the most generally used expression of the divergence of a vector in space.

When we compare with the formula (h) of the preceding section, we see that we can write the equation

$$(j) \qquad \operatorname{div} \mathbf{A} = \frac{\partial A_z}{\partial z} + \operatorname{div}_2 \mathbf{A}$$

where $\operatorname{div}_2 \mathbf{A}$ represents the divergence of that two-dimensional vector which has the components A_x and A_y. Now let the three-dimensional vector **A** be solenoidal, $\operatorname{div} \mathbf{A} = 0$. Then equation ($i$) gives

$$(j) \qquad \frac{\partial A_z}{\partial z} = -\operatorname{div}_2 \mathbf{A}$$

This is a differential equation by which we may determine the third component A_z of a solenoidal vector, of which we know the two components A_x and A_y. This will be our most important diagnostic formula. We shall use it to derive the vertical motion from the observed horizontal motion in the atmosphere.

172. **Curl of a Two-Dimensional Vector.**—Instead of the integral of the normal component A_n we shall now consider that of the tangential component A_s taken along a curve s.

$$(a) \qquad \int A_s ds$$

In the special case of a closed curve we shall call this integral the *circulation* of the vector **A** around the curve s. Lord Kelvin has introduced this name for cases where the vector **A** represents velocity. We shall use it, precisely as the expressions transport and outflow, even for cases of abstract vectors, which have nothing to do with motion. Circulation is a quantity which has a definite sign depending upon the direction which we have chosen as positive for rotating motion around a point or circulating motion around a closed curve (section 155).

Circulations have an additive property similar to outflows. We can join two points of the circuit originally given by a curve. The area limited by the first circuit will then be divided into two areas. We can form the sum of the circulations around the contours of each of them, using in both cases the same direction of circulation. In this sum the line-integral taken along the dividing curve will appear twice with opposite signs in the two cases, and will therefore drop out (fig. 98). Thus the sum of the circulations around the contours of the two parts of an area will be equal to the circulation around the contour of this total area. As we can continue the subdivision, we arrive at the result that the circulation around the contour of any area is equal to the sum of the circulations around the contours of all the areas into which it can be subdivided. We can express this result by the equation

$$(b) \qquad \int A_s ds = \Sigma \int A_s ds$$

extending the integral of the first member to the contour of the primary area and the integrals of the second to the contours of the areas produced by the division.

Now let the primary area be subdivided into elementary areas by two systems of curves, namely, the vector-lines and their positive normal curves n. The elements dn of the contour of these areas will then give no addition to the line-integral. The circulation in positive direction around the contour (fig. 99) will be represented by the difference

$$(c) \qquad -(A'ds' - Ads)$$

A' and ds' will vary as we proceed along the curve n. They can therefore be developed as functions of the length of arc n

$$A' = A + \frac{\partial A}{\partial n} dn \qquad ds' = ds + \frac{\partial ds}{\partial n} dn$$

Introducing this and leaving the term of second order out of consideration, we get for (c)

$$-\left(\frac{\partial A}{\partial n} dnds + A\frac{\partial ds}{\partial n} dn\right)$$

or introducing the area $d\sigma = dnds$ of the element

(c') $$-\left(\frac{\partial A}{\partial n} + A \cdot \frac{1}{ds}\frac{\partial ds}{\partial n}\right) d\sigma$$

The factor of $d\sigma$ then represents the circulation per unit area, or the *curl* of the two-dimensional vector **A**. We shall introduce the notation

(d) $$\text{curl}_2 \mathbf{A} = -\left(\frac{\partial A}{\partial n} + A\frac{1}{ds}\frac{\partial ds}{\partial n}\right)$$

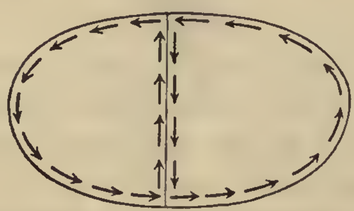


FIG. 98.

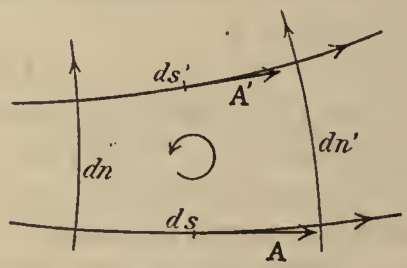


FIG. 99.

the suffix 2 denoting that the operation curl is performed only in two dimensions. We shall see presently that the curl of the three-dimensional vector is a vector. But, precisely as in the case of the vector-product, the vector-nature of the curl does not appear if we confine ourselves to the consideration of two-dimensional fields.

We can now write every term in the sum appearing as second member of equation (b) in the form $\text{curl}_2 \mathbf{A}\, d\sigma$. This sum then takes the form of an integral extended to the area formed by all the elements $d\sigma$. Thus we get the formula

(e) $$\int A_s ds = \int \text{curl}_2 \mathbf{A} d\sigma$$

that is, the line-integral of the tangential component of a two-dimensional vector taken around a closed curve is equal to the integral of the curl of the vector taken over the area bounded by the closed curve.

As the expression $\frac{1}{dn}\frac{\partial dn}{\partial s}$ represented the divergence of the vector-lines, section 170 (f), *i. e.*, the curvature of their positive normal curves, the expression $\frac{1}{ds}\frac{\partial ds}{\partial n}$ will represent the divergence of the positive normal curves, *i. e.*, *the negative curvature* ($-\gamma$) *of the vector-lines* which are the negative normal curves to the curves n (section 168). That is, we can write the expression of $\text{curl}_2 \mathbf{A}$

(f) $$\text{curl}_2 \mathbf{A} = -\frac{\partial A}{\partial n} + A\gamma$$

By the expression (*f*) we can construct the field of curl_2 **A**. The construction will be perfectly analogous to that of the divergence:

(1) We perform the graphical differentiation of the intensity-field of the given vector with respect to the positive normal curves to the vector-lines.

(2) We form the field of curvature of the vector-lines of the given vector (see section 168).

(3) We perform the graphical multiplication of the latter field with the intensity-field of the given vector.

(4) We perform the graphical subtraction of the two fields obtained by the operations (3) and (1).

The expression (*f*) may be used also for forming the curl of any component of the given vector. If we use cartesian coordinates, the vector-lines of each component-field will be straight lines. The curvature γ will be equal to zero and the curl of each component-field will be expressed by the first term only. Observing the rule of signs, we get $-\frac{\partial A_x}{\partial y}$ for the curl of the component A_x, and $\frac{\partial A_y}{\partial x}$ for the field of the component A_y. Forming the sum we get

$$(g) \qquad \text{curl}_2 \mathbf{A} = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$$

When the field of each component is given, we can construct the field of the curl in accordance with this formula. By linear differentiation of the field of A_y along lines parallel to the axis of X, and of the field of A_x along lines parallel to the axis of Y we form the fields of the two derivatives $\frac{\partial A_y}{\partial x}$ and $\frac{\partial A_x}{\partial y}$. Afterwards, by graphical subtraction of the latter from the former, we get the field of the curl.

173. Curl of a Vector in Space.—Now let **A** be any vector in space. We may then define a vector **c** which has the rectangular components

$$(a) \qquad c_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \qquad c_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \qquad c_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$$

By this definition we see that **c** is a vector of which each component is the curl of a two-dimensional vector: c_x of that which has the components A_z and A_y; c_y of that which has the components A_x and A_z; c_z of that which has the components A_y and A_x. We see further that each component of the vector **c** is normal to that plane which contains the two-dimensional vector from which it is derived. We will agree to represent this vector by curl **A**, thus

$$(a') \qquad \mathbf{c} = \text{curl } \mathbf{A}$$

Now let us consider any surface σ in the three-dimensional field. The vector **A** will determine a two-dimensional vector in this surface, for which we can write the theorem (*e*) of the preceding section. But what we have written there as curl_2 **A**, conceiving **A** as the two-dimensional vector contained in the surface, may now be expressed as the normal component to the surface of the vector (*a*), (curl **A**)$_n$. Thus

$$\int A_s ds = \int (\text{curl } \mathbf{A})_n \, d\sigma \tag{b}$$

or in words:

The line-integral of the tangential component of any vector taken around a closed curve is equal to the surface-integral of the normal component of the curl of the vector taken over any surface which has the given closed curve as contour. (Stokes's theorem.)

As long as we deal with two-dimensional vectors only, the vector-nature of the curl does not become apparent, as we have then to deal only with the component of the vector normal to the surface which contains the two-dimensional vector-field. In this respect the case is analogous to that of the vector-product.

The general theorem allows us to demonstrate an important property of every vector which is the curl of another vector. If the surface σ is closed, the contour s will disappear, and thus the line-integral around this be zero. We then get the equation

$$\int (\text{curl } \mathbf{A})_n d\sigma = 0 \tag{c}$$

where the integral is extended to the closed surface. But this equation indicates that the vector curl **A** is a solenoidal vector. This result can also be verified if we substitute the expressions of the components (*a*) of the curl into the solenoidal condition in its differential form. This leads to the identity

$$\text{div curl } \mathbf{A} = 0 \tag{c'}$$

Thus: *The curl of a vector is a solenoidal vector.*

174. Complex Differential Operations.—Divergence and curl may be considered as the intrinsic derivatives of a vector-field. The intrinsic structure of a field is known when we know curl and divergence.

Besides the differential operations leading to these intrinsic derivatives, we shall have to consider also a differential operation of a more complex nature. **A** and **B** being two vectors, we shall consider a vector **F** which has the three components

$$\begin{aligned} F_x &= B_x\frac{\partial A_x}{\partial x}+B_y\frac{\partial A_x}{\partial y}+B_z\frac{\partial A_x}{\partial z} \\ F_y &= B_x\frac{\partial A_y}{\partial x}+B_y\frac{\partial A_y}{\partial y}+B_z\frac{\partial A_y}{\partial z} \\ F_z &= B_x\frac{\partial A_z}{\partial x}+B_y\frac{\partial A_z}{\partial y}+B_z\frac{\partial A_z}{\partial z} \end{aligned} \tag{a}$$

Remembering the definitions of the scalar product and of the ascendant, we see that the expression of each component may be written as the scalar product of the vector **B** and the three ascendants ∇A_x, ∇A_y and ∇A_z, thus

$$F_x = \mathbf{B}.\nabla A_x \qquad F_y = \mathbf{B}.\nabla A_y \qquad F_z = \mathbf{B}.\nabla A_z$$

We will denote the vector which has the components (a) by the sign $\mathbf{B}\nabla\mathbf{A}$, thus

(a') $$\mathbf{F} = \mathbf{B}\nabla\mathbf{A}$$

The vector-equation (a') may be considered as a shortened symbolic expression of the three scalar equations (a).

We shall consider especially the two-dimensional vector **F** in the case when $\mathbf{B} = \mathbf{A}$. This vector will have the two components

(b) $$F_x = A_x\frac{\partial A_x}{\partial x}+A_y\frac{\partial A_x}{\partial y} \qquad F_y = A_x\frac{\partial A_y}{\partial x}+A_y\frac{\partial A_y}{\partial y}$$

and will in accordance with (a') be represented by the vector-formula

(b') $$\mathbf{F} = \mathbf{A}\nabla\mathbf{A}$$

If the fields of the two components A_x and A_y are given separately, we can form the fields of F_x and F_y in accordance with these formulæ, performing for each of them two graphical differentiations, two graphical multiplications, and one graphical addition.

In order to examine more closely the relation of the derived vector **F** to the given vector **A**, we can make a special choice of the system of coordinates (fig. 100). At the considered point the axis of X shall be tangential to the vector-line s of the given vector **A**. F_x will then be the same as the component F_s tangential to the line s. As at the considered point $A_x = A$ and $A_y = 0$, and as ultimately dx will be identical with ds, we get for the tangential component

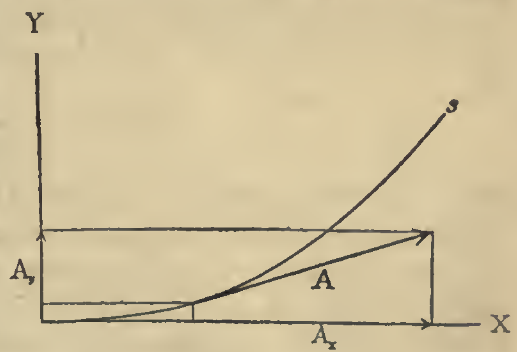


FIG. 100.

(c) $$F_s = A\frac{\partial A}{\partial s} = \frac{\partial}{\partial s}\left(\tfrac{1}{2}A^2\right)$$

As the curve s near the point of tangency forms the infinitely small angle α with the axis of x, we can write here $A_y = A\,\alpha$. Derivation then gives $\frac{\partial A_y}{\partial x} = \alpha\frac{\partial A}{\partial x} + A\frac{\partial \alpha}{\partial x}$. As at the point of tangency α is zero, we get here $\frac{\partial A_y}{\partial x} = A\frac{\partial \alpha}{\partial x} = A\frac{\partial \alpha}{\partial s}$. But $\frac{\partial \alpha}{\partial s}$ represents the curvature γ of the curve s. Instead of $\frac{\partial A_y}{\partial x}$ in the second equation (b) we can thus write $A\gamma$. When we introduce this, and remember that at the considered point $A_x = A$ and $A_y = 0$, we get this expression of F_y or F_n

(c') $$F_n = A^2\gamma$$

Thus the derived vector **F** will have two rectangular components, one which has the direction of the given vector and is equal to the derivative of the half square of the intensity of this vector with respect to its vector-lines, while the other is normal to the given vector and equal to the square of the intensity of this given vector multiplied by the curvature of its vector-lines. Hence we can form the field of this derived vector **F** by the following construction:

(1) We form the half square of the intensity-field of the given vector (section 147) and then the derivative (c) with respect to the vector-lines.

(2) We form the field of curvature of the given vector-lines (section 168) and perform the graphical multiplication of this field by that of the square of the intensity (c').

(3) We perform the graphical addition of two mutually normal vectors (section 157): the vector $\mathbf{F}_s$ which has the same direction as the given vector **A** and the intensity determined by the operation (1); and of the vector $\mathbf{F}_n$ which is normal to the given vector and has the intensity determined by the operation (2).

We can also give another method for determining the vector **F**. We can change the second member of equations (b): in the first of these equations by adding and subtracting the term $A_y\frac{\partial A_y}{\partial x}$; in the second by adding and subtracting $A_x\frac{\partial A_x}{\partial y}$. This gives

$$(d)\qquad \begin{aligned} F_x &= A_x\frac{\partial A_x}{\partial x}+A_y\frac{\partial A_y}{\partial x}-A_y\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)=\frac{\partial}{\partial x}\left(\tfrac{1}{2}(A_x^2+A_y^2)\right)-A_y\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)\\ F_y &= A_x\frac{\partial A_x}{\partial y}+A_y\frac{\partial A_y}{\partial y}+A_x\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)=\frac{\partial}{\partial y}\left(\tfrac{1}{2}(A_x^2+A_y^2)\right)+A_x\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)\end{aligned}$$

These equations represent the vector **F** as the vector-sum of two vectors. The first is the ascendant of the scalar $\frac{1}{2}(A_x^2+A_y^2) = \frac{1}{2}A^2$. The second is the vector-product of the vectors $\text{curl}_2\mathbf{A}$ and **A**. When we remember that the vector $\text{curl}_2\mathbf{A}$ is normal to the surface which contains **A**, we see by the properties of the vector-product that this second vector will be directed along the positive normal to **A**. Thus we can represent the scalar equation (d) by the vector-equation

$$(d')\qquad \mathbf{F} = \nabla(\tfrac{1}{2}A^2)+(\text{curl}_2\,\mathbf{A})\times\mathbf{A}$$

Thus we can also use the following method for constructing the field of the vector **F**.

(1) We construct the scalar field of the half square of the intensity of the given vector (sec. 147), and then the field of the ascendant of this scalar (sec. 169).

(2) We construct the field of the curl of the given vector (section 172) and perform the graphical multiplication of this field by that of the intensity of the given vector. This field is considered as the intensity-field of a vector which has the direction of the positive normal to the given vector.

(3) We form the field of the sum of the two vectors, the fields of which we have found by the first two operations.

In most cases the first method will be preferable, as the two fields the vector-sum of which we shall form are then normal to each other. But still in special cases the second may be the shorter, for instance if some of the partial fields upon which the construction depends are already constructed for other purposes.

175. Pure Time-Differentiations and Time-Integrations of Scalar Fields.—While a pure space-differentiation is performed upon one chart, representing the field of a scalar or a vector at a given moment, the pure time-differentiations will consist in the comparison of two charts, which represent the field at two different moments.

Let α be a scalar which depends upon coordinates and time. Now let α_0 be the value of this scalar at a certain point at a time t_0, and α_1 its value at the *same point* at the time t_1. The quotient

$$\bar{\varphi} = \frac{\alpha_1 - \alpha_0}{t_1 - t_0} \tag{a}$$

will then represent the *average* value which the differential-quotient

$$\varphi = \frac{\partial \alpha}{\partial t} \tag{b}$$

has at this point during the interval of time $t_1 - t_0$. If this interval is sufficiently short we can consider the value of the quotient (*a*) as identical with the value of the differential quotient (*b*) at the time

$$t = t_0 + \frac{t_1 - t_0}{2} \tag{c}$$

If we know the field of the scalar α at two moments t_0 and t_1, which are separated by a sufficiently short interval of time $t_1 - t_0$, we can form the field of the derivative (*b*) at the time (*c*) in this manner:

We form by graphical subtraction the field of the difference

$$\alpha_1 - \alpha_0 \tag{d}$$

and afterwards perform the division of this field by the constant factor

$$t_1 - t_0 \tag{e}$$

The problem is thus reduced to algebraic problems which we have already treated. The only difficulty will be that the fields α_0 and α_1 may too closely resemble each other. Their equiscalar curves may cut each other under too small angles and it may be difficult to get a good drawing of that set of diagonal curves which represents the difference (*d*). It will be important to remark, however, that the errors will take precisely the same character as in the previous cases of differentiation: the curves representing the derivative will get an oscillating course, and these oscillations can be smoothed out afterwards. But in order to avoid these errors from the beginning, it will be important not to choose too short an interval of time (*e*). On the other hand it must not be chosen too long if it is to be allowed to identify, within the margin of allowable departures, the finite difference-quotient (*a*) with the differential-quotient (*b*) at the time (*c*).

The reversed problem, that of the pure time-integration, will be solved with the same ease. Let the field of α be given at the time t_0, $\alpha = \alpha_0$; and let the value of the derivative φ be known at any time t which is subject to the condition $t_0 < t < t_1$. If then the interval of time $t_1 - t_0$ is sufficiently short, we can identify the value of φ at the time t with the average value $\bar{\varphi}$ during the interval of time $t_1 - t_0$. We then find the value of α at the time t_1 by the formula

$$\alpha_1 = \alpha_0 + \bar{\varphi}(t_1 - t_0) \tag{f}$$

Thus we have to perform the following graphical operations: first to multiply the field of the derivative $\bar{\varphi}$ by the interval of time $t_1 - t_0$, and then to perform the graphical addition of the fields α_0 and $\bar{\varphi}(t_1-t_0)$. If we have sufficient knowledge of the derivative φ at different times t, we can repeat this operation and thus find at any time t the field α which is expressed analytically by the integral

$$(g) \qquad \alpha = \alpha_0 + \int_{t_0}^{t} \varphi \, dt$$

The graphical addition (*f*) will cause no such difficulty as that of the graphical subtraction (*d*). The only difficulty connected with the integration will arise from the gradual summing up of small errors from the one partial operation to the other.

