ANNUAL REPORT

OF THE

CHIEF SIGNAL OFFICER OF THE ARMY

TO THE

SECRETARY OF WAR

FOR

THE YEAR 1887.

IN TWO PARTS.

PART 2.

WASHINGTON:
GOVERNMENT PRINTING OFFICE.
1887.

National Oceanic and Atmospheric Administration

Annual Report of the Chief Signal Officer, U.S. Army Signal Corps

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ANNUAL REPORT OF THE CHIEF SIGNAL OFFICER FOR 1887.

APPENDIX 46.

TREATISE

ON

METEOROLOGICAL APPARATUS AND METHODS,

BY

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Prepared under the direction of

BRIGADIER-GENERAL A. W. GREELY,

CHIEF SIGNAL OFFICER,

BY AUTHORITY OF THE SECRETARY OF WAR.

WASHINGTON:
GOVERNMENT PRINTING OFFICE.
1888.

SIG 87, PT 2

ERRATA.

On page 9, line 30, *dele* "force produced by the".

On page 38, at bottom, add "[NOTE, September, 1888.—A full synopsis of recent work at the Bureau International on gas and mercurial thermometers is given by Chappuis, Bibl. Univ., Arch. d. Sci., 1888, XX.]".

On page 76, line 30, between "(" and before "Annalen" insert "Wiedemann".

On page 77, line 12, for "Wiedemann" read "Weidmann".

On page 77, at bottom, add "[NOTE, September, 1888. The first part of Guillaume's Memoir on the Mercurial Thermometer, Bibl. Univ. Arch. d. Sci., 1886, having just now been received by me, is recommended to the student in search of further details.]".

On page 80, line 25, after "and" insert "in 1838 by Bravais, in 1855 by Renou, and".

On page 82, line 5 from the bottom, add "In 1855 Liais and Le Verrier furnished gilded bulbs for determining the air temperature at the new telegraph stations of the Imperial Observatory".

On page 89, at the bottom, add "[In 1856 the problem of determining the temperature of the air by means of a single isolated thermometer was made the subject of the "Bordin prize" by the Academy of Sciences at Paris, but I do not find that any satisfactory memoir was ever presented.]".

On page 312, line 15 from the bottom, a note on the application of photography to the determination of the heights of the clouds is given by Pouillet in Paris, Comptes Rendus, 1855, XL, p. 1157, and it is necessary to modify this text to read as follows: "The application of photography for this purpose was urged by Pouillet in 1855 and was contemplated by myself in 1858. Pouillet's plan was to establish a camera at each of two neighboring points, with the optical axes directed permanently toward the zenith; both simultaneous photographs will necessarily overlap, whence the zenith distances and bearings of many cloud points can be measured and the altitudes computed. Much preparatory work was done by Pouillet showing the practicability of his method. My own plan was to use a modification of the telemeter and photograph upon one plate simultaneously two images of the same cloud, reflected from two slightly convex mirrors a short distance apart."

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PREFACE.

The antiquity of much of our local weather lore shows that meteorological observations and generalizations were among the first acts of an intelligent race; the practical study of meteorology and the local prediction of the impending weather began wherever and whenever man began to consider the possibility of ameliorating his condition on the earth. Records of wind and rain and weather and miscellaneous phenomena date, like the records of eclipses, from the earliest times in the history of Assyria and China. The Phœnician navigator had given a distinctive name to the cloud that indicated rain and storm, but instrumental observations of temperature, pressure, moisture, winds, and clouds could begin only since the days of Galileo and the development of physical science. Accordingly we find that, in 1653, as soon as the thermometer and barometer had become available, Ferdinand II, Grand Duke of Tuscany, organized a system of meteorological stations, each equipped with instruments, and from this date may be reckoned the beginning of modern accurate meteorology. Since that date new instruments, or improvements on the old, have been invented, and the present treatise is intended to set forth both the progress and the present condition of our knowledge of the methods of accurately observing the fundamental data of meteorology. A uniform thought has guided the arrangement of the chapters on the different instruments, namely: there is first given a general description of the object to be attained; second, a development of the formulæ for correcting the errors of the apparatus; and, finally, an indication of the refined methods of making standard determinations, to which all ordinary practical methods are to be considered as approximations. It will be found that satisfactory normal measurements of the temperature, pressure, moisture, and motion of the free air are now quite within the power of the meteorologist, but involve great circumspection and labor, as does every work that lays claim to accuracy.

Meteorology can only be worthy of a place among the exact sciences in proportion to the improvements in its methods of observation, and to the extent to which they cover the field of the phenomena. Thus the comparison between theory and observation requires the amount of solar radiation to be known to within 1 per cent., whereas it is at present uncertain by 15 per cent.; it requires the temperature of the air to be known within 0.5° F., whereas published observations are not always reliable to within one or two degrees; it requires the general movement of the air to be known, whereas we have only very uncertain records of the stratum below 100 feet, nothing of the stratum between 100 and 3,000 feet, and scanty records of the cloud stratum between 3,000 and 30,000 feet.

Every effort to explain the ordinary phenomena of storms is embarrassed by the fact that assumptions as to the temperature, moisture, and wind have to be made because of the absence of actual observations. Weather predictions will undoubtedly be more satisfactory when the present round of observations is enlarged so as to include the condition and movement of the great mass of air above us, while at the same time increasing the accuracy of measuring the lowest stratum. It is hoped that the present treatise will contribute to extend the scope and to increase the accuracy of meteorological observations.

One of the most serious obstacles to exact observation has been the great diversity of standards and instruments disseminated among observers. In this respect the past decade has seen great improvement, especially through the recognition of the International Bureau of Weights and Measures as the ultimate authority in respect to all standards. I have, therefore, quite fully presented the methods adopted in that bureau, and must reiterate my former statements that implicitly it is the legal authority for the standards of temperature and pressure to be adopted by all the nations that contribute to its support.

It is only proper to acknowledge the services rendered me in the preparation of this treatise by the scientific staff of the service, and especially by Junior Professor Frank Waldo and Sergeant George E. Curtis, Signal Corps. Their compilations and suggestions have been valuable and timely, and the completion of this work within the present year was only rendered possible through their interest and assiduity.

The very extensive subjects of optics, electricity, and actinometry remain to be presented in a subsequent volume.

CLEVELAND ABBE.

October, 1887.

INTRODUCTION.

STANDARDS AND UNITS OF MEASUREMENT.

The measurement of meteorological phenomena involves the use of numerous physical units, and, for convenience of reference, such explanations of terms and units as will be of general application in the subsequent part of this work are here collected.

The units adopted in scientific work are preferably those of the Metric System, but the English System is sometimes used, and, therefore, both are defined in the following schedule:

UNITS OF LENGTH:

Metric System:

The meter; an empirically assumed unit intended to be one ten-millionth of the quadrant of a meridian, but actually represented by standard platinum bars preserved by the Bureau des Archives, and by iridio-platinum bars preserved by the International Bureau of Weights and Measures at Paris. These are approximately equal to the one ten-millionth of the quadrant of a meridian with which the meter was originally intended to be identical.

English System:

The foot; the third part of a yard; an empirical unit legalized by English laws, and actually represented by standard bronze bars preserved by the warden of the mint at London.

The mile, 5,280 feet.

UNITS OF AREA:

Metric System: The square centimeter.

English System: The square foot.

UNITS OF VOLUME:

Metric System:

The cubic decimeter; called, also, one liter.

English System:

The cubic foot.

The gallon; equivalent to the volume of 10 pounds avoirdupois of distilled water as weighed in air at a temperature of 62° F., and under a pressure of 30 inches of the barometer at London; this volume has been determined to be equivalent to 277.274 cubic inches.

UNITS OF ANGLE:

(1) The angle subtended by the circumference of a circle, fractions being expressed as decimal parts of this unit.

(2) The radian; the angle subtended by a portion of the circumference of a circle equal to the radius.

(3) The degree; the angle subtended by the 360th part of the circumference, measurements being expressed in degrees and decimals, or in degrees, minutes, and seconds.

The radian equals 57°.29578.

UNITS OF TIME:

The second; the one 86 400th of a mean solar day.

The hour; the 24th of a mean solar day.

The mean solar day.

The mean sidereal day.

The mean solar year, or 365.2422 solar days.

UNITS OF MASS:

Metric System:

The kilogram; represented by a definite mass of platinum preserved by the *Bureau des Archives* at Paris. This is very approximately equal to the mass of one cubic decimeter of pure water at its maximum density with which it was originally intended to be identical.

The gram; the 1000th of a kilogram.

English System:

The pound troy.

The pound avoirdupois.

These are both represented by brass and platinum standards preserved at London.

The legal relation of these units is, one pound troy equals 5,760 grains; one pound avoirdupois equals 7,000 grains.

UNITS OF LINEAR VELOCITY:

Metric System:

A centimeter per second.

English System:

A foot per second.

A mile (5,280 feet) per hour.

UNITS OF ANGULAR VELOCITY:

Metric System:

A radian per second.

English System:

A degree per second; any unit of time suitable to the nature of the motion, as a second, a day, or a year may be used.

UNITS OF LINEAR ACCELERATION:

The unit acceleration is an increase of any given velocity by a unit velocity in a unit of time.

Metric System:

An increase of velocity of one centimeter per second, per second.

English System:

An increase of velocity of one foot per second, per second.

UNITS OF ANGULAR ACCELERATION:

Metric System:

An increase of angular velocity of one radian per second, per second.

English System:

An increase of angular velocity of one degree per second, per second.

UNITS OF MOMENTUM:

In general the momentum of a unit mass moving with unit velocity.

UNITS OF MOMENTUM FOR LINEAR MOTION:

Metric System:

The momentum of a gram moving with a velocity of one centimeter per second.

English System:

The momentum of a pound avoirdupois moving with a velocity of one foot per second.

UNITS OF MOMENTUM FOR ANGULAR MOTION:

Metric System:

The momentum of a gram moving with an angular velocity of one radian per second on a circle of one centimeter radius.

English System:

The momentum of a pound moving with an angular velocity of one radian per second on a circle of one foot radius.

A unit mass having such angular motion has a unit moment of momentum. The measure of any moment of momentum is, then, in general, given by the equation $M.M. = I.W$, where I is the moment of inertia, and W the angular velocity; I is given by the formula $I = \int r^2 \, dm$, r being the distance of any particle dm from the axis of rotation. When I and W are each unity, or when their product is unity, we have a unit moment of momentum.

UNITS OF FORCE:

The existence of force is known only by its effects in producing or destroying momentum. In general, the unit force is defined as that which in a unit of time generates a unit of momentum, or acting on a unit of mass generates in it a unit velocity.

Metric System:

The dyne; that force which, acting continuously on a gram, is sufficient to give it a velocity of one centimeter per second, per second. If a force F, acting on a free mass of m grams for t seconds, will generate a velocity of v centimeters per second, then will $F = \frac{m.v}{t}$ dynes.

English System:

The poundal; that force which, acting on a pound, is sufficient to generate a velocity of one foot per second, per second.

The force of gravity is about 32 poundals.

Gravitation System:

The force produced by the apparent attraction of the earth at latitude 45° and sea-level acting on a unit mass.

UNIT COUPLE:

Two equal and opposite forces applied to a body so as to alter the rate of rotation are called a couple. The distance between their lines of action is called the arm, and the product of the arm into one of the forces is the moment of the couple. In general, a unit couple is one which has a unit moment or the product of a unit force into a unit length of arm.

Metric System:

The unit couple is one dyne acting at a lever arm of one centimeter.

English System:

The unit couple is one poundal acting at a lever arm of one foot.

UNITS OF WORK:

In general, the work done by a unit force acting through a unit distance.

Metric System:

The erg; the work done by a force of one dyne acting through a distance of one centimeter.

English System:

The megalerg = 1 000 000 ergs.

The foot-poundal; the work done by a poundal acting through a distance of one foot. (British and American engineers are in the habit of using the foot-pound, or the work spent in raising one pound one foot against the force of gravity. This would be satisfactory if gravity were constant over the earth's surface, but the acceleration of gravity varies, and hence the foot-pound must be a variable unit.)

UNITS OF WEIGHT:

Apparent weight is the vertical pressure due to the action of all the forces that affect bodies on the earth's surface. After correcting for the buoyancy of the air, the remaining weight, called the weight in-vacuo, is essentially due to the apparent gravity, or the attraction of the earth diminished by centrifugal force. The unit of weight is the weight *in vacuo* of the unit of mass under the action of apparent gravity at latitude 45° and sea-level.

Metric System:

The weight *in vacuo* under standard gravity of the kilogram preserved at Paris.

English System:

The weight under gravity at London at the office of the warden of the standards, and in air (whose density corresponds to a temperature of 62° F. to a relative humidity of 65 per cent., and to a pressure of 30.00 inches of mercury at standard density under the gravity at London at sea-level) of a brass Troy pound preserved by the warden of the mint.

UNITS OF HEAT:

In general, the amount of heat required to change by one unit the temperature of a unit mass of pure water.

Metric System:

A calorie: the amount of heat required to raise the temperature of a kilogram of pure water from 0° to 1° C. (Pouillet.)

A gram-degree or small calorie. 0.001 of a calorie (Everett.)

English System:

A pound-degree: the amount of heat required to raise a pound of water from the temperature 50° F. to 51° F. (Tait.)

SECTION A.

THE MEASUREMENT OF TEMPERATURE.

CHAPTER I.

GENERAL HISTORY OF THERMOMETERS.

1. IMPORTANCE OF HISTORICAL STUDY.

The temperature of the air is measured by means of the thermometer, and is the most important meteorological phenomenon. Observations of the thermometer have been recorded in the United States since 1738, in France since 1670, in England since 1661, and in Italy since 1654; but the thermometers employed, as well as the methods of exposure, have been so various, that it is only by careful study of the history of the thermometer that the older records can be rightly used in discussing questions as to secular changes in climate. For this purpose, especially, the progress of the improvements leading to modern accurate thermometry must be presented in some detail.

2. ITALIAN ORIGIN.

The expansion of air and liquids by heat was known in ancient times, but the first application of this knowledge to the determination of relative temperatures dates from the time of Galileo. (Galileo was born 1564, and died 1642.) Owing to the loss of the unpublished letters and treatises of Galileo, we have to rely on the obscure statements of a few cotemporary authors, and the conflicting views of later writers, for information as to the origin of the thermometer. Apparently an elementary form of the air thermometer was known to Galileo, and used by him in his lectures at the University of Padua during the years from 1592 to 1609. This thermometer is described in an Italian edition of Porta's Pneumaticorum, published in 1606. It was adopted by the physician Sanctorius after becoming professor of surgery at Padua,* where he lectured during the years 1611 to 1624, after which he removed to Venice, where, in 1624, he published his commentary on Avicenna, in which its use is referred to. Although called Sanctorius' thermometer, he did not invent or claim it, but merely introduced its use into medical practice. As used by him, it was a simple thermoscope for showing the excess in the temperature of a feverish patient above that of persons in normal health. It consisted, essentially (see Fig. 1), of a glass globe (*a*) opening into a narrow tube (*b*), a little water having been first poured into the globe through the tube; the latter is then inverted and dipped into the vessel of water (*c*),

* Apparently Sanctorius had not adopted it previous to 1611, when he published the first edition of his commentary on Galen.

whereupon the contained water flows down and partly fills the tube up to some point (*d*). If now a person in ordinary health grasps the globe (*a*), the expansion of the contained air will depress the water below the point (*d*), and a feverish patient having a higher temperature will produce a still greater depression. This primitive form of the thermometer became known throughout Europe through the crowds that thronged the lectures of Galileo and Sanctorius.*

In the first form of the thermometer or thermoscope, and in many of its early modifications, the varying pressure of the atmosphere has so large an influence on the length of the column of water in the tube that the observations with a given instrument must all be made in the short time during which the atmospheric pressure is constant; moreover, the instrument can not be carried any distance without danger of completely changing its condition.

From this crude form of thermoscope an important advance in accurate thermometry was made by Galileo in adopting a bulb full of liquid as the thermometric material. Sagredo in his correspondence with Galileo, May 9, 1613, and also Viviani in his biography of Galileo, written in 1654, ascribe to him alone the invention of this improved thermometer evidently earlier than 1611 or 1612; Libri inclines to the date 1596. This form was definitely described by Jaen Rey in a letter to Father Mersenne, dated January 1, 1632, as quoted by Hoeffer in his History of Chemistry, wherein Rey speaks of the thermometer made of a small vial with a long neck filled with water as used by him for observing the temperature of feverish patients and other objects.

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* This thermometer soon became the subject of description by many writers to some of whom it has occasionally been attributed as an independent invention. The following early works contain a reference to it or a description:

Telioux (b. ——, d. ——), Matematica Mara Vigliosa, 1611; Solomon De Caus (b. 1576, d. 1630), Raison des Forces, 1615; Lord Francis Bacon (b. 1561, d. 1626), Novum Organum, 1620; C. Drebbel (b. 1572, d. 1634), Tractatus de Naturæ-Elementorum; S. Sanctorius (b. 1560, d. 1636), Commentary on Avicenna, Venice, 1624, and Commentary on Galen, 2d ed., Leyden, 1631; P. Sarpi (b. 1572, d. 1623) used the thermometer in 1617; Laurechon (b. ——, d. ——), Recreations Mathematique, 1624; Caspar Ens (b. ——, d. ——), Translation of Laurechon, 1636; R. Fludd (b. 1574, d. 1637), Philosophia Moysaica, published in 1638 (says it was known five hundred years ago); Van Helmolt (b. 1577, d. 1644), Ortes Medicini, 1648; Otto Guericke (b. 1602, d. 1686), in lectures delivered in 1654–1663. Guericke erected a very large calendarium in the open air, introducing a floating figure as an index in the tube, and used it as a weather glass, by which he predicted the approach of a storm in 1660. On account of its incessant variations he called it *perpetuum mobile*, or one form of perpetual motion. Sturm (b. 1655, d. 1703), Collegium Experimentale, 1685, describes this with other forms, including the differential thermometer of Leslie.

Even as late as 1700 this form of thermometer was re-invented and used by Geoffrey in some investigations communicated in 1700 to the Royal Academy of Science at Paris, and in 1701 to the Royal Society of London, "On the cold of solutions and fermentations." Geoffrey had, in fact, been using the Florentine thermometers, but finding them not sufficiently sensitive had returned to the older Galilean form.

The records of the Accademia del Cimento* show that Galileo and his pupils, especially Ferdinand II, Grand Duke of Tuscany, and a patron of the academy, made further improvements, and that, as early as 1641, the Florentine philosophers were using the modern form of thermometer, namely, a bulb filled with spirit of wine or other liquid, with its tube sealed and graduated to accord with some standard by marks placed on the glass stem itself—all which improvements are to be credited to Ferdinand II. Spirit of wine was the liquid ordinarily employed, colorless spirit being preferred because of its less liability to molecular changes; mercury was used by the members of the academy only for a few special investigations on the capacity of bodies for heat, or on the sensitiveness of different substances. Spirit thermometers of this construction were made in great numbers by the expert glass-blowers in the pay of the Grand Duke of Tuscany, and were distributed widely in Italy and southern Europe, where systematic meteorological records began to be kept at a large number of stations under the auspices of Ferdinand II. In 1653 this enlightened prince distributed thermometers to several cloisters, beginning with that of Angelus, near Florence, and organized a system of daily records, observations, and reports, the general oversight of which was committed to Father Luigi Antinori. Spirit thermometers were used at these stations, and the graduations extended from zero up to fifty; one subdivision corresponded closely to fiftieths of the volume of the bulb.†

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* The Accademia del Cimento of Florence was founded by Prince Leopold, brother of Ferdinand II, Grand Duke of Tuscany, and consisted of nine members, all of whom were former pupils of Galileo, namely, Borelli, C. Buono, P. Buono, Magalotti, Marselli, Olivia, Redi, Renaldini, and Viviani. The sessions of the academy were held in the palace of the prince and began June 19, 1657. The object of the academy was to further the study of those exact sciences in the pursuit of which Galileo had spent his life. Its meetings were strictly private and were devoted to the experiments and discussions in which all took part. Whatever results were clearly arrived at were the property of the academy as a whole; a minute diary was kept, and is still preserved in manuscript, showing the exact part taken by each; but in the official publication no person is mentioned by name. It is not known that any of its results were published until the first volume appeared, just before the abolition of the academy and some ten years after its founding; such thoroughness was required in order to meet every objection put forth by the opponents of scientific progress, and such secrecy was necessary in order that the hatred of the Jesuits should not fall upon any one individual. But the academy eventually suffered severely from the personal dissensions of its members, and was dissolved by Leopold when he accepted the place of cardinal that had been vacant since the death of his cousin Carl. It is most probable that the decree of abolition was not, as stated by some, an immediate consequence of his elevation, but was due rather to the internal condition of the academy. The publication, on June 10, of the first and only volume of its transactions "Saggi di naturali sperienze; fatte nell' Accademia del Cimento, Firenze 1667," marks an epoch in the history of science; this volume consists of thirteen chapters devoted to experiments in the physical sciences, except the first, which deals with the instruments and methods employed.

† This style of thermometer was used by Ferdinand II in 1644 in experiments on artificial incubation. Monconys states that it had been shown him by Torricelli in 1646. It is also that referred to by Sturm as having been sent from Florence to Rome in 1649.

By 1657 the barometer, wind-vane, and hygrometer were furnished each station, and thus the first complete meteorological system in Europe was inaugurated.

How closely the thermometers of that time agreed among themselves is not known; much complaint was made in England of the disagreements of the Florentine thermometers, but it is certain that great care was taken at Florence to adjust properly the sizes of the bulbs and tubes. Two centuries later, in 1829, V. Antinori, superintendent of the Physical Museum at Florence, found a chest containing a large number of these old thermometers, such as had been furnished by Ferdinand II to his observers, apparently in good condition. In 1830 Libri made comparisons between some of these and the modern standard thermometers, with the general result that the Florentine scale can be converted into Réaumur degrees by a table of equivalents of which the following is an abstract:

Florentine scale.	Réaumur degrees.	Centigrade.
°	°	°
0.	−15.	−18.9
13.5	0.	0.
50.	+44.	+55.

Libri then reduced the meteorological observations made at the cloister at Angelus, near Florence, for the sixteen years, 1655–1670. A comparison of the resulting temperatures with the modern observations at the same place showed that there had been no appreciable change during the past two hundred years.

These old observations have been ascribed by some to Renieri, the pupil of Galileo, but as he died in 1647 or 1648 it is evident there must be some mistake.

3. ENGLISH THERMOMETERS.

In the early part of the seventeenth century there were but few places in Europe where glass tubes could be properly constructed, and nowhere could the workmen vie with those in Florence. Among the first scale thermometers made outside of Italy were those made at Oxford in 1661 by Sir Robert Boyle, who states that having seen a small sealed Florentine thermometer which had been brought to England by Sir Robert Southwell, then president of the Royal Society, he immediately set about constructing one like it.

* Robert Hooke (b. 1635, d. 1703), who was an assistant to Boyle and "curator of experiments to the Royal Society," states that he was the first to make these thermometers in England, whence I infer that he did this at Boyle's request in Boyle's laboratory. At that time the Royal Society was essentially a friendly association of the promoters of the new experimental philosophy. Like the Florentine Academy, this society also held the establishment of a new truth in philosophy to be a matter in which the academy had a common interest; any advance was ratified or indorsed by the society by a common vote, and the question of individual credit for a discovery was held as a matter of minor importance. In this way it happened that the per-

Boyle's method of graduation was similar to that of the Florentine makers. From his description it would appear that he assumed the volume of the spirits of wine at the temperature of freezing water to be unity, and then marked on the tube graduations corresponding to expansions by successive ten-thousandths of that volume instead of fiftieths. Although he thus imitated the sealed Florentine instrument, yet there is presumptive evidence that the thermometer ordinarily used by him was not sealed, as indeed was not necessary, except in order to preserve the liquid from gradual evaporation. Following his precedent, open tubes, although discarded in Florence, yet remained in common use in England as late as 1740, by which time the sealed Fahrenheit thermometers had begun to displace them. During these eighty years, from 1660 to 1740, the one improvement in thermometry originating in England was that by Sir Isaac Newton, published anonymously in 1701. This improvement consisted in what is now known as calibration, or an accurate method of allowing for irregularities in the bore of the tube.

SUBSEQUENT PROGRESS IN THERMOMETRIC DETAILS.

4. THE MATERIAL.

The thermometric substance first adopted was air. But, as first used by Galileo, the compression due to varying atmospheric pressure introduced a serious source of irregularity in its indications. Various methods of eliminating this were subsequently suggested by Hooke, Amontons, La Hire, and others, or are derivable from their apparatus, leading to the modern or Joly air thermometer, in which the volume of air remains constant, and its temperature is given by measuring the variations of its pressure. The first thermometer in which the expansion of volume of a liquid is used was due to Galileo, and became the parent of the famous Florentine thermometers. In these a fine tube was expanded at one end into a bulb which was filled with spirit of wine, and the upper end was hermetically sealed. The

sistency of Hooke in claiming priority of invention, frequently on unsatisfactory grounds, notwithstanding his remarkable versatility, gave rise to much unpleasant feeling; his dislike of such controversies moved Sir Isaac Newton to delay publishing some of his most interesting works, including his paper on the graduation of the thermometer, which appeared anonymously in 1701, although it must have been completed, and the substance possibly delivered in his lectures at Cambridge, as early as 1668, and was certainly in his hands in 1680. (See Cotes's Lectures, published in 1738 by his successor, Robert Smith.)

* This statement is made in view of the fact that Halley, in 1693, proposed the use of mercury and then rejected that liquid, apparently not knowing that mercurial thermometers had been used in Florence; again, Halley knew and proposed to utilize the boiling-point of spirit and possibly water; Hooke, also, as early as 1664, proposed the use of the constancy of the freezing-point of water, and in 1684 the constancy of the boiling-point; but none of these ideas were adopted, and there was really no advance made on the Florentine system of graduation, although, owing to the latter being but little understood, others subsequently claimed to have pointed out methods of improving thereon.

only other liquids employed to any extent have been linseed oil, used by Sir Isaac Newton and his successors at Cambridge down to 1740; oil of anise seed, used by Hooke; ordinary wine, frequently used instead of spirit of wine; and mercury, which did not come into general use until it was adopted by Fahrenheit in 1724.

5. THE FORM.

The expanding material has been utilized in the following methods:

(*a*) Air expanding against the pressure of the outer atmosphere, as in Galileo's, Drebbel's, Fludd's, and all similar forms.

(*b*) Air expanding against the pressure of a column of mercury in a sealed tube, as in the form devised by Hubin (1725), and by D'Alencé (1688), and slightly modified forms by Hermann, Balthasar, J. Bernoulli, and Amontons. In both these the efforts to nullify the variations of external pressure are quite successful, but other disturbing causes are introduced, rendering the apparatus too complicated to be accurate or convenient.

(*c*) Air expanding against air of a constant temperature in a sealed bulb. This constitutes the so-called differential thermometer and with various slight modifications is presented in the inventions of Sturm (1766) and Schott (1657).

(*d*) Air expanding against the pressure of a column of mercury in addition to that of the external air, the mercurial column being so adjusted that the original air volume is always retained. This is the modern air thermometer as presented in the construction adopted by Regnault, Joly, etc.

(*e*) A bulb full of liquid expanding into a tube whose upper end is open to external atmospheric influences of all kinds. This is the form of the liquid thermometer invented by Gallileo, and used by Boyle, Hooke, Newton, Halley, and in England generally until the Fahrenheit thermometer superseded it.

(*f*) A bulb full of liquid expanding into a sealed tube with air above it. This was the case with most of the thermometers called Florentine, whether constructed in Florence or elsewhere in Europe. Although the presence of the air may have been at first accidental or unintentional, yet in the mercurial thermometers constructed by Wolff (1709), and in the spirit thermometers made by Réaumur, Fahrenheit, and some others, a little air is intentionally left in the upper part of the tube where its expansion and pressure tends to raise the boiling-point of the contained liquid so that these thermometers would indicate temperatures somewhat above the boiling-point of the liquid of which they are made.

(*g*) A bulb full of liquid expanding into a tube which is, as nearly as possible, exhausted of air, and of all gases except the vapor of the contained liquid. This form was adopted by Ducrest, Christins, Deluc, and Celsius, and is that of the modern thermometer. The desirability of expelling the air seems to have been first appreciated by Boyle, who, however, states that he was unable to persuade his co-laborers that a thermometer of this sort could possibly be made. The objections to a vacuum in the tube of spirit thermometers were fully considered and refuted by Ducrest. The necessity of a vacuum chamber in mercurial thermometers in order to prevent their bursting at high temperatures was appreciated by Fahrenheit. The importance of expelling all air from the glass tube to the sides of which air strongly adheres, in order thereby to increase the sensitiveness and diminish the resistance to the movement of the mercurial column, is a matter that has been only during the present century appreciated.

(*h*) One of the forms given to the thermometer, but one not at all desirable for delicacy or accuracy, was for a time popular in Italy, and is mentioned by the Accademia del Cimento. It consists essentially of a tall jar of water in which are floating hollow glass globes terminating below in short tubes. As the temperature of the water rises and its density diminishes, these globes, which are partly filled with air, change their own density more rapidly than does the water; they consequently rise or fall to new positions of equilibrium. If the globes and their tubes are hermetically sealed, their motions depend almost wholly on the changes in the density of the water, and not on the expansion of the air within them, so that they will fall instead of rise for rising temperature. If the bulb and tubes are open they are sensitive to changes in the atmospheric pressure, which, acting through the water, alters the density of the air within them. To offset this the jar containing them may be hermetically sealed, in which case a rise of temperature, by expanding the air near the bottom of the jar, acts like an increase of atmospheric pressure, thereby condensing the air within the open globes and causing them to descend in the jar, while for closed globes the rise of temperature will cause them to descend in a less degree. These four combinations, namely, closed or open globes within closed or open jars of water, forming what is known as "Ferdinand's globes," or the "Florentine," and the "Stuttgart" experiments with the Cartesian divers, were exhibited throughout Europe, and served to stimulate the study of the phenomena of expansion due to heat and pressure. But, considered as thermometers, they were properly condemned by the Florentine academicians as unnecessarily complex and sluggish.

6. CALIBRATION.

In the early thermometers care was taken to select glass tubes of apparently uniform bore, but as it became important to render thermometers more thoroughly comparable among themselves, methods were devised of allowing for inequalities in the diameter of the bore. The first of these was that executed by Sir Isaac Newton, who poured equal weights of mercury into his tube until it was full, marking for each addition the height to which the tube was filled, thus obtaining divisions corresponding to equal increments of volume.

Réaumur adopted a similar method. A second method was invented by Joseph N. De Lisle (b. 1688, d. 1768) and used also by Ducrest. This consists in observing the length of a drop of mercury contained within the bore of the tube, which, as it slips from one end of the bore to the other, has various lengths according to the diameter of the tube; these lengths, therefore, show how the tube should be divided so as to secure uniform volumes for each division of the scale. This method, in various modern improved forms, as given by Gay-Lussac, Rudberg, Egen, Bessel, Oettingen, P. A. Hansen, F. E. Neumann, Thiesen, and Broch, is that now commonly used.

7. FIDUCIAL POINTS AND NUMERATIONS OF DEGREES.

The construction of a thermometer that shall be a reliable indicator of relative temperatures implies the adoption of a standard temperature, a method of subdivision, and a system of numeration. The latter are arbitrary, the first should conform to the laws of nature. Beginning with Galileo, there was an active search for a natural standard of temperature, easily reproduced at any time, by means of which to secure comparability, and to ascertain the reliability of thermometers. At first, physicists and physicians alike considered the temperature of the healthy human body as a natural standard, and for a century this was adopted as one fixed point by almost all thermometer-makers, including Galileo, Borelli, Newton, and Fahrenheit. In the first editions of Sanctorius's Commentaries there is nothing definite as to any divisions on his thermoscope, but his second edition (1631) speaks of measurements as though by that time he had modified the instrument. Neither does Porta, in 1606, say anything about the division of his thermometers. These were, therefore, at first simply rude thermoscopes, but afterwards a few graduations were added in order to measure differences of temperature; subsequently the Drebbel or Fludd form of the primitive air thermometer had its zero point placed at the normal temperature of a healthy man, as determined by placing the thermometer in the hand or in the axillæ; the departures from this temperature were measured above and below, and the extremes were numbered 3, 7, or 12, according to the mystic or religious ideas of the physicians.

Galileo and Sagredo adopted other fixed points, such as the temperature of cold spring water, considering this to represent another natural standard, namely, the temperature of the ground. The first graduated "bulb-and-stem" thermometer was that referred to in the letter of Sagredo to Galileo, in 1613, wherein the arbitrary division of 360 degrees was adopted, apparently from the divisions of a circle, and where zero represented the temperature of a mixture of snow and salt, and 360 that of the hottest summer day. On this scale 100 degrees was the temperature of melting snow. It seems plausible that Sagredo arrived at this division and numeration by simply curving the long stem of his thermometer around a graduated circle, very much like the thermometers that now frequently accompany aneroid barometers. This adoption by Sagredo of two fixed points or temperatures with a definite intermediate graduation seems to have been soon forgotten, but the adoption of the 360 circular degrees sufficed to fix the application of the word "degree" to the thermometric spaces. With this exception thermometers were for a long time divided by volumes in accordance with the idea that the expansion is in exact proportion to the amount of heat, and that, therefore, the degrees should represent equal volumes or portions of the original volume of the liquid. Ferdinand and his associates continued the search for a constant and easily verified point of reference. They determined the temperature of human beings and of animals; for example, that of the fowl during incubation; the temperature of the earth as shown by spring water, and by the air in caves. For a hundred years the cave temperatures at Paris were used by some as standards, but Boyle, in 1667, clearly showed that such a standard was illusory. The freezing mixtures, especially ice and salt, gave the temperature that was adopted in Florence as the zero of the numeration of the scale, as this was at or below the lowest temperature likely to occur; but the melting-point of ice was used as a fiducial point at which to compare and correct different thermometers. Thus the original zero of Galileo and Sagredo was perhaps intentionally retained in the accurate thermometers used by the Accademia del Cimento. These were graduated from near the bulb upwards in tenths of inches to the temperature of 50° or 100° or 300°, depending upon the object upon which they were to be used. On these thermometers, and after graduation, the point for melting ice was carefully determined, and was 20 degrees on the 100-scale thermometer, and 13.5 on the 50-scale thermometer; all other thermometers of this size had the same divisions and point of melting ice. The artisan who made these for the Grand Duke "stated that he could guaranty to make at least two or three of the 50 degree scale that would be perfectly comparable." It is evident that each of these degrees must have corresponded to some definite fractional part of the volume of the bulb, although that fraction was not adopted in advance or represented by any known number.

During the rest of the seventeenth century observers in other countries each adopted a zero point at his own convenience; thus D'Alancé adopted freezing weather (namely, weather in which frost was formed) as his zero, and melting butter as his 20-degree point; he extended his scale 5 degrees above and below these limits. De l'Isle at first found that with an accurately calibrated tube, ten thousand parts at freezing became 10150 at boiling; he therefore divided the space into 150 equal parts, and as his zero adopted the temperature of the caves of the observatory at Paris. Afterwards, on removing to Russia, he

* The thermometer hung outside his window; the frost was formed on the ground near by; therefore, in general, the zero of his thermometer was probably above the freezing-point.

retained the same value of 1 degree, but adopted as zero 100 degrees below the Paris cave temperatures. Sir Isaac Newton, having first carefully calibrated his tube, then graduated it and numbered it by volumes from 1000, at freezing, up to 1412 as his highest point; he then added to this a second scale of "degrees of heat," in which he took as his two arbitrary fixed points freezing water and the temperature of the human body, which he calls, respectively, zero and 12, and which agree with the volumes 1000 and 1026. Assuming that equal volumes correspond to equal volumes of degrees of heat, this gave him 33 or 34 degrees for boiling water, and 192 degrees for his highest temperature, or 1416 volumes. By taking steps in geometrical progression (0, 12, 24, 48), he found his upper limit, 192, to be the fifth step in this series, which he calls his geometrical scale of degrees. To these Newton added a series of careful determinations of numerous natural temperatures, an abstract of which is given in the following table:

Newton's scale of degrees of heat.

Volumes.	Degrees of heat.		Natural temperatures.
	Arithmetric.	*Geometric.*	
1416	192	5	Hot fire.
1208	96	4	Melting lead.
1104	48	3	Melting alloy.
1074	34	2.5	Boiling water.
1071	33	....	Boiling water.
1051.5	24	2	Melting wax.
1026	12	1	Human body.
1000	0	0	Freezing water.
990	....	....	Lowest air temperature.

The above work was probably done by Newton mostly about 1668 and was fully published by him in his lectures at Cambridge—thirty years before its appearance in 1701, in the Transactions of the Royal Society.

In 1694 C. Renaldini published in his Philosophia Naturæ a method that, had it been practicable, would have constituted an improvement on the methods of graduations previously in use. Renaldini's method was based on the principle that the temperature is directly proportional to the quantity of heat in a given volume of water, or, in modern phraseology, that the specific heat is the same at all temperatures. He proposed to call 12 the temperature of boiling water, and zero that of freezing water, and to make mixtures of 1, 2, 3, etc., parts of boiling water with 11, 10, 9 parts of freezing water, and by plunging the thermometers into these mixtures to divide the space from zero to 12 into twelve equal parts. In this method two fixed points are used, and the subdivisions represent equal proportions of heat. But, notwithstanding the excellencies of this method, the Florentine method of equal volumes of expansion remained in general use.

In 1702 Amontons announced the constancy of the boiling-point of water, as had been done before by Newton, Renaldini, and Halley; but Amontons, like the others, did not actually use this boiling-point in graduating any thermometer, and the constancy of the boiling-point of water was not generally credited until redetermined by Fahrenheit in 1721.

The definite adoption of a graduation like that of Galileo and Sagredo *between two fiducial points*, instead of by volume, is due to Fahrenheit (b. 1686, d. 1736). Fahrenheit seems to have begun making thermometers in 1701, and at first used as his upper limit the temperature of the human body, which he called + 90°, as being the assumed highest possible natural temperature of the air, while for his lowest point he adopted the Florentine zero, or the mixture of salt and ice, as the assumed lowest natural temperature of the air in Europe, which he called — 90°; the mean of these two points he called 0° on his scale and gave it the designation "temperate." His division of the entire range into 180°, like the 360° of Sagredo, was evidently taken from the graduations of ordinary circles, and as Fahrenheit was a maker of all forms of philosophical apparatus it is possible that he may have had some special convenience for dividing a linear distance into 90 or 180 parts. His adoption of this special subdivision was in accordance with the custom then prevailing, which sanctioned the introduction of national, local, and individual peculiarities into the apparatus of the various makers. But the important point of difference between Fahrenheit and previous makers consists in the fact that he adopted two distinct fiducial points, unconsciously following the precedent of Sagredo, so that, although his 180 subdivisions still represented distinct volumes of expansion, yet their precise value in terms of some initial volume was no longer a predetermined matter, but each merely represented the $\frac{1}{180}$th part of the expansion of the liquid between the limiting temperatures of the freezing mixture and of the human body. Fahrenheit's mean temperature or zero, must have closely corresponded to 9° Cent.

Fahrenheit soon after adopted a second scale in which +12° represented the temperature of the body, —12° that of snow and ice. In 1714 he adopted a third scale, namely, +24° for the temperature of the body, and zero for the lowest temperature observed by him at Dantzic in the winter of 1709; this latter temperature which he must have preserved by means of a standard thermometer, is now known to correspond very closely to —18° Cent. Subsequently Fahrenheit graduated thermometers on a fourth scale, suggested by Roemer, the Danish astronomer. In this scale, zero was the lowest cold of 1709, and +24° the temperature of the human body; wherefore, +8° must have been about the temperature of melting ice. But in this latter scale the degree spaces were so large that for convenience he subdivided them into quarters, and very soon issued thermometers on a fifth system, in which these quarters were counted as whole degrees, so that 96° became the temperature of the human body, zero the greatest cold of 1709, and 32° the temperature of melting ice. Through all these changes in his arbitrary scale divisions, numerations, and fiducial points, it would seem that Fahrenheit held fast to his most important idea, namely, that of two natural fixed points, one or both of which could be tested at any time if he had reason to suspect a change in his instrument.

In 1724 Fahrenheit published a communication in the Transactions of the Royal Society of London informing the scientific world of still further improvements. He stated that he had long desired to verify Amontons' assertion as to the constancy of the boiling-point of water by using a mercurial thermometer. In 1721, having made a satisfactory thermometer of this substance, he found that the boiling-point of water was constant at 212° on the scale, whose lower portion agreed with the indications of his spirit thermometer graduated according to his fifth system, and whose upper portion was a simple extension of this scale upwards. It does not appear that Fahrenheit at any time graduated his mercurial thermometers by assuming 32° as the freezing and 212° as the boiling point of water, but the divisions of the mercurial scale for these readings were originally transferred from a standard spirit thermometer, whose fiducial points were determined at zero and 96°. It is sufficient to claim for him the adoption of two fiducial points, and a subdivision of their distance into equal parts, as a substitute for the one fixed point and the division by equal volumes previously in vogue, thereby making possible the construction of accordant thermometers. By the one-point or volumetric method thermometers containing liquids whose rates of expansion are unequal can agree at one point only of the scale, whereas by the two-point method only irregularities in the rate of expansion can affect the accordance of the thermometers. The relative ease of manufacture by the method of Fahrenheit was a material advantage, by reason of which it gained immediate favor. On account of the reputation for accuracy and the convenience as to size of Fahrenheit's mercurial thermometers they became standards, and other makers copied his scale numbers, but instead of adopting his standard temperatures and the corresponding fiducial points they, like De l'Isle, soon learned to adopt the freezing and boiling points of water, partly because of the ease with which melting ice and boiling water could be obtained and partly because of their confidence in the constancy of these points.

Fahrenheit also discovered that the apparently irregular variations of the boiling-point depended on the barometric pressure; in fact he constructed a thermo-barometer which could be used like the hypsometric thermometers of the present time; the lower portion of this instrument was an ordinary thermometer, just above whose highest temperature the bore was enlarged sufficiently to hold the expanded mercury until a temperature of 200° Fahrenheit was reached; the bore then contracted again to its former dimensions and was so graduated that the readings gave inches of barometric pressure on the assumption that the thermometer was then dipped in boiling water. Fahrenheit's knowledge of the connection between the temperature of boiling water and the atmospheric pressure led him (in the absence of numerical corrections) to determine his own boiling-points, or 212°, only when the barometric reading (uncorrected for the temperature of the barometer and under the gravity at Dantzic) was 28 Paris inches, or 29.84 English inches. The boiling-point adopted by Ducrest corresponds to a pressure of 27 Paris inches 9 lines; that of Celsius to a pressure of 25 Paris inches and 3 lines; that of Deluc to 27 Paris inches; that of Lambert to 28 Paris inches; each of these, therefore, depending as they did upon the pressure that was prevailing at their residences, constituted a purely local boiling-point; in each of them the thermometer bulb was supposed to be dipped slightly under the surface of boiling water, and no regard was had to the fact that is now well recognized, that the boiling-point varies with the substance of the vessel that is holding the water.

In 1724 De l'Isle established in the underground caves of the Paris Observatory some thermometers on which he had engraved zero as freezing water and 100 as the approximately very constant temperature of the cave itself. After his removal (in 1726) to St. Petersburg, and before 1736, he adopted mercurial thermometers on which zero represented the boiling-point and 150 the freezing-point of water. He, therefore, next after Fahrenheit, adopted mercury as expanding liquid, and two fixed points with an intermediate graduation. He must be especially credited with the direct adoption of the freezing and boiling points of water. His thermometers were extensively used in Russia.

In 1730 Réaumur (b. 1683, d. 1757) calibrated his tubes and graduated them to the one-thousandth part of the volume of the liquid, as Newton had done before; he filled the tubes with alcohol and sealed them. He found that 1,000 volumes in freezing water became 1,080 in boiling water, and like others marked the freezing-point of water as zero. He put 80 at the highest point reached by his liquid when the thermometer was plunged into boiling water, which point we now know must have corresponded very nearly to the boiling-point of alcohol, but which he mistook for the boiling-point of water. He afterwards discovered that he had not observed the boiling-point of water, as he had intended to do, but the boiling-point of alcohol under the pressure of its own vapor.

In 1732 Nollet persuaded Réaumur to remedy some of the defects of his first thermometer by putting zero at the point of melting ice, not that of freezing water, and 80° as the boiling-point of water, thus bringing the number 65° at the temperature of boiling spirit itself. The first thermometer made in this way (1732) was still in existence in 1767, and gave 9.5° as the temperature of the caves of the observatory at Paris, whereas a thermometer made by Réaumur's first scale had given 10.25°. Réaumur himself, when subsequently publishing his own meteorological observations, rejected those made with the thermometers of his first manufacture.

Thermometers of the Réaumur scale, as manufactured by him in Paris, were extensively used by continental observers, one of whom, Deluc, of Geneva, demonstrated the influence of altitude by observing the lower temperature of boiling water, and, in graduating his thermometers, adopted as the upper point the temperature of boiling water under the normal pressure of 27.0 Paris or 28.75 English inches of mercury at Geneva. The adoption of this new boiling-point under the low pressure at Geneva introduced a new source of discrepancy and was a step backward, since the pressures and boiling-points adopted by Fahrenheit, De l'Isle and Réaumur had corresponded very closely; fortunately the discrepancy introduced by Deluc was diminished by the fact that he boiled his water in a copper vessel, which gave him too high a temperature.

Up to the death of Fahrenheit the thermometers of different makers differed as to where the zero should be put and how the degrees should be counted, rather than in any fundamental new method introduced in their construction. Each maker, in a wholly arbitrary manner, introduced unnecessary diversities, contributing to the confusion already existing, and perpetuated his own name and fame; undoubtedly many who did thus were ignorant of the various scales already existing, while others were actuated by national or local pride, personal fancy, or the needs of some special investigation.

After the general recognition of the fact that thermometers with two fixed points were simpler and better than those with one, the diversity of scales in ordinary use gradually diminished and only a few more were added to the systems of Fahrenheit and Réaumur. In 1736 Celsius proposed zero for boiling water and 100° for melting ice, thus avoiding plus (+) and minus (—) degrees in ordinary temperatures, and used this scale in the meteorological observations of the Swedish polar expedition. In 1743 a similar thermometer was used by Christin, of Lyons, and with it meteorological observations were made at Toulouse in 1747–1756.

Linnæus, the celebrated Swedish botanist, states that he first constructed mercurial thermometers, in which he placed zero at freezing water and 100 at the boiling-point, but whether this was consciously a modification of the numeration used by Celsius, or whether it was suggested by other considerations, is not stated. The fact remains, however, that the modern centigrade scale is due to Linnæus rather than to Celsius.

In 1766 Lambert proposed to restore the zero of the Florentine thermometers, namely, the temperature of freezing mixture of ice and salt; but this was not generally adopted, and, in fact, since the middle of the eighteenth century only three types of thermometer scales have been in common use, namely, the Réaumur, Fahrenheit, and Linnæus (or centigrade), and makers have, since then, universally adopted for ordinary thermometers the boiling and freezing points of water as the two natural standards of temperature. For other thermometers intended to measure remarkably high or low temperatures, other fixed points, such as the freezing and boiling points of mercury, have occasionally been used. The Meteorological Congress of Rome (1879) recommended that the centigrade thermometer of the International Bureau be used in connection with the metric system of measures, and the Fahrenheit thermometer in connection with the English system.

The fact that the temperature of boiling water varies appreciably with the depth of the bulb in the water was fully investigated by a committee of the Royal Society in 1777, who recommended that the bulb and stem be immersed in steam only; the object was to avoid the effect of pressure on the boiling-point. The influence of the material of the containing vessel was first definitely established by Gay-Lussac in 1817; and the rule at present followed, of placing the thermometer bulb only in the steam above the water, in order to be entirely free from the influence of both vessel and water, was first established by Thenard in 1813, although it was not generally followed until some years after.

As the temperature of steam from boiling water varies with the barometric pressure, although it is free from the influence of the containing vessel, therefore the so-called boiling-point is now defined as that which results from observing in steam under a pressure of one standard atmosphere. This standard atmosphere, which was formerly a certain height of mercury in the barometer uncorrected for temperature, gravity, capillarity, etc., has been successively more and more accurately defined. The Royal Society committee of 1777 adopted 29.3 inches of mercury at 50° F. at London; Sir George Shuckburgh (Phil. Trans., 1779) adopted 30.0 inches at 50° F. at London.

The correction for gravity was introduced into hypsometry by La Place, and was generally used by geodesists and meteorologists of England and France in the definition of the normal boiling-point, but not so generally by physicists; it was, however, officially adopted in 1885 by the International Bureau of Weights and Measures, according to which authority the standard atmosphere is the pressure due to the weight of 760 millimeters of mercury at 0° centigrade and under standard gravity at 45 lat. and sea level. This corresponds to a pressure of 759.7462^{mm} at Regnault's laboratory, to 759.54^{mm} at Kew observatory and at the Standard's office in London, to 759.49^{mm} at Cambridge, and to 759.02^{mm} at 60° N. lat. (Stockholm and St. Petersburg). The temperatures of boiling under 760^{mm} at these places are therefore 100.0093716°; 100.016°; 100.018°, and 100.036°, respectively.

CHAPTER II.

MODERN THERMOMETRY—THE AIR THERMOMETER.

8. THE PROBLEMS OF THERMOMETRY.

Modern science corroborates the view of Sir Isaac Newton, that heat or "caloric," is not a fluid nor an imponderable agent, nor a new force, but is the energy due to the internal motions relative to each other of the molecules of all solids, liquids, or gases, and that the determination of the absolute amount of heat contained in a given mass is simply the measurement of the sum total of such molecular energy. Therefore, if the molecules could be brought to rest among themselves, the mass would possess no heat. Knowing, then, that heat is a form of energy, we may dynamically measure a given quantity of it by the number of units of work to which it is equivalent. The art of measuring quantities of heat is termed calorimetry.

The popular conception of temperature is based on the different sensations produced by bodies when termed hot, warm, or cold, the hotter body being said to have the higher temperature. Thus the temperature of a body is the degree of its sensible heat, and thermometry the art of its measurement. But the units adopted in thermometry have necessarily been based on some of the measurable physical effects of heat, rather than on the difference of physiological sensations. Thus the phenomena of linear expansion, volumetric expansion, change of gaseous elastic pressure, rotation of the plane of polarization, change in the index of refraction, change in the electric resistance, have, among others, been employed as a measure of temperature. None of these phenomena vary exactly in a simple proportion to successive increments of heat, but the volumetric expansion of liquids and gases, and the change of gaseous pressure, which have been principally used in thermometry, approach closely to a linear relation. The departures from this relation have been carefully studied for the differential expansion of alcohol, mercury, water, and several gases when contained in glass or platinum bulbs.

The thermometric unit at present temporarily used by the International Bureau of Weights and Measures is one centigrade degree, or the hundredth part of the fractional increase of pressure of a volume of pure dry gas originally at a pressure of one standard atmosphere and heated from the standard freezing to the standard boiling point of water.

Sir William Thomson has shown that the changes in either volume or pressure of an ideal gas would give an absolute scale of temperature which would serve as a true relative measure of absolute amounts of heat. On this scale the temperature t is defined by the equation $E=kt$, in which E is the average kinetic energy per molecule of a perfect gas which has that temperature, and k a constant.

A method for converting a unit of temperature on the absolute scale into a unit of energy has been proposed by Mr. E. V. Burton,* according to whom the mean kinetic energy of the molecule of an ideal gas at a temperature of 0° C., or 273 centigrade degrees above the absolute zero, may be roughly estimated at

$$2.5 \times 10^{-16} \text{ ergs.}$$

The Bureau of Weights and Measures has stated that their provisional temperatures with gas thermometers will be reduced to the absolute thermometric scale when the peculiarities of their gases have been satisfactorily determined.

Thermometry presents three problems: (1) the determination of equality of temperature of different bodies at the same time and place; (2) of the same body at different times; and (3) the measurement of differences of temperature of two bodies at the same or different times and places.

A large part of the difficulties of accurate thermometry consist in eliminating the effects of the changes in the thermometers depending on the circumstances and the times in which it is used; difficulties that have only very recently been satisfactorily overcome.

The following sections present a brief summary of the present condition of researches relating to the errors and methods of mercurial and air thermometers; further details will be found in the recent works of Marek, Thiesen, Broch, Grunmach, Pernet, Guillaume, Pickering, Mill, etc.

Until recently it has been customary for meteorologists as well as physicists to adopt a normal mercurial thermometer as the standard to which, by means of comparison, to reduce any ordinary thermometer used in meteorological observations; the comparisons between the two instruments being made by immersing them in a bath of water. But normal mercurials differ among themselves, if made of different kinds of glass, and differ from the gas thermometer. In order, therefore, to reduce meteorological observations to a standard that represents our present knowledge of physics, the International Congress of Meteorologists at Rome, in 1879, decided to follow modern physicists in adopting the air or gas thermometer as the temporary standard with which to compare all mercurial and spirit thermometers. As soon as practicable the further step recommended by Sir William Thomson should also be adopted, and the air thermometer be reduced to the absolute thermo-dynamic scale,

*Philosophical Magazine, XXIV, 1887, p. 96.

or, as it is sometimes erroneously called, the ideal perfect gas thermometer, to which hydrogen closely corresponds.

For meteorological purposes the subject of thermometry may be divided into the following sections:

1. The air thermometer and the corrections to reduce it to the thermodynamic or absolute scale.
2. The normal mercurial and its reductions to the air thermometer.
3. The ordinary station thermometers and their reduction to a normal mercurial thermometer.
4. The determination of the temperature of the free air at any place and time.

THE AIR THERMOMETER.

9. DESCRIPTION.

The air thermometer consists of a quantity of pure dry air inclosed in an envelope such that the differential expansion or contraction with heat or pressure can be properly observed. The measurement of the increase in pressure under a constant volume constitutes the ordinary form of air thermometer; the measurement of the increase in volume under a constant pressure, although less convenient, has been used by Regnault as a control or check, and by Berthelot (Ann. de Chimie, 1868) for measuring high temperatures; in the form lately devised by Sir William Thomson this may eventually be found most accurate and convenient (see § Heat; Ency. Brit., 9th ed.), but the following treatment applies to the ordinary form in which the pressure on the inclosed gas varies so as to keep its volume constant.