176. Pure Time-Differentiations and Time-Integrations of Vector-Fields.—The principles for the pure time-differentiations will be precisely the same for a vector-field as for the scalar field.

Let **A** be a vector which depends upon both coordinates and time. Let it have the value $\mathbf{A}_0$ at a certain point at the time t_0, and the value $\mathbf{A}_1$ *at this same point* at the time t. The vector

$$(a) \qquad \bar{\mathbf{F}} = \frac{\mathbf{A}_1 - \mathbf{A}_0}{t_1 - t_0}$$

will then represent the average during the interval of time t_1-t_0 of the vector

$$(b) \qquad \mathbf{F} = \frac{\partial \mathbf{A}}{\partial t}$$

which is the pure time-derivative of the vector **A** at the considered point. If we use sufficiently small intervals of time we can identify the vector $\bar{\mathbf{F}}$ with the value of **F** at the time

$$(c) \qquad t = t_0 + \frac{t_1 - t_0}{2}$$

By these formulæ we see at once that if we know the field of the given vector **A** at two moments t_0 and t_1, which are separated by a sufficiently small interval of time t_1-t_0, we can form the field of the derivative at the time (*c*) in this manner:

We form the field of the vector-difference

$$(d) \qquad \mathbf{A}_1 - \mathbf{A}_0$$

and afterwards perform the division of this field with the constant factor

$$(e) \qquad t_1 - t_0$$

We have thus reduced the pure time differentiation of a vector-field to algebraic problems already treated. The only difficulty connected with this differentiation will consist in the formation of the vector-difference between two vector-fields which are very like each other. For this reason we must not choose too short an interval of time (*e*), just as we must not choose it too long if we are to be able to identify the two vectors (*a*) and (*b*).

For the formation of the vector-difference (*d*) we can use any of the methods which we have developed in vector-algebra. We can use the method of section 158 or the graphical tables (section 160), or finally the complete resultantometer (section 161). If we wish to use either of the first two methods, the field representing the difference of angle is first drawn as accurately as possible. The curves, as they are obtained directly, will always have more or less of the oscillating course which is characteristic of curves obtained by a process of graphical differentiation. These oscillations should be carefully reduced. Then all results concerning singular points, etc., which can be obtained by use of the simple scalar addition or subtraction (section 159), must be worked out with the greatest care. The rest of the work

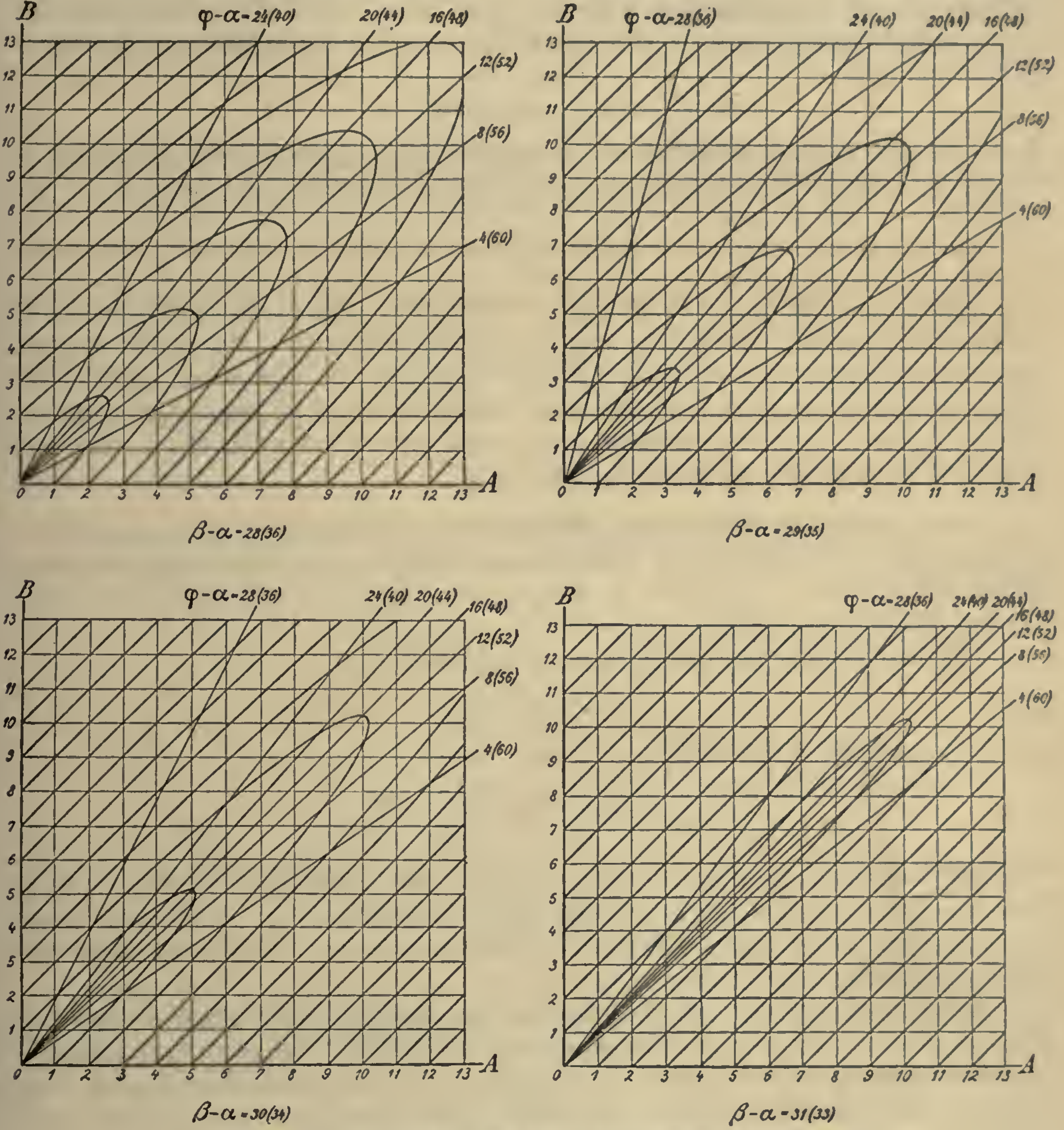


FIG. 101.—Graphical tables for time-differentiation of a vector-field.

will then mainly consist in forming the vector-sum of vectors which form angles differing very little from 32. If for this we wish to use graphical tables those of fig. 101 will serve the purpose. But in many cases the method of section 158 seems to be the best in spite of the greater number of separate operations.

When the field of the vector-derivative **F** is given at a series of epochs, and the field of the vector **A** at the initial epoch t_0, we can perform the pure time-integration, which is the inverse operation to the pure time-differentiation considered. We have then to identify the value of the derivative **F** at a moment t with the average derivative $\overline{\mathbf{F}}$ during a finite but short interval of time $t_1 - t_0$ when $t_0 < t < t_1$. We then perform the multiplication of the average derivative $\overline{\mathbf{F}}$ with the constant factor $t_1 - t_0$, and afterwards perform the addition of the two vector-fields according to the formula

$$(f) \qquad \mathbf{A}_1 = \mathbf{A}_0 + \overline{\mathbf{F}}\,(t_1 - t_0)$$

This operation may be repeated any number of times, and will lead to the field of the vector **A** at the time t, which is expressed analytically by the integral

$$(g) \qquad \mathbf{A} = \mathbf{A}_0 + \int_{t_0}^{t} \mathbf{F}\,dt$$

The delicate point in this process of integration will be the addition of the generally very small vector $\overline{\mathbf{F}}(t_1 - t_0)$ to the finite vector **A**. But as the isogons and the intensity-curves of the two fields will usually cut each other under finite angles, we shall not meet with the same difficulties as those connected with the differentiation. The only difficulty will be the gradual summing up of the small errors which enter at each partial operation.

177. **Complex Time and Space Differentiation.**—Besides the pure space-differentiations and the pure time-differentiations we shall also meet with complex space-time-differentiations. They will be seen to occur in all investigations concerning moving continuous media.

Let f be any function of coordinates and time,

$$(a) \qquad f(x, y, z, t)$$

It has four partial derivatives

$$(b) \qquad \frac{\partial f}{\partial x} \qquad \frac{\partial f}{\partial y} \qquad \frac{\partial f}{\partial z} \qquad \frac{\partial f}{\partial t}$$

The last is what we have called above the pure time-derivative. In order to form it we have to consider x, y, z as constant, and let only time vary; *i. e.*, we compare the values of f in the same locality at two different epochs. We shall therefore also call it the *local* time-derivative.

But on other occasions we shall have to compare the values which the function f has at two epochs at one and the same physical particle. What we keep constant in this comparison will then be not the locality x, y, z, in which the values of f are observed, but the *individuality* of the particle at which the values of f are observed. Now let v_x, v_y, v_z be the velocity-components of the particle. If at the time t it has the coordinates x, y, z, it will at the time $t+dt$ have the coordinates $x+v_x dt$, $y+v_y dt$, $z+v_z dt$. We have then to compare

$$(c) \qquad f(x+v_x dt,\ y+v_y dt,\ z+v_z dt,\ t+dt)$$

with $f(x, y, z, t)$. For this we can develop (c) according to Taylor's theorem, and leave quantities of the second order out of consideration. (c) then takes the form

$$f(x, y, z, t)+\frac{\partial f}{\partial x}v_x dt+\frac{\partial f}{\partial y}v_y dt+\frac{\partial f}{\partial z}v_z dt+\frac{\partial f}{\partial t}dt$$

The excess df of the value of f at the point $(x+v_x dt,\ y+v_y dt,\ z+v_z dt)$ at the time $t+dt$ over its value in the point (x, y, z) at the time t, will then be

$$df=\frac{\partial f}{\partial t}dt+\frac{\partial f}{\partial x}v_x dt+\frac{\partial f}{\partial y}v_y dt+\frac{\partial f}{\partial z}v_z dt$$

If we divide this equation by dt, we get a derivative which gives the rate of change of the value of f at one and the same moving material individuum. We shall call this the *individual* derivative, and denote it by $\frac{d}{dt}$. Its expression in terms of the four partial derivatives (b) will then be

(d)
$$\frac{df}{dt}=\frac{\partial f}{\partial t}+v_x\frac{\partial f}{\partial x}+v_y\frac{\partial f}{\partial y}+v_z\frac{\partial f}{\partial z}$$

or in vector-notations

(d')
$$\frac{df}{dt}=\frac{\partial f}{\partial t}+\mathbf{v}.\nabla f$$

A case of special importance is when f represents one component of a vector **A**. The individual time-derivative of the vector **A** will then be expressed by the three equations

$$\frac{dA_x}{dt}=\frac{\partial A_x}{\partial t}+v_x\frac{\partial A_x}{\partial x}+v_y\frac{\partial A_x}{\partial y}+v_z\frac{\partial A_x}{\partial z}$$

$$\frac{dA_y}{dt}=\frac{\partial A_y}{\partial t}+v_x\frac{\partial A_y}{\partial x}+v_y\frac{\partial A_y}{\partial y}+v_z\frac{\partial A_y}{\partial z}$$

$$\frac{dA_z}{dt}=\frac{\partial A_z}{\partial t}+v_x\frac{\partial A_z}{\partial x}+v_y\frac{\partial A_z}{\partial y}+v_z\frac{\partial A_z}{\partial z}$$

or, using the vector-notations introduced in section 174

(e)
$$\frac{d\mathbf{A}}{dt}=\frac{\partial \mathbf{A}}{\partial t}+\mathbf{v}\nabla\mathbf{A}$$

An important case is that in which the vector **A** is the velocity of the moving particle. The rate of change of its velocity gives its *acceleration*, for which we thus get the equation

(f)
$$\frac{d\mathbf{v}}{dt}=\frac{\partial \mathbf{v}}{t}+\mathbf{v}\nabla\mathbf{v}$$

In order to form the field of acceleration we have thus to perform pure time-derivations and pure space-derivations, which we have investigated already.

The distinction which we have here introduced between *local* and *individual* time-derivations will be of great importance in our continued work. The difference between them can be very well illustrated in connection with the different methods of observing the meteorological elements. The instruments of the ordinary meteorological stations give the local variation of the meteorological elements. When we determine from the records of the barograph the rise of pressure per second, we get the local derivative of the pressure, $\frac{\partial p}{\partial t}$. In the same manner the thermograph of the station will give the local derivative of temperature $\frac{\partial \tau}{\partial t}$. By use of the windfane and the anemometer of the stations we can in the same way determine the local time-derivative of velocity $\frac{\partial \mathbf{v}}{\partial t}$. We may call this the *local acceleration*, to distinguish it carefully from the acceleration without further specification, which gives the rate of change of velocity of one and the same moving individuum.

Instead of considering the stationary instruments of a common station, we can now consider the moving instruments in a balloon, and let the balloon be in perfect equilibrium. It will then move along within one and the same mass of air. The barograph will then register the pressure of this mass of air, the thermograph will register its temperature. Forming from the records the rates of change, we get the individual time-derivatives $\frac{dp}{dt}, \frac{d\tau}{dt}$ . . . . If finally the velocity $\mathbf{v}$ of the balloon itself be registered, we should be able to determine the acceleration $\frac{d\mathbf{v}}{dt}$ of the mass of air in which it moves.

At the moment when this balloon with its moving instruments passes the station with its stationary instruments, the moving and the stationary instruments will show the same instantaneous values of the recorded quantities, but different rates of their change. Formula (*d*) will give the relation between the derivatives found from the records of the moving and the stationary instruments.

CHAPTER X.

THE FORCED VERTICAL MOTION AT THE BOUNDING SURFACES.

178. **Hypsometric and Bathymetric Maps.**—Having now developed the mathematical methods to be used, we can proceed to the accomplishment of the kinematic diagnosis. Chapters II–VII gave the direct methods for working out, from the observations, a complete diagnosis of the horizontal motion in atmosphere or hydrosphere. We shall now see how the correlated diagnosis of the vertical motion should be worked out.

The vertical motion begins at the bounding surfaces. Here the solenoidal surface-condition, section 115 (E), must be fulfilled; *i. e.*, both velocity and specific momentum must be tangential to the surface. The moving masses will be forced up or down according as the motion in horizontal projection goes against the slope or with it. We shall call the vertical motion which is produced in this way the "forced" vertical motion, to distinguish it from the "free" vertical motion to be considered in the next chapter.

In order to investigate this forced vertical motion, we must have complete topographic charts representing the configuration of the bounding surfaces; *i. e.*, we must have a complete representation of the topography of the world above as well as below sea-level. We have referred to such charts before, using them to define the spaces taken up by atmosphere and sea, and thus to give the extent of the fields representing the atmospheric or oceanic states. But the main influence which the bounding surfaces exert upon the internal structure of these fields comes through the forced vertical motion which arises as a consequence of the boundary condition. In view of this kinematic application we have worked out a representation of the topography of the world which is given on the first twenty-four sheets of the collection of plates which accompanies this work.

Our knowledge of the configuration of the bottom of the sea is still very incomplete; but fortunately most of the knowledge acquired has been made accessible by the bathymetrical map on a scale of 1 : 10 000 000 edited by the Prince of Monaco.* This map represents the topography of the earth below sea-level on 16 plates in Mercator and 8 in polar projection. We have for the main part copied our bathymetrical curves as well as the coast-lines from this chart, the most important changes being the following: Corrections and completion of the coast-lines in the Arctic and Antarctic regions have been performed according to the results of the well-known later Arctic and Antarctic expeditions. Changes in the course of the bathymetrical lines have been introduced, for the northern Atlantic according to Helland-Hansen and Nansen,† for the eastern Pacific according to the results

* Carte Générale Bathymétrique des Oceans, dressée par l'ordre de S. A. S. le Prince de Monaco.

† B. Helland-Hansen and Fridtjof Nansen: The Norwegian Sea. Christiania, 1909.

of the American *Albatross* Expedition* for different parts of the Indiàn Ocean and the western Pacific according to the results of the German *Planet* Expedition.†

While it has thus been easy to bring a bathymetrical chart representing in a tolerably satisfactory way our present knowledge of the configuration of the bottom of the sea, we have not been able to produce anything in the same manner satisfactory for the configuration of the crust of the earth above sea-level. The literature of cartography is remarkably poor as regards topographical charts of greater parts of the world. As it would have been quite impracticable for us to collect and utilize all primary material of topography in detail, which is accessible in the cartographical and geographical literature, we have chosen a limited number of sources. The most important of them has been the height numbers contained on Stieler's Atlas‡ used in connection with the course of the rivers and the shadings representing the orographical features of the countries. Besides these we have used a map of the world on a smaller scale edited by the German Marine Authorities,§ which contains the height-curves for 300, 1000, and 2000 meters. During our work Romer's Atlas|| appeared, containing on a small scale charts of the continents, with height-curves corresponding to the interval of 1000 meters. The topography for the United States has been taken from the chart of the Geological Survey, the curves being changed from feet to meters. Special attention has been paid to the latest results of Sven Hedin in Central Asia.¶ The short pieces of height-curves drawn on the chart of the Antarctic continent are derived from Shackleton's chart.** For the drawing of the height-curves in the Arctic regions, we are indebted to Nansen, Isaachsen, and Amundsen for valuable hints.

The chart which we have thus produced must not be considered as a geographical document, and it is to be hoped that better charts may soon be produced by professional geographers. But it will serve our special purposes very well.

Our chart is on the scale of 1 : 20 000 000, and like that of the Prince of Monaco it is distributed on 16 plates in Mercator's projection and 8 in polar projection. It gives the height above and the depths below sea-level precisely in the same way. The curves for the height, respectively the depth, of 200 meters are dotted, those for 500 stippled, and then continuous curves are drawn for every 1000 meters of height or depth. It will be equally legitimate to interpret the meter indicating these heights or depths as the common geometrical meter or as the dynamic meter (compare Statics, section 15).

179. Charts of Idealized Topography.—If we were to proceed with perfect rigor, we should have to apply the surface-condition to the true surface of separation between the moving medium and the bounding surface. This would require the

*Memoires of the Museum of Comparative Zoology at Harvard College, vol. 33. Cambridge, 1906.

†Forschungreise S. M. S. *Planet* 1906–1907. T. 3, Oceanographie. Berlin, 1909.

‡Stieler's Hand-Atlas, Neunte Auflage. Gotha, 1907.

§Weltkarte zur Uebersicht der Meerestiefen & Höhenschichten, herausgegeben von dem Hydrographischen Amte des Reichs-Marine-Amts. Berlin, 1893.

||Lemberg, 1908.