This is usually constructed as shown in Fig. 2, where *A* is the bulb full of air; *A*, *B*, *C*, *D*, a fine tube extending from it and ending at *D* in a broader tube *DE;* a pointer, *P*, within or other index mark on the tube near *D*, is so placed that the mercury in *DE* may be raised up to a constant level (shown by its contact with this pointer) by raising or lowering or squeezing the flexible pipe *EFG*, by which operation the mercury is also raised in the upper glass tube *GF;* the difference of level between the mercury at *P* and *O* indicates the pressure acting at *P* to counteract the elastic pressure of the gas in the space *ABCP*. If the bulb *A* is warmed, this elastic pressure depresses the mercurial surface at *P*, which can be brought back to its standard position only by raising the surface *O; PDEFGOF* constitutes a siphon, by means of which the upward pressure at P is the atmospheric pressure at *O* plus the pressure due to a column of mercury, whose height is the difference of level between *O* and *P*. If now the bulb and portion of the stem *AB* is dipped in a vase of water, or other liquid, *VV*, and several mercurial or spirit thermometers are placed therein beside it, and the mass of liquid is kept thoroughly stirred, it may be assumed that these thermometers and the bulb all have the same temperature; if we vary the temperature of the water, the expansion or contraction of the air in *A* will necessitate corresponding changes in the level of the surface *O* in order to maintain the constant level at *P*. The observations, therefore, consist in reading the thermometers in the water; one or two thermometers for determining the temperature of the air in *BCD*; one or two for determining the temperature of the mercury in *DEFGO*, and the measurement of the difference of level in millimeters between *P* and *O* by means of a vertical graduated scale, or cathetometer. If the thermometers in *VV* have been accurately graduated, and if the experiments be made under the pressure of a standard atmosphere at sea-level and 45 degrees of latitude, and if the differential capillarity of *P* and *O* is allowed for, the thermometers will show zero temperatures when *VV* is full of melting ice, and 100° when it is full of steam, and the measured height, *PO*, and the corresponding pressure at *P*, after applying small corrections, will increase proportionally to the temperature of *A*. If the volume of contained air at the temperature 0° is 1, then at the temperature of 100° it should be nearly 1.3670. If, therefore, the volume is to be kept constant, the pressure, which was one atmosphere or 760 millimeters, must now be increased by the ratio 1.3670, or to 1038.9 millimeters; this increase of pressure of 278.9 millimeters will, therefore, be the height of *O* above *P* when *A* is immersed in steam. In so far as air expands regularly, any other pressure, or the corresponding elevation of *O* above *P*, will represent an exactly proportionate temperature of the bulb *A* and the surrounding liquid; thus the pressure 829.65 millimeters will correspond to the temperature 25° centigrade, and the pressure 690.35 millimeters, or a depression of *O* below *P*, will represent a pull, instead of a pressure, at the point *P*, and correspond to the case where *A* has a temperature −25°. The corresponding readings of mercurial and spirit thermometers immersed in the bath of *A* will give the differences required to correct their indications in order to obtain the temperature given by the air thermometer. It is unimportant in what way these subsidiary thermometers have been graduated, since the application of the corrections thus obtained at once gives us the true temperature, the same as if the air thermometer had been directly employed.

10. METHOD OF OBSERVATION AND REDUCTION.

The bulb and stem of the thermometer (Fig. 2) up to the fixed point *P* are assumed to be full of dry air whose original weight is *W* and tension one atmosphere, and the changes in pressure (p), volume (v), and absolute temperature (T) of this air are assumed to follow the law of Boyle and Mariotte, $pv=RWT$, where the constant R is 29.272 in the metric system of measures.

Whenever an observation is to be made the open end of the tube *HG* is so adjusted that the level of the mercury at *D* rises to the fixed point *P*. Thus the gas is kept at a constant volume except the slight change in volume of bulb by means of a variable pressure, which latter is the current barometric pressure plus that corresponding to the height *HG*. Having determined this internal pressure of the gas for two fixed temperatures, that of freezing and boiling, the observer, so long as he retains unchanged the same amount of air in his bulb, may use this data for determining any other temperature by the following formula, as given by Grunmach in Metronomische Beitrage, No. 3, Berlin, 1881:

Let V equal the value at 0° C. of the volume of Ab, the bulb and tube within the bath.

T equal any temperature of the preceding volume, V.

v equal the volume at 0° C. of the external tube $bBCD$.

t equal any temperature of the preceding volume, v.

H_0 equal the pressure of the air in the bulb and the tube as it would be if all were at the temperature 0° C.

H equal the observed pressure of air in the bulb at temperature T.

α equal the coefficient of cubic expansion of air in volume for 1° C., or 1-100th of the whole expansion from freezing to boiling point.

β equal the coefficient of cubic expansion of the glass of the tube and bulb for 1° C.

W equal the weight of a cubic centimeter of air at 0° C., and under a pressure of one atmosphere.

The weight of the total amount of air which is at a temperature T in the bulb and at a temperature t in the tube is

$$V(1+\beta T)\frac{W}{1+\alpha T}\frac{H}{760}+v(1+\beta t)\frac{W}{1+\alpha t}\frac{H}{760} \quad . \quad . \quad . \quad (1)$$

The weight of the same air, however, when that in the bulb is at 0° C. and that in the tube at temperature t' is expressed as follows:

$$VW\frac{H_0}{760}+v(1+\beta t')\frac{W}{1+\alpha t'}\frac{H_0}{760} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

But as the weight is not changed by these changes in temperature, it is allowable to equate these two values, and after some transformations there results the equation

$$\alpha-\beta=\left(\frac{H'}{H_0'}-1\right)\frac{1+\beta T}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$T=\frac{\frac{H'}{H_0'}-1}{(\alpha-\beta)-\left(\frac{H'}{H_0'}-1\right)\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where

$$H'=H+\Delta H=H+H\frac{v}{V+v}\frac{(\alpha-\beta)(T-t)}{(1+\alpha t)(1-\beta t)} \quad . \quad . \quad . \quad (5)$$

and

$$H_0'=H_0+\Delta H_0=H_0+H_0\frac{v}{V+v}\frac{(\alpha-\beta)(-t')}{(1+\alpha t')} \quad . \quad . \quad . \quad (6)$$

The same bulb and air are used unchanged for a long series of determinations of temperature or comparisons of thermometers, so that β and α are assumed to remain constant.

The method of using the air thermometer is as follows: from observations when $T=0°$, and, again, when $T=100°$ (namely, at the fixed freezing and boiling points), the value of $(\alpha-\beta)$ is to be found from the equation No. 3, assuming an approximate value of β. These values are then used in equation No. 4 to determine the temperature corresponding to any other observed pressure H. Intermediate temperatures thus determined with the air thermometer are corrected for an assumed uniform expansion of glass with temperature, and its return to the same volume at the same temperature at any time. This assumption, however, is not true for glass bulbs unless treated in a uniform manner as to changes of temperature; the best method adopted by physicists is as follows: (1) Determine $(\alpha-\beta)$ from pairs of observations in which the freezing-point is determined immediately after the boiling-point; (2) immediately after determining any temperature, make a new determination of H_0, any appreciable change in which is to be interpreted as implying a change in the quantity of the air or the volume of the bulb, and necessitates the redetermination of $(\alpha-\beta)$. This method of procedure, which is especially necessary in the use of mercurial thermometers, as explained hereafter, is important for the air thermometer to a less degree, but should not be neglected. The accuracy of the resulting temperature, T, depends primarily on the accuracy of α and β; assuming the tube and bulb to have the same temperature, then the effect, $d_\alpha T$, of an error of one unit in the sixth decimal place in the value of α is given by the formula $dT=-\frac{d\alpha}{\alpha-\beta}T$ which gives the numbers in the following table:

T	$d_\alpha T$
°	°
− 10 C	±0.003 C
+ 10	0.003
+ 30	0.008
+ 50	0.014
+ 70	0.019
+ 90	0.025
+110	0.030

The effect of an error of a unit in the sixth place of decimals in the value of β is to change the apparent $\alpha-\beta$ or α, which depends upon it, and by this means a double influence is exerted upon the ultimate temperature, unless the value of α is independently determined; if β is assumed and $\alpha-\beta$ determined from boiling and freezing points, then $d\alpha=1.366d\beta$, and the influence on any observed temperature of an error in β is

$$dT=-\frac{T(100-T)}{1+100\beta}d\beta$$

This error attains a maximum at 50° C., where it is 0.003°. In case α is determined independently, without recourse to the observation of the boiling-point as above described, the effect of an error in the value of β is given by the formula

$$d_\beta T=\left(\frac{1+\alpha T}{\alpha-\beta}\right)Td\beta$$

and in this case the effect of an error in β of one unit in the sixth decimal place is given in the accompanying table:

T	$d_\beta T$
°	°
100.C	+0.038
50.	+0.019
0.	.000
—50.	—0.019

The effect of an error in the determination of H is given by the formula

$$dT=\frac{dH}{(\alpha-\beta)H_0}$$

Thus for ordinary values of α and β an error of 0.27^{mm} in the determination of H or H_0 will produce an error of 0.1° C. A constant error, ΔH, in all pressures including the freezing and boiling points will produce a systematic error, dT, in the determination of other temperatures, as given by the formula

$$dT=-\frac{H-H_0}{(\alpha-\beta)H_0^2}\Delta H$$

As the air within the bulb has a different temperature from that in the further portion of the tube connecting the bulb with the mercurial column, therefore the correction for the external portion becomes important, even when the space is diminished to its smallest practicable amount. Thus in the thermometer of the Deutsche Normal Eichungs Commission, as given by Grunmach, this correction is equivalent to a change in the pressure of -0.19^{mm} when the air temperature is 25° and the observer is determining the freezing-point, and to $+0.80^{mm}$ when the air temperature is 25° and the observer is determining the boiling-point. The right-hand term of equations (5) and (6) contain this correction

$$\Delta H=H(\alpha-\beta)\left(\frac{v}{V+v}\right)\left(\frac{t}{1+\alpha t}\right)$$

from which it will be seen that the correction is diminished by diminishing the ratio $\frac{v}{V}$.

11. THE REDUCTION OF THE AIR THERMOMETER TO ABSOLUTE SCALE.

As already stated, the increase of pressure or volume for ordinary gases does not take place with perfect uniformity for equal increments of heat, and these irregularities increase above the boiling and below the freezing point. To correct for this irregularity, Sir William Thomson, in 1854, proposed an absolute scale which should represent the increased pressures or volumes of an ideal gas. To preserve as much harmony as possible between this absolute and the ordinary centigrade scale, he assumed his degrees of the same size as the centigrade and the temperature of freezing water to be 273.7° on the absolute scale; whence the temperature of boiling water becomes 373.7°, within a small fraction of a degree. The corrections to the air thermometer for intermediate points as originally deduced by him are given in the following table:

Corrections to reduce air thermometers to absolute scale.			
Absolute scale.	Centigrade scale.	Corrections to air thermometers— Of constant volume.	Of constant pressure.
° 273.7+ 0	° 0	0.0000	0.0000
+ 20	20	— .0298	— .0404
+ 40	40	— .0403	— .0477
+ 60	60	— .0366	— .0467
+ 80	80	— .0223	— .0277
+100	100	.0000	.0000
+120	120	+ .0284	+ .0339
+140	140	+ .0615	+ .0721

According to Weinstein (Metronomische Beiträge, Berlin, 1881, page 82), the true corrections for the air thermometer are about one-half of the above, and the latter conform more nearly to those for a thermometer filled with dry carbonic gas. He finds that for the dry air thermometer the observed temperatures (τ) are converted into absolute temperatures (T) by the formula.

$$T=273.4758\,(1+0.00364071\,\tau)^{1.003756}$$

and concludes that the correction $T-\tau$ does not exceed 0.02 between 0° and 100°, but that the data for its exact determination are not yet very satisfactorily known. This formula gives the following values:

τ	Correction= $T-(273.476+\tau)$.	τ	Correction= $T-(273.476+\tau)$.
0	0.000	60	—0.018
10	—0.007	70	— .015
20	— .013	80	— .012
30	— .016	90	— .006
40	— .018	100	0.000
50	— .019		

On account of the careful investigations that have been made into the properties of carbonic acid gas it is possible to more accurately reduce to the absolute scale the indications of a thermometer in which this gas is used; for this purpose Weinstein gives the formula

$$T=274.060\,(1+0.0035913\tau)^{1.01377}$$

from which the following table is computed:

τ	Correction= $T-(274.06+\tau)$.	τ	Correction= $T-(274.06+\tau)$.
0	0.000	60	−0.050
10	−0.020	70	− .043
20	− .034	80	− .032
30	− .045	90	− .018
40	− .051	100	− .000
50	− .053		

The absolute scale has been adopted officially by the International Bureau of Weights and Measures and by meteorological conventions as the ultimate standard, but authoritative accurate numbers have not yet been published, except as above; therefore, for the present, this correction can be stated only approximately as applicable to all current work in air thermometry. It would appear, however, that the International Bureau has temporarily adopted the hydrogen, instead of the ordinary air, thermometer, for which gas the corrections are smaller than those for air. The International Bureau has also in progress a study of thermometers consisting of glass and of platino-iridium bulbs filled with pure dry hydrogen, nitrogen, carbonic acid, etc. Systematic differences between these and between the air thermometers of constant pressure and constant volume, show the effects of the nature of each gas, and when they are properly elucidated the reduction of the gas thermometer will be definitely established. At present the principal information on this subject is to the effect that the reduction of hard glass mercurial thermometers to the hydrogen and also to the nitrogen gas thermometer shows a close accordance. The correction to reduce the mercurial to the hydrogen has its maximum value at 25° C., where it is —0.11° C., and that for the reduction of the mercurial to the nitrogen thermometer has its maximum at 35° or 40° C., and is —0.09° C. These corrections evidently depend almost wholly upon the peculiarities of the glass, so that the reduction of either gas thermometer to the absolute thermodynamic scale will be effected by systems of corrections similar to, but possibly smaller, than those given for air in the above table. (See Procès-Verbaux Comité International de Poids et Mesures, 1885, pp. 161–165.)

12. COMPARISON OF INTERNATIONAL STANDARDS.

The considerable discrepancies of normal mercurials among themselves and from the air thermometer render it very important to ascertain what outstanding differences, depending partly upon the material and partly upon the methods of use, may exist between the mercurial or air thermometers used as standards by the meteorological institutions and those of the International Bureau of Weights and Measures. As air thermometers can not be safely transported with their auxiliary apparatus, the intercomparison becomes practicable only through the intervention of normal mercurial thermometers; and the following embraces most of what is now accessible to me on this subject:

The mercurial Baudin No. 7832, used by Professor Rowland in his thermo-dynamic work, was compared, in 1882, with two Signal Service standards at Baltimore, the result being that the temperatures by the Signal Service air thermometers are on the average 0.01° C. lower than those of Professor Rowland's air thermometer. Eight thermometers made for the Signal Service were compared in 1884 at the International Bureau at Sevres with normal mercurial Tonnelot No. 4263, whose corrections to the air thermometer of the International Bureau have, however, not yet been applied. On arriving at Washington these were compared with the Signal Service mercurial Tonnelot No. 4207 and No. 51. The mean difference (Sevres — Signal) is zero at 0° C., and increases steadily up to −0.08° C., at temperatures from 20° to 35° C. This difference corresponds closely to the ordinary reduction of mercurial to air thermometers. Six thermometers were compared for the Signal Service at the Kew Observatory at temperatures from 0° to 50° C., in 1884-'85, and eight thermometers at the temperature of melting mercury, or —37.9° F. On receipt at Washington these were compared with the Signal Service mercurials Tonnelot Nos. 4207 and 51, from which it resulted that the Signal Service air standard is lower than the Kew mercurial standard by quantities increasing uniformly from 0.00°, at 0° C. up to 0.07° C., at temperatures from 35° to 50° C. For temperatures below freezing no sensible difference exists between the Signal Service and Kew mercurial thermometers for temperatures ranging from +32 F. to —38 F.; but for Signal Service and Kew alcohol thermometers, at a temperature of —38 F., the Signal Service standard is 0.6° F. below the Kew standard. For temperatures above freezing, therefore, the Kew mercurial, corrected for reduction to the air thermometer, will agree with the Signal Service air standard.

The relations between the mercurial normals at the International Bureau and the gas thermometers at that institution have not been definitely published, but may be inferred approximately from the report of the work done by Pernet and Chappuis during 1885. According to this the hydrogen and nitrogen gas thermometers give results closely accordant; the special figures for thermometers of various kinds of glass are as follows:

Hydrogen thermometer at Sevres.	Baudin 6654.	Baudin 7605.	Tonnelot 4250.	Alvergniat 22883 B.
°	°	°	°	°
2.5	—0.013	—0.009	—0.007	—0.011
5	— .032	— .020	— .022	— .030
10	— .046	— .029	— .040	— .050
15	— .063	— .044	— .049	— .082
20	— .088	— .059	— .073	— .108
25	— .103	— .062	— .078	— .114
30	— .095	— .054	— .071	— .113

Tonnelot 4207 (of the same glass as Tonnelot 4250), as compared at Washington with Signal Service air thermometer, needed the following corrections:

Signal Service air thermometer.	Tonnelot No. 4207.
°	°
0.0	.00
5.6	—.01
11.1	.00
16.1	—.02
22.2	—.05
25.2	—.07
30.1	—.09

The agreement of these corrections with those of Tonnelot 4250 shows that the Signal Service air thermometers and the Sevres hydrogen thermometer probably agree within ±0.01 C.

CHAPTER III.

THE MERCURIAL THERMOMETER.

13. GENERAL PRINCIPLES.

The mercurial thermometer consists essentially of a thin glass bulb full of pure mercury, with an attached receptacle containing additional mercury, so that a change in temperature causes a flow out of or into the bulb. This exchange of mercury may be measured by weighing the bulb before and after submitting it to changes of temperature, as in the mercurial weight thermometer; or the volume of the expansion or contraction may be measured directly, as in the ordinary mercurial stem thermometer. The former method has been frequently used, notably by Regnault in his determination of some of the fundamental data of meteorology. The weight thermometer is not affected by the variable errors due to the temperature of the stem of the stem thermometer, and it has, therefore, sometimes been assumed as the standard form of mercurial. It is adapted to measurement of temperature in locations where the readings of a stem thermometer can not be made, but its use is more laborious than the latter, and it is now rarely employed.

In both these forms the adopted measure of temperature has generally been taken to be not the absolute expansion of the mercury, but the differential expansion of the mercury and the glass. But ever since the irregularities in the expansion of these substances have been known it has been apparent that the resulting temperatures do not represent exactly equivalent increments of heat, and, consequently, the absolute expansion of air has been adopted as a standard by means of which to correct the readings of mercurial thermometers.

In all mercury-glass thermometers the temperature is given by the relative change of differential volume at freezing, boiling, and any other temperature. The general formulæ are as follows:

Let V^m_0, V^m_{100}, V^m_t be the volumes of the mercury, and V^g_0, V^g_{100}, V^g_t the volumes of the glass, at the temperatures 0°, 100°, and $t°$. Then will

$$V^m_0 = V^g_0$$

$$\frac{V^m_{100} - V^g_{100}}{V^g_{100}} = \text{the differential expansion from freezing to boiling;}$$

$$\frac{V^m_t - V^g_t}{V^g_t} = \text{the differential expansion from freezing to } t°\text{;}$$

then the temperature on the centigrade scale will be

$$t° = 100 \cdot \frac{V^g_{100}}{V^g_t} \times \frac{(V^m_t - V^g_t)}{(V^m_{100} - V^g_{100})}$$

In order to realize this equation in practice, numerous corrections must be applied to the actual observations.

14. THE NORMAL MERCURIAL WEIGHT THERMOMETER.

In this form of thermometer a glass bulb with very short open stem is filled with mercury to the top of the opening, while the whole is immersed in melting ice. The bulb is dried, weighed, and then transferred to the steam of boiling water. After the bulb has attained the boiling temperature it is again dried and weighed with its contained mercury. Any other temperature is now measured by filling and weighing the bulb at the freezing-point, then transferring it to the bath, and again weighing when it has attained the temperature which is to be measured. The weighing apparatus, namely, the balance, is independent of, and unaffected by, the temperature of the bath.

Let the weighings be W_0, W_{100}, W_t, and A the weight of the glass; then $\frac{W_0-W_t}{W_t-A}$ is the differential expansion from 0° to t°; $\frac{W_0-W_{100}}{W_{100}-A}$ is the differential expansion from 0° to 100°; whence

$$t=100\,\frac{W_0-W_t}{W_0-W_{100}}\times\frac{W_{100}-A}{W_t-A}$$

THE NORMAL MERCURIAL STEM THERMOMETER.

15. DESCRIPTION.

The normal mercurial stem thermometer consists of a stem with capillary bore ending in a glass bulb containing mercury. A change of temperature produces a difference in the volumes occupied by the mercury and the glass, which is shown by the rise and fall of the mercury in the capillary tube. The stem of the thermometer is divided longitudinally, or is fastened to a separate divided scale and thus serves as the apparatus for measuring in standard units the differential volumetric change in the mercury and glass. The stem is long enough for both freezing and boiling points of water to appear on it.

Thermometer scales are of two kinds, according as they are made by one or the other of the two following processes, (*a*) and (*b*):

(*a*) The stem is first graduated empirically into equal linear parts (*e. g.*, millimeters); then the uniformity of the volume of the capillary tube is tested by calibration, which gives the values of the volumes corresponding to the spaces on the divided stem in terms of a standard unit. The scale readings corresponding to the temperatures of boiling and freezing are then determined, the latter immediately after the former, the difference between these readings is the fundamental length; finally each space in the scale is given its value in terms of the hundredth part of the whole volume of the capillary tube between the fiducial points.

(*b*) The second method reverses this process, and proceeds as follows: First, the boiling, then the freezing point (and sometimes intermediate points determined by comparisons or calibration) are marked on the undivided stem; next the spaces between these points are subdivided into the proper number of equal parts, and these graduations are extended above and below the fiducial points; finally the tube is calibrated and the outstanding errors of the 0° and 100° points are determined anew.

The second method gives corrections limited to small fractions of a degree; the first method gives corrections that may have any amount, and, in fact, are called conversions of the arbitrary scale. This latter method is considered more elegant, but is more troublesome to use, because of the resulting large conversions.

Whichever of these processes is adopted for ordinary thermometers the normal mercurial should show the freezing and boiling points; the scale should be divided into equal parts throughout, without allowing for the minor inequalities of bore; and the upper end of the thermometer should have a reservoir, called the calibrating chamber, similar to the bulb. The use of the mercurial thermometer necessitates the study of the following subjects:

(1) Corrections for parallax and refraction.
(2) Corrections for errors in scale division and calibration.
(3) Determination of the boiling-point and freezing-point.
(4) Determination of the fundamental distance and the value of one scale division.
(5) Corrections for temperature of the scale and stem.
(6) Corrections for inequalities in expansion of glass and mercury.
(7) Corrections for exterior and interior pressure on the bulb.
(8) Correction for sluggishness.
(9) Correction for capillarity.
(10) Thermal and elastic reaction; temporary zero point and resulting temperatures.

16. CORRECTION FOR PARALLAX AND REFRACTION.

The thermometer scale when divided on the glass stem is in front of the thread of mercury, but when divided on an auxiliary strip of brass or enamel it is behind the mercury. In either case the reading of the top of the mercurial thread will vary with the direction of the line of sight. In Fig. 3, let the normal incidence be *Eem*, and the line of sight $I\,m'\,m$; let $s = s_1\,s_2 =$ one degree on the scale expressed in linear units; let $a = s_2\,m_2$ be the distance of the scale in front of the mercury expressed in linear units; $n =$ the index of refraction of the glass; i and r the angles of incidence and refraction, respectively; m' the point at which the line of sight intersects the scale; then em' is the linear distance by which m appears lower than for normal incidence, and is therefore the correction to be applied to the apparent temperature read off at m'. The diagram gives the relation $e\,m' = \tan r$, which, expressed in units of one degree of temperature, becomes $\frac{a}{s} \tan r$, where r is to be found from the optical relation $\sin r = \frac{1}{n} \sin i$.

This correction should, of course, be avoided by using only normal incidence, and where this is impracticable we may, by using nearly normal incidence, avoid the computation of the second equation by using i as an approximate value of r in the expression $\frac{a}{s} \tan r$. But in many cases this is not allowable. For a thermometer in which $\frac{a}{s} = 1.1$ and $n = 1.6$ we have the following table:

i	r.		Correction.
°	°	′	°
0	0	0	0.000
10	6	14	0.120
20	12	21	.241
30	18	13	.362
40	23	41	.482
50	28	36	.600
60	32	46	.708

Instead of a computed correction it is still better in all cases to take the mean of two readings, one with the scale on the side towards the observer, and the other with it on the opposite side, thus eliminating refraction; another but inferior method consists in reading with the eye equally distant above and below the scale division.

17. CORRECTIONS FOR ERRORS OF THE SCALE.

(*a*) *Errors in scale division.*—If the maker has furnished the thermometer with a divided scale, any errors in the subdivisions will introduce corresponding errors in the observed temperature. An examination of any ordinary thermometer will disclose errors of this kind. The general method (used by the International Bureau of Weights and Measures) of determining these errors in the subdivision of any scale is given in Chapter VIII of this work.

In thermometry, however, the determination of the corrections for the errors in the subdivisions of the scale is usually combined with the determination of the corrections due to the irregularities of the bore.

(*b*) *Calibration.*—The glass tubes used in thermometers can not be made exactly cylindrical, and therefore a given length of tube represents different volumes at different parts of the scale. For good thermometers tubes are selected whose caliber varies but slightly and in a fairly uniform manner; this is determined by placing a short column of mercury, at successive positions, in the tube and measuring the variations of length which it undergoes. If tubes thus selected be graduated between any reference points, such as the freezing and boiling points, into any number of equidistant parts, and the graduations be extended to portions of the tube beyond these points, the interior capacities at any part of the bore may be expressed in divisions of this scale by applying to every graduation line of the scale small corrections, called calibration corrections. Numerous methods for determining these have been devised, the first being due to Newton. The method of Gay-Lussac is now in general use; of its various modifications that by Hansen is presented by Broch, as follows:

Calibration corrections (considered as including the corrections for errors in scale division) are determined by comparing between themselves the interior capacities of the tube in the same manner as one determines the corrections of a divided scale. To compare these interior volumes among themselves a thread of mercury, which very nearly coincides with the volume to be compared, is separated in the stem of the thermometer from the remainder of the column. The excess (positive or negative) of the length of this thread over the interval between the graduation marks is then measured. The temperature of the mercurial thread should remain unchanged during the series of measures made upon it.

Let x_k and x_i be the calibration correction at any scale divisions, k and i and L the corrected length of the column of mercury; that is to say, the length which it would have had if the tube were exactly cylindrical. Disposing this column between the two scale divisions, i and k, let $i+\Delta i$ and $k+\Delta k$ be the observed readings of the extremities of the column, where Δi and Δk are small quantities expressed in divisions of the scale. We shall have then

$$L=(k+\Delta k+x_{k+\Delta k})-(i+\Delta i+x_{i+\Delta i})$$

and placing $\lambda=L-(k-i)$ we shall have

$$\lambda+x_i-x_k=(\Delta k-\Delta i)+(x_i-x_{i+\Delta i})-(x_k-x_{k+\Delta k})$$

In a first approximation we may assume that the calibration corrections at the points $i+\Delta i$ and $k+\Delta k$ are equal to those of the neighboring points i and k, and the preceding equation reduces to

$$\lambda+x_i-x_k=\Delta k-\Delta i=a$$

From all the equations of this type given by the observations the values of the first approximations to the corrections x are deduced.

In a second approximation the values of Δk, $\Delta i=a$ are corrected by adding $(x_i-x_{i+\Delta i})-(x_k-x_{k+\Delta k})$, calculated by using the first approximation. The whole calculation is then to be repeated with the corrected values of a.

For good thermometer tubes the corrections obtained in the second approximation are negligible, but in accurate work and for the best thermometers the repetition of the computation is necessary. The following example, illustrative of the method of computation, is taken from an actual calibration made by the International Bureau of Weights and Measures in 1885:

The calibration was performed by first dividing the scale from 0° to 100° into 5 parts, and so finding the calibration errors for the points 20°, 40°, 60°, and 80°. Having found the corrections for these points, each of these twenty-degree divisions is subdivided in 5 subdivisions, and the calibration corrections found for every 4° by an identical process. The example of the calculation of the first calibration in 5 parts of 20° each is here given. The quantities given by direct observation are Δk and Δi. The differences of the means of six readings of Δk and Δi give the numbers for $\lambda_5+x_1-x_2$, etc., as found in Table I. The columns contain successively the observations with columns of mercury of 20°, 40°, 60°, and 80° in length. The five successive lines in the computation represent the observations when the left-hand end of the mercury column is in approximate contact with the successive scale divisions $0°=x_1$, $20°=x_2$, $40°=x_3$, $60°=x_4$, and $80°=x_5$. The unit is the ten-thousandth of a degree.