¶Sven Hedin: Transhimalaya. Stockholm, 1909.

**B. E. H. Shackleton: The Heart of the Antarctic. London, 1908.

construction of topographic maps of a completeness which can not be attained Taking the case of the atmosphere, the chart should give the configuration of every irregularity of the ground, every stone, every tree, every house. And the use of the map would require wind-observations taken all around these irregularities.

Just as we have been obliged to consider an idealized wind (section 97), we must use an idealized topography, corresponding to the placing of the fanes and the anemometers in open places, above that sheet of air which has the most irregular motions.

It will therefore be perfectly legitimate to use an idealized topography like that which is represented by the common contour-lines. And in most cases it will be not only legitimate, but necessary, to go still further in the idealization than on common charts. Even the map of the world as we have drawn it on the plates I–XXIV contains far too much detail for meteorological work as long as the phenomena are to be studied on a large scale, and not in minute details.

For our practical work we have therefore been obliged to derive from this map *special maps of idealized topography*. All these special maps have been drawn on a scale of 1 : 10 000 000. We have found this scale convenient for the performance of our constructions, and all our graphical auxiliaries have been made with this scale in view. All these special maps have been drawn in a conical projection corresponding to the latitude. Our reasons for preferring this projection to one with curved meridians have been given already; all kinds of auxiliary graphical instruments (sections 143, 161, 163) are easily applied when the chart is in conical projection. The idealizations have been performed step by step. First we have drawn a map where all the smallest irregularities of the contour-lines have been removed, then a new map where greater irregularities have been removed, and so on. The simplified curves are always drawn so that the volumes of the great mountain-chains and of the continents have retained their value. In this manner correct values will be found for the average intensity of the forced ascending or descending motion, while the small irregular motions up and down, which are only of local importance, will drop out. But it should be remembered that the drawing of the idealized charts has no unique solution. The same degree of idealization can be attained in different ways as regards details. It will be a question of experience to find out the proper degree of idealization and the best solution of dubious questions of detail. In practical work we have used two degrees of idealization, represented by the "moderately idealized" charts of the United States and of Europe given on plates XXV and XXVIII, and the "greatly idealized" charts of plates XXVI and XXIX. We have used the moderately idealized charts more for qualitative purposes, drawing on them the charts of the horizontal motion (section 135), while we use the charts of greatly idealized topography for the rigorous quantitative work.

We have given no examples of idealized bathymetric maps. As we have had no observations from which we could work out a kinematic diagnosis of sea-motions, we have had no opportunity of examining the question of such charts for hydrographic purposes. It should be remembered, however, that the bottom of the sea is, generally speaking, less irregular than the ground above sea-level, and at the same time our knowledge is less detailed. When in spite of this further idealizations have to be performed, great care should be taken, for small irregularities of the bottom may influence the motion of the sea much more than corresponding irregularities of the ground are able to influence the motion of the air.*

When in the following we speak of the ground, we always mean the ideal surface which is represented by our charts. We shall consider the wind-observations obtained at the meteorological stations as representing the air-motion at this surface itself. This will be perfectly legitimate from a kinematic point of view. But the real removing of all irregularities would of course have great dynamic consequences. We shall therefore be obliged later to consider this ideal surface as offering a frictional resistance which a smooth surface would not offer in reality.

180. **The Motion in the Lowest Surface of Flow.**—The particles of the moving medium which are in contact with the bounding surface will move tangential to it in virtue of the solenoidal surface-condition. Therefore a hypsometric map represents directly the topography of the lowest surface of flow in the atmosphere; and in the same manner a bathymetric map represents the topography of the lowest surface of flow in the sea.

When we shall represent the motion in this lowest surface of flow, we must remember its exceedingly minute inclination. Even on our charts of moderately idealized topography hardly any place will be found where contour-lines corresponding to a difference of level of 1000 meters approach each other as closely as 1 mm. On a chart on a scale of 1 in ten millions, this will give an inclination which is smaller than one in ten. The cosine of the angle of inclination will therefore be greater than 0.995, and when we set this cosine equal to unity, we shall never make errors as great as 0.5 per cent. Such errors will be insignificant compared with the errors of observation. *We need therefore make no difference between the numerical values of the horizontal component of the motion and the resultant motion itself which is parallel to the ground.*

For this reason we shall get a representation of the motion along the bounding surface simply by drawing the lines of flow and the curves of equal intensity on outline-maps which contain the contour-lines. The three sets of lines, contour-lines, lines of flow (respective isogonal curves), and intensity-curves give a complete representation of the surface of flow and of the motion in it (compare fig. 43 A and fig. 45 A).

181. **Charts of Vertical Velocity at the Ground.**—From a chart containing these three sets of lines we can easily draw a special chart of the vertical component of the motion. When s is a line of flow in the atmosphere and z its height above sea-level its angle of inclination will be

$$(a) \qquad i = \frac{dz}{ds}$$

*Cf. the notes, pp. 58 and 59.

v being the resultant velocity, the vertical component v_v will then be given by the formula

$$(b) \qquad v_v = v\frac{dz}{ds}$$

In accordance with this expression we can construct the field of v_v. In the case of motion along the bottom of the sea we should have to use the depth below sea-level instead of the height above it. But we shall henceforth consider exclusively the case of the atmosphere. As soon as the observations are at hand, it will be easy to adapt the same methods to the investigation of sea-motions.

Formula (*b*) reduces the drawing of a chart of vertical velocity to a simple problem of graphical differentiation and of graphical algebra.

A rough sketch of the field (*b*) can easily be made by the discontinuous method. Evidently the field (*b*) will contain a zero-line $v_v = 0$, which separates from each other the windward and the leeward sides of the mountains. The general course of this line is seen at once and can be drawn by eye-measure in those parts of the country where the slope is strong enough to produce a vertical motion of any importance. By use of the differentiating sheet of fig. 81, we can then make a few determinations of v_v in the places where it is seen to have its greatest positive and negative values. Afterwards the curves $v_v = \text{const.}$ can be drawn by eye-measure. It will not be difficult in this way to draw such charts in the daily meteorological service.

For more detailed investigations we can bring the continuous graphical methods into application. The method of proceeding will be this:

We construct first the chart of the angle of inclination (*a*). The construction is that which has been exemplified in fig. 83. In this figure we can interpret the lines $a = \text{const.}$ as contour-lines, and the lines s as the lines of flow of the wind. The stippled curves will then be curves for equal values of the angle of inclination i. Of these curves we first draw that for the angle of inclination zero. This curve will pass through all the points of tangency of the lines of flow and the contour-lines. A zero-curve must therefore pass the summit of every mountain as well as the highest point in every pass. Inasmuch as the wind does not travel precisely along the chain, but has a component across it, the zero-line will follow near the highest ridge of the chain, passing all the summits and the highest point of the passes. In the same manner, when the wind does not travel precisely along a valley, but has a component across it, a zero-line will run along it, near its bottom.

As soon as the zero-line is drawn, we determine the course of the curves for integer values of the angle of inclination by making continuous use of the differentiating sheet of fig. 81 as described in section 165.

Finally we perform the graphical multiplication (section 150) of the field of the angle of inclination i with that of the scalar value v of the velocity of the wind. The chart resulting will then represent the field of the vertical velocity v_v.

182. Ascendant-Charts.—From a theoretical point of view the drawing of the charts of vertical velocity is exceedingly simple. But still, when it is to be done with care for the details, it will prove to be the most laborious operation of kinematic diagnosis. The reason is that in spite of all idealizations, the topographic chart will remain more complicated than the charts which represent the field of the meteorological quantities observed.

In order to simplify the work another way may be suggested: From the topographical map we could derive a chart representing the ascendant of the ground, and print it as a blank. The process of differentiation would then be performed once for all; for it is easily seen that the vertical velocity may be expressed as the scalar product of this ascendant and the horizontal velocity. Each chart of vertical velocity could then be derived by a simple algebraic process (section 156). But when this method does not work as well as might be expected, it is due to the great complexity of the isogons and the intensity-curves representing the ascendant. The control due to direct intuition is lost, and keen attention will be required to avoid mistakes; but this method may be considered if extensive detailed investigations on the vertical motion at the ground are to be performed.

A method which also might be considered in such a case would be the consistent use of rectangular components. If the W.–E. and the S.–N. components of the wind were observed, we might draw and print as blanks two special auxiliary charts, one of the W.–E. component and one of the S.–N. component of the ascendant. By a simple graphical multiplication we should then be able to derive a chart of the vertical velocity due to each component of the wind, and afterwards a chart of the total vertical velocity by graphical addition.

183. Change of Velocity into Specific Momentum. Charts of Density at the Ground.—If we have a chart which represents the density of the air at the ground, we can at once by graphical multiplication change a chart of velocity into one of specific momentum. It will be sufficient if the chart of density has an accuracy corresponding to that of the wind-observations. We can then ignore the influence of humidity on density and consider density as a function only of pressure and temperature. When we know the topography of the isobaric surfaces in free space, we can draw their curves of intersection with the ground. These curves will give a chart of the pressure at the ground. By this chart, together with a corresponding chart of temperature at the ground, we can draw a chart of the density at the ground, using one of the two auxiliary tables N.

Table N, A, contains density and pressure as argument, and temperature as the tabulated quantity. It gives the temperature of the point where the equiscalar curves for the required field of density cut the given isobaric curves. Table N, B, contains density and temperature as arguments and gives the pressure of the points where the required curves for equal density cut the given isothermal curves.

A density-chart drawn by one of these tables will possess an accuracy far exceeding that of the observations of velocity. In most cases we can therefore still further simplify the method, treating pressure at the ground as if it depended only upon the height above sea-level and ignoring its variations from day to day. We can then get the density of the air as function of height and temperature.

When we use the average relation between pressure and height given in Statics, table A, p. 29, we get the tables O. The first gives the temperature at points where the contour-lines of the topographic map are cut by the required curves of equal density, while the second gives the height of the points where the required density-curves cut the given isotherms. As density is proportional to pressure, and as pressure at a place will as a rule differ only a small percentage from its average value, the use of this simpler method will generally give only a small percentage of errors of the values of density. This will be of no importance when the general inaccuracy of the observations of velocity is considered.

TABLES N.

A. *Temperature at points where isopycnic curves cut isobaric curves.*

ρ	p								
	1100	1050	1000	950	900	850	800	750	700
0.0008								54	32
09						57	37	18	− 1
10				58	40	26	6	−12	−29
11			44	28	12	− 4	−20	−36	−51
12	46	32	17	3	−11	−26	−40	−55	
13	22	8	− 5	−18	−32	−45	−59		
14	1	−12	−24	−36	−49				
15	−18	−29	−36	−53					

B. *Pressure at points where isopycnic curves cut isothermic curves.*

ρ	t								
	−40	−30	−20	−10	0	10	20	30	40
0.0008				604	627	650	673	696	719
09			653	679	705	731	757	783	808
10	669	697	726	755	784	812	841	870	898
11	736	767	799	830	862	893	925	957	988
12	802	837	871	906	940	975	1009	1044	1078
13	869	907	944	981	1019	1056	1093		
14	936	976	1017	1057	1097				
15	1003	1046	1089						

Instead of drawing a chart of temperature at the ground, we can take an average value of the temperature, and draw the density-chart by use of that column in table O, B, which corresponds to this temperature.

TABLES O.

A. *Temperature at points where isopycnic curves cut contour-lines.*

ρ	z					
	0	200	500	1000	2000	3000
0.0008						25
09					31	−7
10			54	35	− 2	−34
11	44	36	25	7	−27	−47
12	17	10	0	−16	−46	
13	− 5	−11	−21	−36		
14	−24	−30	−39	−53		
15	−41	−47	−56			

B. *Height of points where isopycnic curves cut isotherms.*

ρ	t								
	−40	−30	−20	−10	0	10	20	30	40
0.0008						3400	3100	2900	2600
09			3400	3100	2800	2500	2200	2000	1700
10	3200	2900	2600	2300	2000	1700	1400	1100	900
11	2400	2100	1800	1500	1200	900	600	400	100
12	1800	1400	1100	800	500	200			
13	1100	800	500	200					
14	500	200							
15									

We can even simplify still further, and neglect also the variations of temperature. We then consider density as given only by the height above sea-level, for instance, by the column for 0° C of table O, B.

In this case we should then always use the same density-chart, which could be derived from the contour-lines of the topographic chart by use of the numbers in this column.

In all cases when the density-chart is found, we have simply to perform the graphical multiplication of the charts of vertical velocity by that of density in order to get the chart of vertical specific momentum.

184. Direct Method of Determining Vertical Specific Momentum from Horizontal Velocity at the Ground.—If the chart of vertical velocity is drawn already, the method given in the preceding section will give the easiest construction of the chart of vertical specific momentum. But we can also use a direct method without passing through the vertical velocity. As the contour-lines of our charts can be interpreted as lines of equal dynamic height H, we can write equation 181 (*b*)

(*a*) $$v_v = v\frac{dH}{ds}$$

Now, according to the fundamental equation of hydrostatics, we have $dH = -\alpha dp$, where pressure p is measured in decibars when dynamic height H is measured in dynamic meters. When we introduce this and divide by the specific volume α, *i. e.*, multiply by the density ρ, we get on the left side the vertical component V_v of specific momentum,

(*b*) $$V_v = -v\frac{dp}{ds}$$

This equation gives the following rule for drawing the chart of vertical specific momentum at the ground: We first draw the chart which represents the field of pressure at the ground; then we perform the graphical differentiation of this field with respect to the length of arc along the lines of flow; finally we perform graphical multiplication of the field thus obtained by the field of the scalar value v of velocity.

This method is precisely like that which we have developed for the velocity except that we use the chart of pressure at the ground instead of the topographic chart. But it will give more work, inasmuch as the topographic map always remains the same, while that of pressure changes and must be drawn again in each case. If we ignore, however, the variations in time of the pressure, we can draw a chart representing the average pressure at the ground and use this chart consistently for the determination of vertical specific momentum, precisely as the topographic map for the determination of the vertical velocity. Then it will be as easy to draw charts of vertical specific momentum as of vertical velocity. The errors in the determination of vertical specific momentum caused by the use of the average pressure will amount to a small percentage and thus always be small compared to those which arise from the imperfectness of the observations of the wind. Therefore in general there will be no objection to using this simplified method.

We have therefore drawn the charts of plates XXVII and XXX, which give the average pressure at the ground in the United States and in Europe. As to the degree of idealization, they correspond to the strongly idealized topographic maps of plates XXVI and XXIX. The coast-line is to be considered as an isobaric line of pressure about 1013 m-bar. Then the curves for 1000, 900, 800, . . . m-bar have been drawn as continuous lines, while a curve for the pressure of 980 m-bar is dotted and a curve for 950 m-bar is stippled.

CHAPTER XI.

VERTICAL MOTION IN FREE SPACE—COMPLETE KINEMATIC DIAGNOSIS.

185. Free Vertical Motion.—As the distance from the bounding surface increases, the forced vertical motion produced at this surface will gradually be modified. An additional vertical motion will arise in the free space and conjoin with the forced vertical motion. We shall for the sake of brevity call it the *free* vertical motion. It can be investigated by the solenoidal condition in space, precisely as the forced vertical motion by the solenoidal surface-condition.

We have done it already from a qualitative point of view (Chapter V). We had to take the free vertical motion into consideration in order to explain the features of the horizontal motion. The vertical motion existing above centers or lines of convergence and of divergence gives typical examples of this free vertical motion and shows its connection with the horizontal motion. It will therefore be understood at once that from a given horizontal motion we can derive the correlated vertical motion by making quantitative use of the solenoidal condition.

The vector which fulfils the solenoidal condition with the highest degree of approximation is specific momentum. Both in atmosphere and in hydrosphere the field of mass can be considered as stationary in space (section 117). Therefore the mass-transport leading into a stationary volume through one part of the bounding surface will be equal to that leading out of it through other parts of this surface. The solenoidal nature of specific momentum is a consequence of this property of the mass-transport. In the hydrosphere the moving masses can be considered as incompressible. Then the volume-transport obtains the same property as the mass-transport, and even velocity will be a solenoidal vector. But in developing our methods we shall consider only atmospheric motions. Their adaptation to sea-motions will cause no difficulty as soon as the observations to be used are at hand.

186. Diagnostic Use of the Solenoidal Condition.—We shall consider an atmospheric sheet limited by two horizontal or quasi-horizontal surfaces. dz will be their vertical distance. The average horizontal motion in this sheet will be represented by the specific momentum **V**. A chart will be given containing the lines of flow (or the isogons) and curves for equal intensity $V =$ const. of this vector. By using the solenoidal condition we shall derive from this chart the correlated data regarding the vertical motion. We will give three different methods of deriving these data, each leading to a special form for the representation of the vertical motion.

(A) *Areas of equal vertical transport.*—The simplest plan will be to draw a chart of the horizontal transport T in the sheet. By the solenoidal condition this chart must necessarily give an indirect representation also of the correlated vertical transport T_v. Let dn be a horizontal element of line which is normal to the lines of flow. The expression

$$(a) \qquad T = Vdndz$$

will then give the horizontal transport through the area $dndz$, which extends from the bottom to the top of the sheet. Thus we have to draw a chart representing the expression (a).

In order to do this we shall first consider the expression

$$(b) \qquad T_1 = Vdn$$

which represents the transport in a sheet of the thickness of $dz = 1$. The curves T_1 = const. will be the curves of equal transport for the two-dimensional vector **V**. In order to draw these curves we may proceed as we have developed already (section 167): On the chart which represents **V** we first draw an arbitrary initial curve C' and divide it into elements which give equal values of the two-dimensional transport; *i. e.*, for each element we shall have

$$(c) \qquad V'dn' = c'$$

dn' denoting the projection of the element of the curve C' upon the normal to the lines of flow. c' is an arbitrarily chosen constant, equal either to the unit of transport used in practice or equal to a simple multiple or fraction of this unit. The essential point is to choose the constant so that we get bands of flow of suitable breadth for the construction. Through the points of division we draw lines of flow which will then define the bands of flow to which the transport T is to be referred. Using the divided sheet of fig. 86, we then draw curves for equal values of the breadth dn of these bands of flow. Finally we perform the graphical multiplication of this field by that of V. The field resulting will be that of T_1, which represents the horizontal transport in a sheet of unit thickness, $dz = 1$.

In order to get a chart of T we have finally to perform the multiplication by the thickness dz of the sheet. If dz is constant this will lead to a simple change of the intervals between the curves T_1 = const. In the general case, where the thickness of the sheet is variable from place to place, dz will be represented by a chart which gives the topography of the upper limiting surface of the sheet relatively to the lower. We have then to perform the graphical multiplication of this field by that of T_1. The result will be the field of T represented by curves for integer values

$$T = \ldots 11, 10, 9, 8, \ldots$$

This field directly represents the average horizontal transport in the sheet, but indirectly it will also represent the correlated free vertical transport. Let us suppose, for the sake of simplicity, that the lower limiting surface of the sheet is a surface of flow. The bands of flow in the two-dimensional drawing will then represent tubes, the bottom and the two lateral walls of which are surfaces of flow, while a transport goes through the top. The curves T = const. will represent vertical walls which are sections of these tubes. When we proceed along a tube from one section to the next, we have unit change of horizontal transport. By the solenoidal condition we must therefore have unit vertical transport through that area of the top which is contained between these two sections. Thus the curves T = const. will divide the bands of flow into areas for each of which we have unit vertical transport through the upper limiting surface of the sheet. In the case of decreasing horizontal transport the vertical transport will go up, and in case of increasing vertical transport it will go down through the top of the sheet.