TABLE I.—*Observation-equations and calculation of s and Σ.*

	$n=5$	$\Sigma_5=2\frac{s_5}{5}$	$\Sigma_4=2\frac{s_4}{4}+2\frac{s_2}{2}$
$L_1=\underline{20°}+\lambda_5$	$L_2=\underline{40°}+\lambda_4$	$L_3=\underline{60°}+\lambda_3$	$L_4=\underline{80°}+\lambda_2$
$\lambda_5+x_1-x_2=+82$	$\lambda_4+x_1-x_3=-735$	$\lambda_3+x_1-x_4=-737$	$\lambda_2+x_1-x_5=-434$
$+x_2-x_3=-1765$	$+x_2-x_4=-2550$	$+x_2-x_5=-1028$	$+x_2-x_6=+675$
$+x_3-x_4=-1754$	$+x_3-x_5=-978$	$+x_3-x_6=+1934$	
$+x_4-x_5=-179$	$+x_4-x_6=+1048$		
$+x_5-x_6=+1168$			
Sum, $s_5=-2448$	$s_4=-2315$	$s_3=+169$	$s_2=+241$
Divisor, 5	4	3	2
$2\frac{s_5}{5}=-979$	$2\frac{s_4}{4}=-1157$	$2\frac{s_3}{3}=+113$	$2\frac{s_2}{2}=+241$
$\Sigma_5=-979$	$+251$ $\Sigma_4=-916$		

TABLE II.—*Calculation of t.*

x_1	x_2	x_3	x_4	x_5	x_6
+82	−1765	−1754	−179	+1168	0
−735	−2550	−978	+1948	+434	−675
−737	−1028	+1934	+737	+1028	−1934
−434	+675	+735	+2550	+978	−1948
0	−82	+1765	+1754	+179	−1168
−1824	−4750	+1702	+6810	+3787	−5725
t_1	t_2	t_3	t_4	t_5	t_6

Check.

$t_1 =$ −1824
$t_2 =$ −4750
$t_3 =$ +1702
$t_4 =$ +6810
$t_5 =$ +3787
$t_6 =$ −5725
+12299 −12299

Check.

$4s_2 =$ +964
$3s_3 =$ +507
$2s_4 =$ −4630
$s_5 =$ −2148
+1471 −7078
−5607
2
−11214

$+5t_1 =$ −9120
$+3t_2 =$ −14250
$+t_3 =$ +1702
$-t_4 =$ −6810
$-3t_5 =$ −11361
$-5t_6 =$ +28625
+30327 −41541
−11214

TABLE III.—*Calculation of S and P.*

$S_6 = t_6 + t_1$ $\quad 6P_6 = S_6 + 2X$
$S_5 = t_5 + t_2$ $\quad 6P_5 = S_5 + 2X$
$S_4 = t_4 + t_3$ $\quad 6P_4 = S_4 + 2X$

$P_6 = 0 \quad 2X = -S_6 = +7549$
$X = +3775$

$t_6 + t_1$	$t_5 + t_2$	$t_4 + t_3$
−5725	+3787	+6810
−1824	−4750	+1702
−7549	−963	+8512
	+7549	+7549
	6) +6586	6) +16061
	+1098	+2677
	P_5	P_4

Check: $P_5 + P_4 = +3775 = X$

TABLE IV.—*Calculation of D and R.*

$$D_6 = t_6 - t_1$$
$$D_5 = t_5 - t_2$$
$$D_4 = t_4 - t_3$$

$$R_5 = \tfrac{1}{6}(D_5 - D_6 - \Sigma_5)$$

$$R_4 = \tfrac{1}{6}(D_4 - D_5 - \Sigma_4)$$

t_6-t_1	t_5-t_2	t_4-t_3
−5725	+3787	+6810
+1824	+4750	−1702
−3901	+8537	+5108
	+3901	−8537
	+979	+916
	6)+13417	6)−2513
	+2236	−419
	R_5	R_4

TABLE V.—*Calculation of Q.*

$$Q_6 = 0$$

$$Q_5 = R_5 + Q_6 - \frac{2}{1.5}Q_6 = +2236$$

$$Q_4 = R_4 + Q_5 - \frac{2}{2.4}(Q_6 + Q_5) = -419 + 2236 - 559 = +1258$$

TABLE VI.—*Calculation of x.*

$$x_n = \tfrac{1}{2}(P_n + Q_n) \qquad x_2 = \tfrac{1}{2}(P_n - Q_n)$$
$$x_3 = \tfrac{1}{2}(P_{n-1} - Q_{n-1})$$
$$x_{n-1} = \tfrac{1}{2}(P_{n-1} + Q_{n-1})$$

		Sums.	Differences.	Calibration correction. ½ sum.	Calibration correction. ½ difference.
$P_6=0$	$Q_6=0$	0	0	$x_6=0$	$x_1=0$
$P_5=+1098$	$Q_5=+2236$	+3334	−1138	$x_5=+1667$	$x_2=-569$
$P_4=+2077$	$Q_4=+1258$	+3935	+1419	$x_4=+1968$	$x_3=+709$

Check: $\Sigma x = X = +3775$

TABLE VII.—*Calculation of λ.*

$2\lambda_4 = s_5 + (Q_6 + Q_5)$	$= +241 + 2236 = +2477$	$\lambda_2 = +1239$
$3\lambda_3 = s_4 + (Q_6 + Q_5 + Q_4)$	$= +169 + 3494 = +3663$	$\lambda_3 = +1221$
$4\lambda_2 = s_3 + (Q_6 + Q_5)$	$= -2315 + 2236 = -79$	$\lambda_4 = -20$
$5\lambda_5 = s_2 + Q$	$= -2448 = -2448$	$\lambda_5 = -490$

TABLE VIII.—*Calculation of residual errors.*

Cal.	Obs.	C.–O.	Cal.	Obs.	C.–O.	Cal.	Obs.	C.–O.	Cal.	Obs.	C.–O.
+ 79	+ 82	– 3	– 729	– 735	+6	– 747	– 737	–10	–428	–434	+6
–1768	–1765	– 3	–2557	–2550	–7	–1015	–1028	+13	+670	+675	–5
–1749	–1754	+ 5	– 978	– 978	0	+1930	+1934	– 4			
– 189	– 179	–10	+1948	+1948	0						
+1177	+1168	+ 9									

The sum of the squares of the residual errors is $[\Delta\Delta]=655$

$$r=0.67449\sqrt{\frac{[\Delta\Delta]}{14-8}}=\pm 7 \text{ or } \pm 0.0007^{\circ}$$

The calibration corrections are, therefore, as follows:

Scale readings.	Calibration corrections.
°C.	°C.
0	0.0000
20	–0.0569
40	+0.0709
60	+0.1968
80	+0.1667
100	0.0000

In the above example the bore was of unusual regularity.

The following table gives, by way of illustration, the calibration and scale division corrections of the thermometer, Tonnelot No. 51, as determined at Washington by Prof. Thomas Russell. Columns of 5°, 10°, ... 95°C. in length were used, and the computations repeated for a second approximation according to Thiesen's method:

Scale readings.	Calibration corrections.	Scale readings.	Calibration corrections.
°C.	°C.	°C.	°C.
0	0.000	55	–0.556
5	–0.104	60	–0.572
10	–0.134	65	–0.558
15	–0.173	70	–0.553
20	–0.250	75	–0.528
25	–0.321	80	–0.471
30	–0.389	85	–0.383
35	–0.428	90	–0.281
40	–0.464	95	–0.154
45	–0.491	100	–0.000
50	–0.522		

The probable error of one of these corrections is not greater than ±0.002° C.

The glass tubes, like glass bulbs, are liable to a permanent change of dimensions, depending on the temperature to which the glass has been subjected. Therefore the calibration should be repeated at intervals of a few years.

The corrections above given are expressed in terms of the average length of one scale division, and the scale readings, as well as the calibration corrections, can not be converted into thermometric degrees until the value of a scale division has been determined. The above calibration corrections are based on the assumption that the standard value of one division of the scale is $\frac{1}{100}$ of the total volume between certain limiting divisions; but these limiting divisions may be either at the extremities of the scale, or at any other intermediate position. In the latter case the successive calibration measurements with each mercurial thread may be extended above and below, so that the relative value of each part of the scale becomes known in terms of one standard portion of the tube. The conversion into thermometric degrees can be effected as soon as it is known how much of the calibrated portion of the tube is occupied by the expansion of the mercury from the freezing to the boiling temperature. This portion of the tube is called the *fundamental length.*

18. DETERMINATION OF THE BOILING-POINT.

(*a*) The boiling-point of pure water is determined by means of an apparatus which allows of hanging the thermometer entirely within a double casing, so that the whole is surrounded by steam of uniform temperature. The scale divisions are read either through a glass window or by allowing them to project above the double casing into a glass tube also full of steam. The thermometer bulb hangs a little above the surface of the boiling water, and therefore gives the temperature of the vapor, which may differ largely from that of the liquid. A vent is provided for the free escape of the steam from the outer casing, and a delicate manometer, with which to measure any difference of pressure between the steam within the apparatus and the air without. The external pressure is determined by the barometer, and the pressure of the steam is obtained by adding the manometer reading. The water should boil until its steam has expelled all air from the inner casing. The barometer and manometer should be read before and after the boiling-point is observed. If the reading of the boiling-point be made when the thermometer has reached its highest reading, and the boiling be continued for a long time, the mercury will be observed to fall slowly. This is due to a gradual change in the capacity of the bulb; the lowest point reached is that which is desired, and is called the depressed boiling-point. In order to expedite his observation of the depressed boiling-point the observer should then make a determination of the freezing-point, and immediately after a new determination of the boiling-point. Apparently the rapid alternations of temperature facilitate the molecular changes of the glass. When two successive determinations agree it is probable that the permanent depressed boiling-point has been attained. In order to diminish the very appreciable conduction of heat from the thermometer to the surrounding air the thermometer must be wholly immersed in steam at 100° C.; if the apparatus does not allow this then the observer should secure uniformity by performing all determinations in a room whose temperature is from 6° to 12° C.

(*b*) *The effect of pressure on the temperature of boiling water.*—The observed temperature of steam depends upon the barometric pressure existing at that moment at the surface of the boiling water. Therefore, the standard boiling-point under a pressure of one standard atmosphere is obtained by reducing the actual observations according to the relation between the temperature and pressure of vapor as experimentally determined by Regnault and Magnus. The International Bureau of Weights and Measures has published a table containing Broch's reductions of Regnault's observations, in which the standard boiling-point is that corresponding to the pressure of a column of 760^{mm} of mercury at 0° C., and under the normal gravity at 45° latitude and sea level. In reducing observations of the boiling-point this table is to be entered with the external barometric pressure plus the pressure given by the manometer, all reduced to normal gravity, as the argument; the resulting temperatures are in "normal degrees." The following is an extract therefrom:

Pressure.	Temperature of steam.	Pressure.	Temperature of steam.
	°C.		°C.
750	99.6310	760	100.0000
751	.6681	761	.0367
752	.7051	762	.0733
753	.7421	763	.1099
754	.7791	764	.1465
755	.8160	765	.1830
756	.8529	766	.2194
757	.8897	767	.2559
758	.9265	768	.2923
759	99.9633	769	100.3286

The correction for gravity implies that all instrumental errors in the barometer have been corrected for.

Example.—At latitude 52° and altitude 150 feet the observed pressure of the vapor surrounding the thermometer is 757.0^{mm}, and the observed temperature of the steam is 99.90°; reduce the observed pressure to standard gravity, and with this corrected pressure, 757.5^{mm}, the preceding table gives 99.91° for the correct boiling-point under the prevailing pressure; whence the thermometer reads 0.01° too low. This correction, +0.01, strictly holds good for the scale reading 99.9°.

The standard pressure of the atmosphere adopted formerly by Bird Troughton and other makers, who used the English system of Fahrenheit degrees, was 30 inches of mercury at 62° F. under gravity at London, which is 29.905 inches of mercury at 32° F. at London, which becomes 29.924 under standard gravity; that adopted by Clarke in his standards of length coincided with the International Bureau, and that adopted by Welshe for Kew was 760^{mm}=29.922, at 32° F., under gravity at Kew, or 760.46^{mm} under normal gravity, for which the normal boiling-point is 100.016°; that adopted by Regnault for his centigrade thermometer was 760^{mm}, at latitude 48° 50′ 14″, and altitude 60^{mm} above sea level.

The degrees observed on these various systems therefore require a slight correction to reduce to normal degrees of the scale of the International Bureau of Weights and Measures. Thus, if t be a Regnault temperature and T a corresponding normal temperature, the relation as given by Broch is

$$T = t \times 1.000093176$$

This is deduced from the following considerations, *i. e.*, 760^{mm} at 45° latitude becomes 759.7462 at 48° 50′ 14″. A diminution of pressure of 0.2538^{mm} diminishes the boiling-point 0.0093176° C., or 0.000093176 of the whole fundamental length.

(*c*) *Influence of pressure on the bulb at the temperature of boiling.*—Not only does the atmospheric pressure affect the temperature of boiling but it also exerts a small influence on the bulb, whose glass is subjected to the interior hydrostatic pressure of the contained mercury and to the exterior pressure of the atmosphere. The effect of these internal and external pressures will be considered in a subsequent section. It is sufficient to note here that, for accurate thermometry, the boiling point should be determined with the thermometer horizontal as well as vertical, in order to have a direct measure of the effect of internal pressure, which is a minimum when the thermometer is horizontal.

19. DETERMINATION OF THE FREEZING-POINT.

(*a*) *Method of operation.*—The normal freezing-point (or the zero of the international centigrade scale) is the temperature at which ice from distilled water melts under the standard atmospheric pressure. It is determined by placing a thermometer in a vessel of melting ice. The ice may be in the shape of freshly fallen snow or small chips or shavings of ice made from pure water, or very small pieces. The purity of the ice and water is important, as a slight mixture of salts, acids, or alkalies sensibly alters the freezing-point. It is important that the resulting water should flow freely over the bulb. The stem of the thermometer should be immersed in ice and water to a distance far enough above the freezing-point to annul any conduction of heat. As the glass bulb cools more rapidly than the larger mass of mercury within it the mercurial column may rise momentarily, but will then fall rapidly; in a few minutes the column will attain a minimum called the depressed freezing-point, or the zero that obtains for the thermometer in the condition of its bulb just before immersion in the freezing mixture. This is called the temporary zero, or Z_t; if the thermometer has just come from a bath of boiling water the depressed freezing-point will be that for the temperature of boiling water, or Z_{100}. If the thermometer is retained in the cooling mixture for some time the reading slowly increases, and in the course of from one to four days attains a point known as the raised zero point, or zero relève. This slow rise is due to a corresponding slow contraction of the glass bulb, and can be hastened if the thermometer is warmed up and cooled slowly several times; a rapid cooling seems to produce a more rigid setting of the exterior layers of the molecules of the bulb, as in the similar case of steel tempered by being plunged in cold water.

(*b*) *Depressed and temporary zero points.*—If the thermometer is taken out of the freezing mixture immediately after reaching the depressed zero point and kept at ordinary air temperature for a long time, say from six to twenty-four months, it will also contract and show a higher reading. If the depressed zero point is then determined it will be found to differ but little from the above-described raised zero point, and is known as the secular zero point, or the zero of long repose. These slow changes in the zero point, due to slow residual contractions in the glass bulb, take place more rapidly during the first few minutes after the thermometer is placed in the freezing mixture. They also take place more slowly and to a less extent the higher the temperature at which the bulb is maintained; thus a thermometer kept at the boiling-point would not perceptibly change its zero from the original value of Z_{100}.

If the thermometer has been originally heated to some temperature lower than that of boiling water the corresponding depressed freezing-point for that temperature (Z_t) will differ from that found for boiling water; in general, the zero is lower the higher the temperature to which the bulb was exposed immediately before placing it in the freezing mixture. The temporary zero of a thermometer, therefore, depends upon the highest temperature to which it has recently been exposed and upon the time that has elapsed since that exposure. Therefore, after any observation of temperature has been made the proper zero of the thermometer must be determined either by calculations based upon previous studies of this thermometer, or still better by immediately making a direct determination of the actual freezing-point. The former method is that adapted to ordinary meteorological work where the zero point can be directly determined only at stated intervals. The formula for computing the temporary zero will be given in a future section.

For accurate thermometry the depressed zero corresponding to any temperature must be observed immediately after the measurement at that temperature; and as this observation must be repeated very frequently it is best that the freezing mixture be not merely a packing of the snow and ice about the thermometer, as is still very commonly used, but rather a thoroughly-stirred mixture of about one part water and five parts of slowly-melting ice. The rapid stirring facilitates the rapid cooling of the bulb. The bulb and stem should be gently cooled for one or two minutes before plunging into the cold bath. The depressed zero should be obtained by taking numerous readings just before and after the minimum is reached, from which the true minimum can be deduced by some graphic process. The work of observation is easily completed in five or ten minutes after immersing the bulb, and requires quick dexterity on the part of the observer. It has been shown by Pernet that when determined in this way the depressed zero for any temperature varies as the square of the temperature above 0° C., so that, if not observed directly, the zero may be computed very approximately by the formula

$$Z_t = Z_0 - (Z_0 - Z_{100})\frac{t^2}{100^2}$$

where Z_{100} is the depressed zero, determined just after heating to the boiling-point. Z_0 is the zero of long repose or its equivalent, the zero relève.

(c) *The effect of temperature of the room.*—The observed freezing-point is found to depend slightly on the temperature of the room. It varies by 0.01° C. for temperatures of the surrounding air that are as high as 15° or 20° C. This is probably the general result of conduction of heat down the stem, and can be absolutely annulled only by determining the freezing-point when the surrounding temperatures are near zero; but if there be uniformity in respect to the temperature of the room the resulting error will be nearly constant for all observations. The custom of the International Bureau is to make the determinations of zero when the temperature of the room is from 6° to 12° C.

(d) *The effect of pressure on the temperature of melting ice.*—Theory and observation show that an increase of pressure lowers the temperature at which ice melts. This effect is barely appreciable in very delicate work. The standard freezing-point t_0, under a pressure of one atmosphere, is obtained from the observed temperature t_n of the freezing-point under the pressure of n atmospheres by the formula

$$t_0 = t_n + 0.0073^{\circ}\text{ C.}\,(n-1)$$

(e) *Influence of pressure on the bulb at temperature of freezing.*—As the hydrostatic pressure of the vertical column of mercury in the bore, although small, has a perceptible influence on the bulb, it is necessary to determine this by methods to be hereafter given. For the present it is only necessary to say that the freezing-point should be determined for both vertical and horizontal positions of the thermometer in order to obtain the data necessary for the pressure correction.

20. THE METHOD OF COMPUTING TEMPERATURES.

(*a*) *Fundamental distance.*—In the early forms of the stem thermometer each division represented a simple aliquot part, say 0.0001, of the volume of the liquid at 0°. In the modern forms a degree was at first considered to represent an aliquot part of the distance between the fixed points corresponding to the boiling and freezing temperatures. A more exact study, as indicated in the last two articles, has shown that these points themselves vary independently of each other, depending on time, temperature, the treatment of the thermometers, and the nature of the glass. The changes are largely in the glass of the bulb, yet they may be due in part to changes in the bore of the tube, especially when, as shown by Pernet (Travaux et Mémoires, T. IV), the lower part of the stem has alone been frequently heated. It has been shown, however, that the depressed boiling-point and the corresponding depressed freezing-point, determined within an hour after the boiling, show a great degree of constancy, especially when alternate observations of each are made, in order to determine the maximum depressions. Guillaume states that the fundamental distances determined according to these rules can be relied upon in the *mean of several* observations to 0.002° C.; Pernet finds the fundamental distance from a *single* determination reliable to 0.02°, and Grunmach finds that for a thermometer made of the best glass the *individual determinations* can be relied upon to 0.01°.

Based upon these considerations, the distance between the depressed boiling-point and the depressed freezing-point corresponding thereto is adopted as the constant and definite length or fundamental distance for the purpose of ascertaining the thermometric value of each division of the scale. The constancy of the fundamental distance thus determined has been demonstrated by Guillaume (Travaux et Mémoires, Tome V, page 67), who finds that for hard glass the observed variations are really smaller than the errors of observation.

It is interesting to note that in the centigrade mercury-glass thermometer 1° corresponds to about $\frac{1}{6500}$ of the volume of the bulb at zero instead of the fraction $\frac{1}{100}$ or $\frac{1}{10000}$, as adopted by the physicists of the seventeenth and eighteenth centuries.

(*b*) *Transformation of scale readings into temperatures.*—Very important irregular differences have for a long time existed in the methods of passing from scale readings to degrees, and equally in the passage from degrees to temperatures. These methods have been revised by Pernet in view of the laws just stated with regard to the changes in the fundamental points. He shows that the following is decidedly the most rational and best method of calculating temperatures. It is substantially that now adopted by the International Bureau. Let x be the scale reading for the temperature t; z_t the depressed zero, determined immediately after observing the scale reading x; S_{100} the depressed boiling-point, determined immediately after z_t; Z_{100} the depressed zero point, determined immediately after observing S_{100}; then will $F=S_{100}-Z_{100}$ be the fundamental distance, and $\frac{100}{F}$ the value of one scale degree in centigrade degrees. The temperature t will therefore be given by the following formula:

$$t=(x-z_t)\frac{100}{S_{100}-Z_{100}}$$

This method requires that z_t, S_{100}, and Z_{100} be successively determined immediately after any important observation of temperature; but when extreme accuracy is not demanded the fundamental distance, and consequently the value of a scale division, may be assumed constant, as known from previous determinations, and only the value of z_t need be determined. Even this latter may be approximately computed by the formula subsequently given.

The other methods of computing t, as used by Berthelot, Henrici, Oettingen, Rechnagel, Muncke, and others, are based upon assumptions as to the changes in the fundamental points, or on the determination of these points before the observation of the temperature x, and have been shown by Pernet to introduce discrepancies amounting to 0.2° C., whereas different thermometers, used according to the above method and formula, give results agreeing to within a few hundredths of a degree. Thus, by taking for the fundamental interval the distance between the depressed fixed points, and by applying to any thermometric reading the correction for the actual or temporary zero corresponding to the moment of observation Pernet has made the first step toward a standard method of measuring temperatures with the mercurial thermometer.

21. CORRECTIONS FOR TEMPERATURE OF SCALE AND STEM.

When a stem-graduated thermometer is wholly immersed in a liquid, or, as in meteorological observations, in the air, the whole stem and bulb may be supposed to be at a uniform temperature; but when the bulb only is immersed in the liquid, or when the thermometer has a metallic scale (usually fastened to the stem at the upper end), the stem, the scale, and the capillary column of mercury may have a very different temperature from that of the bulb. This is obviously the case with wet-bulb, dew-point, and solar radiation thermometers. The thermometer corrections obtained by comparisons with a standard when immersed in a bath of water, therefore, need to be supplemented by additional corrections, reducing an observed height of the mercurial column to that which would have been read off were the temperature of the scale and stem and capillary column uniform with that of the bulb. These corrections were applied so long ago as 1777 by the committee on thermometry of the Royal Society of London (see Phil. Trans., 1777, LXVII, p. 816).

(*a*) *Corrections for temperature of brass scale.*—If the temperature of the scale is higher than that of the bulb, the scale is pushed downward by expansion from its upper fixed end, and the reading is therefore too high. Let τ be the reading corresponding to the fixed point; t_3 be the temperature of the scale; t be the correct temperature of the bulb and mercury up to the lowest division τ_0 corresponding to the reading at the lowest point to which the scale extends; $\frac{1}{3}\delta=0.0000183=$ the coefficient of linear expansion of brass; then this correction will be given by the expression $(\tau-t)\frac{1}{3}\delta \cdot (t_3-t)$, and is subtractive.

(*b*) *Correction for temperature of glass stem in thermometers with brass scales.*—If the temperature of the glass stem is higher than that of the bulb the bulb and mercury in it are pushed downward on the divided brass scale by the linear expansion of the stem from the fixed point τ. Let t_2 be the temperature of the glass stem between τ and the upper end of the mercurial column; t_1 the temperature of the glass stem from the upper end of the mercurial column to τ_0; $\beta=0.000025$, the coefficient of volumetric expansion of glass. The upper portion of the stem, whose length is $\tau-t$, has an excess of temperature t_2-t, the correction for which may be called B_1; the lower portion of the stem, whose length is $t-T_0$, has an excess of temperature t_1-t, the correction for which is B_2. The total correction, B_1+B_2, is additive, and is given by the expression

$$B_1+B_2=\tfrac{1}{3}\beta[(t_2-t)(\tau-t)+(t_1-t)(t-\tau_0)]$$

(*c*) *Correction for temperature of capillary column.*—The upper end of the column of mercury has approximately the temperature t_1, while the lower end has the temperature of the bulb t. Its average temperature, t_m, may be determined by the laws of conduction of heat, but will be approximately the mean of these two. If t_m is warmer than t the volume of the mercurial column will therefore be increased above its proper volume by the quantity $(t-\tau_0)\gamma(t_m-t)\alpha$, where $\gamma=0.00013$ is the coefficient of cubic expansion of mercury, and α is the volume of one degree of the capillary tube. The volume of the capillary tube is also larger than its value at the temperature t by the quantity $(t-\tau_0)\beta \cdot (t_m-t)\alpha$. The excess of the former over the latter of these two expressions divided by α gives the quantity expressed in scale degrees, by which the reading is too high. Therefore the correction is given by the expression

$$(\gamma-\beta)(t-\tau_0)(t_m-t)$$

and is subtractive. For $\gamma-\beta$ the approximate value 0.000155 may be used. If t_m-t is negative the correction becomes an additive number. The preceding corrections are presented in the following tables, in order that the reader may appreciate the necessity of avoiding them or of estimating their magnitude for such cases as may possibly arise in meteorological experience, especially in observations of the dew point and wet bulb. Detailed tables are given in Landolt and Bornstein; Physikalisch-Chemische Tabellen, Berlin, 1883, but special tables for any thermometer should be based on direct measures made with it:

A.—*Correction for temperature of brass scale.*

Correction $= -(\tau - t)\frac{1}{3}\delta(t_3 - t)$
Assuming $\delta = 0.000055$

$\tau - t$	$t_3 - t$		
	60° C.	40° C.	20° C.
°C.	°	°	°
160	−0.176	−0.117	−0.059
140	−0.154	−0.102	−0.051
120	−0.132	−0.088	−0.044
100	−0.110	−0.073	−0.037
80	−0.088	−0.059	−0.029
60	−0.066	−0.014	−0.022
40	−0.044	−0.029	−0.015
20	−0.022	−0.015	−0.007
0	−0.0	−0.0	−0.0

For Fahrenheit thermometers the above corrections become numerically larger than the tabular figures in the ratio $\frac{9}{5}$. The magnitude of this correction shows the necessity for the steady progress that has been made toward the adoption of stem-graduated thermometers.

B_1.—*Correction for the temperature of the upper portion of glass stem.*

Correction $= +(\tau - t)\frac{1}{3}\beta(t_2 - t)$
Assuming $\beta = .000025$.

$\tau - t$	$t_2 - t$			
	40° C.	30° C.	20° C.	10° C.
°C.	°	°	°	
100	0.033	0.025	0.017	0.008
80	.026	.020	.013	.007
60	.020	.015	.010	.005
40	.013	.010	.007	.003
20	.007	.005	.003	.002

B_2.—*Correction for the temperature of the lower part of the stem.*
Correction $= +(t - \tau_0)\frac{1}{3}\beta(t_1 - t)$

This can be taken from the above table for B_1 by using $t - \tau_0$ and $t_1 - t$ as arguments.

C.—Correction for temperature of capillary column of the lower portion of the stem.

Correction $= -(\gamma-\beta)(t-\tau_0)(t_m-t)$ for mercurial thermometers.
Assuming $\gamma-\beta=0.00015$.