If there is a vertical transport through the lower limiting surface of the sheet, the areas will represent that addition to the vertical transport which arises on account of the horizontal motion in the sheet.

We thus see that we have a method of arriving at a representation of vertical motion like that illustrated by figs. 43 C and 45 C.

(B) *Topographic method.*—We shall retain that division of the given chart of **V** into bands of flow which we have performed as an introduction to the construction of areas of equal vertical transport. The curve C' represents a vertical wall of the given constant height dz'. The bands of flow on the chart represent tubes of flow in space, which at this wall have the given transport $T' = V'dn'dz'$. In case (A) we have examined the change of transport T as we proceeded along tubes, which were limited below and above by given surfaces. Now only the lower limiting surface will be given. The upper will be subject to this condition, that it shall pass through the upper edge of the wall C'. We will determine its height dz above the lower surface so that the tubes retain in all sections the transport T' which they have in the section formed by the wall C'.

For this we have to introduce into (a) the value $V'dn'dz'$ for T, and to solve with respect to dz,

$$(d) \qquad dz = \frac{V'dn'}{Vdn}\, dz'$$

and construct a chart of this height dz. This will be a topographic chart which gives the height of the upper limiting surface relatively to the given lower surface.

The construction will be very like the preceding one. We first perform the construction for the case of a wall C' of unit height. Setting $dz' = 1$ and remembering that $V'dn'$ has been determined to be equal to the number c', we have

$$(e) \qquad dz_1 = \frac{c'}{Vdn}$$

As c' is equal either to unity or to a simple multiple or decimal fraction of the unity, we can determine the field of the quantity $\frac{c'}{dn}$ in one operation, using the divided sheet of fig. 81. Then we perform the graphical division of this field by that of V. The field resulting will be a topographic chart representing the upper limiting surface when the initial wall C' has unit height.

Performing the multiplication by the constant height dz' we get the field of dz, *i. e.*, the topographic chart representing the upper limiting surface for any given constant height of the initial wall C'.

The interpretation of the chart will be easiest in the case where the given lower limiting surface of the sheet is a surface of flow. The transport in each tube being constant, we conclude by the solenoidal condition that the upper limiting surface will also be a surface of flow.

We have thus obtained a method of constructing the topography of one surface of flow relatively to another, and thus of arriving at those representations of vertical motions which are illustrated by the figures 43 A and B and 45 A and B.

If the lower limiting surface of the sheet is not a surface of flow, the upper surface (the topography of which we have determined) will not be one either. But still it will characterize that part of the vertical motion which arises as a consequence of the horizontal motion within the sheet.

(C) *Vertical component of specific momentum.*—If we wish to find the vertical component of specific momentum, we have simply to use the solenoidal condition in its differential form. By equation (*j*) of section 171, we have

$$(f) \qquad \frac{\partial V_z}{\partial z} = -\text{div}_2\mathbf{V}$$

or, when we multiply by dz,

$$(g) \qquad dV_z = (-\text{div}_2\mathbf{V})dz$$

By this equation we can draw a chart of the increase dV_z of vertical specific momentum within a sheet of any thickness dz within which we know the horizontal specific momentum **V**.

As in the preceding cases, it will be convenient to begin with the case of a sheet of unit thickness $dz = 1$. The corresponding increase of vertical specific momentum will be

$$(h) \qquad dV_{1,z} = -\text{div}_2\mathbf{V}$$

From the given chart which represents the field of the horizontal vector **V** we derive the field of the divergence $\text{div}_2\mathbf{V}$, using the method developed in section 170. This field of divergence will, after change of sign, represent the increase $dV_{1,z}$ of vertical specific momentum from bottom to top in a sheet of unit thickness.

In order to get the increase dV_z for a sheet of any thickness we have to perform the multiplication by the thickness of dz. If dz is constant, this will simply be a change of the interval between the curves for constant values of $dV_{1,z}$. In the general case where dz is variable, and is represented by a chart which gives the topography of the upper limiting surface of the sheet relatively to the lower, we have to perform the graphical multiplication of the fields of $dV_{1,z}$ and of dz.

187. **Change of Variables.**—The horizontal mass-transport was given by the formula

$$T = Vdndz$$

It is the dz appearing here which brings in the vertical dimension in the formulæ of the preceding section and allows us to describe the motion in reference to this dimension.

Instead of expressing the vertical dimension in the direct way by the length dz measured along a vertical line, we can express it indirectly by the decrease of pressure $-dp$ along this line. For when the field of pressure is known, the indication of a pressure will be equivalent to that of a height. In order to bring in pressure we can first substitute dynamic height H for geometric height z. This can be done with sufficient accuracy by the relation

$$dz = 1.02\, dH$$

dz being expressed in meters and dH in dynamic meters. Then we can pass from dynamic height to pressure by the equation of hydrostatics

$$dH = -\alpha dp$$

where pressure p is to be expressed in decibars and H in dynamic meters. When we introduce this in the expression of T and remember

$$v = \alpha V$$

we shall get as a new expression of the horizontal mass-transport

$$(a) \qquad T = (1.02\, v)\, dn\, (-dp)$$

or, when we leave out the practically insignificant factor 1.02

$$(a') \qquad T = v\, dn\, (-dp)$$

When we compare this expression with the original, $T = Vdndz$, we conclude that in the formulæ of the preceding section we are entitled to introduce the decrease of pressure $-dp$ instead of the increase of height dz on condition of introducing at the same time horizontal velocity v instead of horizontal specific momentum V. This change of the formulæ leads at once to the following general rule:

The constructions described in the preceding section may be performed upon charts of horizontal velocity v *instead of upon charts of horizontal specific momentum* V. *The charts resulting will then describe the vertical motion in reference to the pressure decreasing upward instead of in reference to the height increasing upward.*

Thus to mention the special cases:

(A) *Areas for equal vertical mass-transport.*—We start with a chart representing horizontal velocity, and propose to draw a chart representing the transport (a').

For this we first draw a chart of the expression

$$(b) \qquad T_1 = vdn$$

which represents the horizontal mass-transport in a sheet of a thickness defined by unit decrease of pressure from bottom to top, $-dp = 1$. In order to get this chart we first draw an initial curve C' and divide it into elements which give

$$(c) \qquad v'dn' = c'$$

where dn' denotes the projection of the element of the curve C' on the normal to the lines of flow, and c' is a constant chosen so as to get proper breadths of the bands of flow. Through the points of division we draw lines of flow dividing the field into the bands of flow to which the transport T_1 is to be referred. Then we draw curves for equal values of the breadths dn of these bands of flow and perform the graphical multiplication of this field by that of the scalar value v of the velocity. This gives the field of T_1.

The field of T_1 will represent the final result if the thickness of the sheet is defined by unit decrease of pressure. If it has a thickness defined by any variable decrease of pressure, a chart of this decrease of pressure $-dp$ must be given.

This chart will give in terms of pressure the topography of the upper limiting surface of the sheet relatively to the lower one. If we perform the graphical multiplication of this field of pressure $-dp$ by that of T_1, we get the field of T.

The direct interpretation of the chart of T is this: it gives the *horizontal* mass-transport in the sheet the thickness of which is defined by the decrease of pressure $-dp$ from bottom to top. But at the same time it represents the *vertical* mass-transport through the top of this sheet in an indirect way: The curves $T=$const. divide the bands of flow into elementary areas; for each of these areas we have unit mass-transport through the upper limiting surface of the sheet.

(B) *Topographic method.*—We retain that division of the given velocity-chart into bands of flow which we have performed by drawing the curve C' and dividing it into elements. The curve C' will now represent a vertical wall the height of which is given by the condition that there shall be constant decrease of pressure $-dp'$ from bottom to top. At this wall the tubes will then have the given mass-transport $T' = v'dn'(-dp')$. We propose to draw a chart of that decrease of pressure

$$(d) \qquad -dp = \frac{v'dn'}{vdn}(-dp')$$

which must define the thickness of the sheet if the tubes are to have everywhere the same mass-transport as they have at the wall C'.

We perform the construction first for the case in which the wall C' has the height which is defined by unit decrease of pressure from bottom to top, $-dp'=1$. This is done according to the formula

$$(e) \qquad -dp_1 = \frac{c'}{vdn}$$

where c' is the value of the two-dimensional transport $v'dn'$ at the curve C'. In order to find the field of $-dp_1$ we first draw the field of $\frac{c'}{dn}$ by use of the differentiating sheet of fig. 81. Then we perform the graphical division by the field of the scalar value of the velocity v. The resulting field will be a chart which gives in terms of pressure the topography of the upper limiting surface of the sheet relatively to the lower one in the case $-dp_1 = 1$. If the wall C' has a height defined by another constant decrease of pressure $-dp'$, we have finally to perform the multiplication of the field of $-dp_1$ by this constant $-dp'$. The field resulting (d) represents in terms of pressure the topography of the upper limiting surface of the sheet relatively to the lower one. If the lower is a surface of flow, the upper will also be a surface of flow in virtue of the solenoidal condition. We thus have a method of drawing charts of surfaces of flow in the atmosphere, giving the topography of these surfaces in reference to the field of pressure.

(C) *Vertical component of specific momentum.*—When we make the change of variables in the solenoidal condition in its differential form we shall come to the equation

$$\frac{\partial V_z}{-\partial p} = -\text{div}_2\text{v} \qquad (f)$$

or, solving with respect to the increase dV_z of vertical specific momentum, we get

$$dV_z = -(\text{div}_2\text{v})\ (-dp) \qquad (g)$$

By use of this equation we can find the increase dV_z of vertical specific momentum in a sheet the thickness of which is defined by the decrease of pressure $-dp$.

The practical work will begin by drawing a chart for the case in which the sheet is defined by unit decrease of pressure, $-dp=1$. The increase of vertical specific momentum in this sheet will be

$$dV_{1,z} = -\text{div}_2\text{v} \qquad (h)$$

That is, it will be found if we draw the field of divergence of the given field of horizontal velocity **v**, and then change the sign.

From a sheet defined by unit decrease of pressure we can pass to one for any decrease of pressure by multiplication by that pressure $-dp$ which defines the thickness of the sheet. If $-dp$ is constant, the result will simply be a change of the interval between the curves which represent $dV_{1,z}$. If $-dp$ is variable from place to place, it must be represented by a chart, which will then represent the topography of the upper limiting surface of the sheet relatively to the lower one, topography being expressed by decreases of pressure instead of by increases of height. By graphical multiplication of the chart of $-dp$ by that of $dV_{1,z}$ we shall then arrive at the chart of dV_z, which represents the increase of vertical specific momentum in a sheet of any variable thickness.

188. Example. Cyclonic Center, United States of America, November 28, 1905.—As the two sets of parallel methods which we have developed in the two preceding sections lead to precisely the same formal constructions, it will be sufficient to exemplify one of these sets. We shall take that of section 187, as we can then apply directly the chart of observed horizontal velocity without changing it first into a chart of specific momentum.

In all cases we have to start with the chart of fig. 102, which represents the observed horizontal velocity at 8 a. m., 75th meridian time. The fine lines are curves for equal wind-velocity, expressed in meters per second. The thick lines with arrow-heads are the lines of flow, which are seen to run into a marked center of convergence. For further data regarding the meteorological conditions at the epoch of observation see plates XXXV and XXXVI.

The chart of fig. 102 is on a scale of 1 : 10 000 000. Thus 1 centimeter on the chart represents 100,000 meters. As the centimeter is the unit length on our divided sheets, we see that by using them for measurements on our charts we express horizontal distances in a unit length of 10^5 meters.

(A) *Areas of unit vertical transport.*—We draw the curve C', fig. 103, and divide it into elements which give $v'dn' = 5$. (The value $v'dn' = 1$ would have given too narrow bands for a good construction.) Through the points of division we draw new lines of flow which define the bands of flow to which the transport shall be referred. On the chart which represents these bands we have also copied the curves of

FIG. 102.—Lines of flow and curves of equal wind-intensity, U. S. A., 1905, Nov. 28, 8 a. m.

equal wind-intensity from the preceding chart. We then perform the measurement of the breadth dn of the bands, using the divided sheet of fig. 86. The chart of fig. 104 gives the curves for equal values of these breadths, together with the lines of flow copied from the preceding chart. The graphical multiplication of the field of dn by that of v finally gives the field of transport T_1, which we have represented on the chart of fig. 105 by the following curves

$$T_1 = \ldots 6, 5, 4, 3, \ldots$$

The chart which we have obtained in this manner will represent the horizontal transport in a sheet the thickness of which is given by unit decrease of pressure from the ground to the upper limiting surface of the sheet, and at the same time the vertical transport through this upper limiting surface.

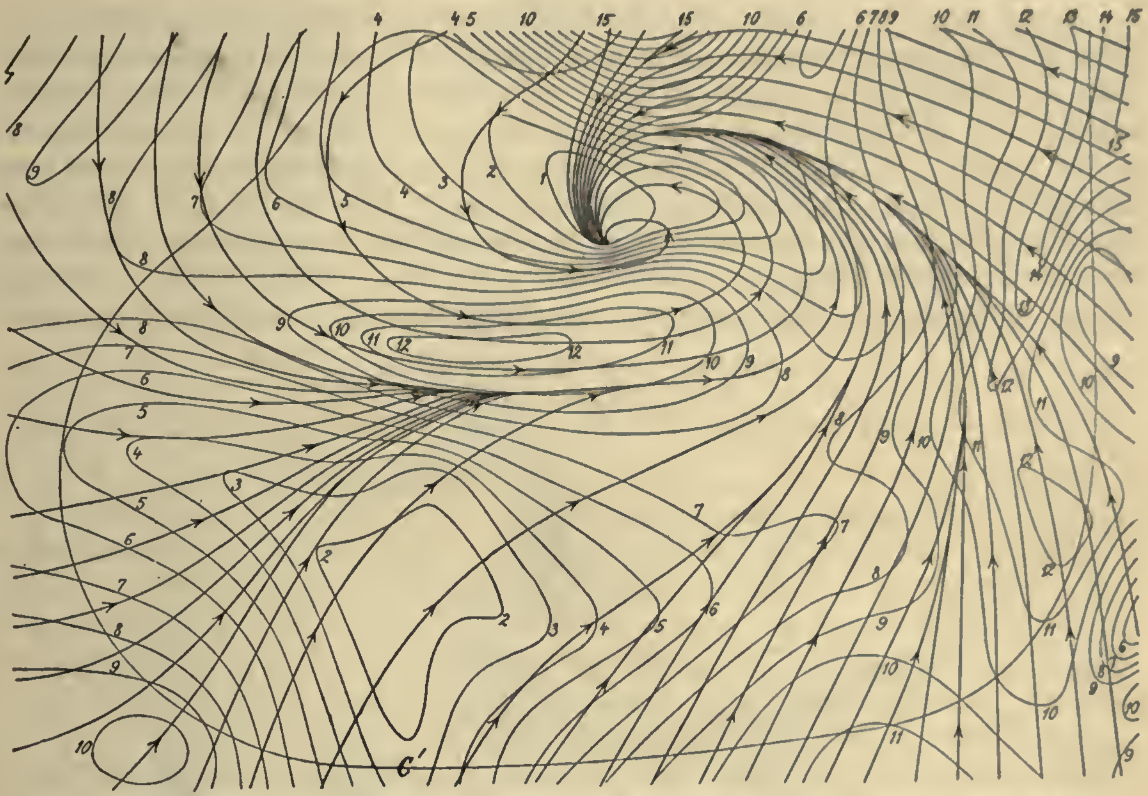


FIG. 103.—Bands of flow which have equal transport at the initial curve C'. U. S. A., 1905, Nov. 28, 8 a. m.

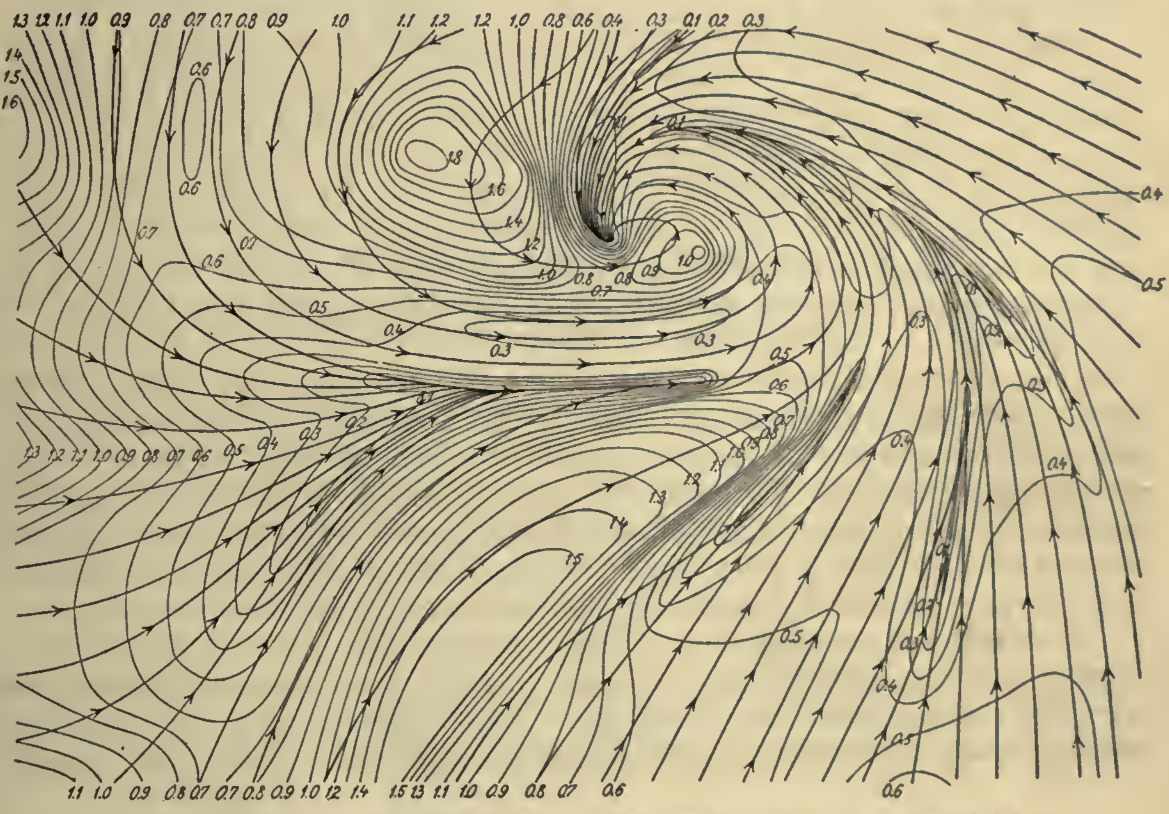


FIG. 104.—Curves for equal breadth dn of the bands of flow. U. S. A., 1905, Nov. 28, 8 a. m.