$t-\tau_0$	t_m-t		
	+40° C.	+20° C.	−20 C.
°C.	°	°	°
100	−0.600	−0.300	+0.300
80	−0.480	−0.240	+0.240
60	−0.360	−0.180	+0.180
40	−0.240	−0.120	+0.120
20	−0.120	−0.060	+0.060

For alcohol thermometers $\gamma-\beta$ becomes $\varepsilon-\beta$, where ε, the coefficient of expansion of alcohol, is 0.00105; for ether thermometers the factor is $\eta-\beta$, where η, the coefficient of expansion of ether, is 0.00150. In these cases we have approximately $\varepsilon-\beta=.0010$, and $\eta-\beta=0.0015$. The corrections therefore are respectively seven and ten times as large for alcohol and ether thermometers as the figures given in the above table.

The above values of γ, β, ε, and η are average values. Their precise values in any case can be known only by special determinations of the coefficients of expansion of the materials actually employed. This has only rarely been performed; thus in a class of thermometers studied by Thorpe he found $\gamma-\beta=0.000143$, instead of the value above assumed.

(*d*) *Determination of the temperature of the lower portion of the stem in stem-graduated thermometers.*—The evaluation of the correction *C* requires the determination of t_1, the mean temperature of that portion of the stem containing the mercurial column. The usual approximate method for obtaining t_1 is to assume that when the bulb only is immersed in a bath the temperature changes uniformly from t at the bottom to t_a or the surrounding air temperature at the top of the stem, whence

$$t_1=\frac{t+t_a}{2}$$

But this quantity is sometimes needed with greater accuracy, and it can be determined more nearly in accordance with the law of the conduction of heat in long glass stems. Let l be the length of the protruding stem up to the top of the mercury column; ν, the number of scale degrees in a unit's length; $n=\frac{l}{\nu}$, the number of degrees in the length l; β, the coefficient of expansion of glass; $a=\sqrt{\frac{hP}{kq}}$, quantities depending on the conductivity and emissivity of the glass; t, the temperature of the bath; τ_1, the temperature of the stem at the top of the mercury column;

t', the scale reading at the top of the mercury column. Then the laws of conduction of heat give

$$\frac{\nu}{an} \cdot (t-t_a) = \tau_s - t_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Again, the uniform change of temperature along l gives

$$t-t' = n_l\beta(t-\tau_s) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The coefficient $\frac{\nu}{a}$ in (1) may be determined experimentally for any special thermometer by using two auxiliary thermometers for obtaining the temperatures of the bath and of the air, t and t_a, while the thermometer under examination is so placed that it shall have the largest practicable value of n and of $t-t_a$. For this case, therefore, t' t_a, and t are known, and the remaining unknown quantities in (1) and (2) are τ_s and $\frac{\nu}{a}$. By eliminating between (1) and (2) we obtain

$$\frac{\nu}{a} = n - \frac{t-t'}{\beta(t-t_a)} = B$$

$\frac{\nu}{a}$ is thus determined once for all for the special thermometer, and the values of τ_s, as given by the formula

$$\tau_s = t_a + \frac{B}{n} \cdot (t-t_a)$$

may be tabulated for convenience in use.

(*e*) *The Poggendorff correction.*—In the stem thermometer the changes in the volume of the liquid are measured in the capillary tube. But the capillary tube by expansion and contraction at different temperatures undergoes variations in capacity, whence it results that the differential expansion of the bulb and its contained mercury is measured by a variable standard. Ordinarily the bulb and stem both have the temperature of the air or liquid. A correction, therefore, needs to be applied to reduce the readings to what would have been given by a tube maintained at a standard temperature, while the bulb only has the temperature that is to be measured. This correction is called the Poggendorff correction from the name of the physicist who first applied it in 1826; its neglect has given rise to discrepancies in the determination of fundamental meteorological data; the equivalent of this correction is always introduced into the reduction of observations with air thermometers. The correction may be determined for mercurial thermometers either instrumentally, by comparisons with an air thermometer, or by direct numerical computation.

The formula for computing the correction is obtained as follows:

Let V=volume of mercury at 0° C.; this is the same as that of the bulb and tube up to the freezing-point when at the temperature of 0° C.

v=volume of tube from the freezing to the boiling-points when at the temperature 0° C.

β=coefficient of cubical expansion of glass.

γ=coefficient of expansion of mercury.

T=thermometer reading when the true temperature is t.

Then since at any temperature the volume of the bulb and of the stem up to the top of the mercury column is equal to the volume of the contained mercury, we have the equation

$$V(1+\beta t)+v(1+\beta t)\frac{T}{100}=V(1+\gamma t) \quad . \quad . \quad . \quad . \quad (1)$$

For $t=100$; T also equals 100, being a fiducial point of the thermometer, and the equation becomes

$$V(1+\beta \,.\, 100)+v(1+\beta \,.\, 100)=V(1+\gamma \,.\, 100) \quad . \quad . \quad . \quad (2)$$

Eliminating V and v between (1) and (2)

$$t=T \,.\, \frac{1+\beta \,.\, t}{1+\beta \,.\, 100} \text{ or } t-T=\beta\frac{t-100}{1+\beta t}t$$

The corrections above given, A B C, give the temperature that would be observed with stem and scale at the same temperature as the bulb. The Poggendorff correction reduces this to the temperature that would be given by a thermometer whose stem is maintained at a uniform standard temperature of 100°, since at this temperature the stem has the volume that determined the standard fundamental length. This correction is given in the following table for the extreme values of β that may occur in practice:

T	Poggendorff correction.	
	β=0.000020	β=0.000030
	°C.	°C.
−40	+0.112	+0.168
−30	0.078	0.117
−20	0.048	0.072
−10	+0.022	+0.033
0	0.000	0.000
+10	−0.018	−0.027
20	0.032	0.048
30	0.042	0.063
40	0.049	0.072
50	0.051	0.075
60	0.049	0.072
70	0.042	0.063
80	0.033	0.048
90	−0.019	−0.027
+100	±0.000	±0.000

22. CORRECTIONS FOR IRREGULARITIES IN THE EXPANSION OF GLASS AND MERCURY.

The Poggendorff correction pertains only to an error in the capacity of the measuring apparatus, namely, the capillary tube. But since glass and mercury do not expand with perfect uniformity, the quantity to be measured, namely, the differential expansion of the bulb and its contained mercury does not bear an exact linear relation to the temperature. The quantitative effect of the irregularities in expansion must, therefore, be determined in the use of either the mercurial weight or the mercurial stem thermometer.

If in equations (1) and (2) of the preceding section for the stem thermometer we substitute for β and γ, β_t and γ_t, the mean coefficients of expansion between 0° and t°, the volumes of the mercury at t° and 100° will be respectively

$$V(1+t\gamma_t) \text{ and } V(1+100\gamma_{100})$$

the volumes of the bulb and stem up to the freezing-point when at tem-atures t° and 100° will be

$$V(1+t\beta_t) \text{ and } V(1+100\beta_{100})$$

respectively. The volumes of the stem above the freezing-point will be

$$v(1+t\beta_t)\frac{T}{100} \text{ and } v(1+100\beta_{100})$$

at t° and 100°, respectively. Hence we have

$$V(1+t\beta_t)+v(1+t\beta_t)\frac{T}{100}=V(1+t\gamma_t)$$

$$V(1+100\beta_{100})+v(1+100\beta_{100})=V(1+100\gamma_{100})$$

Eliminating V and v there results

$$T=t\cdot\frac{1+100\beta_{100}}{1+t\beta_t}\cdot\frac{\gamma_t-\beta_t}{\gamma_{100}-\beta_{100}}$$

The first factor of this equation is, as before, the Poggendorff correction; the second factor takes account of irregularities in expansion.

The laws of expansion have been carefully investigated for pure mercury and for glass bars, but the actual direct determination of the expansion of the thin unannealed glass bulbs of thermometers is still a desideratum. There is reason to believe that appreciable, systematic, and irregular variations exist therein other than those known to exist in glass rods. These must be investigated by studying an existing thermometer without injuring it, and without subjecting it to a variable internal or external pressure.

In place of direct measures Von Oettingen and Rechnagel have computed the combined Poggendorff and expansion corrections on the assumption that the laws of expansion for ordinary glass hold good for thermometer bulbs, and that β_t and γ_t vary with the first, second, and third powers of the temperature. The following table, computed by the above formula for T, using the accepted formula for variable expansion, as indicated by the tabulated values of β_t and γ_t, gives the resulting corrections for various parts of the scale:

Combined correction for capacity (Poggendorff correction) and irregular expansion.

t	β_t	γ_t	$t-T$
°C.			°C.
0	0.00002531	0.00018028	0.000
10	2554	18038	−0.085
20	2577	18047	−0.147
30	2600	18057	−0.186
40	2623	18068	−0.207
50	2646	18079	−0.207
60	2669	18091	−0.189
70	2692	18104	−0.154
80	2715	18119	−0.111
90	2738	18135	−0.058
100	2761	18153	0.000

If the above corrections do not suffice to convert temperatures by the normal mercurial into temperatures observed simultaneously with the normal air thermometer it is reasonable to assume that the outstanding discrepancies may be accounted for by supposing that these are due to slight errors in the adopted coefficients for the expansion of glass, since the expansion of mercury is certainly very accurately known; this method of treating the outstanding differences between the air thermometer and a normal mercurial has been followed by Prof. Thomas Russell. He finds that the mercurial, Tonnelot No. 4207, can be made to agree with the air thermometer by assuming the coefficient of cubical expansion of the glass bulb to be

$$\beta = 0.000\,026 + 0.000\,000\,021\,859t + 0.000\,000\,000\,099\,512t^2$$

This value is sufficiently near to the best formula for the expansion of glass bars to show that the special expansion of the glass bulb will probably, in all cases, explain the outstanding differences between normal mercurial and air thermometers.

In conclusion, it is evident that any normal mercurial can in general be accurately reduced to the standard air thermometer only by means of direct comparisons between them, and that, in the absence of these, the temperatures given by the mercurial must be unreliable to the extent indicated by the variations in the corrections above given.

The magnitude of the combined correction for capacity (Poggendorff) and irregular expansion is illustrated by the following system of corrections obtained by Professor Russell for thermometers made of different kinds of glass:

Temperature.	French crystal glass.		Special new glass.	
	Baudin, 9704.	Baudin, 9705.	Green, 7375.	Green, 7376.
°F.	°F.	°F.	°F.	°F.
−38	−0.49	−0.43	....	....
−28	−0.39	−0.35	+0.23	+0.26
−18	−0.31	−0.28	+0.15	+0.20
− 8	−0.24	−0.22	0.00	+0.11
+ 2	−0.17	−0.15	+0.03	+0.06
12	−0.11	−0.10	−0.03	+0.02
22	−0.05	−0.05	0.08	−0.05
32	0.00	0.00	0.00	0.00
42	+0.16	+0.13	+0.02	+0.05
52	+0.20	+0.13	0.09	+0.10
62	+0.27	+0.20	0.06	+0.06
72	+0.29	+0.23	0.05	+0.05
82	+0.29	+0.22	0.03	+0.02
92	+0.33	+0.27	0.03	−0.01
102	+0.27	+0.22	0.02	−0.06
112	+0.30	+0.21	0.02	−0.06
+122	+0.24	+0.22	0.08	−0.08

The close similarity between the corrections for bulbs of the same kind of glass confirms the conclusion of European physicists that glasses of different chemical composition have different coefficients of expansion, and that to this is partly due the outstanding differences in normal mercurial thermometers.

23. CORRECTION FOR PRESSURE ON THE BULB AND FOR VERTICALITY OF STEM.

A mercurial thermometer gives different readings according as the stem is vertical or horizontal, the readings being higher in the latter case. Apparently the difference is due principally to the effect of the heavy mercury in distorting the shape of the thin glass bulb. If the vertical be adopted as the standard position the correction may be expressed as being very nearly the difference between the vertical and horizontal readings multiplied by the sine of the inclination of the thermometer. It does not exactly follow this or any other simple law, but depends upon individual peculiarities in the bulbs. A table for this correction may be prepared by determining the correction for several inclinations to the vertical and interpolating by some graphic process. When possible this source of error should be avoided by keeping the thermometer strictly vertical, but as it is sometimes necessary to tip one in order to get good illumination or to avoid parallax, therefore a careful consideration of this correction has been made.

The distension of the delicate glass bulb varies with the hydrostatic pressure to which it is subjected. This pressure may be external or internal. The internal pressure both compresses the mercury and expands the glass bulb; the external pressure compresses the glass bulb. The former depends upon the vertical height of the column of mercury in the tube, and upon the capillary resistance which increases the pressure of rising columns and diminishes the pressure of falling columns. No precise results have been yet attained by means of which to separately correct for the variations of capillary pressure within the thermometer tube. These variations, however, can become appreciable in tubes of very small bores, and for this reason such are to be avoided. According to Guillaume the coefficient of pressure for the thermometer bulb is a quantity which, multiplied by the pressure exerted on the bulb, gives the correction to be applied to the observed temperature to reduce to what would have been observed under standard pressure of one atmosphere internally and externally. The usual units are a centigrade degree for temperature and a millimeter of mercury under standard gravity for pressure.

According as the pressure is internal or external β_i and β_e are used to indicate the coefficients of pressure. Since β_i, as above stated, is equal to β_e plus the compressibility of mercury, therefore these two coefficients are connected by the following relation:

$$\beta_i-\beta_e=K(\chi_m-\chi_g)$$

where χ_m and χ_g designate the coefficients of cubic compressibility of mercury and glass, and K is a factor for converting a unit volume into a unit of the thermometer scale, χ_m, χ_g, and β_e can be determined with great accuracy by actual experiment on the thermometer.

(*a*) *External pressure.*—To determine β_e the thermometer is inclosed in the short leg of a flexible siphon tube filled with mercury, and is subjected to pressures varying from zero to any desired limit by raising and lowering the free end of the siphon. In order to maintain a constant temperature the siphon is placed in a bath of water. The observations should always be made with a rising column in order to avoid changes in capillarity. The following table illustrates the amount of this pressure correction, for a few high-class thermometers. One-tenth of the numbers given in the last column are the values of β_e, according to the preceding definition. The effect of a pressure of 0.6 of an atmosphere is about that experienced at the highest meteorological stations:

Mercurial thermometer description.	Scale.	Rise of thermometer for an increase in pressure of—		
		One atmosphere.	0.6 of atmosphere.	10^{mm}
		°	°	°
Tonnelot, 4207 (cylindrical)	Cent.	0.0912 C.	0.055 C.	0.0012 C.
Tonnelot, 2587 (cylindrical)	Fahr.	.1370 F.	.082 F.	.0018 F.
Baudin, 9704 (cylindrical)	Fahr.	.3420 F.	.205 F.	.0045 F.
Green, 1034 (spherical)	Fahr.	.3876 F.	0.233 F.	0.0051 F.

External pressure generally affects cylindrical bulbs less than spherical, because the glass of the former is thicker.

In general the correction is given by the expression $-(p-p_0)\beta_e$, where β_e is the rise in thermometric reading for unit change of pressure and p_0 the standard pressure adopted as the zero of reference. The large values of β_e above given, for delicate normal thermometers, show that ordinary station thermometers may experience an appreciable change when carried to high stations.

When thermometers are used for the determination of the temperature of liquid baths, and especially when comparisons are made with the air thermometer, the pressure of the external liquid on the bulb should be allowed for. This is also necessary in determining the temperatures of the waters of rivers and harbors. In determining the temperature of greater depths at sea it is customary to avoid this correction by surrounding the bulb by some device that will counteract the great pressure of the ocean, although the sensitiveness of the thermometer is greatly impaired thereby.

Spirit thermometers and air thermometers have larger bores for the same size of bulbs than mercurials, and their indications are therefore proportionately less affected by pressure. The correction for pressure sensibly affects all thermometric work with ordinary thermometers at high stations, except in the few cases where the fiducial points have been determined under the prevailing low pressure current at the station.

The effect of external pressure on the bulb is modified by whatever affects the elasticity of the glass, and as this diminishes with temperature a slight increase in the pressure effect with an increase of temperature has been found; but for ordinary ranges of temperature and pressure in meteorological work this is insensible. For accurate physical work Guillaume finds that the coefficient β_e increases slightly with temperature, so that its value becomes

$$(\beta_e)_t = \beta_e(1+0.000016t^2)$$

(*b*) *Internal pressure.*—In order to determine β_i, the coefficient of internal pressure, Guillaume made direct experiments upon several hundred thermometers. His results confirmed the value of β_i computed directly from the observations made by Decamps and Amaury. These latter observations gave Guillaume the relation

$$K(\chi_m-\chi_g) = +0.0000154$$

which substituted in the preceding formula gives for the correction to the thermometric reading due to an increase of p millimeters in internal pressure

$$+(\beta_e+0.0000154)\ .\ p$$

This correction, in so far as it depends on the hydrostatic pressure p of the capillary column, will vary with its height in millimeters, or with the temperature (*i. e.*, with $1+Kt$), and with the inclination of the tube to the vertical (*i. e.*, sin θ); in fact, the principal part of the correction is determined directly when we compare a thermometer in its horizontal and vertical positions. Guillaume states that the determination of the boiling-point with the thermometer placed alternately horizontal and vertical gives the best check on the computation of β_i.

Pernet (Travaux et Mémoires, *t.* IV) gives $\beta_e=+0.000194$ to 384 and $\beta_i=+0.000195$ to 377 as the extremes for six delicate Baudin thermometers.

The corrections for internal and external pressure may, finally, be expressed as follows:

$$C_p=\beta_e(p_e)_t+\beta_i p_i$$

$$=\beta_e \cdot p_e(1+0.00016t)+(\beta_e+0.0000154)(1+Kt)\sin\theta$$

where θ is the inclination of the tube to the vertical and K the length of 1° on the scale expressed in millimeters.

Any differences in the elastic properties of different kinds of glass will cause differences in their records depending on pressures as distinct from peculiarities depending on the temperature of the glass. This point has been investigated by Guillaume and Tornoe in Paris, whose results with regard to "elastic reaction" agree with the results of the similar work on "thermal reaction" done by the German physicists. This latter work, as summarized by Wiedemann, shows that thermometers of the same chemical composition, whether of soft glass (*crystal*) or hard glass (*verre-dur*), give identical results among themselves, but that those of hard glass, between 0° and 100°, give lower readings than those of soft glass. Finally, the differences between hard and soft glass can, in general, be represented by a simple formula which, with varying constants, will apply to glass of any composition. This formula for the two types of glass generally used in France, and investigated by Guillaume, is as follows:

Let t_d be the temperature given by a thermometer of the glass "verre-dur," whose composition is silica, 71.5; aluminum, 1.6; lime, 14.5; sodium, 10.8; let t_c be the temperature given by a thermometer of the glass "crystal," whose composition is silica, 60.7; oxide of lead, 15.1; aluminum, 0.9; lime, 5.4; sodium, 10.6; potash, 6.6; then will the differences in the indications of thermometers made of these kinds of glass be

$$t_c-t_d=t(100-t)(14.126-0.0311t)10^{-6}$$

Thus, by considering the peculiarities due to the composition of the glass, results are obtained demonstrating that the temperatures given by mercurial thermometers, far from being subject to numerous anomalies, such as many have imagined, really follow very simple laws, whose form is as above prescribed.

It will thus become possible to convert the earlier physical measurements into the modern standard whenever we know the composition of the glass of their thermometers and the manner in which they have themselves taken account of the variations of the zero.

24. CORRECTION FOR CAPILLARITY.

Capillarity is a phenomenon of surface tension. When a liquid, a gas, and a glass are in contact at any point there is a triplet of tensions, the resultant of which, if not neutral, tends either to draw the liquid towards the glass or to repel the liquid from the glass; in the former case the glass is said to be wetted by the liquid. The contact of clean glass with water or alcohol is an example of the first case, and that of glass with mercury is an example of the second case. In an open capillary tube, or between two plane surfaces at capillary distances, the surface tension is sufficient to support or depress a column of liquid to an appreciable amount above or below its level, as determined by gravity or other forces. For tubes the height of this column is inversely proportional to their radius.

The angle between the surface of the liquid and the glass is called the angle of capillarity. This angle is frequently stated in physical text-books to be constant for any given combination of substances. It is known, however, that the angle varies with the slightest change in the condition of the surfaces, and with the temperature. In computing this correction it is consequently necessary that direct measures be made for each thermometer.

The fundamental relations involved in the evaluation of the capillary corrections are given in the following analysis:

Let f=area of section of a tube standing in a basin of liquid.

s=weight of unit volume of the liquid.

h=height which the bottom point of meniscus attains above the position of the level surface outside the tube and determined by other forces; h is positive or negative according as the capillarity makes the surface concave, as in alcohol, or convex, as in mercurial thermometers.

m=weight of liquid meniscus above its bottom surface.

q=upward (or downward) pressure per unit of area in the bore, due to the resultant surface tension.

θ=capillary angle between the tube and liquid.

Then the laws of hydrostatics give

$$sfh+m=qf \quad \ldots\ldots\ldots\ldots \quad (1)$$

In very narrow bores m may be neglected in comparison with sfh, in which case

$$h=\frac{q}{s}$$

If ρ and ρ_1 are respectively the smallest and largest radius of curvature of the meniscus in an oval bore, and H is a constant depending on the liquid, the laws of surface tension give

$$q=\frac{H}{2}\left(\frac{1}{\rho}+\frac{1}{\rho_1}\right)$$

If r and r_1 be the radii of the tube corresponding to ρ and ρ_1, and i be the measured height of the meniscus, then (see Fig. 4)

$$\rho^2=r^2+(\rho-i)^2 \qquad \rho_1^2=r_1^2+(\rho_1-i)^2$$

$$2\rho i=r^2+i^2 \qquad 2\rho_1 i=r_1^2+i^2$$

$$\frac{1}{\rho}+\frac{1}{\rho_1}=\frac{2i}{r^2+i^2}+\frac{2i}{r_1^2+i^2}$$

If θ be measured instead of i we have

$$\rho=r\sec\theta \qquad \rho_1=r_1\sec\theta$$

whence

$$\frac{1}{\rho}+\frac{1}{\rho_1}=\left(\frac{1}{r}+\frac{1}{r_1}\right)\cos\theta$$

In a circular bore $r=r_1$ and $\rho=\rho_1$, whence

$$\frac{1}{\rho}+\frac{1}{\rho_1}=\frac{4i}{r^2+i^2}=\frac{2\cos\theta}{r}$$

Therefore for narrow elliptical bores

$$h=\frac{H}{2s}\left(\frac{1}{\rho}+\frac{1}{\rho_1}\right)$$

and for circular sections

$$h=\frac{H}{s}\cdot\frac{\cos\theta}{r}=\frac{H}{s}\cdot\frac{2i}{r^2+i^2}=a^2$$

The approximate values of H, s, and θ being known, the value of h may be computed: θ is obtained by direct measures. H may be determined, as by Poseuille, by observing directly the pressure required to force mercury through a capillary tube, for which case θ becomes 0, and $H=rhs=rq$, q being expressed in millimeters of mercury. The factor $\frac{H}{s}$ may be termed the capillary constant and is usually represented by the symbol a^2. For this constant Bravais has used the value $a^2=\frac{H}{s}=6.528$ in computing his tables, which give the capillary correction for observed values of θ. Delcros has used the value $\frac{H}{s}=6.5278$ in his table, which has been republished in the Smithsonian Tables, and which gives the correction when r and i are known. For a mercurial thermometer whose bore is 0.10 millimeter in diameter, and for a a meniscus whose height is $i=0.05^{mm}$, there results $h=6.528\times20=130.56^{mm}$, or the downward pressure due to capillarity is about one-sixth of an atmosphere.

The pressure thus computed as experienced by the rising thread of mercury is to be added to the weight of the vertical mercurial column to obtain the total internal pressure acting on the bulb. To the formula of article 23, therefore, must be appended an additional term containing a correction for capillary pressure. This term will be simply

$$h_{i}\beta_{i}=h(_{i}\beta_{e}+0.000154)$$

In an extreme case this total effect has been found to amount to 0.15° C.; it always tends to diminish the readings. When the capillary column is rising and the meniscus full this pressure on the bulb is a maximum, and for good tubes is nearly uniform, and the correction to the reading is additive; but when the column is falling the meniscus flattens and the capillary pressure diminishes. When all temperatures have been uniformly observed with rising columns the results are free from both total and differential capillary effects.

If the frictional resistance of the tube to the flow of mercury is at any point equal to the hydrostatic minus the total capillary pressure, a falling mercurial thread, will break at that point, and the observations must stop unless the mercury can be driven down by a blow from the outside. Such a break is intentionally effected in various forms of self-registering or mercurial maximum thermometers, and is liable to occur when not desired in ordinary thermometers.

The molecular condition of the surface of the bore is subject to considerable change, depending upon its own temperature and also upon the deposit of mercurial oxides, and partly upon the supposed disengagement by heat of gases adhering to its sides. To obviate this latter as far as possible pure mercury should be used, and the tube should be heated before filling sufficiently to expel the adhering air, or, the filling may be conducted in a vacuum, as in Wild's process for barometers.

Corrections for the effect of variable capillary pressure are to be applied also to the readings of the manometer and the barometer used in the boiling apparatus, and to those used with the air thermometer.

A direct determination of the differential correction for capillary pressure with rising and falling columns can be made at any point of the scale in the following way:

Let any *stationary temperature* be determined with the mercury column falling and again with the column rising; the difference in the two readings is approximately twice the correction. It is evident that when in a bath of variable temperature the thermometer changes from a rising to a falling column; an actual change of temperature in the bulb to the extent of twice the differential capillary correction will have taken place before the column begins to fall, owing to the fact that the expansion of the bulb, due to the change in pressure, is balanced by the expansion in mercury, due to change in temperature. This failure to indicate the change of temperature is similar to the dead motion of a micrometer screw.

Guillaume has determined the rate of change in temperature of the bath necessary, in order to cause the meniscus to change from least to maximum convexity; for his standard mercurial it was about 0.005° C. per minute; he also found that in determining the boiling-point with rising or full and falling or flat meniscus a difference of 0.04° C. existed (see Travaux et Mémoires, Tome V, 1886).

Pernet (Trav. et Mém., IV) states that for tubes of 0.03^{mm} diameter the "dead space" or lost record of temperature may amount to 0.07° C.

In order to avoid this source of error thermometer tubes should be of as large bore as possible. The use of a narrow flat bore should be avoided in a normal thermometer, as its meniscus is apt to have an irregular curvature.

25. CORRECTION FOR SLUGGISHNESS.

The thermometer gives the temperature of the outside surface of its bulb only when the whole mass of the thermometer has a uniform temperature. Owing to the slow conduction of heat through glass and mercury and the resistance to the ascent of the mercury in the stem of the thermometer, as well as to the fact that conduction proceeds more slowly in proportion as the inner temperature of the thermometer approaches that of its surface and the next outer layer of substance, it follows that when the initial temperature of the thermometer is different from that of the liquid in which it is immersed an appreciable time is required to attain complete equality between the thermometer and the adjacent layer of particles. If the temperature of this layer varies with the time, then the surface of the thermometer is continually endeavoring to attain the same temperature, but lags always behind it. The whole retardation due to internal and external causes is called the sluggishness or want of sensitiveness of the thermometer.

If by the rapid circulation of external fluid the external surface of the thermometer is kept at a constant temperature, to which the interior soon attains, then the retardation is due to sluggishness proper as a purely instrumental defect, which is measured by the time required by the whole interior mass of the thermometer to attain a temperature uniform with that of its external surface. This time depends principally on the interior conductivity of the thermometric substance. But if the particles of a liquid surrounding the thermometer be supposed absolutely quiet the time required by the thermometer to attain a temperature uniform with them will differ from that required when the liquid is in motion, and the temperature which it attains will also be different by an amount depending not only on the sluggishness proper of the thermometer but also on the conduction and convection of heat by the liquid.

Three cases may therefore be considered: (1) That in which the surface of the thermometer is kept at a uniform temperature, usually by rapid convection; (2) that in which the surface has a varying temperature, due to the fact that the convection is not sufficient to keep the surface at a uniform steady temperature; (3) that in which the surface has a varying temperature, due to the fact that convection is continually bringing forward particles of liquid of different temperature.

In the first case any difference between the temperature as read off and the surface of the bulb is due to internal sluggishness proper. This is essentially the case of a thermometer dipped into a bath of water which is rapidly stirred. The second case is exemplified when the surface temperature of the thermometer is different from that of a mass of still air in which it is suddenly placed. By the warming or cooling of the adjacent layer of air gentle vertical convection currents arise, which gradually diminish the difference of temperature between the surface and the surrounding air, but are not at any time adequate to keep the surface at the uniform steady temperature of the mass of air. The third case is that of a thermometer exposed, as in ordinary meteorological observations to air currents of varying temperatures.

(*a*) In the first case the laws of the conduction of heat, if known with perfect accuracy, would give directly the time required by a solid bulb to attain throughout a temperature uniform with that of the surface; but the computation has not been effected for liquid bulbs, within which convection currents may exist; to such convection currents is attributed the observed greater sensitiveness of thermometer bulbs over solids of the same dimensions. Cylindrical bulbs, notwithstanding their thick glass walls and greater mass of mercury, are decidedly more sensitive than spherical bulbs. This is to be attributed both to their shape and to the increased facility with which convection currents are set up. The combined effect of interior conduction and convection, when the exterior interchange of heat between the outer surface of the thermometer and the adjacent water is supposed to be so perfect as to maintain the surface of the bulb at uniform temperature, may be represented as follows:

Let u be the excess of temperature at the time θ, let u_0 be the excess when $\theta=0$, and let e be the Napierian base, then $u=u_0e^{-k_1\theta}$. The coefficient k_1 represents the total internal sluggishness, and is to be determined by observing the rate of cooling of the thermometer when transferred from a temperature $t_0=T+u_0$ to a very rapidly stirred bath of water at a temperature T.

(*b*) In the second case the internal sluggishness proper is combined with convection currents that must vary with the roughness of the bulb, with the mobility of the air, and especially with the difference of temperature between the bulb and the air. An approximate experimental investigation of this case, where the temperature of the surface is varying under self-induced convection, was made by Dr. Gyldèn and myself in 1865, at Poulkova, in connection with questions in meteorology and atmospheric refraction (see Bulletin Imperial Acad., St. Petersburg, X, page 462, 1866). The thermometers used by Mr. Hammon in his balloon observations for the Signal Service in 1885 were also carefully investigated in this respect.

The features of this case are presented in the following analysis:

The air temperature T is a function of the time θ, so that we may place $T=f(\theta)$.

The temperature shown by the thermometer may be expressed as $t=T+x$, and for small bodies Fourier's laws of heat give for small values of x

$$\frac{dx}{d\theta}=-kx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where k is a coefficient for each thermometer increasing with its sensitiveness, and can be called the coefficient of sensitiveness.

In order to find the value of k experiments are made at a constant air temperature, for which case $\frac{dT}{d\theta}=0$ and the integration of the preceding equation gives

$$x=\Delta e^{-k\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Let v_0 and v be the observed temperatures at the time 0 and θ, and V the constant temperature of the air, the preceding equation becomes

$$v-V=(v_0-V)e^{-k\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If, now, a thermometer is warmed up to some high temperature and allowed to cool freely, and its readings v_0, v_1, v_2 at the times 0, θ_1, θ_2 be observed, we shall have

$$v_0-V=(v_0-V)e^{-0} \qquad v_1-V=(v_0-V)e^{-k\theta_1} \qquad v_2-V=(v_0-V)e^{-k\theta_2}$$

whence, by subtraction,

$$v_0-v_1=(v_0-V)(1-e^{-k\theta_1}) \qquad v_0-v_2=(v_0-V)(1-e^{-k\theta_2})$$

Dividing these we obtain

$$\frac{1-e^{-k\theta_1}}{1-e^{-k\theta_2}}=\frac{v_1-v_0}{v_2-v_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If we do not know the temperature of the air, which has been assumed to be constant, the value of k must be found from this equation, either by successive approximations or by development in a series like that given by Gyldèn; but ordinarily the constant temperature V can be determined with sufficient approximation by direct observation with an additional sensitive thermometer, in which case k can be found by means of two observations, v_0 v_1, at the times θ_0 and θ_1, from the equation

$$k = -\frac{1}{\theta_1} \frac{\log (v_1 - V) - \log (v_0 - V)}{\text{mod.}} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Owing to a variety of circumstances the sensitiveness may be different for rising and falling temperatures. The following values of k, determined for five thermometers in air at constant temperature, assume one minute as the unit of time:

Thermometer.	k for rising temperature.	k for falling temperature.
Girgensohn, No. 345....	0.221	0.255
No. 333....	0.114	0.249
No. 406....	0.210	0.242
Brauer, No. 235........	0.117	0.135
Anonymous...........	0.720	0.923

The rate of cooling computed with the value of k thus determined, or the outstanding error at the end of a given time, is based upon the assumption that no wind is blowing past the thermometer, except the current produced by its own excess of temperature; these results, therefore, apply to the open air during calms. In such cases the effect of sluggishness upon the air temperature given by the thermometer becomes appreciable, and abundance of time must be given for the thermometer to attain the surrounding temperature; if this is not done the correction is given by the above formulæ. For example, if a thermometer whose temperature is v_0 is exposed at the time θ_0 to the air whose unknown temperature is V, and if at the time θ the observed thermometer temperature is v, then the true air temperature will be

$$V = v - (v_0 - V)e^{-k\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In computing the last term we may assume V approximately equal to v, and if necessary make a second approximation.

(c) In the third case the temperature of the air is also changing. In this case the preceding equation (2) becomes

$$\frac{dx}{d\theta} = -kx - \frac{dT}{d\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

by substituting in the right-hand member the approximate value of x, already found, and then integrating, we get

$$x = e^{-k\theta}\left(\Delta - \int \frac{dT}{d\theta} e^{+k\theta} d\theta\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where the right-hand member expresses the effect of the change in the temperature of the air during the interval θ.

An approximate value for this change $\left(\frac{dT}{d\theta}\right)$ can easily be obtained by assuming an arbitrary value such as might occur in extreme cases.

But the effect of the average value of the maximum hourly change in air temperature is by Gyldèn computed in the following way: The mean diurnal air temperature may be expressed by the formula

$$T=a_0+a_1 \sin (A_1+n\theta)+a_2 \sin (A_2+2n\theta) \quad . \quad . \quad . \quad (10)$$

where θ is counted from noon and n depends on the unit of time; if this is one minute, then

$$n=\frac{\pi}{12\times 60}=0.0043632$$

Substituting this value of T in equation (9) and developing the integral in a series of terms, of which Gyldèn retains only the first, there results

$$x=\Delta_1 e^{-k\theta}-nma_1 \cos (A_1-M+n\theta) \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where

$$\Delta_1=\Delta+nma_1 \cos (A_1-M)$$

and m and M are determined by the relations

$$m=\frac{1}{\sqrt{k^2+n^2}} \qquad \tan M=\frac{n}{k}$$

The first term of this expression for x becomes zero if the initial temperature of the thermometer is the same as that of the air, and in any case becomes inappreciable when the time is very long. The second term shows how the error x will vary with the diurnal change in air temperature; the maximum error will be the coefficient nma_1, the thermometer reading too low when the temperature is rising and too high when the temperature is falling.

As an example of the application of this formula, take the thermometers for which the coefficient k has already been given; assume that in the expression for the diurnal change of air temperature a is 5° C. and that m is given by the preceding formula. Then the resulting maximum error for each thermometer will be

Thermometer.	Maximum corrections, nma_1	
	For rising temperatures.	For falling temperatures.
	°	°
Girgensohn, No. 345....	+0.10	−0.09
Girgensohn, No. 333....	+0.19	−0.09
Girgensohn, No. 406....	+0.11	−0.09
Brauer, No. 235	+0.19	−0.16
Anonymous	+0.03	−0.08

These appreciable corrections, applicable to calm or quiet air, are greatly diminished when a light breeze springs up; but they enforce the importance of using sensitive thermometers, and of maintaining by artificial means a uniform and rapid circulation of air.

In case the temperature must be determined by a single stationary thermometer whose coefficient of sluggishness is unknown, the effect of sluggishness may be allowed for by the following method, proposed in 1865 by Volpicelli, in the Paris Comptes Rendus, also in the same year independently practiced by me at Poulkova, and by Dufour in Switzerland, and theoretically developed by Gyldèn in 1866, all apparently unaware that Fourier had also proposed the same plan.

This method consists in determining the temperature from three direct observations, t_1, t_2, t_3, at the times θ_1, θ_2, θ_3, at equal intervals. Let the true temperature at the time θ_2 be T, then will three equations, like equation No. 4, give by elimination

$$\frac{t_3-T}{t_2-T}=\frac{t_2-T}{t_1-T}$$

whence

$$T=t_2-\frac{(t_1-t_2)\,(t_2-t_3)}{(t_1-t_2)-(t_2-t_3)}$$

26. THERMAL AND ELASTIC REACTION, TEMPORARY ZERO, AND RESULTING TEMPERATURES.

It has already been stated that normal mercurial thermometers show systematic differences from the air thermometer, and exhibit systematic changes in their own readings with time and temperature. These irregularities have their origin in the glass bulb, and are especially noticeable in the mercurial thermometer, because the irregularities in the expansion of the glass bear a large ratio to the expansion of the mercury. They exist to a less degree in the spirit thermometer, and are inappreciable in the air thermometer.

The fact that the zero point is subject to a slow change with time was first clearly stated by Bellani in 1822. Bellani also discovered that the zero point changes with the treatment of the thermometer; but this was not minutely investigated until thirty years later by Faraday, Regnault, Egen, Sheepshanks and other physicists. The amount of these changes depends upon the nature of the glass of which the bulb is made, varying from 0.05° C. for the best glass to 0.8° C. for the ordinary glass, of which otherwise accurate thermometers have generally been made. In all cases, however, the effect of these changes may be practically eliminated by a proper treatment of the thermometer. To this end rules have been formulated by a number of investigators, with different degrees of success. The following method has come to be adopted as the most satisfactory for determining any temperature below the boiling-point:

(*a*) Irrespective of the previous treatment of the thermometer read off the temperature, r_t, of the bath.

(*b*) Immediately thereafter determine the temporary freezing-point z_r. The difference between these two readings, corrected for pressure, capillarity, calibration, inclination, etc., is to be considered as a distance on the scale, which needs, first, to be converted into its equivalent in centigrade degrees; and, second, to be added to the reading of the depressed freezing-point z_{100}, which is the adopted zero of the centigrade scale.

(c) The equivalent in centigrade degrees is found by determining at once the reading of the boiling-point r_{100}, and then of the depressed freezing-point z_{100} immediately thereafter. The difference between these two readings gives the scale value of one degree, namely, $\frac{r_{100}-z_{100}}{100}$, consequently the correct temperature corresponding to r_t is

$$t=(r_t-z_r)\left(\frac{r_{100}-z_{100}}{100}\right)+z_{100}$$

The great labor involved in two determinations of the freezing-point and one of the boiling-point whenever a temperature is to be measured, the fact that frequent boiling changes the fundamental distance, and the necessity of providing for cases which frequently occur where a mercurial thermometer can not be disturbed during a long investigation, has stimulated the study of the gradual changes that take place in the position of the zero point, and the making of glass for bulbs whose molecular structure is such that the changes become as small as practicable.

As before stated, it is found that after heating a thermometer, and especially when it is first filled by the maker, a temporary enlargement of the bulb (and of the stem), followed by a gradual contraction, takes place. The contraction is at first rapid and afterwards slow, and finally in some bulbs ceases altogether, while in many it continues indefinitely at a variable rate. For ordinary glass the changes become inappreciable after two years, by which time the zero point may be 0.2° or 0.3° C. above the point observed when the bulb was first filled. By again heating the thermometer at any time the bulb becomes again temporarily enlarged, the zero point is lowered by an initial depression, whose amount depends on the heating, and the slow process of contraction to its normal volume again begins.

If the initial depressions of the zero point, corresponding to all temperatures of the thermometer, and the rate of change of any depression with time be both known the approximate value of z_r, in the preceding equation may be computed, and thus the first determination of the freezing-point be avoided. The best formula for the computation of the initial depression produced by such rise of temperature is given by Pernet:

Let z_0 represent the zero point determined after a long period of repose by an exposure in ice; let z_{100} be the depressed zero point corresponding to the boiling-point; then for the temperature t the corresponding depressed zero point z_t will be given by the equation

$$z_t=z_0-(z_0-z_{100})\frac{t^2}{(100)^2}$$

This is the quantity called z_r in the previous equation, and Pernet's formula, therefore, enables it to be determined from observations of z_0.

and z_{100}, made at any convenient time, the only precaution being that the temperature t shall be rather higher than any to which it has been exposed during the previous period of rest, whereby the secular effect at lower temperatures is annulled.

The method of observing and computing temperatures given in the preceding paragraphs gives results agreeing always within a tenth of a degree centigrade with the temperatures of Regnault; therefore his work on fundamental data can be applied without change, except in minor refinements, to current observations. This agreement is due essentially to the fact that the glass of Regnault's bulbs was of excellent quality for thermometric purposes and for good determinations of zero points. Manifestly, however, the above method for eliminating the changes in the zero point, implying as it does the possession of laboratory facilities, is in general inapplicable to the work of the ordinary meteorological observer, whose records must therefore remain uncorrected for the variations of the volume of the bulb. Therefore, by reason of the annual and diurnal variations in temperature, thermometers exposed in the air have a periodical as well as a secular error in their readings. After exposure to the cold of winter they give the succeeding spring and summer temperatures somewhat too high, until the contraction of the bulb has been overcome by the maximum summer temperature, after which they give too low temperatures until the greatest cold of the winter has been reached. The seasonal variation may amount to 0.05° C., or, in extreme cases, 0.10° C. for average glass. The most practicable remedy for this error must be the discovery of some glass for which changes of this kind are inappreciable.

During the past few years many varieties of glass have been made and tested at the laboratory of glass technology at Jena, and the laws of the changes due to heat and those due to elastic pressure have been investigated. G. Wiedemann (Annalen der Physik, June, 1886) has given a summary of the results of experiments upon the mechanical or elastic properties of glass, and shows that the elastic reaction is entirely parallel and similar to the thermal, and that in general the formula applicable to elastic reactions may be applied to thermometers.

Let c be any initial deformation; x the outstanding deformation or departure from perfect equilibrium at the time t; then we have

$$x = Ce^{-at^m}$$

where e is the base of the Napierian system of logarithms; a, m, and c are constants that vary with the chemical composition of the glass. For ordinary good glass a is 1; but for the poorest, 0.7; m varies from 0.4 to 0.6; c varies from 0.17 to 0.005.

If by elastic reaction at the time t we understand the ratio of the outstanding distance (x) from equilibrium to the initial departure from equilibrium, then we may consider the following points established:

1. The elastic reaction depends on the amount of preceding flexure and the dimensions of the body.

2. The elastic reaction diminishes with higher temperatures.

3. The thermal reaction is large or small according as the elastic reaction is larger or smaller.

4. The elastic and thermal reactions both depend on the chemical constitution of the glass. A pure potash glass has a larger reaction than a pure soda glass, and a mixed glass of equal parts of soda and potash has a much larger and much slower elastic reaction than a pure soda glass.

5. Probably elastic reaction for various kinds of deformation, such as bending, twisting, stretching, or compressing, is the same, other conditions being identical.

The results of the studies of Wiedemann, Pernet, Schott, and Abbé of Jena, based on a large number of specimens of glass made in the technical laboratory at Jena, have established the following as the best composition of glass for the manufacture of thermometers: *Normal thermometer glass of Jena: Quartz*, 67.5; *soda*, 14.0; *oxide of zinc*, 7.0; *lime*, 7.0; *clay*, 2.5; *borax*, 2.0 *per cent.*

This glass shows a depression of the freezing-point of only 0.05° C. after the thermometer has been exposed to the temperature of boiling water; it has been rapidly adopted by the best German thermometer makers, and those desiring instruments that will not change the zero point more than 0.01° in a long interval of time must provide themselves with thermometers made of this or some equivalent glass.

The glass used by Messrs. J. and H. J. Green in the manufacture of their mercurial thermometers has for a long time been made especially for them, and is of considerable stability.

That made by the French glass makers and used by Tonnelot gives a depression of 0.08°; as to that used by Regnault, we know only that he rejected any thermometer in which the depression was greater than 0.1°. But most of the French makers, and also the English, use a glass containing much lead that gives initial depressions of 0.2° or more.

The preceding remarks relative to the bulb apply also to the glass stem. Thus Pernet (Trav. et Mém., IV) found changes of 0.01° in the calibration corrections of thermometers that had been frequently used at temperatures not exceeding 50° C., and the upper half of whose stems had been kept at ordinary temperatures. He also found that calibrated tubes to which a new bulb had been afterwards applied and fitted had entirely changed their calibration corrections by as much as 0.1° C.

CHAPTER IV.

EXPOSURE OF THERMOMETERS.

27. THE PROBLEM STATED.

The primary use of the thermometer in meteorology is for determining the temperature of the free air at a given place at a given moment of time, and to this end the circumstances of its exposure are of equal importance with the accuracy of the instrument. The bulb of a thermometer exposed in the air receives heat by radiation from surrounding objects in addition to that conducted from contiguous air and the stem itself. Its temperature is therefore dependent on the reflective, absorptive, and emissive powers of the surface of the bulb and the conduction from the stem.

The emission of heat from the bulb takes place by radiation and by external conduction to the adjacent layer of air. The latter combines with the effect of air currents moving past the bulb, giving rise to convection, which is generally of more importance than external conduction.

The consideration of these influences and their effects must lead to a standard method of determining air temperature to which all others may be considered as approximations.

General relations.—A thermometer exposed in the free air may be considered to indicate a temperature that represents a balance between the influences of convection, radiation, and conduction of heat.

The latter two are not negligible, but when the wind is blowing strongly past the thermometer the resulting convection brings the thermometer to a temperature closely approximating that of the air. The general analysis of the combined effect of radiation, absorption, and convection, as applied to thermometers, is due to Fourier, and has recently been given more fully by Prof. William Ferrel (Temperature of the Atmosphere and Earth's Surface; Signal Service Professional Papers, No. XIII). [Two memoirs by Wild, in the tenth volume of his Repertorium, were received after this present work had gone to press.]

When a thermometer attains a stationary temperature there is an equilibrium between the amounts of heat acquired and imparted in a unit of time by virtue of the surrounding influences to which it is subjected.

Assuming such a condition, let H be the total amount of heat radiated to the thermometer, and r the coefficient of absorption; then rH will be the quantity absorbed in a unit of time.

If H_r, H_c, H_v be the quantities of heat lost from the thermometer by radiation, conduction, and convection, respectively, the condition of equilibrium is expressed by the equation

$$rH = H_r + H_c + H_v$$

The right-hand member may be expressed in terms of the temperature by introducing the elementary laws of radiation and conduction.

Under the assumption that the difference of temperature between the thermometer and the air is small, let t_1 be the temperature of the thermometer; t_a that of the air; B the coefficient of radiation; $B' =$ the coefficient of conduction through the air; $B'' =$ the coefficient of convection for a unit velocity of the current; $v =$ the velocity of the current.

The above equation may now be written thus

$$rH = (Br + B' + B''v)\,(t_1 - t_a)$$

from which $(t_1 - t_a)$ may be found whenever H and the coefficients are known.

The true temperature of the air around the thermometer can not be determined unless this equation is solved, or unless the observations are so arranged as to eliminate or correct for these influences.

Among the methods adopted for obtaining correct air temperatures from direct observation the most important are: (1) Methods based on the use of a shelter which cuts off noxious radiation; (2) methods that measure and correct for the effect of radiation; (3) methods of counteracting radiation. These are treated in the following sections:

28. STATIONARY OPEN SHELTER.

The most evident source of error, namely, the direct radiation of the sun was obviated in the earliest meteorological use of the thermometer by hanging it on the shady side of some object, and by changing its position when necessary. In tropical countries the thermometer was suspended beneath a roof, preferably of thatch, open on all sides to the wind. By this shelter the sun's rays are shut off, except possibly for a short time in the morning and evening, and radiation to the sky is largely prevented. Early in the present century special thermometer shelters were introduced, which, under various forms, such as the patterns Glaisher, Stevenson, Stow, Renou, Wild, and the Signal Service, are now in general use. For many years Flaugergues at Viviens (see Zach, Correspondance Astr., 1819, II, p. 434) used a single vertical cylinder of two sheets of silver paper, inclosing a thin layer of non-conducting substance; the thermometer bulb within the open cylinder was sufficiently exposed without experiencing the full force of the wind.

The ideal thermometer shelter, besides screening from rain and injury, prevents the direct radiation of heat between the thermometer and all other objects, except the shelter, without interfering with the free motion of the air; but all forms necessarily fall short of this ideal, and some to a large extent. If, as in some cases, a portion of the screen is left open to allow of the free circulation of air, then injurious radiations are liable to occur. If, on the other hand, these radiations are cut off by a continuous screen, then the exchange of the air within and without the shelter is entirely stopped or reduced to a very small portion of the wind's velocity, and the mass of air within the shelter may come to possess a different temperature from that of the free air. With the use of shelters made of open louver work, such as the present Signal Service pattern, this source of error is scarcely appreciable in any wind above 5 miles per hour. In such winds the slight errors introduced by such a screen arise primarily from the temperature of its own mass, by reason of which the air within a screen of wooden louvers is usually warmer than the surrounding air during sunshine, and during clear nights up to the time when the louvers have cooled by radiation down to or even slightly below the air temperature. This influence of the temperature of the screen upon the thermometer is reduced by the convection of the air, that is, forced by the wind into the screen, and the greater this quantity the less will be the influence of the screen temperature.

It is evident, therefore, that a perfect screen, namely, one that gives the air temperature at any time correct to within some assumed limit, should fulfill the following three conditions, respectively, formulated in 1865 by myself at Poulkova, in 1884 by Professor Wild at St. Petersburg, and in 1885 by Prof. H. A. Hazen at Washington:

1. The screened thermometers should not impart heat to nor receive heat from any object that has a temperature differing by some prescribed limit from that of the external air.

2. When fresh air is drawn into the screen from without the thermometer should show no change in temperature greater than the assumed limit of accuracy.

3. The screened thermometer should agree with the sling thermometer shaded from noxious radiations (especially to sky and sunshine) and whirled in the free air near the shelter.

The degree to which these conditions are attainable is shown in the following paragraphs.

In order to test the quantity of direct or reflected solar radiation that may enter a shelter the following method was adopted by me at Poulkova in 1865–'66, and has recently been used by Koeppen. Two thermometers, whose bulbs have very different radiating and absorbing powers with respect to solar radiation, are placed in the shelter. When the thermometers agree it is a proof that the solar radiation penetrating the shelter is not of sufficient amount to cause either thermometer to differ from the temperature due to conduction from the air flowing through the shelter. If the thermometers do differ the correction due to the effect of this radiation may be obtained by elimination from the two following equations:

$$r_1H=B_1r_1(t_1-t_a)+B_1'(t_1-t_a)+B_1''v(t_1-t_a)$$

$$r_2H=B_2r_2(t_2-t_a)+B_2'(t_2-t_a)+B_2''v(t_2-t_a)$$

whence

$$t_2-t_a=(t_1-t_a)\frac{B_1+\frac{B_1'+B_1''v}{r_1}}{B_2+\frac{B_2'+B_2''v}{r_2}}=C(t_1-t_a)$$

$$t_a-t_2=\frac{C}{1-C}(t_2-t_1)$$

In the course of a year's observations at Poulkova, the largest differences (t_2-t_1) between plain glass and blackened bulbs did not exceed 0.6° C. within a screen of one cubic meter made of louvers of oiled paper. The correction thus obtained must be carefully understood to be that due to the special radiations that penetrate the shelter, for which the thermometers have the very different coefficients B_1 and B_2; of course radiations whose wave lengths are such that they affect both bulbs equally can not be thus determined, and the method is strictly applicable only to the detection of any direct or reflected solar radiation that may penetrate the shelter. Fourier considered that the value of $\frac{C}{1-C}$ would be about ¼ for solar rays and surfaces of glass and of lamp-black, but this mistake was corrected by a criticism of Plana. All ordinary dark heat, *i. e.*, radiations of long wave length such as come from substances at low temperatures, affects the ordinary plain and blackened bulbs and the walls of the shelter so nearly alike that such heat can not be detected in this way.

The test of a shelter for the effect of all heat radiations whatever is best made by means of an artificial current of free air, if it can be assumed that the temperature of the air shall not be sensibly changed in the process. Thus Professor Wild attaches to his shelter (a wooden louver work inclosing a sheet-iron cylinder) a ventilating apparatus for renewing the air at will. He finds that the change of temperature produced in the shelter by the action of the ventilator is rarely more than 0.1° C., and hence concludes that his screen is reliable to that limit. Instead of currents flowing into the shelter two thermometers may be exposed to currents within and without, respectively; thus, experiments (Hazen, H. A., Thermometer Exposure, Signal Service Professional Papers, No. XVIII) made at Washington in 1884-'85 show that the temperature within the wooden louver shelter adopted by the Signal Service agrees to within 0.3° F. with a thermometer whirled outside during the winter half of the year without correcting the latter for any residual effect of radiation to the earth and sky.

During the day time the sun heats one side of a shelter and the air in contact with it far above the local air temperature desired by the meteorologist. The resulting distribution of temperature within the shelter will depend on the direction and strength of the wind. The largest discrepancies between the various parts of the shelter may be expected to occur in calm weather, and have been studied by H. A. Hazen in order to ascertain what size of shelter is needed to secure a good location for the still thermometer fixed within. The result was the adoption of a cube $3\frac{1}{2}$ feet long, 3 feet high, and 3 feet deep, surmounted by a double sloping roof (see Professional Papers, No. XVIII, page 27).

At Signal Service stations provision is now made for whirling the thermometers rapidly in a circle of about 8 inches radius in the central space of the "Hazen" shelter. This does not materially affect the interchange of air within and without the shelter, but produces a thorough mixture of that within, so that the thermometers give the average temperature of the whole. The rapid whirling undoubtedly tends to distort the bulb by centrifugal force, and may alter its capacity so as to cause a temporary change in the reading of the thermometers, but this effect is immediately counteracted by the elasticity of the glass when the whirling ceases.

The temperatures given by these thermometers whirled within the Signal Service shelters do not differ more than 0.2° F. from the temperatures given by the sling thermometers whirled in the shade in the free air near by (see Professional Papers, No. XVIII, page 32).

29. STATIONARY CLOSED SHELTERS.

In this class of shelters the air does not have direct access to the thermometer. Arago's method of thermometer exposure is the earliest example of this kind.

He carried out a suggestion of Fourier's that a silver thimble, if it be so perfectly polished as to reflect *all* radiant heat, must attain by conduction the temperature of the air in contact with it, and therefore a thermometer within it must also have that temperature. The closed silver-thimble shelter was adopted by Regnault in his psychrometer.

The assumption, however, is at fault, because no substance reflects all the rays; the absorption for even the best polished silver thimble in the sunlight amounts to 7 or 8 per cent. of the total intercepted radiation. Therefore, when exposed in still air, the excess of its temperature above that of the surrounding air may amount to several degrees. This shelter gives good results only when there is a rapid convection current.

A second form of closed shelter is due to Joule. The temperature of a mass of well-stirred water is brought to an equality with the temperature of the air, and at the moment when no convection currents exist in the adjacent air the temperature of the water is then observed by thermometers immersed therein; by this means the thermometers are entirely unaffected by any radiation, but the method has little or no direct meteorological application.