When we use the decibar as unit pressure the upper limiting surface of the sheet will be situated at the approximate height of 750 meters above the ground. The lines of flow represent vertical walls which divide this sheet into tubes. At the initial wall C' the transport in each tube is 5.10^5 m.t.s. units, *i. e.*, 500,000 tons of air per second. As we proceed from the curve C' to other curves $T_1 =$ const., we have a loss or gain of horizontal transport of 100,000 tons per second. The areas into which the bands of flow are divided by the curves of equal transport will thus represent a vertical transport of 100,000 tons per second through the upper limiting surface of the sheet. This transport is directed upward or downward according

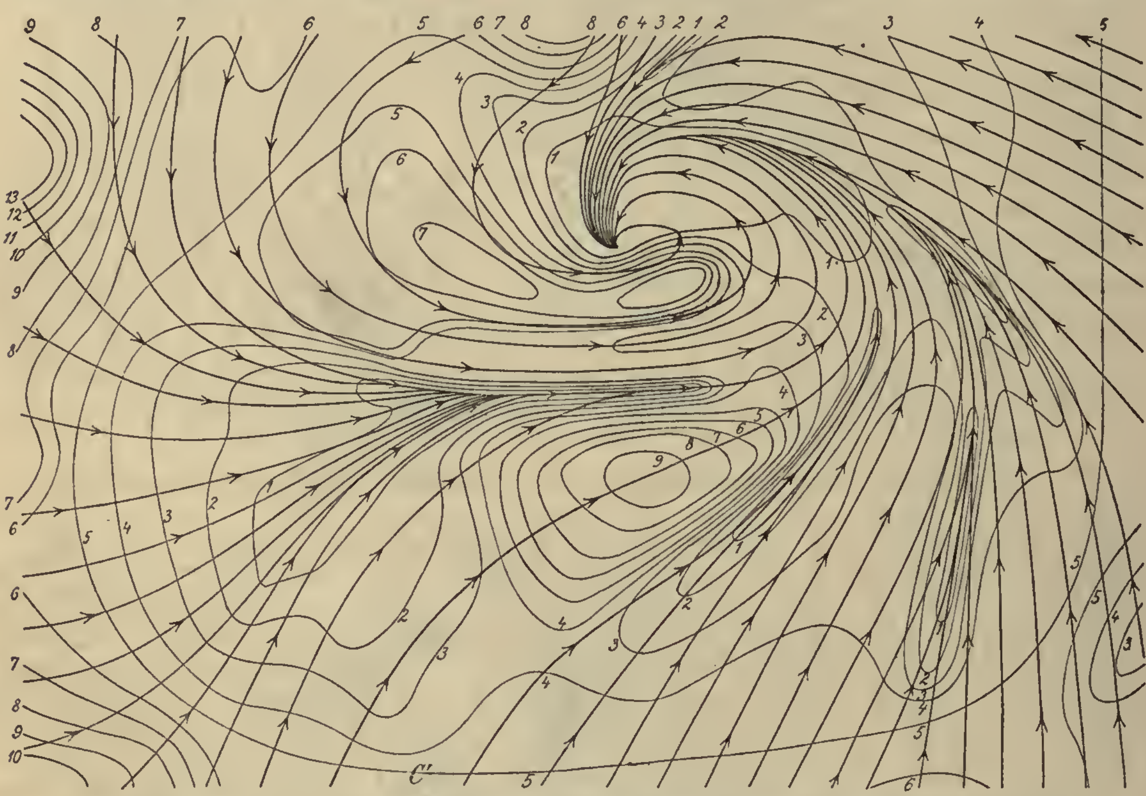


FIG. 105.—Areas of equal vertical mass-transport through a surface where pressure is one unit smaller than at the ground. U. S. A., 1905, Nov. 28, 8 a. m.

as the numbers on the curves $T_1 =$ const. decrease or increase as we proceed in the direction of motion along the tubes. The triangular areas which surround the point and the lines of convergence represent the same vertical transport as the others. As small areas indicate intense vertical motion, we see that we have a powerful ascending motion near the point of convergence, especially on its northern side and along the lines of convergence. But areas of descending motion also occur even very near the point of convergence and between two of the lines of convergence.

If we multiply the pressure of 1 decibar, which defines the sheet, by 0.1 we get a sheet which has the thickness of about 75 meters. The tubes of flow will have a transport of 50,000 tons per second at the wall C', and the areas will represent a vertical transport of 10,000 tons per second through a surface having the approximate height of 75 meters above the ground. If we multiply by 0.01 we get a sheet of an approximate thickness of 7.5 meters; the tubes will have a transport of 5000 tons of air per second at the wall C', and the areas will represent a vertical transport of 1000 tons of air per second through the surface which has the approximate height of 7.5 meters above the ground. Of course it will be legitimate to go up to so great heights as 75 or 750 meters only on condition that the original chart, fig. 102, represents the average horizontal motion between the ground and these heights.

A change in the interpretation of the charts, which will be useful for qualitative purposes, can be obtained in this manner: we multiply the unit pressure which defines the thickness of the sheet by $\frac{10^n}{750}$. We shall then obtain a sheet the thickness of which will be approximately 1, 10, 100, 1000, . . . meters, according to the value given to n. In order to get the mass-transport in this sheet, we must multiply the field of T_1 by the same number. But instead of that we can multiply only by 10^n on condition of interpreting T_1 as *volume-transport* instead of mass-transport. For 750 is the approximate volume in cubic meters of a ton of air in the lower strata of the atmosphere. In other words, for qualitative purposes it will be permissible to give an interpretation like the following of the chart of fig. 105. It represents a sheet of a thickness of 1000 meters. The tubes have a horizontal transport of 500,000,000 cubic meters of air per second at the wall C', and the areas represent a vertical transport of 100,000,000 cubic meters of air through the surface of a height of 1000 meters. When we choose the thickness of 100 or 10 meters of the sheet, we get the proportional reduction of the numbers representing the volume-transport.

From the chart of fig. 105 we can see without difficulty how the tubes of flow go up and down. Let us return to the original interpretation. The areas of 100,000 tons of vertical transport can then be conceived as the sections of the upper limiting surface of the sheet with tubes of this transport. For each element of the curve C' five such tubes rest upon each other, giving the total horizontal transport of 500,000 tons. Each area shows one of these tubes coming up or going down through the upper limiting surface of the sheet. (Compare the schematic examples of figs. 43 c and 45 c.)

(B) *Topographic method.*—In order to follow not only qualitatively, but quantitatively, the course of the tubes up and down, we can pass to the topographic method. We then retain the curve C', its division into elements fulfilling the condition $v'dn' = 5$ and the corresponding division of the chart into bands of flow, fig. 103. Introducing the value $c' = 5$ in formula (c) section 187, we get

$$-dp_1 = \frac{5}{vdn}$$

By use of the divided sheet for reciprocal length-measurements (fig. 81) we draw the field $\frac{5}{dn}$. The curves representing this field will have the same course as those representing the breadth dn (fig. 104), only with other intervals. Finally we perform the graphical division by v. The field resulting is given by the chart of fig. 106, representing in terms of pressure the topography relatively to the ground of a surface of flow formed by those lines of flow which at the initial curve C' have a height above the ground defined by unit decrease of pressure. The contour-lines $-dp_1 =$ const. of this chart have the same course as the curves $T_1 =$ const. of fig. 105, only with changed intervals. The curves 1, 2, 3, 4, 5, . . . show the points where pressure is 1, 2, 3, 4, 5, . . . units smaller than at the ground. According as we use m-bar, c-bar or d-bar as unit of pressure, these curves will represent the approximate

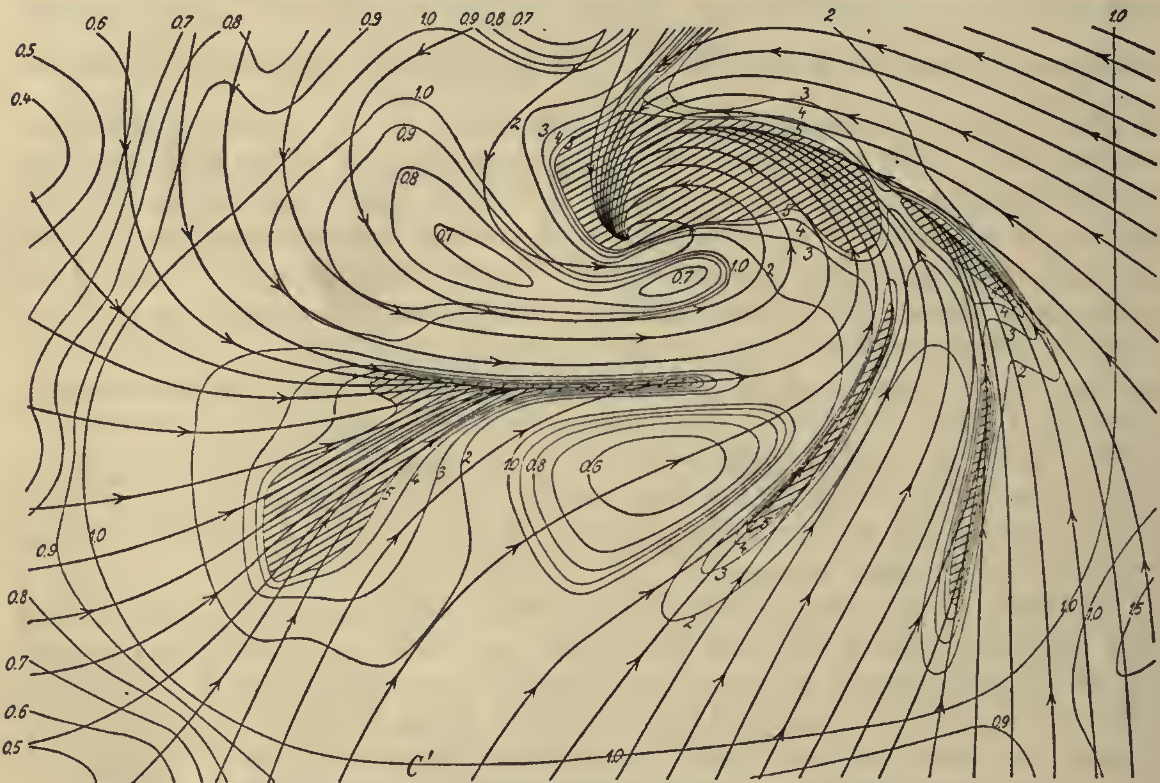


FIG. 106.—Topography of a surface of flow relatively to the earth. U. S. A., 1905, Nov. 28, 8 a. m.

heights of 7.5, 15, 22.5, 30, 37.5, . . . of 75, 150, 225, 300, 375, . . . or of 75, 1500, 2250, 3000, 3750, . . . meters above the ground. Whether it be legitimate to go to greater heights will depend upon whether the given chart gives a true picture of the average horizontal motion between the ground and these heights.

We have drawn no curve inside the curve 5, which, according to the different interpretations, represents an approximate height of 37.5, of 375, or of 3750 meters. But the formal construction, in losing its physical significance, would give an infinity of contour-lines inside this curve, indicating an infinite increase of height of the surface of flow as we approach the point or the lines of convergence. The lowest part of the surface is represented by the curves 0.9, 0.8, 0.7 . . . which are found partly outside the curve C', and partly inside it, especially a little south of the point of convergence and between two of the lines of convergence.

The chart of fig. 106 gives the topography of the surface of flow expressed in terms of pressure; qualitatively we can consider it also as a chart giving topography in terms of height. We have given above the approximate height corresponding to the different integer values of pressure. But if we multiply by $\frac{10^{n}}{750}$ we pass to decimal heights. Thus in rough approximation we can interpret the curves 1, 2, 3, . . . of the chart as contour-lines which give the heights 1, 2, 3, . . . meters or the heights 10, 20, 30, . . . or 100, 200, 300, . . . of a surface of flow.

(C) *Vertical component of specific momentum.*—In order to find vertical specific momentum, we have to draw a chart of divergence of the horizontal motion (see formula (*h*) of section 187). For this we can use directly the given chart of fig. 102, no special division into bands of flow being required. Divergence of the two-dimensional field of velocity **v** will according to formula (*g*) of section 170 be given by the equation

$$\mathrm{div}_2\,\mathbf{v} = \frac{\partial v}{\partial s} + v\delta$$

s denoting the length of arc along the lines of flow and δ the divergence of these lines (see section 168). As we here come across the most important construction of kinematic diagnosis, we will illustrate each of the four separate operations, the last of which gives the result.

(1) We construct the field of the derivative $\frac{\partial v}{\partial s}$ of the intensity of the vector with respect to its vector-lines. This differentiation is performed in the regular way by use of the differentiating sheet of fig. 81 as illustrated in section 165. The resulting field is given in fig. 107. The numbers added to the curves give the values of the derivative obtained when *ds* is measured in centimeters on the chart. In order to get the true values per meter we have to multiply by 10^{-5}, as a centimeter on the chart represents 10^5 meters.

(2) Then we have to draw the field of divergence δ of the lines of flow. We can determine this field by use of the divided sheet for differentiations of the second order, fig. 90, this sheet being placed with the radii tangential to and the circles normal to the lines of flow. But if the isogons of the lines of flow are given, we get a much better determination by using the ordinary differentiating sheet of fig. 81. We then perform the differentiation of the angle represented by the isogons with respect to the normal curves *n* to the lines of flow. The resulting field of divergence of the lines of flow is given on the chart of fig. 108. The numbers give the value of the divergence referred to the centimeter as unit of length and to the scale of the chart. Multiplying by 10^{-5} we get the true divergence of the lines of flow referred to the meter as unit of length.

(3) Then we perform the graphical multiplication of this field of divergence by that of the intensity *v* of the given velocity. The result of this multiplication, which is performed in the regular way (section 150) is given on the chart of fig. 109.

(4) Finally we perform the graphical addition of the two fields of figures 107 and 109, and change the sign in order to pass from divergence to vertical-component of specific momentum. We thus get the chart of fig. 110, which contains the result.

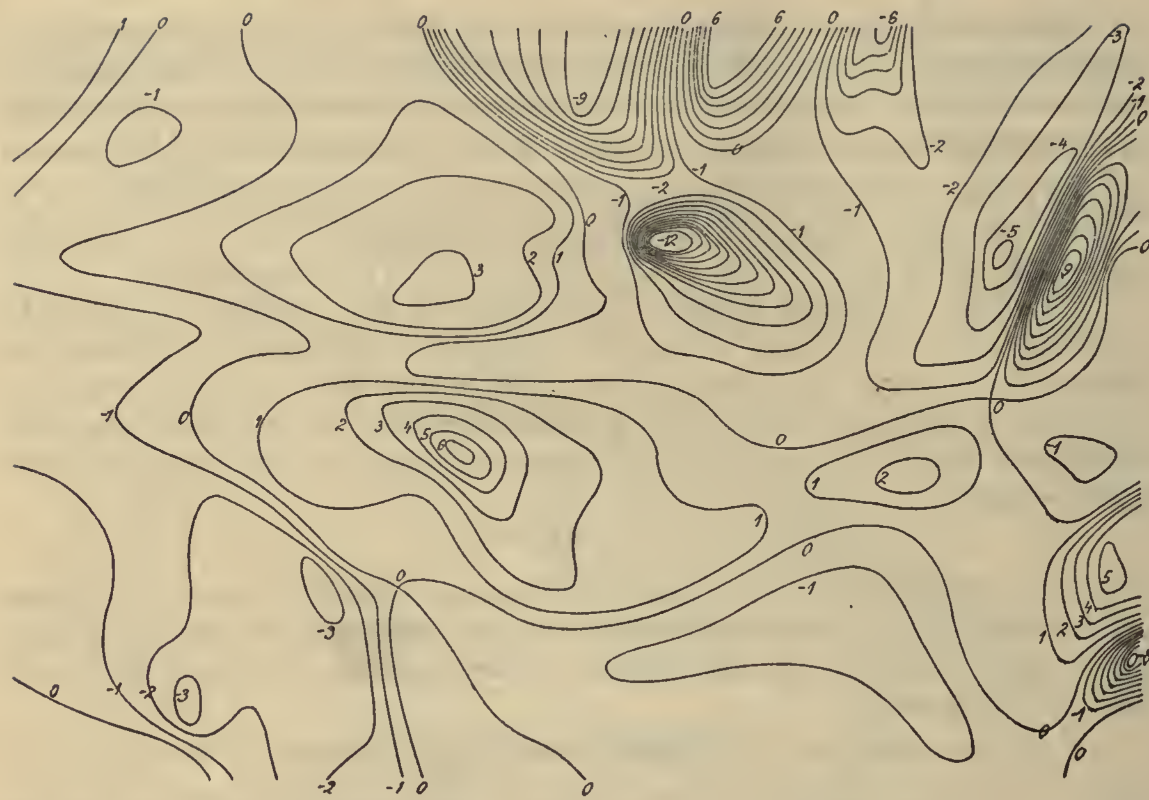


FIG. 107.—Derivative $\frac{\partial v}{\partial s}$ of velocity with respect to the lines of flow. U. S. A., 1905, Nov. 28, 8 a. m.

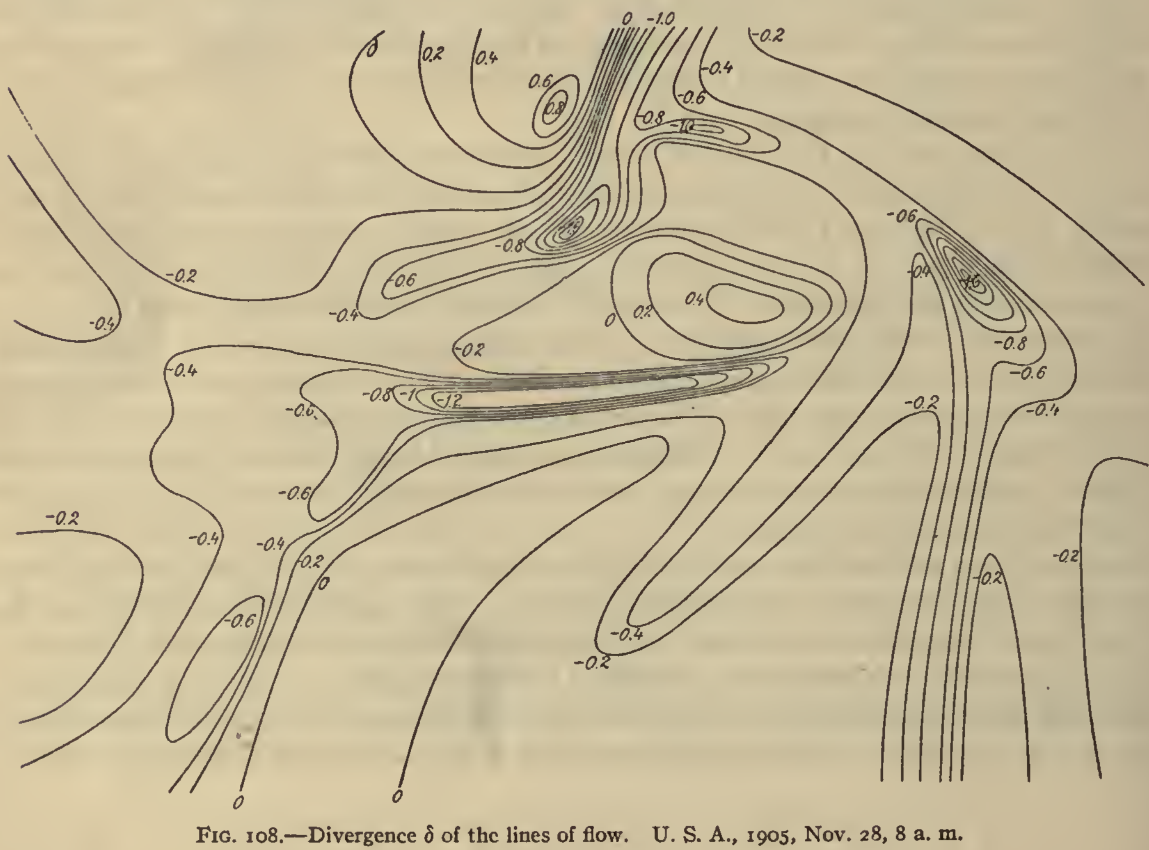


FIG. 108.—Divergence δ of the lines of flow. U. S. A., 1905, Nov. 28, 8 a. m.

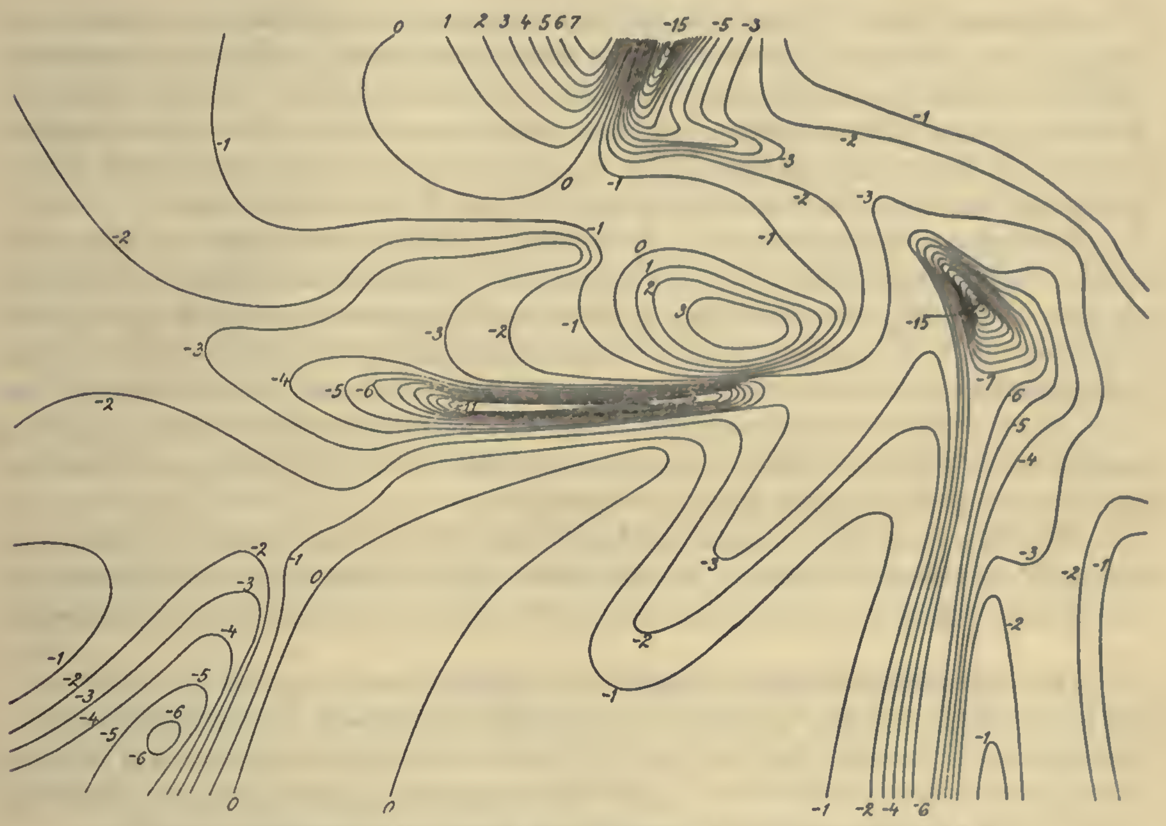


FIG. 109.—Product $v\delta$ of wind-velocity and divergence. U. S. A., 1905, Nov. 28, 8 a. m.

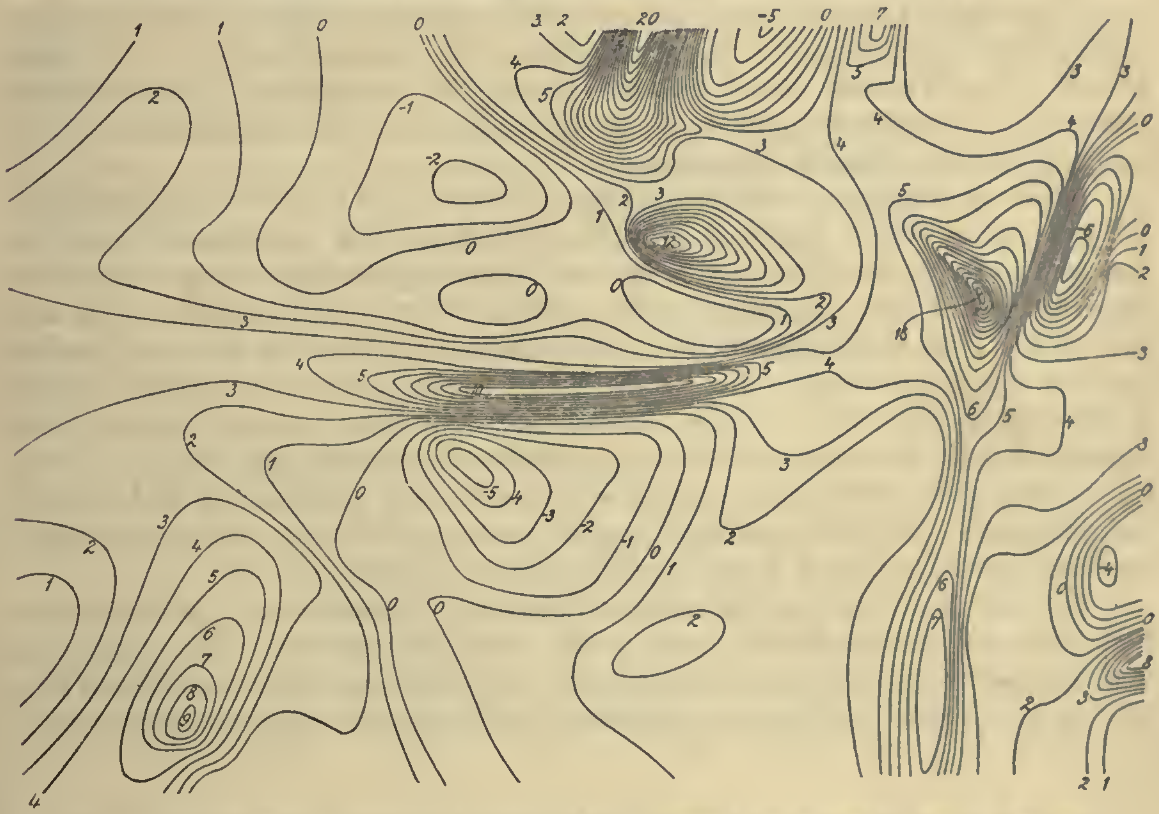


FIG. 110.—Vertical specific momentum at a surface, where pressure is one unit smaller than at the ground. U. S. A., 1905, Nov. 28, 8 a. m.

The chart (fig. 110) gives the vertical component of specific momentum in the height where pressure is one unit smaller than at the ground. The sheet can have a thickness defined by the decrease of pressure of one m-bar, of one c-bar, or of one d-bar. The numbers added to the curves will then represent the vertical specific momentum respectively in the units 0.1 gram per square meter per second, 1 gram per square meter per second, or 10 grams per square meter per second.

Instead of defining the sheets by the decrease of pressure, we can define them as sheets of a thickness of 10, 100, or 1000 meters. The numbers added to the curves on the chart of fig. 110 will then approximately represent *vertical velocity*, in the following units: in tenths of millimeters if the sheet has a thickness of 10 meters, in millimeters if the sheet has a thickness of 100 meters, and in centimeters if the sheet has a thickness of 1000 meters. This rule will be very convenient for getting a qualitative picture of the vertical motion which the chart of fig. 110 describes quantitatively by vertical specific momentum.

The chart is seen to give an ascending velocity which has its greatest values near the point and along the lines of convergence. But areas of descending velocity are also found, even near the point of convergence and between two lines of convergence.

189. **Complete Kinematic Diagnosis.**—Each of the three methods of representing free vertical motion, by areas of equal vertical transport, by topographic representation of surfaces of flow, or by charts of the vertical component, will have its special advantages in special cases. But the question will now be which of them will work best as a link in a complete kinematic diagnosis of atmospheric motions.

The construction of a chart of areas which represent equal vertical transport will be easy for each single atmospheric sheet. But inasmuch as the lines of flow have a different course in the different sheets, the summation of the transports produced in the different sheets will be circumstantial. For this reason we shall not make a general use of this method.

When the topographic method is applied, we shall not meet with this difficulty. We can pass by simple graphical addition from the relative topographies which we find by the solenoidal condition to the corresponding absolute topographies. But the drawback of the topographic method will be the great complication of the surfaces of flow. In the neighborhood of the initial curve C' used to define the surface it will be relatively simple. But the farther we follow it the more complicated will be the course of the contour-lines. Finally we shall always come to places where the surface folds itself so as to be cut by a vertical line at more than one point. The topographic method of representation will then become complicated, and will lose its conspicuity. While the method may do good service for special investigations, we shall not try to take it as the base for a universal method.

We shall therefore base the complete kinematic diagnosis upon the representation of the vertical motion by charts of the vertical components. The production of these charts is a little more laborious than that of the preceding ones, but as soon as they are produced all further operations will be easy to perform upon them.

In order to perform this diagnosis, we must first know the field of pressure, *i. e.*, we must have charts giving the topography of the standard isobaric surfaces and of the pressure at the ground. From the latter we derive a special chart of the difference of pressure between the ground and the lowest isobaric surface in free air. Then we must have a chart of velocity at the ground, and charts of the average horizontal velocity within each of the standard isobaric sheets, as well as of this average velocity in the incomplete sheet between the ground and the lowest standard surface in free air. The kinematic diagnosis will be accomplished as soon as we have found the complete representation of the vertical motion. We shall arrive at this representation by the following operations (compare the example, section 204, below):

(1) From the chart of velocity in connection with that of pressure at the ground we derive the chart of the forced vertical specific momentum at the ground.

(2) From the chart of the average horizontal velocity in the incomplete sheet between the ground and the lowest standard isobaric surface in free air we derive free vertical specific momentum through this surface. The construction is first performed for a unit sheet, and then the result is obtained for a sheet of irregular thickness by graphical multiplication by the decrease of pressure which defines the sheet.

(3) From the charts of average horizontal velocity in the different standard isobaric sheets we derive the vertical specific momenta produced in each sheet. If a sheet is partly incomplete, the limiting surfaces cutting the ground, we use the method (2) for the incomplete parts of the sheet.

By successive graphical additions of the charts (1), (2), (3), . . . we get the charts of the absolute vertical specific momenta in the different standard isobaric surfaces. If it be desired it will be easy afterwards to change them into charts of vertical velocity.

It will be understood at once how a perfectly similar kinematic diagnosis can be carried out based upon the division of the atmosphere into level instead of into isobaric sheets.

CHAPTER XII.

KINEMATIC PROGNOSIS.

190. Determination of Displacements from Given Velocities.—The fundamental kinematic vector, velocity, is by its very definition a quantity of prognostic nature. If the initial position and simultaneously the velocity of a particle is given, it will always be possible to make a certain definite statement regarding its future position. How far in the future this statement will have any value will depend upon the time-variations of velocity. If it does not vary, either in direction or in intensity, the determination can be made for any future time. But if the velocity varies according to an unknown law, the forecast will be of value only for a limited period of time. When we select a sufficiently short period, the variation of velocity will have insignificant influence, and the prognosis of the future position can be based exclusively upon the knowledge of the initial position of the particle and the initial value of its velocity.

This kinematic prognosis will always be the first step when a rational precalculation of future atmospheric or hydrospheric states is to be made. In principle this step will be perfectly simple. The only delicate point will be the choice of proper periods for which the prognosis may be ventured. They can only be found by experience. As regards the case of the hydrosphere our experience is still quite insufficient. As we have not been able to produce any example of kinematic diagnosis, we can not give any of kinematic prognosis either. As regards the atmosphere, our preliminary experience seems to indicate that periods of a few hours may be used, say from one to six hours. If three hours are used, this period will be convenient also because it is in rough approximation a decimal multiple of our unit of time, the second, viz, 10,800 seconds, or in the mentioned rough approximation 10,000 seconds.

191. Synoptical Representation of Horizontal Displacements.—When a chart of horizontal velocity is given, the tangent to a line of flow gives the direction in which the displacement of any particle takes place, and the scalar value of velocity multiplied by 10,800 gives the length of the displacement in three hours. On the velocity-chart we can thus easily mark the initial and the final situation of any number of points, marking, for instance, the initial position by a little circle with a dark area, and the final position with a corresponding circle with a white area. In order to show which points belong to each other we can draw a line from each black circle to the corresponding white one.

In order to make conspicuous the chart of horizontal displacements, it will be advantageous to choose systematically the initial situations of the points. They can be chosen so that they belong to a set of isogons, or so that they belong to a set of intensity-curves. In the first case the points situated on the same curve will be displaced in the same direction, in the second along the same length. This will at the same time make the construction easy and the figure conspicuous. A complicated picture will, however, appear in places where one series of points is displaced beyond the initial places of another series.

This difficulty may be completely avoided if we choose the points according to another principle, namely, so that the final situation of one point shall be the initial situation of another. In this manner we get chains of points (fig. 111) which have a certain similarity with the lines of flow and would coincide with them if we drew the displacements for infinitely short intervals of time.

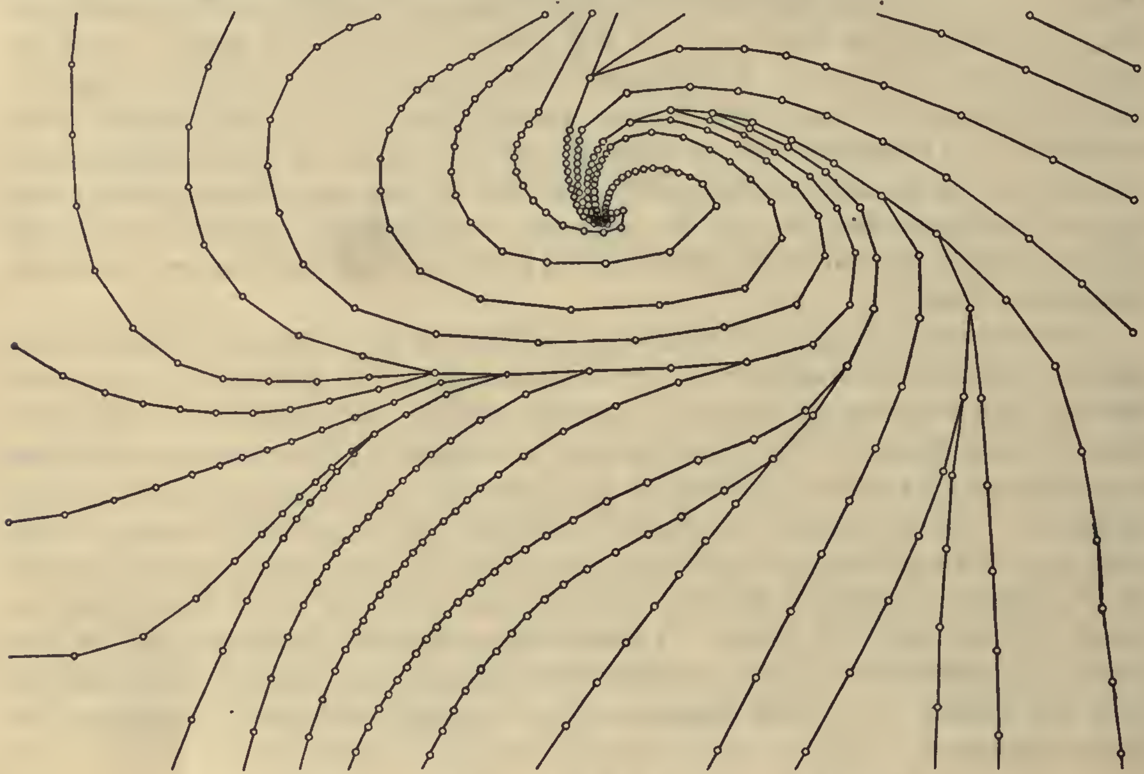


Fig. 111.—Displacements in 3 hours. U. S. A., 1905, Nov. 28, 8 to 11 a. m.

It will be understood at once that from the corresponding charts of vertical velocity we can derive the correlated vertical displacements, but it will be of no use to enter into details before we come to the more general problem of dynamic prognosis. It will be sufficient that we have indicated here the general principle of kinematic prognosis.

192. Different Forms of the Equation of Continuity.—Before we leave the question of kinematic prognosis we have to examine the prognostic value of the equation of continuity. We have already alluded to the prognostic nature of this equation, but we have used it hitherto exclusively for diagnostic purposes. For the more general purpose we have first to give the complete mathematical formulation of the equation of continuity. The two theorems, section 114 (A) and (B), correspond to two different mathematical forms of the equation. The first theorem deals with the velocity of expansion of a given mass and states its identity with the integral of the normal component of velocity taken over the limiting surface of the mass. K being the volume of the mass, the velocity of expansion will be expressed by the *individual* time-derivative of K. Thus we can write the equation

$$(a) \qquad \frac{dK}{dt} = \int v_n \, d\sigma$$

The theorem 114 (B) deals with the varying mass M which is contained within a stationary volume, and states that the diminution of this mass per unit time is equal to the mass-outflow through the limiting surface of the volume. Evidently the diminution of the mass M per unit time in a stationary volume is expressed by the negative *local* time-derivative of M. When we identify this derivative with the well-known expression of the mass-outflow, we get this other form of the equation of continuity

$$(b) \qquad -\frac{\partial M}{\partial t} = \int V_n d\sigma$$

In order to bring the equations to forms more easily used we can apply them to infinitely small volumes K. The integrals appearing in the second member of (a) or (b) will then be expressed by the product of this volume K into the divergence of the vector. When at the same time we express the volume K of the moving mass by the product of its mass M into its specific volume, $K = \alpha M$, and remember that this mass M is constant, we get equation (a) in the form

$$M\frac{d\alpha}{dt} = K \operatorname{div} \mathbf{v}$$

Dividing by K, and remembering that the ratio $\frac{M}{K}$ is the reciprocal specific volume, we get

$$(c) \qquad \frac{1}{\alpha}\frac{d\alpha}{dt} = \operatorname{div} \mathbf{v}$$

In the same manner, when in (b) we express the mass M as the product of its density ρ into its volume K, and remember that here the volume is stationary in space and therefore constant, we get

$$(d) \qquad -\frac{\partial \rho}{\partial t} = \operatorname{div} \mathbf{V}$$

When we use the relation existing between local and individual derivative (section 177) as well as the relations existing between density and specific volume and between velocity and specific momentum, we can verify at once the fact that (c) and (d) are merely different forms of the same equation.