30. METHOD OF DETERMINING AIR TEMPERATURE BY MEASURING AND CORRECTING FOR THE EFFECT OF CONDUCTION AND RADIATION.

Instead of attempting by the use of a shelter to protect the thermometer from all noxious radiation, Liais (Comptes Rendus, Paris, XXXIII, 1851, 207) proposed to measure it, and correct for its effect in the following manner:

Let three thermometers of similar size, shape, and sensitiveness be exposed side by side to a uniform radiation to the earth and sky, but screened from the direct and from any intense reflected solar rays. Let f, f', f'' be the emissive powers of the three bulbs, supposed to be as different as possible from each other in this respect. Let A be the total quantity of radiant heat that reaches the bulbs in a unit of time; then, because of the equality of the coefficients of absorption and emission, the total quantity of radiant heat absorbed by the bulbs will be, respectively, Af, Af', and Af''.

Let t, $t+a$, $t+b$ be the resulting excess of the thermometric temperatures above the air temperatures.

mft, $mf'(t+a)$, $mf''(t+b)$ the quantities of heat lost by radiation by reason of this excess of temperature.

nt, $n(t+a)$, $n(t+b)$ the quantities of heat lost by the conduction and convection of the air.

When the thermometers indicate stationary temperatures the absorbed radiant heat will equal that lost by radiation, conduction, and convection, whence

$$Af=mft+nt$$

$$Af'=mf'(t+a)+n(t+a)$$

$$Af''=mf''(t+b)+n(t+b)$$

By elimination we obtain

$$t=\frac{abf(f''-f')}{bf''(f'-f)-af'(f''-f)}$$

which is the correction to be applied to the temperature of the first thermometer in order to get the true air temperature. This correction will be positive or negative, according as the surrounding objects exchanging radiation are cooler or warmer than the air.

A similar method, using two thermometers, was early proposed by Fourier and applied, but soon forgotten until independently revived and used by myself at Poulkova in 1865. The following presentation of this method is given by Professor Ferrel (Signal Service Professional Papers, XIII, 59-60):

Let t_a be the true air temperature.

t_1 and t_2, the temperatures read off from two thermometers exposed side by side, whose bulbs differ largely as to their coefficients of radiation and absorption.

H, the total amount of heat radiated to the thermometers.

a_1 and a_2, the respective absorbing powers of the two thermometers.

r_1 and r_2, the respective radiating powers of the two thermometers.

k, the rate of cooling by convection for unit velocity of the current, to be determined for each thermometer by observations at different velocities, after making allowance for internal sluggishness. For different surfaces the value of k changes little when the bulbs are of the same size, shape, and material; the same value may be assumed to hold good for each thermometer.

Finally, let v be the velocity of the convection current.

Assuming a continuous rapid convection current, there is no conduction proper, and the equations of thermal equilibrium are, respectively,

$$a_1H=r_1(t_1-t_a)+kv(t_1-t_a)$$

$$a_2H=r_2(t_2-t_a)+kv(t_2-t_a)$$

Eliminating H we obtain

$$t_a=t_2-C(t_1-t_2)$$

where

$$C=\frac{a_2(r_1+kv)}{a_1r_2-a_2r_1+(a_1-a_2)kv}$$

This expression shows that by increasing the ventilation the relative influence of the terms containing r diminishes when compared with the terms in kv; consequently for high velocities the terms in r may be neglected, and the value of C becomes

$$C=\frac{a_2}{a_1-a_2}$$

In the Ann. de Chimie, November, 1817, Fourier had proposed the use of a plain glass bulb and a blackened glass bulb and the computation of the air temperature by the formula $t_a=t_1-C(t_a-t_1)$. He estimated that C would be about $\frac{1}{4}$, but Plana (see Zach, Astron. Corresp., 1818, I, p. 545) showed that the absorption of glass and lamp-black are too nearly alike to allow this method to give good results. [This important point has been dwelt on by Wild in his Memoir of 1887, received too late to be more fully quoted here.]

31. METHODS OF ANNULLING THE EFFECTS OF RADIATION.

The use of ordinary shelters gives the temperature of the air within the shelter, without assurance that this air has the same temperature as that of the free atmosphere outside. To remedy this defect the following methods have been used:

(*a*) *Method by ventilation.*—The Italian physicist Belli, in 1837, published the description of the psychrometer used by him, in which a ventilator is used to draw in the outside air and make it flow rapidly over the thermometers contained within the air duct. In this apparatus the thermometer is stationary, its changes and final stationary temperature can be easily read, and the apparatus for producing the flow of air may be simply the inflation of the lungs. But there is a slight doubt as to whether the air may not change its temperature in its flow through the tube before reaching the thermometer. This liability to a change in temperature of the air itself increases with the velocity of the indraught, but by a proper manipulation the error may be reduced to a small quantity, and the method has great merit. In general, however, it is easier to use the thermometer in the free open air and to adopt some method of eliminating the errors of exposure. [Since the above was written Dr. Assmann has described a slight modification of Belli's method for dry and wet thermometers that is adapted to give fairly good results with very little trouble.]

(*b*) *Method by whirling.*—The idea of whirling the thermometer as a method of obtaining the air temperature is said to have been suggested by Arago, but the whirled thermometer was first used for this purpose in 1836 by A. Bravais (Comptes Rendus, Paris, XXXVIII, 1077, 1854). It had previously been used, however, to obtain the wet-bulb temperature by Saussure (Voyages dans L'Alps, T. VII, Chap. VIII).

This method consists simply in whirling rapidly in the free air a thermometer attached to a string or swivel, held in the hand; for constant use in a fixed location the thermometer is attached to the Arago whirling table.

The usage of Bravais and the French observers has been to whirl the thermometer in a shaded spot, so that radiations only from terrestrial objects, clouds, and sky can affect it.

It is evident that by a rapid whirling the effect of convection must bring the temperature of the surface of the thermometer into close accordance with the temperature of the air, precisely as when the water is rapidly stirred in a thermometric bath.

The general relation between temperature and ventilation for the case of a small spherical body and small differences of temperature, *after the bulb has come to a stationary temperature*, t, has been expressed by Ferrel in the following formula:

$$t_1 - t_a = \frac{I}{0.0077\,B\mu^{t_1}} \cdot \frac{s}{\sigma} \cdot \frac{a}{r+kv}$$

where I is the intensity of the heat radiation that falls on the thermometer.

a, the coefficient of absorption of these radiations by the surface of the bulb.

r, the coefficient of radiation for the special surface and temperature of the bulb.

s, the area of a section of the bulb.

σ, the total effective radiating surface of the thermometer toward the source of the radiation I; this will be less than the total surface of the bulb in proportion as its radiations are reflected back to it by surrounding objects.

B, for black bulb thermometers; the amount of heat radiated at a temperature 0° C. in a unit of time from a unit of lamp-black surface whose thickness is sufficient to give the maximum radiation due to lamp-black. According to Dulong and Petit B=1.146 calories per minute per square centimeter; the value of a is approximately 0.80 for plain glass for direct solar rays and 0.10 for polished silver and gold and iridium surfaces.

μ the radiation constant; 1.0077, according to Dulong and Petit.

k the coefficient of external conduction for a unit veloc ty of air.

The formula shows that when the convection current is zero the difference between the thermometer and the air temperature may become quite large. If v=0 and the bulb is of such material that a=r (the case in a well-covered black bulb) the correction becomes

$$t_1 - t_a = \frac{Is}{.0077B\mu^{t_1}}\,\frac{s}{\sigma}$$

for ordinary plain glass bulbs. The ratio $\frac{a}{r}$ for solar rays is $\frac{0.16}{0.5}$=0.3.

The above correction varies directly as I, the intensity of radiant solar heat, direct or reflected, falling upon the bulb; for a thermometer swung in the shade, protected from radiation to space and surrounded by non-reflecting surfaces, experiment shows that the correction is inappreciable.

(c) *The whirled shelter.*—The protection of the thermometer from radiation and the production of a continuous current of fresh air were combined by Renou in his whirled shelter (Comptes Rendus, Paris, XL, 1855, p. 1083) and by Wild in his ventilated shelter, as introduced into the stations of the Russian meteorological service.

In Renou's method a small triple shelter of concentric thin metallic cylinders cuts off all exterior radiation. By rotating the shelter about an axis the air is forced to flow rapidly between the concentric cylinders and past the thermometer. For meteorological observations the shelter is mounted on a post in the open air.

Since the whirling of the shelter would have a tendency to rarefy the inclosed air, whence the inflowing air by expansion might be slightly cooled, Renou examined this possible source of error, and concluded that it is inappreciable for velocities ordinarily used.

This is substantially the method independently introduced by Dr. B. F. Craig, in 1867, into the practice of the observers of the Medical Department, U. S. Army; Dr. Craig's device consisted of a single thin brass box about 14 inches long, and having a section of 2 square inches, inclosing the thermometers (dry and wet). The box had openings at each end, so placed that when whirled by means of a swivel air is drawn in at the upper end of the thermometer and driven out past the bulb. The rapid convection brings the thin metallic box and the inclosed thermometer very quickly to the air temperature.

In Wild's method a fixed louver shelter has an attached ventilator that causes currents of fresh external air to circulate rapidly through the shelter when the wind fails to do so.

32. CORRECTION FOR THE FRICTION AND COMPRESSION OF THE AIR.

In determining the air temperature by a whirled thermometer there is introduced a possible source of error due to the relative movement of the air and the bulb. If this correction be appreciable its amount should be determined and applied to the observations. Manifestly the same source of error also arises when the wind or an artificial current blows past a fixed thermometer, for which case the correction will likewise be applicable. When the thermometer moves rapidly in the still air, or when a current of air blows past the thermometer, the temperature of the thermometer tends to be raised both by the friction on its surface and by the dynamic heating of the air due to its compression on the front or windward side. Experiments made by Joule and Thomson show that on the windward side of an obstacle temperatures are slightly raised by the stoppage of the wind, and the consequent conversion of a portion of its energy into heat; while on the leeward side the temperature is a little lower than on the windward side, due to the fact that the warmed air expands and cools slightly on reaching the rear. They found that a thermometer placed in the middle of the windward side of a wall, but sheltered from the wind itself, gave a higher temperature than when exposed to the full blast and isolated from the effect of the wall. This result shows that the heating of the air by compression, when blowing against a normal surface of considerable size, is greater than the effective heating produced by the friction and compression of air against the small surface of an isolated thermometer.

Bravais made experiments to determine the amount of any such heating of the whole thermometer, and states that it does not exceed 0.046° C. for glass or gilded bulbs whirled at a velocity of 10 meters per second.

The same subject was investigated very accurately by Joule and Thomson in 1856–'57 (Proceedings Royal Society, London, VIII, 1856–'57, X, 1860). They found that the thermal effect of impinging air depends upon the compression in front, the rarefaction in the rear, and the friction on the surface. Experiments to determine the amount of the thermal effect upon thermometers whirled at different velocities showed that it is approximately proportional to the square of the velocity. The following quantitative results are a sample of those given by them for ordinary glass bulbs:

Relative velocity of bulb and air.		Thermal effect.	
Miles per hour.	Meters per second.	Observed.	Computed.
		°C.	°C.
10	4.5	....	0.01
20	8.9	....	0.02
25	11.2	0.06	0.05
30	13.4	0.09	0.07
40	17.9	0.14	0.13
50	22.4	0.22	0.20
60	26.8	0.28	0.28
70	31.3	0.32	0.38
80	35.8	0.44	0.50

The last column contains the heating effect computed for different velocities upon the assumption that the observations may be represented by the expression 0.0000782 v^2, where v is given in miles per hour.

By altering the surface of the bulb so as to largely increase the frictional effect the total thermal effect was considerably augmented.

From these observations we may conclude, therefore, (1) that for velocities of whirling up to 10 meters per second the reading of the thermometer observed to the nearest tenth of a degree (centigrade) needs no correction for the whirling; (2) that a thermometer exposed to a high wind (over 30 miles per hour) needs a negative correction, whose amount is given in the preceding table; (3) that the thermometer should not be exposed and observed near an obstacle where, on account of dynamic heating or cooling, the temperature may not be that of the free air current.

33. CONCLUSION.

From the preceding analysis of the methods of observation proposed and in use it is evident that the temperature of the free air at any spot can be directly obtained only by taking care to avoid or allow for any injurious effects upon the temperature of the thermometer from its radiation into space, *i. e.*, to the upper air, its absorption of radiant heat, its friction with the air, its dynamic heating or cooling, and the changes of air temperature, all of which are matters additional to the instrumental corrections treated in Chapter III.

An examination of different methods shows that these various sources of error may be rendered inappreciable and the temperature of the free air at a given place and time obtained to the nearest tenth of a degree with an ordinary glass bulb thermometer, sheltered from direct or reflected solar radiation and well ventilated by whirling or by indraught, the relative motion of the air and the thermometer not necessarily exceeding 10 meters per second. Finally, the temperature of the air may be obtained even in the sunshine, as recommended by me (Bulletin of the Philosophical Society of Washington, March, 1883, and on earlier occasions), by rapidly whirling, simultaneously, a black bulb and a polished silvered or gilded bulb thermometer, and computing the temperature from the formula

$$t_{\text{air}} = t_{\text{bright}} - C\,(t_{\text{black}} - t_{\text{bright}})$$

where C is a factor depending on the coefficients of absorption, radiation, and convection for each thermometer for the special wave lengths that come into consideration. In both these methods, however, there must remain a slight error, due to radiations that are common to both thermometer bulbs in their interchange of heat with surrounding objects. [These conclusions are well illustrated and confirmed by the excellent latest work of Wild, already referred to.]

CHAPTER V.

MISCELLANEOUS FORMS OF THERMOMETER.

Numerous forms of thermometer have been introduced to accomplish special objects and to meet the demands of special investigations. Such of these as have been widely used in meteorology are here mentioned. In all these forms it is assumed that the instruments are kept carefully compared with the standard air or mercurial thermometer. The principles upon which these instruments are based and the nature of the errors to be guarded against in their use will be briefly stated.

34. THE ALCOHOL THERMOMETER.

Owing to its fluidity at low temperatures the alcohol thermometer has been relied on especially for temperatures below the freezing-point of mercury. Notwithstanding every effort to obtain accurate results by careful calibration of the tube, and by the study of the great variation in the coefficient of contraction of alcohol, it has been found that an equally serious source of error exists in the adhesion of the liquid to the sides of the glass bore, and no independent determination of the amount of the correction for adhesion seems possible; neither is it practicable to calibrate tubes when full of alcohol. The alcohol thermometer can not, therefore, be regarded as an independent standard instrument, but must be compared with a normal, and its corrections determined for a large number of points on the scale from which, by a graphic process, the corrections may be interpolated for each degree. The corrections thus obtained are the algebraic sum of the corrections for freezing-point, scale division and calibration, irregular expansion, and adhesion.

The special subject of adhesion has been studied by Recknagel (München: Bericht, 1866, II, 327–408). The following illustration of the amount of adhesion correction in one case is taken from Recknagel's work: An alcohol thermometer whose other errors are all known, and which agreed exactly with the air thermometer at 0° C. and 15° C., in a vertical position, gave an adhesion correction of +1.7° C. at −40.1° C., +1.6° at −41.1°, and +1.4° at −79.0°, the correction for the irregularity of expansion having been allowed for.

The effect of adhesion proper can be determined only after waiting a long time for the spirit to drain down the sides of the tube as much as possible. If this is not done there arises an unknown correction for imperfect drainage. When the thermometer is used in the vertical position the drainage error is a minimum, but when used almost horizontally, as in the self-registering minimum thermometer, the error thus introduced may become quite appreciable, and depends upon the rapidity with which the temperature changes. The comparison with the normal, therefore, should be made with the minimum thermometer in the position in which it is to be used.

35. THE MAXIMUM THERMOMETER.

The mercurial thermometer is made to give a record of the maximum temperature by means of several devices. The earliest is that of Cavendish (1757). In this form the thermometer lies horizontally, and differs from common thermometers only in having the top of the stem drawn out into a capillary tube, which enters into a glass ball joined to the stem at the place where it begins to be contracted. The upper part of the tube above the mercury and a portion of the glass bulb are filled with spirit of wine. When the temperature rises the spirit of wine is driven out of the tube into the glass ball, where it is left when the temperature falls. The length of the empty part will be proportional to the fall of the thermometer, and by means of a proper scale the top of the spirit of wine will show how many degrees the temperature has been higher than when observed. This early and crude form was followed by three types, which have come into general use, Rutherford's, Phillips's, and Negretti and Zambra's.

(*a*) *The Rutherford maximum* (1794).—This has a light movable steel index at the top of the mercurial column. The instrument is placed horizontally, and as the temperature rises the mercury pushes the index before it; when the temperature falls the index is left *in situ* to mark the position of the maximum. After an observation is made the thermometer is set by bringing the steel index to the top of the mercury column by means of a magnet.

(*b*) *The Phillips maximum, or the Walferdin* (1854).—In this a small bubble of air makes a permanent break in the upper part of the mercurial column; with small bore and short detached column the tube may be placed vertical. As the temperature rises the column moves up the tube; when the temperature begins to fall the lower column contracts toward the bulb, but the detached upper portion is left behind to register the greatest heat, the pressure of the air bubble and the frictional resistance to motion combining to prevent its descent. To set the thermometer the detached column is forced back by the centrifugal force generated by whirling the thermometer held at arm's length. The size and location of the air bubble may vary, especially during transportation, thus introducing a variable correction for the reading of the top of the mercurial column; consequently the thermometer should be frequently examined and compared.

If the mercury moves easily in the bore the tube should be kept nearly horizontal, otherwise the detached column will be in danger of sliding down by its own weight before the observer has made his daily reading. Such an instrument should not be used. In spite of its defects this form of maximum is in many respects the best for meteorological work.

(c) *The Negretti and Zambra maximum.*—The Negretti and Zambra maximum has the bore of the tube constricted near the bottom of the scale, so that mercury once pushed up past this point can not easily slide back into the bulb.

After reading the maximum temperature the column of mercury is forced back into contact with that in the bulb by whirling. This form of maximum is exclusively used at the stations of the U. S. Signal Service and is otherwise widely diffused. Its merit consists in the fact that the break always occurs at a definite point. For accuracy it is necessary to apply a correction for the effect of the lower temperature at which the observer subsequently reads the registered column, whose volume was fixed at the maximum temperature of the day. This correction is similar to that given in previous section on the corrections for difference of temperature between the stem and the bulb, and may easily become quite appreciable. In practice this correction has hitherto always been neglected.

The pressure required to force mercury up through the constriction is such that the bulb is subject to an unknown increase of internal pressure, and the liquid ascends, when the column rises, by one or two-tenths of a degree at each step; therefore the maximum is subject to a deficiency amounting to the whole of this quantity, and is on the average too low by about one-half of it. Both the preceding sources of error give temperatures too low; the following, however, gives an opposite tendency. If the constriction is, as usual, of a long conical form above and below the narrowest point, then the lower end of the mercurial column has a smaller diameter, and therefore a greater capillary curvature than the upper end of the thread. There is therefore an unbalanced capillary pressure tending to raise the whole column. Its effect is greater at low temperatures, when the weight of the column is proportionally diminished. This effect can be observed by reading the lower end of the mercurial column as well as the upper end. When the temperature falls very slowly it can happen that a continuous mercurial connection is maintained between the upper and lower columns, by virtue of which the upper is drawn down into the bulb, so that the record is falsified or wholly lost. The Phillips maximum is free from all these defects.

36. THE MINIMUM THERMOMETER.

The alcohol minimum thermometer is now universally used. This instrument was devised by Rutherford in 1794, excepting that he used spirit of wine instead of alcohol as his expanding liquid. The registration is effected by a light steel or glass index, enlarged and rounded at the end, and wholly immersed in the column of alcohol. When the alcohol contracts with a fall of temperature the surface tension of the top of the alcohol column is sufficient to carry with it the index, and then, when the temperature rises, the alcohol flows around and past the index, leaving it to mark the lowest temperature. The instrument manifestly gives also the temperature at the hour of observation. The thermometer is set by drawing the index to the top of the alcohol column by simply tipping the thermometer if the index moves easily, or otherwise by a magnet, or by jarring or whirling the instrument. Care should be taken, however, to avoid driving the index out of the column of alcohol. In most thermometers of this kind the index moves easily, and the stem must be kept very nearly horizontal, so that any vibration of the support shall not cause the index to slip down the tube.

Baudin has succeeded in making the index fit the bore so closely that it maintains its position even when the stem is vertical. To set the instrument he incloses in the bulb a slender bit of glass (the hammer), which upon inverting the thermometer slides down and pushes the index to position. No error is introduced by the fact that the minimum is subsequently read at temperatures higher than that at which the record was originally made.

37. COMBINED MAXIMUM AND MINIMUM THERMOMETER.

The oldest form of combined maximum and minimum thermometer is one described in the Acts of the Academy del Cimento, and next in age to this is that devised by J. Bernoulli in 1693, in both of which forms the horizontal thermometer tubes have auxiliary branches or depressions, into which mercury falls at the maximum and minimum readings.

The most widely used combined thermometer is that invented in 1781 by J. Six, of Colchester. This consisted of a U-shaped tube, as in Fig. 5, ending in a large bulb, *K*, and a smaller bulb, *R*. The mercurial column fills the lower portion of the tube. The main bulb *K* is full of alcohol; the lesser bulb *R* has both air and alcohol. A rising temperature causes the alcohol in *K* to expand downwards, thus pushing the mercurial column up the short end of the tube and compressing the air in *R*. The index B' shows the maximum temperature and stays at any point in its tube when the mercurial column begins to fall. Similarly, when the bulb *K* cools and its alcohol contracts the air pressure in *R* pushes the mercury down away from B' and raises B'', which finally rises to a point indicating the lowest temperature. After reading the maximum and minimum for the day the steel index pieces are "set" or drawn back to their starting places at the ends of the mercurial column by means of a small magnet.

This form of thermometer has been modified by Kappeller, as shown in Fig. 6, in which form he employs a much shorter column of mercury in a long horizontal tube. The indices B' and B'' are therefore not liable to fall down from their proper positions. Kappeller's form is used in Austria, while the early form by Six and a modified form by Hicks is widely diffused in England and its colonies. Both forms are graduated by comparison with a standard. The bores are larger and the degrees are shorter than the ordinary thermometers. Owing to the care given to the construction of other forms of thermometers Six's form has been left in the background as to accuracy; but it can generally be relied upon to within 1° F., especially in the form made by Hicks.

38. THERMOMETERS REGISTERING AT ANY MOMENT AT WILL.

In 1874 Negretti and Zambra introduced their new upsetting thermometer (Fig. 8*a*). This consists of a bulb and wide tube, at the bottom of which is a slight constriction sufficient to break the column when the thermometer is upset. The upper end of the thermometer tube is curved like an inverted **U**, the other leg of which constitutes the measuring tube. If at any time it is desired to record the temperature the thermometer is simply rotated in a vertical plane. The result of the first half of the rotation is to upset the **U** and break off the mercurial column at the constriction; the second half of the rotation throws the column thus broken into the measuring leg of the **U**, where it stays until the observer can make the reading and restore it to its place. This upset may occur at any time, and a series of such thermometers attached to clock-work constitutes an arrangement by which hourly readings of the temperature may be obtained.

39. METALLIC THERMOMETERS.

The differential expansions of metals have been used as thermometers since the earliest times; but of the very many forms devised the following are the most promising for accurate results.

(*a*) *Breguet* (1817).—In this form two thin spiral bands of different metals are soldered together; one end of the compound spiral is fixed; the other free to move. Differences of temperature, with reference to an initial condition, cause the spiral to coil or uncoil, and the free end with an attached index moves over a graduated circular arc. This apparatus may be made much more sensitive than the ordinary mercurial thermometer, but its use requires that the air or liquid, whose temperature is to be measured, shall not move the coil by virtue of its own motion. This condition necessitates inclosing the bi-metallic ribbon in an outer case by which the sensitiveness of the whole to outside temperatures is diminished; but very delicate portable instruments of this class are now made.

(*b*) *Hermann and Pister* (1865).—In this form the two spirals are so soldered together that the free end unrolls in one plane and the needle point by which the record is made has a rectilinear motion; the coil is made by them of brass and steel, and is annealed in boiling oil. This form is made sufficiently substantial to be unaffected by ordinary air currents and is used for self-registration at Swiss, German, and Austrian stations.

(c) *Jurgensen* (1841).—By reversing the temperature compensation of the balance of a chronometer, Jurgensen constructed a chronometer whose rate was so largely affected by the temperature that it was styled a chrono-thermometer. Its rate during any interval depended upon the average temperature during that time. This, therefore, constitutes an integrating thermometer. The spiral spring is very sensitive to changes of temperature, but as the whole has to be inclosed for protection from currents of air, it is therefore useful only when great sensitiveness is not desired. Its finest applications have been in the determination of the daily and weekly average temperature of rooms, ovens, and chronometer cases.

(d) *Tremeschini* (1875).—The metallic thermometer constructed by this maker consisted of two bars of different metals standing vertically side by side and otherwise unconnected. A lever extends from the top of one over and beyond that of the other, resting by its weight upon the top of both. The differential expansion of the two bars causes the end of the lever to describe a circular arc, and its motion is converted into a thermometer scale by comparison with a standard. This method, which is not wholly inconvenient for work in a permanent meteorological observatory, has, however, found its most important application hitherto in the determination of the temperature changes and the equality of temperature in the horizontal bars used in geodesy for measuring the lengths of base lines.

(e) *Krecke* (1860?)—In this form (see Fig. 7) a glass tube, *gg*, has attached to its two ends the zinc bars $z_1 z_1'$ and $z_2 z_2'$, the length of each being about half of that of the glass tube; the bar *ab* connects the two ends z_1' and z_2', and as these ends are by their expansion moved past each other the motions of the bar give a magnified presentation of their changes in length relative to that of the glass tube. If in place of the bar *ab* we substitute a mirror, attached to and between the zinc bars, a reflected beam of light may be used to read off the changes in relative position, and the sensitiveness of the apparatus thus increased. This arrangement is convenient for determining the temperature of the air, if the latter can be drawn rapidly through the glass tube.

40. OPTICAL THERMOMETERS.

When polarized light passes parallel to the optical axis of certain crystals, as quartz, the location of the plane of polarization is rotated about the axis of the beam of light. The amount of the rotation depends, among other things, upon the wave length of light, the thickness and the temperature of the crystalline plate. For definite thickness and wave length the rotation angle of the plate of quartz increases with the increase of temperature. Thus for a plate whose thickness is one millimeter at the temperature of 0° C., the rotation, expressed in centigrade degrees, is given by Sohncke's formula

$$\rho = \rho_0(1 + 0.0000999t + 0.000308t^2)$$

The coefficient ρ_0 is about 16.4° for the lithium line, whose wave length is 0.0006745; it is 21.6° for the natrium line, whose wave length is 0.0005888, and is 26.5° for the thalium line, whose wave length is 0.0005347.

As quartz can be submitted to a wide range of temperature the determination of the rotating power of equivalent plates at an unknown temperature for a given ray of light allows one to calculate that temperature from the rotating power with considerable accuracy. On account of the great stability of natural quartz crystals this method has been supposed by Cornu to offer special advantages in the study of high temperatures.

41. ELECTRIC THERMOMETERS.

The utilization of electrical properties for the purpose of measuring differential temperatures has been accomplished in several ways, of which the principal are the the thermo-electric and the thermo-resistance methods.