193. **Equation of Continuity as a Prognostic Equation.**—Equation (d) of the previous section directly tells us that if we know the field of specific momentum at any moment, we can find a field representing the rate of decrease of density $-\frac{\partial \rho}{\partial t}$

simply by forming the field of divergence of the specific momentum. Then we could multiply the field of $-\frac{\partial \rho}{\partial t}$ by a suitable interval of time dt, and add it to the field of density at the time t. We should then get the field of density at the time $t+dt$.

This method could be formally carried out if we had sufficiently complete and exact observations of specific momentum **V**. But as we have to form the divergence in space, we need observations not only of the horizontal, but also of the vertical component of specific momentum. Therefore, as long as we can get an idea of the vertical motions only in the indirect way by making a diagnostic use of the equation of continuity, supposing simply the field of density to be stationary in space, $\frac{\partial \rho}{\partial t} = 0$, every prognostic use of the equation of continuity in this direct way will be excluded.

But we could also think of a prognosis of a more summary character, which would also be of great value if it could be carried out practically. We shall return to the equation of continuity in the integral form

$$(a) \qquad -\frac{\partial M}{\partial t} = \int V_n d\sigma$$

and apply it to a vertical cylinder going from the ground to the limit of the atmosphere, or at least to a height in which the density of the air is so low that it can cause only an insignificant mass-transport. It will then be sufficient to integrate the horizontal specific momentum over the cylindrical surface, and our ignorance of the vertical motion will cause no difficulty.

Now the ground carries the weight of the mass of air M in this cylinder. σ_0 being the area of the base and p the pressure, we have $p\sigma_0 = Mg$, g representing an average value of acceleration of gravity. Multiplying equation (a) by g, introducing $p\sigma_0$ instead of gM, and remembering that the cylinder is stationary and therefore its base σ_0 constant, we see that the equation can be written

$$(b) \qquad -\frac{\partial p}{\partial t} = \frac{g}{\sigma_0}\int V_n d\sigma$$

Therefore, if we know horizontal specific momentum **V** sufficiently well up to sufficient heights, we should be able by this equation to forecast the change of pressure at the ground. Evidently this would be of high practical value.

The question whether this will succeed will depend on the degree of completeness and of accuracy required in the knowledge of **V**, or of the corresponding velocity **v**. In order to estimate it, we can express the vertical dimension by pressure and at the same time substitute velocity for specific momentum. Thus we have first $d\sigma = dz\, ds$, dz being a vertical and ds a horizontal element of line. Then we can express dz approximately by pressure, writing $dz = -0.1\, \alpha\, dp$, where z is measured in meters and p in the m. t. s. unit of pressure, centibar. Thus

$$\int V_n d\sigma = \int\int V_n dz ds = \int\int v_n(-0.1\, dp)\, ds$$

Here we can first perform the integration with respect to p, denoting by $\bar{v}_n$ the average value of v_n along a vertical line of the cylindrical surface, and by $p-p'$ the difference of pressure between bottom and top of the cylinder. Then

$$\int v_n d\sigma = 0.1\ (p-p') \int \bar{v}_n ds$$

Finally we can by $\bar{\bar{v}}_n$ denote the average value of $\bar{v}_n$ when we integrate with respect to s, *i. e.*, around the base of the cylinder. The circumference being s, we get

$$\int v_n d\sigma = 0.1\ (p-p')\ s\bar{\bar{v}}_n$$

Equation (b) thus takes the form

$$-\frac{1}{p-p'}\frac{\partial p}{\partial t} = 0.1\frac{s}{\sigma_0}g\bar{\bar{v}}_n$$

If the cylinder is circular, the ratio of its circumference to the area σ_0 of its base will be $\frac{4}{D}$, D being the diameter of the base. As p' is the pressure in very great height, and thus very small, we can leave it out without essentially changing the formula. Thus we get for a circular cylinder of sufficient height

$$-\frac{1}{p}\frac{\partial p}{\partial t} = 0.4\frac{g}{D}\bar{\bar{v}}_n$$

In order to estimate the exactitude required in the observations of velocity if it should be possible to forecast pressure at the ground by this formula, we solve with respect to $\bar{\bar{v}}_n$

$$\bar{\bar{v}}_n = -\frac{1}{p}\frac{\partial p}{\partial t}\frac{D}{0.4\ g}$$

In this formula we can express pressure in any unit. We shall then use m-bars. Passing at the same time from second to hour as unit of time, calling m the change of pressure in m-bars per hour, setting $g = 9.8$, and calling d the diameter of the cylinder expressed in kilometers, $d = 0.001\ D$, we get

$$\bar{\bar{v}}_n = -0.00007\ md$$

For the change of pressure of 1 m-bar per hour, $m = 1$, we shall then have

$d = 1000$ km.	$\bar{\bar{v}}_n = 7$ cm. per second.
$d = 100$ km.	$\bar{\bar{v}}_n = 7$ mm. per second.
$d = 10$ km.	$\bar{\bar{v}}_n = 0.7$ mm. per second.

Thus, even if we take areas of a diameter of 1000 km., the observations of the wind-velocity would have to be correct to a centimeter over the whole area of a cylinder having this diameter and extending up to heights where pressure is imperceptible. Of course observations of wind-velocity of this exactitude and completeness can not be thought of in the present state of development of meteorological observations.

Kinematic prognosis could therefore hitherto only give the displacements of the masses of air as developed in sections 190 and 191.

CHAPTER XIII.

REVERSAL OF THE PROBLEM OF KINEMATIC PROGNOSIS. KINEMATIC DETERMINATION OF ACCELERATION.

194. On the Reversed Problems.—According to our general plan (section 96) we shall now consider the problem of kinematic prognosis in its reverse form. Knowing from the observations the initial and the final state of motion, we shall investigate the change of motion which has led from one state to the other. This will involve a determination, on pure kinematic principles, of the acceleration of the moving particles.

If we ever succeed in giving the complete solution of the problem of prognosis, we shall have to determine the accelerations not by kinematic but by dynamic methods. This should be theoretically possible because the observations should allow us to derive the forces which produce the accelerations. But passing to the practical performance, we shall meet with a great difficulty. Though we know from laboratory experiments the coefficient of the friction of the air, we shall not be able to use it practically for determining the influence of friction on acceleration. The reason is obvious. Friction depends upon true motion, while we are forced to work with an idealized motion, disregarding all the small irregularities of the motion rsection 97) and of the ground (section 179). If we were to determine the frictional resistance in the rational way we should have to examine the motion from millimeter to millimeter, and not only at stations which may be hundreds of kilometers from each other. As this will not be possible, we shall be obliged to find other ways for determining the influence which, as an indirect effect of friction, modifies the idealized motion which we consider. We must develop methods for determining, by pure kinematic principles, the acceleration of the idealized motion to the consideration of which we are confined, and by comparing it with the accelerating forces find empirical rules for taking the effect of friction into account.

As an introductory problem to the kinematic determination of accelerations, we shall first treat the problem of the identification of particles on two successive charts of motion or (what comes to the same thing) the determination in the second approximation of the displacement of these particles.

195. Determination of Displacements in the Second Approximation.—Let a chart be given which represents the state of motion at the epoch t_0. We shall consider a particle which at this epoch is situated at the point A (fig. 112). According to the chart it has a certain velocity $\mathbf{v}_0$. During the short interval of time $t_1 - t_0$ its displacement will then in the first approximation be

$$(a) \qquad AB' = \mathbf{v}_0 (t_1 - t_0)$$

Thus the point B' will give in the first approximation the situation of the particle at the epoch t_1.

Now let a second chart be given, which represents the state of motion at this second epoch t_1. According to this chart we shall find not the velocity $\mathbf{v}_0$ but a certain velocity $\mathbf{v}_1'$ at the point B'. We may therefore expect to get a better determination of the displacement if we suppose that the particle has moved not with the velocity $\mathbf{v}_0$, but with the velocity $\frac{1}{2}(\mathbf{v}_1'+\mathbf{v}_0)$. During the time t_1-t_0 this velocity would give the displacement

(*b*) $$AB''=\tfrac{1}{2}(\mathbf{v}_0+\mathbf{v}_1')(t_1-t_0)$$

The point B'' would then more correctly than the point B' give the situation of of the particle at the epoch t_1.

The point B'' and the corresponding displacement AB'' can be found by a direct continuation of the construction which led to the point B'; *i. e.*, we measure off from this point the displacement

(*b'*) $$B'C''=\mathbf{v}_1'(t_1-t_0)$$

The point B'' will then be the central point of the line AC'' which represents the vector-sum of the two displacements AB' and $B'C''$.

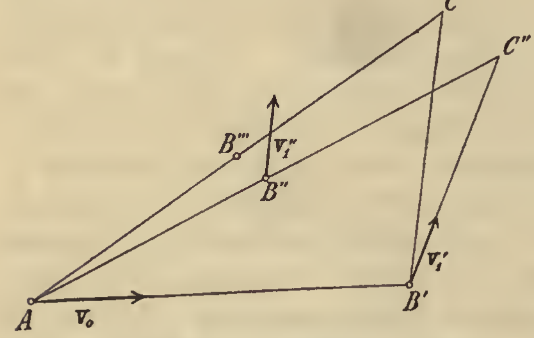


FIG. 112.—Construction of displacement in second approximation.

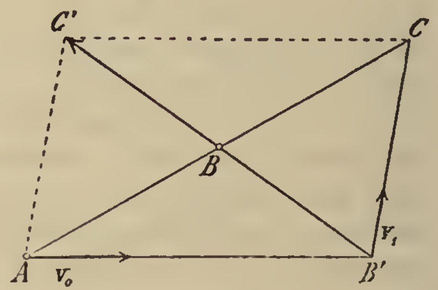


FIG. 113.—Displacement and acceleration.

But according to the second chart the velocity at the point B'' will not have the value $\mathbf{v}_1'$ but a certain other value $\mathbf{v}_1''$. Therefore we may expect to get a still better determination of the displacement required if we suppose that the particle has moved during the time t_1-t_0, not with the velocity $\frac{1}{2}(\mathbf{v}_0+\mathbf{v}_1')$, but with the velocity $\frac{1}{2}(\mathbf{v}_0+\mathbf{v}_1'')$, which would have given the displacement

(*c*) $$AB'''=\tfrac{1}{2}(\mathbf{v}_0+\mathbf{v}_1'')(t_1-t_0)$$

The point B''' at which the particle would then arrive can be found by a construction similar to that which led to the point B''. From the point B' we set off the displacement

(*c'*) $$B'C'''=\mathbf{v}_1''(t_1-t_0)$$

The point B''' will then be the central point of the line AC''' which represents the vector-sum of the two displacements AB' and $B'C'''$.

Evidently these constructions can be continued indefinitely. Two cases may then present themselves:

(1°) The distances between the points B', B'', B''' . . . may converge toward zero. The process of constructions will then be convergent and will lead ultimately to the determination of a definite point B. This point will then represent the situation of the particle at the epoch t_1 with the highest degree of approximation which can be attained when the determination is to be made by the use of *two* charts of velocity instead of by the use of only one. Or in other words: AB will represent the displacement in the second approximation.

(2°) The distances between the points in the series B', B'', B''' . . . may remain finite. The process of constructions will then be divergent and will lose every physical significance. Examples of this divergence can easily be given. We should meet with it, for instance, in the case of atmospheric wave-motions (see fig. 51) if the interval of time $t_1 - t_0$ was selected of such a length that the displacement (a) obtained in the first approximation was of the same order of magnitude as the wave-length. This case of divergence must be avoided, and can always be avoided if the selected interval of time $t_1 - t_0$ be sufficiently short. The periods which from this point of view may be used must be found gradually by experience.

We shall consider henceforth exclusively the case of convergence, and of convergence so rapid that already the point B'' or the point B''', will define with sufficient approximation the situation of the required point B. According to our experience the interval of three hours which we have used seems always to give convergence, and as a rule of a satisfactory rapidity.

196. Discontinuous Method for Constructing Charts of Acceleration.—Let A be the situation of the considered particle at the epoch t_0, and B its situation at the time t_1 as we find it in the second approximation by the construction of the preceding section. v_0 being the velocity at the point A at the epoch t_0, and v_1 the velocity at the point B at the epoch t_1, we shall then have (fig. 113)

$$(a) \qquad AB' = v_0(t_1 - t_0) \qquad B'C = v_1(t_1 - t_0)$$

the half vector-sum of these displacements AB' and $B'C$ defines the displacement AB.

The vector-difference of the same two displacements (a) will be represented by the line $B'\,C'$, for which we shall thus have

$$(b) \qquad B'C' = (v_1 - v_0)\,(t_1 - t_0)$$

Now let us divide this equation by $(t_1 - t_0)^2$. We shall then get

$$(c) \qquad \frac{v_1 - v_0}{t_1 - t_0} = \frac{B'C'}{(t_1 - t_0)^2}$$

But the first member in this equation represents the average acceleration of the particle during the time $t_1 - t_0$. The equation therefore expresses the fact that the vector $B'C'$ which we have constructed will, after division by the square of the interval of time $(t_1 - t_0)^2$, represent the acceleration required. If this acceleration should be attributed to a definite place in the field, it would of course be to the central point between the points A and B.

We have thus arrived at a discontinuous method of constructing charts of acceleration: For a sufficient number of particles we perform the construction giving the displacement of the particles in the second approximation. This construction at the same time gives the vector $B'\,C'$, which gives the direction and (after division by the square of the interval of time) the intensity of the acceleration. When we have constructed this vector at a sufficient number of points, we can afterwards draw its vector-lines or its isogons and its intensity- curves.

197. Continuous Method for Constructing Charts of Acceleration.—We can base a continuous method of constructing accelerations upon the analytical representation of this vector as a complex time-derivative and space-derivative. By formula *f* of section 177 we have

$$\frac{d\mathbf{v}}{dt}=\frac{\partial\mathbf{v}}{\partial t}+\mathbf{v}\nabla\mathbf{v}$$

We have already called the first term of the second member the *local* acceleration. If this term be zero

$$\frac{\partial\mathbf{v}}{\partial t}=0$$

the wind-fanes of each station will show invariable direction and the anemometers invariable intensity of the wind. The velocity-chart will remain unchanged as long as this condition is fulfilled. The particles of air will then move along a system of lines of flow which remain unchanged. The lines of flow will be the paths of the moving particles. During this motion the particles will accelerate or retard so as to take at every place precisely the velocity which is characteristic of the place. We shall call a motion which is defined by this condition a *stationary* motion. The acceleration which the particles of air must have in the case of stationary motion is obtained if we set the local acceleration equal to zero in equation (*f*) of section 177; *i. e.*, the term $\mathbf{v}\nabla\mathbf{v}$ represents the acceleration which the particles must have if the motion is stationary.

We can therefore state: The acceleration of the moving particles may be represented as the vector-sum of two partial accelerations:

(A) Stationary acceleration which is given by the space-derivative

$$\mathbf{v}\nabla\mathbf{v} \tag{a}$$

(B) Local acceleration which is given by the time-derivative

$$\frac{\partial\mathbf{v}}{\partial t} \tag{b}$$

We have already treated the construction of fields representing derivatives of the forms (*a*) and (*b*). We can thus construct the fields of the two partial accelerations, and form their vector-sum

$$\frac{d\mathbf{v}}{dt}=\frac{\partial\mathbf{v}}{\partial t}+\mathbf{v}\nabla\mathbf{v} \tag{c}$$

which will then give the field of the true acceleration.

(A) *Stationary acceleration.*—When a velocity-field is given, the field of stationary acceleration (*a*) can be found in the following way (see section 174).

(1) We perform the derivation of the half square of velocity with respect to the lines of flow, *i. e.*, we form the field

$$\frac{\partial}{\partial s}\left(\tfrac{1}{2}v^2\right) \tag{d}$$

It will easily be seen that this field represents the tangential component of the acceleration in the stationary motion.

(2) We form the field of curvature γ of the lines of flow, and perform the multiplication of this field by that of the square of the velocity v^2. The field

(e) $$\gamma v^2$$

which we get in this way evidently represents the normal component of acceleration in the stationary motion of the particles along the lines of flow.

(3) We perform the vector-addition of the vector (*d*) which is directed along, and the vector (*e*) which is directed normally to the lines of flow.

Another method of constructing the field of stationary acceleration in which the single operations will not have the same simple physical significance, but which may still under special circumstances be advantageous, will be this (see formula (*d′*) of section 174):

(1′) We construct the ascendant of the half square of the velocity

(d′) $$\nabla(\tfrac{1}{2} v^2)$$

(2′) We construct the two-dimensional curl of the velocity (section 172) and form the vector-product of this vector and the velocity. This vector

(e′) $$(\text{curl}_2\mathbf{v})\times\mathbf{v}$$

will be directed along the positive normal to the lines of flow.

(3′) We perform the vector-addition of the two vectors (*d′*) and (*e′*).

(B) *Local acceleration.*—While stationary acceleration is derived from *one* chart which represents the given field of velocities at the given time, local acceleration must be derived from *two* charts which represent velocity at the two different epochs. The method will be that of the regular vector-subtraction and subsequent division by the interval of time as we have developed for the case of pure time-derivations of vector-fields (section 176).

198. Special Remarks.—The chart of local acceleration which we derive from the charts of velocity for the epochs t_0 and t_1 will correspond to the epoch $t_0+\frac{1}{2}(t_1-t_0)$. On the other hand the chart of stationary acceleration, which is derived from one of the given charts of velocity, will correspond either to the epoch t_0 or to the epoch t_1. If the interval between these epochs is sufficiently short, the circumstance that the charts of local and of stationary acceleration correspond to slightly different epochs will cause no trouble. But in order to get a satisfactory construction of the local acceleration, we are obliged to select the interval of time t_1-t_0 with as great a length as possible. For this reason it will be rational to derive the stationary acceleration from both given charts of velocity. The best method will then be this:

By vector-addition and division by 2 we form the chart of the average velocity

$$\tfrac{1}{2}(\mathbf{v}_1+\mathbf{v}_0)$$

during the time from t_0 to t_1. From this chart of average velocity we derive the chart of stationary acceleration, which will then correspond to the epoch $t_0+\frac{1}{2}(t_1-t_0)$. Then we form the vector-difference of the same two fields of velocity

$$\mathbf{v}_1-\mathbf{v}_0$$

divide by the interval of time $t_1 - t_0$, and thus find the local acceleration at the epoch $t_0 + \frac{1}{2}(t_1 - t_0)$. Then the sum of the two partial accelerations will give the best value of the acceleration at this epoch.

As even the determination of stationary acceleration involves a vector-addition, the complete determination of the field of acceleration will involve the performance of no less than two vector-additions and one vector-subtraction. The work will therefore continue laborious. But as this kinematic determination of accelerations will never enter as a link in the chain of operations which must be performed for the solution of the problem of prognosis, a practical demand for special rapid methods will not be required.

199. Return to the Problem of Prognosis.—It may be useful to consider a little more closely what could be done for the problem of prognosis as soon as we can determine by dynamic methods the accelerations of the moving particles.

To the displacement AB' (fig. 113) found in the first approximation we should then be able to add the displacement $B'B$ due to the acceleration. In this manner we should be able to forecast the displacements of the particles with a higher degree of approximation, retaining the length of time for which we make the forecast; and, dispensing with the greater accuracy, we could make the forecasts for longer periods.

But in addition to this we should also be able to prognosticate the new field of motion. For we know the velocities which the particles have when they arrive at their new positions, and we can then draw the field of these velocities. Instead of this discontinuous method we could also use a continuous one. From the field of velocities observed we should have to derive that of stationary acceleration. Subtracting this field from that of the true accelerations, which we calculated by dynamic methods, we will get the field of local acceleration. Multiplying this by a suitable interval of time $t_1 - t_0$, and adding to the field of velocity at the time t_0, we get the field of velocity at the time t_1. Thus, as soon as the dynamic method has given us the field of accelerations, kinematic methods, which we have treated already, allow us to determine the future field of horizontal motion. From this we may again, by kinematic methods which we have developed, derive the correlated vertical motions.

CHAPTER XIV.

EXAMPLES OF ATMOSPHERIC MOTIONS.

200. Indian Southwest Monsoon in July.—In giving a few examples of the kinematic diagnosis of atmospheric motions, we shall begin with a case of great simplicity, namely, the Indian Southwest Monsoon in July, at the time of its highest development.

Plate XXXI gives the discontinuous representation of this air-motion taken from plate 17 of Sir John Eliot's Climatological Atlas of India. The arrows represent the average wind-directions for the month, and the numbers the corresponding average intensities, changed from miles per hour to meters per second. In this case the distribution of arrows is a regular one, and it causes no difficulty to draw the lines of flow from them. The moderately idealized contour-lines of the blank map on which the construction is performed are a good help for the understanding and correct drawing of these lines. The chart representing the lines of flow and curves of equal wind-intensity is given on plate XXXII.

On the peninsula of India the motion represented by these two systems of curves is of great regularity. A striking effect of the topography of the land is seen, inasmuch as the lines of flow make a bend around the southern projection of the peninsula in order to avoid going across the mountains of the west coast. In the places where the wind must still travel directly toward the shore and the slope of the mountains, decided minima of velocity are seen to exist.

In the northern part of the chart the most marked peculiarity is the long line of convergence which goes up the whole length of the Ganges valley, in order to end in a constellation of a point of convergence and a neutral point situated above the Punjab plains. This long line of convergence is evidently an effect of the Himalaya chain. The observations do not go to a sufficient height in the mountains to let us see the complete character of the motion. But in all probability a correlated line of divergence must exist higher up on the slope of the chain. These two parallel lines of convergence and of divergence will then give the limits in horizontal projection of a rolling mass of air, which is kept in rotatory motion by the Monsoon-current passing across the mountain in greater height (cf. fig. 52B, p. 59).*

Plate XXXIII shows the forced vertical velocity at the ground. This chart has been derived from the preceding one by the method described in section 181. The shaded parts are the areas of ascending and the unshaded ones those of descending motion, the shaded ones on the windward and the unshaded ones on the leeward sides of the mountains. The ascending motion reaches its greatest values on the west coast, where it has a maximum amounting to 15 cm. per second.

Plate XXXIV gives the free vertical motion derived by use of the solenoidal condition as described in section 188 (C). Properly adjusting the units, we can

*Mr. E. Gold has arrived at a similar conclusion. Nature, Feb. 1908, p. 355.

interpret the chart as representing vertical specific momentum in the height where the pressure is one unit smaller than at the ground, or vertical velocity in unit height above the ground. If we use the latter interpretation, the numbers added to the curves give the vertical velocity in millimeters per second at the height of 100 meters above the ground, and in centimeters per second at the height of 1000 meters above the ground. The chart will give the correct picture of this part of the vertical velocity, provided that the chart of plate XXXII represents the average horizontal motion for the sheet between the ground and these heights. For a wind which has the regularity of the monsoon, it is not improbable that the observations at the ground give the character of the motion up to considerable heights. But decided exceptions exist. Thus, if the line of convergence in the Ganges valley existed unchanged to the height of 1000 meters it should give here a vertical velocity of 9 cm. per second, and a corresponding greatly localized precipitation might be expected. But as Sir John Eliot's chart of precipitation for July does not show any sign of this, we have a strong reason for believing that the line of convergence is a local phenomenon limited to the lower layers. (Compare section 134.)

Comparing the plates XXXIII and XXXIV, we see that the free vertical motion has a certain tendency to be of an opposite sign to the forced vertical motion existing at the ground. The addition will therefore in most places give a reduced vertical motion. Lower down the forced vertical velocity is the stronger of the two. But at the height of 1000 meters both are of about the same order of magnitude and as we proceed farther upward the influence of the ground will constantly recede to the background.

From the two charts XXXIII and XXXIV we can derive charts for the total vertical velocity at any constant height above the ground by graphical addition. If we wish to have the total vertical velocity at a given height above sea-level we must, before the addition, perform the graphical multiplication of chart XXXIV by a chart which represents the height from the ground to the given level. It is interesting to draw such charts of total vertical motion and to compare them with charts of average precipitation like those found in Eliot's Atlas. But in a case like that before us no complete accordance should be expected. We have referred already to one departure, the reason of which is easily understood. Another cause of departures is this: In spite of its great regularity the monsoon-wind shows changes from day to day, causing corresponding changes from day to day in the distribution of the vertical motion. For this reason there will from time to time appear ascending motion and consequently precipitation in places where the average motion is descending and where no precipitation would appear if there were no departures from the average motion.

201. **North America, 1905, November 28, 8 a. m.**—Instead of average motions we shall henceforth consider actual motions.

Plate XXXV represents the field of pressure and of mass in the lowest atmospheric sheet above North America, November 28, 1905, 8 a.m., time of 75th meridian. The single lines give the absolute topography of the 1000 m-bar surface, and the double lines, consisting of a thick line and a thin one, give the relative topography of the 900 m-bar surface relatively to the 1000 m-bar surface, and thus the average specific volume of the air in the sheet between these two surfaces. The thin line is on the side where the sheet is thinner. All lines are stippled where they have their course below the ground. It will be seen that the 1000 m-bar surface has a strong depression, going down to 100 dynamic meters below sea-level in southern Minnesota, with a secondary depression in Colorado. The great area of depression is surrounded by high areas situated in New England, in Montana and the adjacent parts of Canada, and on the southern part of the coast of California. Another depression is situated farther north on the Pacific coast.

Plate XXXVI gives the representation of the observed wind-directions in the common way by arrows. The corresponding numbers, according to the dial of fig. 32, are also inscribed, and another set of numbers give the wind-intensities in meters per second. A glance at the arrows at once shows the unfortunate consequences of the observation of only eight wind-directions. If the lines of flow were drawn strictly tangential to the arrows they would get polygonal form, with a great number of lines of convergence and of divergence separating from each other the areas of different wind-directions. It must therefore be highly recommended to observe at least double the number of wind-directions. Provisionally we can only remove the discontinuities in the drawing of the isogonal curves or the lines of flow by eye-measure.

Plate XXXVII gives the continuous representation of the motion by isogonal curves and curves of equal wind-intensity. Plate XXXVIII gives the same representation by lines of flow and intensity-curves. The isogonal curves have a remarkably simple course: only two singular points appear, one in southern Minnesota and one in California—the former positive, the latter negative. The lines of flow show a marked point of convergence in southern Minnesota, near the point of the lowest depression, and several lines of convergence which run into this point. A line of divergence connects the two high areas in Montana and California, and this line has a neutral point where the isogonal curves had the negative singular point. The lines of flow make a very striking bend in order to go around instead of across the Allegheny mountains. While the lines of flow have a relatively simple course, the distribution of wind-intensity is very irregular, with a great number of maxima and minima. North of the cyclonic center winds go up to 28 meters per second.

Chart XXXVIII is drawn upon a blank surface which gives the topography of the land greatly idealized. By the method of section 181 we have derived from it the chart of plate XXXIX, which gives the vertical velocity at the ground. The shaded areas on the windward slopes are those of ascending motion, the unshaded ones on the leeward slopes those of descending motion. The greatest vertical velocities amount to 20 cm. per second. The fact that higher values are never reached is of course due to the idealization of topography. The true local values may be much greater, while our chart gives only the average values for greater areas.

From either of the two plates XXXVII and XXXVIII we can derive by the solenoidal condition the free vertical motion, which is represented by the chart of plate XL. We can interpret the chart as giving vertical velocity in millimeters per second at the height of 100 meters above the ground, or in centimeters per second at the height of 1000 meters above the ground. If we venture to extrapolate to the latter height we get free vertical velocities of the same order of magnitude as the forced vertical velocity derived by the surface-condition. This free vertical velocity is seen to have a very irregular distribution. A certain tendency to be opposite to the forced one is manifest in different places. But as to its general features free vertical motion is seen to be governed by pressure. Generally speaking the area of depression is an area of ascending motion, except in the details, inasmuch as smaller areas of descending motion exist even in the immediate neighborhood of the cyclonic center.

We can in this case make the simple experiment of compounding free and forced vertical velocity for the height of 1000 meters above the ground. The result is given on plate XLI. When this chart is compared with the simultaneous data regarding the distribution of precipitation, of cloudiness, and of blue sky, a considerable accordance will be seen to exist in spite of the great extrapolation involved in the estimation of vertical velocities at so great heights from observations taken only at the ground.

202. Practical Applications of the Charts of Motion.—It will involve no difficulty to introduce the drawing of charts like that of plate XXXVIII, representing the horizontal motion by lines of flow and curves of equal intensity, into the daily meteorological service for the forecast of the weather. When the chart is to be drawn only for qualitative purposes, it will not be required to use the more circumstantial method to draw first the isogonal curves. The main course of the lines of flow can be sketched directly. When the drawing of the lines of flow and of the curves of equal intensity is distributed between two workers, and these have acquired some experience, it will cause them no difficulty to have the chart of motion ready in a space of time comparable to that required for drawing the common charts representing pressure, temperature, and other data.

These charts of motion possess many characteristic features in the form of singularities which are in an obvious relation to the conditions of the weather. Therefore we have reason to believe that experience will gradually lead to practical rules for weather-forecasts based upon the examination of the charts of motion in themselves or in connection with the other charts.

When the draftsman has acquired sufficient experience, a rapid examination of the chart of horizontal motion will show him the places for the strongest forced vertical motion and for the strongest free one. By making a few measurements in these places he will be able rapidly to sketch charts of the vertical motion, and there is hardly any reason to doubt that these charts would prove useful for the forecast of precipitation.

The charts of motion may also be useful for aerial navigation. For instance, a glance at the chart of plate XXXVIII will show at once that an air-ship which moves, *e. g.*, 15 meters per second will not be able to go in a straight line say from Bismarck in North Dakota to the southern coast of Lake Superior, for here it would have a head-wind of 28 meters. But it would easily accomplish the voyage by the circuit south of the center of the cyclone. If we were able to estimate the degree of persistency of the state of motion, and the direction in which the changes are to take place, it would be possible by use of such charts to plan the course of aerial ships so that they will reach their destination in the shortest time.

203. Charts of Acceleration.—Charts XXXVII to XLI exemplify the kinematic diagnosis as far as it can be carried out by use of observations taken at one epoch only, and only at the stations at the ground. When we have observations from two successive epochs, we can go one step farther and determine the acceleration of the motion kinematically.

The charts of plates XXXVII or XXXVIII were taken from the registered values of wind-intensities and wind-directions during the hour from 7 to 8 a. m., time of 75th meridian. The charts of plates XLII and XLIII show the corresponding representation of the motion derived from the values registered during the hour from 10 to 11. As will be seen, the point of convergence has been displaced a little more than 200 kilometers toward northeast, but otherwise the general features of the chart are unchanged.

In order to determine the average acceleration during the interval of time between the two epochs, we first form the chart of the average velocity for this interval of time. This is done by addition and division by 2 of the two vector-fields represented by plates XXXVII and XLII. The result as obtained directly, represented by isogonal curves and curves of equal intensity, is shown on plate XLIV; plate XLV shows the corresponding representation by lines of flow and curves of equal intensity.

By the subtraction of the same two vector-fields and division by the interval of time, 3 hours or 10,800 seconds, we form the chart of *local* acceleration. Plate XLVI contains the representation of this vector by isogonal curves and curves of equal intensity, and plate XLVII gives the representation by vector-lines and curves of equal intensity.

From charts of average motion (plates XLIV or XLV) we derive the chart of stationary acceleration as described in section 197 (A). The result is given on plate XLVIII by isogons and intensity-curves, and on plate XLIX by vector-lines and intensity-curves.

The true acceleration of the moving particles is finally obtained by the addition of the vector-fields representing local and stationary acceleration. The result is represented by the charts of plates L and LI, on the first by isogonal curves and intensity-curves, on the second by vector-lines and intensity-curves.

Much more experience than we have at present must be gained before we can estimate the degree of objective reliability of a chart of acceleration like that given on these plates. In the western mountainous parts, where in many cases great doubt may arise as regards the charts of velocity from which the chart of acceleration has been derived, the values found for the acceleration must of course be used with great reserve. The same should be the case along the borders of the chart. But in the more central part, in the Mississippi valley, we have every reason to believe that the chart gives a good approximation to the truth. The question of attaining the same reliability of the chart of acceleration in the other districts will, as will be understood at once, simply be a question of further developing the net of stations and improving the methods of observing the wind.

204. Main Example of Kinematic Diagnosis, Europe, 1907, July 25, 7 a. m. Greenwich.—In the preceding examples we have exclusively used observations from the common meteorological stations at the ground. We shall now consider a case where observations, though in quite insufficient number, are at hand also from the higher strata, namely, the aerological observations on the morning of July 25, 1907.

To begin with the observations from the ground, plate LII gives the distribution of pressure and of mass in the lowest atmospheric sheet. The single lines give the absolute topography of the 1000 m-bar surface and the double lines the relative topography of the 900 m-bar surface. It will be seen that pressure is rather uniformly distributed. A relatively high ridge goes from Iceland over Scotland and the North Sea toward the Balkan Peninsula. East of this ridge a great number of local maxima and minima are seen. Plate LIII gives the winds observed, represented by arrows, by numbers of direction, and by numbers of intensity. The winds are generally faint and irregularly distributed.

Plate LIV gives the corresponding continuous representation by use of isogonal curves and curves of equal intensity. The diagram of isogons shows a great number of positive and negative singular points. Plate LV gives the representation by lines of flow and intensity-curves, with the corresponding great number of neutral points as well as of points and lines of convergence and of divergence. The comparison with the chart of pressure plate LII shows that the points of convergence with great regularity coincide with the small depressions, and the points of divergence coincide with the corresponding heightening of the isobaric surface of 1000 m-bar pressure.

The chart of plate LIV is drawn upon a blank which represents the average pressure at the ground (see plate XXX). From this chart we therefore easily derive that of vertical specific momentum at the ground, plate LVI.

When we pass to air-motion in the higher strata, we must limit our considerations to the small area where we have the closest network of aerological stations. We have done this on plates LVII to LX, which correspond respectively to the standard sheets X, IX, VIII, and VII (section 107). The incomplete sheet XI is so thin that we have left it out of consideration. The wind-observations are represented on the charts A of these plates by arrows and intensity-numbers, as we have mentioned already. As the arrows and numbers are too few in number for the drawing of the charts of horizontal motion, we have used in anticipation dynamic principles for giving at least a tolerably probable reconstruction of the horizontal motions as developed in section 139. The reconstructed horizontal motions are represented by lines of flow and curves of equal intensity on the charts B of plates LVII to LX. Of course we must leave open the question regarding the degree to which we have thus succeeded in reconstructing the true horizontal motions. We shall therefore use them only to illustrate the *formal* methods of a complete kinematic diagnosis of atmospheric motions.

From the charts B we then derive auxiliary charts representing the contribution of each sheet to the vertical component of the specific momentum. These auxiliary charts have not been reproduced. But the method of drawing them is that developed in section 187 (C), and for the incomplete parts of the sheets in section 189.

As soon as these auxiliary charts are drawn we find the true vertical specific momenta at different standard isobaric surfaces by successive graphical additions

By graphical addition of the vertical specific momentum at the ground (plate LVI) to that produced in the incomplete sheet X we get the chart LVII C, which gives the vertical specific momentum at the isobaric surface of 900 m-bar pressure.

By graphical addition of this vertical specific momentum to that produced in the sheet IX we get the chart LVIII C, which represents vertical specific momentum at the standard isobaric surface of 800 m-bar pressure.

By graphical addition of this vertical specific momentum to that produced in sheet VIII we get in the same manner the chart LIX C, representing the vertical specific momentum at the standard isobaric surface of 700 m-bar pressure.

By graphical addition of this vertical specific momentum and that produced in sheet VII we get the chart LX C, which represents the vertical specific momentum at the standard isobaric surface of 600 m-bar pressure.

The plates LVII to LX thus give the complete result of the static and the kinematic diagnosis of atmospheric conditions on July 25, 1907, about 7 a. m., Greenwich time, on the basis of the aerological soundings performed about this time. The charts LVII A to LX A give the field of pressure and of mass as the result of the static diagnosis; the charts LVII B to LX B give the horizontal motion within each of the four isobaric sheets defined by the charts LVII A to LX A. The charts LVII C to LX C represent the vertical transfer of mass from one to the other of these isobaric sheets.

We have exemplified the diagnosis by taking the four lowest atmospheric sheets. Some of the observations on the day under consideration go much higher. But it will have no interest to extend the diagnostic work further before the observations have attained the completeness required for working out diagnoses which have an unquestionable objective value. Till then we can only exemplify the formal methods.

If the observations were performed according to the plan which we have developed in Chapter I, we should be able to work out complete diagnoses at epochs which were sufficiently near each other to allow us to derive also the fields of acceleration within all atmospheric sheets. This would be the first step in opening the way for serious investigations in atmospheric dynamics.