(*a*) *Thermo-electric currents.*—In Melloni's thermo-electric junction the ends of short pieces of two different metals are soldered together in an alternating series, and the extremities communicate by wire with a needle galvanometer. According as one set of junctions is warmed or cooled an electric current traverses the wire, and the galvanometer needle is deflected by an angle varying with the strength of the current, which latter is directly proportional to the difference of temperature of the alternate junctions. The formulæ for computing the strength of the current, and therefore the temperature from the observed deflection of the needle, are given by the laws of electro-magnetics. This apparatus is usually considered as an interpolation or differential method only, and the value of a deflection of the needle is expressed in thermometric degrees by means of a comparison at stated intervals with thermometers. The most ordinary use of the junction is to ascertain whether the radiations from two bodies have the same temperature or not. On the other hand Wrobleffski finds that at very low temperatures, such as −200° C., the thermo-electric thermometer gives more uniform indications than the hydrogen gas thermometer, and that it is therefore an important auxiliary at these temperatures, and may possibly be the best means of determining the reduction to the absolute thermo-dynamic scale.

(*b*) *Resistance coils or the galvanic differential thermometer.*—The resistance experienced by an electric current in its flow through a delicate insulated wire ordinarily increases with the temperature of the wire. The fact of a change in resistance is shown by the change in the deflection of a galvanometer needle placed in the circuit; therefore a resistance coil, whose coefficients have been determined by immersing it in water at several successive temperatures, as measured by a standard thermometer, can become a means of redetermining both those and all intermediate temperatures at any time. A convenient form of this apparatus for ordinary measures is given by Mendenhall (American Journal of Science, August, 1885, page 114). The most delicate form of the apparatus is the bolometer of Langley, as applied to the examination of the distribution of heat in the solar spectrum. Ordinarily two equal resistance coils are balanced against each other, one of them being in a bath of water, whose temperature can be varied and measured by a thermometer. When the needle deflection is zero the coils have the same temperature. This form is the differential resistance thermometer, and an idea of its delicacy is obtained from the fact that Knut Angstrom, in his researches on the diffusion of heat from unpolished surfaces, used a differential resistance thermometer for which one scale division corresponded to a transfer of heat to the extent of 0.000033 gram-colories per minute per square centimeter of the surface.

42. THERMOMETERS FOR WATER AND EARTH TEMPERATURES.

The arrangements for the observation of earth and water temperatures relate not so much to the peculiarities of the thermometer as to those of their exposure and use.

(*a*) *Temperature of the surface water.*—The essential features in an apparatus for this purpose are that the thermometer case, as it is lowered to a slight depth below the surface, shall allow the water to flow freely through it, but shall then close and bring up from a given depth a sufficient amount of water with the inclosed thermometer, so that no change of temperature can possibly take place before the thermometer is read off. The ordinary process of raising a bucketfull from the surface of the water and dipping the thermometer into it for a minute or less is not adapted to giving temperatures correct to within less than 0.5° F.

(*b*) *Protected thermometers for deep-sea temperatures.*—The temperatures recorded by self-registering thermometers when they are immersed under water are largely affected by the external pressure to which they are subjected, and the proper correction for this pressure must be carefully investigated by subjecting them to the pressures produced by hydraulic pumps. The correction, even to a very thick glass bulb, may amount to many degrees when it is subject to the great pressure that prevails deep in the ocean. This pressure effect is wholly annulled by adopting a special protection for the bulb (see Fig. 8). The whole thermometer is placed within a strong bottle or cylinder, which is then partly filled with water or mercury, above which some air remains; the protecting cylinder is hermetically sealed, and when lowered to the ocean depths the external pressure, compressing the cylinder somewhat, causes the water to rise and compress the air slightly. This latter slight increase of pressure is the only one that affects the thermometer bulb; the upsetting device of Negretti and Zambra makes the thermometer self-registering. The sluggishness of the thermometer is certainly increased by the necessity of inclosing it in liquid and thick glass; but this is remedied by simply allowing the whole to remain a little longer time in the sea water. Such protected thermometers were used by Sabine in 1822, and William Thomson in 1850. Protected Six's thermometers were made by Negretti and Zambra in 1857 for Fitz Roy, and by Casella in 1869 for Dr. W. A. Miller. The protected upsetting thermometers, as made by Negretti and Zambra, is the best form, and is shown in Figs. 8*a* and 8*b*.

(*c*) *Earth temperatures.*—For the purpose of measuring the temperature of the earth the special method adopted in the early part of the century was to construct thermometers of large bulbs and wide tubes, and so long that, although buried many feet in the earth, yet the top of the liquid column was visible above the earth. Thermometers whose stems were 20 feet long have been used. In a second form, introduced by Lamont, the thermometer is of ordinary size and inclosed in a wooden tube and other non-conducting packings, and is pushed down into a hole in the ground, where it is left at the proper depth. The temperature of the thermometer is supposed not to change during the short time required to pull up the tubes and make the readings. Pobably a tenth of a degree would express the average result of the accidental errors introduced by raising the tube; but the systematic errors of this method of observation are apparently often as large as 0.5°.

Electricity has been applied with great convenience to the measuring of the temperature of the earth or water at any spot. In Becquerel's method two equivalent thermo-electric junctions are used; one is placed in the ground or water to be measured; the other in a vessel of water in the observer's laboratory. Both are in the same circuit with a galvanometer needle. The laboratory bath is warmed or cooled until the current from one junction neutralizes that from the other. The temperature of the bath is then read off from a thermometer placed therein, and must be the same as that of the buried junction. In the electric-resistance method a well-calibrated resistance coil is buried in the earth and a similar one immersed in a bath of water. They are brought to the same temperature, as shown by the fact that their resistances are equal. The temperature of the bath is then the desired earth temperature.

In Mendenhall's differential resistance method the laboratory bath and coil are kept at a uniform known temperature, and the earth temperature computed from the observed differential resistance.

43. RADIATION THERMOMETERS.

(*a*) *Black-bulb thermometers.*—In order to obtain a thermometer whose balb absorbs all the heat that falls upon it recourse has been had to a covering of finely-divided pure carbon, such as lamp-black, bone-black, etc. The thermal qualities of a layer of lamp-black depend upon the fineness and purity of the carbon (which varies with the material and mode of manufacture) on the thickness of the layer, and the manner of making it adhere to the bulb. In exact work it is therefore necessary that the absorbing and radiating power of any black bulb should be specially determined.

These details have been studied perhaps most thoroughly by Angstrom (Wiedemann Annalen, 1885, XXVI, page 253) and by Godard (Journal de Physique (2), 1887, VI, page 157). The latter finds that the use of gum shellac, varnish, glue, etc., must be avoided, because they introduce appreciable reflection, special absorption, and radiation. He therefore attaches the layers of carbon to the glass bulb by means of very slowly evaporating pure water or alcohol, and finds that the black coating, though delicate, will adhere so long as the bulbs are not exposed to the wind or sudden shocks. Godard finds that a layer whose thickness is at least 0.346 or 0.350 millimeter is needed in order to give the maximum amount of absorption of the solar rays; when the film is less than this it becomes partly diathermanous, and the reflections from the glass bulb are appreciable.

Experiments with various substances show that the limit of thickness increases when the compression of the powder increases; it also increases as the grains of powder are larger, and varies with the nature of the substance and the nature of the source of heat. With layers whose thickness is equal to or greater than the limit, whatever be the sources of heat, solar or terrestrial, and whatever be the wave-length, the law of diffusion is the same, namely: the intensity of the diffused ray is equal to the intensity of normal diffusion multiplied by the sine of the angle of the inclination of the ray to the surface. The normal diffusive power increases as the wave length diminishes, and therefore varies with the temperature of the source of heat in so far as higher temperatures are accompanied by shorter wave lengths. As radiation can not exist except in connection with material molecules, and as the radiation that emanates from a special kind of matter is absorbed readily by other masses of the same matter it is easily shown to be impossible that any one kind of matter should absorb all kinds of radiation with equal facility.

Lamp-black apparently absorbs with great approach to equality all those radiations that bring to the earth appreciable quantities of heat, namely, all below the violet end of the spectrum, and has therefore been generally adopted for use in radiation thermometers. In respect to the various forms of lamp-black Provostaye and Desains state that sufficiently thick layers of pine or resin soot reflect less than 0.0067 of the incident heat. Christiansen states the absorptive coefficient to be 90 per cent., or reflective power 10 per cent., of the incident ray. K. Angstrom finds for the soot from a flame of oil of turpentine the absorption coefficient 97.6, or the reflection 2.4 per cent.; this is based on the assumption that all the rays are equally reflected and absorbed.

This coefficient is by Angstrom found to vary with the kind of soot and method of preparing it; thus the soot from stearine candles has but one-half the diffusive powers of that from turpentine oil. He finds the thickness of the layer affects the results very much, as stated by Godard. He recommends first coating the metallic or glass surface with a galvanic deposit of platinum black, which makes the surface itself as strongly absorbent as is any way possible; he then applies successive thin layers of soot. Tyndall suggests that the glass of the blackened bulb should be of a kind (green) that itself absorbs nearly all the radiation. In the use of these thermometers for radiation purposes the law of radiation enunciated by Stefan in 1879 should be adopted, since it has received important confirmation by the observations of Schneebeli (Wiedemann Annalen, 1884, XXII, page 450) and Schleiermacher (Wiedemann Annalen, 1885, XXVI, page 287). According to this law the quantity of heat radiated by any body in a vacuum is proportional to the fourth power of the absolute temperature.

(*b*) *Nocturnal radiation thermometers.*—In order to observe the approximate temperature of the surfaces of leaves and other bodies exposed by radiation at night time, the so-called radiation thermometers have been used. The object of the maker has been to secure great sensitiveness, and the bulbs are therefore sometimes made of hollow cylinders or a long spiral coil; however, these thermometers give the temperature of the mass of cooling air in contact with them, rather than the temperature of the radiating surface, which latter can only be obtained by calculations based upon actinometric measures.

(*c*) *Bright-bulb thermometers.*—A thermometer that reflects most of the radiation that falls upon it and absorbs only a small percentage of the heat is made by covering the bulb with a thin layer of polished silver, gold, iridium,* etc. This layer should be in contact with the glass of the bulb, so as to allow a perfect conductivity at all temperatures. To secure such solid contact an elastic layer of some substance would seem to be necessary between the glass and the polished metal, in order to allow for different expansibility of metal and glass. But in place of this a thin film of graphite, lamp-black, or even mercury may be used, or the layer may be deposited electrically at a temperature lower than will occur in subsequent use, and the effect of any compression of the bulb by the layer at low temperatures can be experimentally allowed for. The initial electrolytic compression has been carefully studied by E. J. Mills (Proc. Roy. Soc. London, 1877, XXVI, p. 504) and by E. Bouty (Journal de Physique, 1879, VIII, p. 289), who find that each successive layer of metal deposited electrolytically compresses more tightly those beneath it, the total pressure on the bulb amounting to 108 atmospheres in one case. Therefore the thinnest layers allowable for radiation thermometers should by a scratch or crack be made to relieve the glass bulb of any pressure before being used.

* These are very beautifully executed by Brashear, optician, of Pittsburgh, Pa.

CHAPTER VI.

THERMOGRAPHS.

The continuous record of atmospheric temperature has frequently been sought to be attained during the past century, and is an important feature in modern meteorology. In general the record of fixed thermographs must be converted into true air temperatures by comparison with standard instruments and the application of resulting corrections.

The force required to make a continuous record is an obstacle preventing the use of very delicate thermometric apparatus. The present chapter describes the forms of apparatus that attain sufficient accuracy and are in any extensive use, classifying them according to the principles involved in registration.

44. THE PHOTOGRAPHIC METHOD.

This form of registration, which is applicable both to wet and dry-bulb thermometers, can undoubtedly be made the most accurate, but it has the objection of all photographic records, that it requires much time and expense to keep it in operation. The plan of this apparatus, as designed by Beckley, and made by Casella, of London, is as follows (see Fig. 9): *l* is a dry-bulb thermometer and *m* the wet bulb. These thermometers have long bent tubes with two bends, as shown; near the top of the mercury column in each stem there is an air bubble, which separates the column. *G* is a screw for regulating the height of thermometers; the light from the lamps is thrown on the mirrors *n* and *k* by the condensers *f* and *i*, and is then thrown through the slits *o o*, and through the air bubbles in the thermometer tubes, so that the images of the bubbles are made to appear on the surface of the revolving drum *C* by means of the lenses *e e*. This drum turns by clock-work, and has prepared paper wrapped around it; *b* is a shutter which is made to pass automatically before the light every two hours and cut off the beam for a few moments; these breaks in the photographic record are used as guide points in making the reductions.

45. THE METALLIC THERMOGRAPH.

The use of the metallic thermometer is the most common on account of its simplicity and the small liability to get out of order. There are several forms of this instrument in use, the Hipp, Höttinger, Wild-Hassler, Draper, Richard, etc. The Wild-Hassler thermograph is arranged as follows: The thermometer consists of a compensated spiral, made of a strip of steel and brass, about a meter in length. The inside end of this coil is fixed firmly to a stand, and the outside free end (which moves by contraction or expansion, due to changes of temperature) has a steel magnet screwed to it in a radial, horizontal direction; this latter gives a spiral motion to a vertical rod of soft iron. The motions of this latter are communicated to a parallel pointer by means of a common axis, both rod and pointer being in rigid connection with this axis, which is supported on hardened steel surfaces. The vertical pointer has a horizontally-placed needle point, and this point is pressed against the paper at stated intervals by an arrangement similar to that employed in the Wild-Hassler barograph (see Fig. 28). A change of 1° C. causes the pointer to move over a space of 3^{mm}.

This linear expansion of metal bars due to increase of heat can be recorded, and the instrument may be rendered very sensitive by a proper combination of such rods and levers.

46. ELECTRIC CONTACT THERMOMETER.

The mercurial thermometer with electric contacts is used in the instruments of Theorell, Hough, and Secchi.

A thermometer with large bulb, which gives considerable range to the mercury in the tube for small changes of temperature, is chosen. This bulb has a platinum wire sealed into it, so as to make metallic connection between the mercury in the bulb and the outside of the bulb. The upper end of the tube is left open, and a fine platinum wire descends into the open tube; these two platinum wires are a portion of an electrical circuit, and when the platinum wire within the tube is made to touch the mercury surface in the tube the circuit becomes closed.

The platinum wire in the tube is made to descend by clock-work at regular intervals; when the wire comes in contact with the top of the mercury in the thermometer tube the resulting closing of the circuit causes to be registered automatically the distance which the platinum point descends in order to touch the mercury surface. By this means the thermometer reading may be recorded in degrees and tenths in printed figures, or a curve of points may be recorded, which must be reduced to figures by interpolation by means of direct observation. This method has the one objection, that the mercury does not always show the same condition of surface, and it is difficult to keep the surface clean, even when a covering of oil is used to prevent the oxidation.

So large a bulb is necessary that the thermometer is somewhat sluggish in action.

47. MANOMETER THERMOGRAPH.

Numerous forms of pressure thermographs have been proposed, but in general they can be divided into two kinds: First, gas thermometers, and second, those in which some sort of liquid is used in the closed vessels instead of air. The general construction of the former is as follows (see Fig. 10):

An air-tight vessel, *a*, made of sheet metal has a capillary tube, *b*, connecting it with a manometer tube, *D*, which contains mercury. The tubes *b* and *D* are connected by a rubber tube, *c*, to permit motions of the tube *D* in the vertical. The vessel *a* is located at the place whose temperature is to be determined. The gas in *a* takes the temperature of the place, and is either contracted or expanded as the place gets colder or warmer. The change of pressure resulting from these changes of temperature of confined air are communicated to the mercury in *D* and cause fluctuations in the position of the surface of the mercury in the manometer tube. Lamont, Osnaghi, Regnault, Schreiber, Sprung, etc., have proposed various forms of this instrument, some of which have been constructed. Probably the best that have been put into actual use are those of Schreiber and Sprung. The former is in use at the Deutsche Seewarte in Hamburg, and the latter at Magdeburg and Spandau.

(*a*) *The Schreiber Thermo-barograph.*—The Schreiber thermograph (and barograph) is described in the "Oester. Zeit. für Meteor.," Vol. X; "Carl's Repertorium," Vol. XIV, and also in the annual volume (Aus dem Archiv) of the Deutsche Seewarte, Vol. I. The registration apparatus for temperature is on the principle of the balance barograph (see Fig. 11). The copper air vessel *A*, which is in a shelter on the north side of the building, contains 5 liters of dry air, and the lead-connecting tube *B* is 4^{mm} inner diameter and 5 meters long.

The glass tube *D* is suspended from the wheel *F* pivoted at *E*, and is immersed at its lower end in the mercury in the cup *C*. The weight *G* counterpoises *D* and carries the marking pencil, which makes a record at stated times by automatic action of clock-work.

The tube *B* passes through the mercury in *C* and has its opening within *D*. It is readily seen that an increase in the temperature of *A* will cause *D* to rise and *G* to fall, and the curve of these fluctuations is used for determining the true temperature at *A*.

A somewhat similarly arranged self-registering thermometer is used for obtaining the temperature inside of the baro-thermograph case to be used in the reduction of the barometric observations. In this latter case a temperature change of 1° C. in the case causes a motion of 15^{mm} on the barometer record and a change of 27^{mm} on the inside thermometer record. A change of 1° C. in the temperature of *A* causes a motion of 7.4^{mm}, while a change of 1° for the inside of the thermometric registering case causes a motion of only 0.5^{mm} on the registering thermometer.

Schreiber's theoretical formula for the reduction of his thermograph curves or thermograms is

$$t=t_0+0.1393x-0.4\Delta b+0.07\tau$$

which is obtained *a priori* by a study of the relations and dimensions of his apparatus.

In this formula t_0 is a constant, which is dependent on the position of the axis of abscissas of the thermogram; x is the ordinate; Δb, the change in barometer height as recorded by the barograph. (For the complete theory of Schreiber's thermograph see Carl's Repertorium, Bd. XV.)

In practice, by inserting A in melting ice and water of various known temperatures, from 0° C. to 34° C., the following formula has been experimentally obtained by Schreiber:

$$t=t_0+0.1334x-0.422\Delta b+0.07\tau$$

whence the accuracy of the apparatus is evident.

(*b*) *The Sprung thermograph.*—As early as 1878 (Oester. Zeit. für Meteor., 1878) Sprung proposed a form of self-registering air thermometer with balance registration; a more definite form was given to his idea in 1881 (Zeit. f. Inst., 1881), and he then thought of closing the open end of the manometer and thus excluding the change of atmospheric pressure. He finally decided to use the open manometer, however, in the instruments finally constructed by Fuess for Magdeburg and Spandau.

In the "Zeitschrift f. Instrumentenkunde," June, 1886, Sprung gave a description of his baro-thermograph, and the theory of the working of the instrument. The construction of this instrument can best be explained by aid of the following diagram (see Fig. 12). Its complete mathematical analysis is reserved for Chapter X, in connection with the description of the barographic registration.

A tube, P', has its upper end in communication with a gas reservoir, A, by means of a fine lead tube. The lower end of P' is immersed in an iron cistern, P_1', containing mercury; this cistern is supported from a knife-edge on the right-hand end of a balance beam. An expansion of the gas in the reservoir A causes the mercury to flow from P' to P_1', and consequently causes the right arm of the balance beam to descend; this descent is immediately counteracted by the rolling counterbalance-wheel u'. A decrease in air pressure also causes the weight of P_1' to be increased, but this is counterbalanced by the decrease in weight of the barometer tube P, which is also suspended from the right arm of the balance beam. The changes of temperature of the gas in the reservoir are thus recorded directly by recording the motions of the rolling wheel u'. (For further details see Chapter X.)

In place of a gas a liquid may be confined in the reservoir of an air thermometer or in a tube, and its expansion or contraction communicated to a registering apparatus. For example, at the Montsouris Observatory a long copper tube is filled with alcohol and arranged for registration. In the Richard thermograph a Bourdon-pressure tube (see Chapter X) is filled with alcohol and the differential expansion produces a change in shape of the tube precisely similar to that produced by air pressure.

In all cases where large vessels of air are used, as thermometer bulbs, there is a very appreciable sluggishness, which is usually diminished by using cylindrical vessels. The theory of the conduction of heat in the air within a spherical bulb is given by Maurer (Schweiz. Met. Beob., XIX, 1882). He finds that if a copper sphere whose radius is 10 centimeters has its surface temperature suddenly raised 5° C., then in four minutes the average temperature of the whole sphere will be raised 4.9°. For a sphere filled with a fluid the effect of interior convection hastens the attainment of a uniform interior temperature, and the time required is only about one-third of that necessary for conduction alone to equalize the temperature of the mass. If the air temperature is changed very gradually, instead of suddenly, then, as in the mercurial thermometer, the bulb will follow the air at a longer interval.

48. AUTOMATIC POSITION THERMOMETERS.

(*a*) *Negretti and Zambra's upsetting thermometer.*—A simple and satisfactory hourly register of temperature was in 1874 introduced into use in England by Negretti and Zambra. This apparatus consists of twelve thermometers on a stand, each thermometer being in such a connection with a clock-work that it can be inverted at the proper hour or at any chosen time.

The construction of the thermometer is shown in Fig. 8. Near the bulb is a double contraction of the tube with a little chamber between the narrowings, and at the upper end of the tube there is a chamber similar to the calibrating chamber of an ordinary standard thermometer.

In the ordinary position of the thermometer the mercury fills the capillary tube and partly fills the little chamber. If, now, the thermometer is inverted the column breaks at the contraction near the bulb, flows through the small chamber down into the measuring tube beyond, leaving a clear space between the bulb and the top of this inverted column.

The graduations on the measuring tube are numbered backward from its end, and the length of the column representing the temperature at time of inversion can thus be read off when the instrument is in this inverted position.

(*b*) *Kreil's balanced thermometer.*—The Kreil thermograph is a thermometer with a long large cylindrical bulb. The thermometer is balanced on a knife-edge; when the temperature is increased the long end of the thermometer becomes heavier and falls, the bulb end rising. Of course, the opposite takes place when the temperature decreases. The rise and fall of the long end of the thermometer is registered by mechanical means, and can be made on any convenient scale. The apparatus is very sensitive but can not be used where the currents of air can affect it mechanically.

49. REDUCTION OF THERMOGRAPH RECORDS.

All of the forms of self-registers require more or less preliminary calculation before the instrument can be constructed; but these computations are usually rather in the nature of limits than definite results. For instance, in the case of the open-top thermometer the relative size of the bulb and the diameter of the tube must be carefully determined, as the tube must be large enough to admit the platinum wire, but still it must not be so large that the degree divisions will come too close together. So with the other forms of instruments various points must be considered.

Few inventors have given much study to the errors peculiar to the instruments, and in no case, except that of Sprung and Schreiber, have they been able to assign constants to an instrument so as to use it without comparison with an ordinary thermometer for determining the actual constants of the self-register from day to day.

Schreiber (Carl's Repertorium, Vol. XV), Sprung (Zeitschrift für Instrumentenkunde, 1886), and Maurer (Annalen der Schweizerischen Meteorologischen Central-Anstalt, 1882, Bd. 19) have, however, given computations based on theoretical considerations as to the action of the thermographs, the two former treating of their own inventions, and Maurer of the spiral metal thermometer.

The Sprung and Schreiber forms require essentially the treatment necessary for a manometer and gas thermometer, in which, however, the measuring is done by a balance instead of a cathetometer.

In the metal thermometer the coil, which is nearly an Archimedian spiral, must be investigated for the effects of elasticity of the metals, and this is by no means an easy matter, and at best computation can give only very approximate results.

(*a*) *Methods of reduction.*—The records of the Wild-Hassler thermograph are reduced as follows:

Measure the length of the ordinate of the curve for the times when direct control observations are simultaneously made (three times daily) on any ordinary mercurial thermometer, mounted in the same shelter as that of the thermograph. This gives ninety control observations for a month. Arrange the ordinates and corresponding thermometer readings according to the length of the ordinates, and divide the series into nine groups of ten each. Take the mean of each group.

The observed temperatures t and the measured ordinates x are connected by the simple equation

$$t=a+bx$$

where a and b are the constants to be determined.

Substitute the mean t and the corresponding mean x in this equation for each group and we have nine equations. These are to be solved for a and b by the method of least squares.

When it is desired to find the temperature corresponding to any ordinate of the thermogram, insert the constants a and b and the numerical value of x, and compute the required value of t from the above formula.

In the reduction of a series of observations much time will be saved by computing t for various values of x, constructing a graphical table therefrom, and using this graphical table for the interpolation of intermediate values. The reduction must be gone through with for each month of the year.

The reduction formula of the Schreiber thermograph has already been indicated (page 104); Sprung for his thermograph uses the empirical formula

$$\tau = -0.03^{\circ} + 1.222^{\circ} t$$

where τ is the mercurial thermometer reading in centigrade degrees, and t that of the thermogram.

(*b*) *The accuracy of thermographs.*—Let the registrations of a thermometer for some months or a year be reduced by means of the direct tri-daily control observations. Then find the outstanding deviations of the registrations, at the time the control observations are made, from the direct observations made by means of mercurial thermometers. The average of these deviations, taken without regard to sign, or, more accurately, the square root of the average square, may be assumed to show the accuracy of the registrations, regarding the mercurial thermometer as a standard.

For the best instruments, such as the Sprung and Wild-Hassler thermographs, careful reduction gives an average deviation of about $\pm 0.15^{\circ}$ C.; but single cases occur where the barogram differs by 0.5° C. from the direct observations. For the photographic registration it is safe to say the average deviation is not more than 0.1° C. In a recent letter* Professor Carpmael states that for the Toronto photographic system the maximum difference (barograph — eye) during three months was $\pm 0.3^{\circ}$ F., and the average deviation $\pm 0.066^{\circ}$ F.

Sprung has remarked, however, that in his system the closed metal gas reservoir must be carefully sheltered, and perhaps in a different manner from the mercurial thermometer, in order to give the true air temperature. He found in the experiments with his thermograph that two mercurial thermometers exposed side by side showed frequent differences of 0.4° C., or as much as the deviation of his registering instrument from the standard; therefore his instrument possibly has an actual accuracy closer than that indicated above, as the standard may have been influenced by the shelter used.

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* To Professor Waldo, who has assisted in the preparation of this chapter.

SECTION B.

THE MEASUREMENT OF ATMOSPHERIC PRESSURE.

CHAPTER VII.

THE BAROMETER IN GENERAL.

50. ITALIAN ORIGIN.

Galileo first taught that the air has weight; his pupil, Torricelli, brilliantly demonstrated this truth by showing that the weight of the air will uphold a column of water, and, in the course of his first series of observations, the occurrence of variations in the weight of the air became apparent. Subsequently it was seen that the air is an elastic fluid; that its vertical weight is transmitted in all directions by it as an elastic pressure; that the barometer measures an elastic pressure by means of the vertical pressure due to the weight of a vertical column of liquid.

The pressure of the air will not only uphold a liquid column, but will compress elastic bodies, including the air itself; upon this pressure depends the refractive index of the air, which varies with its density, the boiling-points of liquids, and other pheuomena. Upon these and other properties various forms of the barometer have been based. The mercurial barometer is adopted as the standard, while other liquid barometers and barometers depending upon the compression of elastic bodies, and upon other properties, are used for differential measures.

The mercurial barometer, in its elementary form (see Fig. 13), consists of a vertical glass tube, *A*, hermetically sealed at the top and opening at its lower end into a basin of mercury. If by any means the air is entirely exhausted from the tube the mercury will rise to a height of about 30 inches (or less if the instrument is far above sea level) and remain there nearly stationary, leaving the upper portion of the tube entirely vacuous.

The elementary laws of hydrostatics show that the downward and upward pressure in a section of the tube at *B* must balance each other, and therefore the pressure due to the weight of the mercurial column *BA* must equal the pressure of the layer of air contiguous to the free surface of the mercury in the reservoir. Barometry is the art of measuring the height of this column of mercury, and deducing therefrom the pressure prevailing in the free air at a given place and time, as expressed in standard units of force. A normal barometer is one by which this object is accomplished with the greatest attainable accuracy, while ordinary standard barometers giving less accuracy with greater expedition are to be considered as substandards, whose errors have been determined by direct or indirect comparison with the normal.

The forms in which the barometer has been made may be classified as follows:

(1) Cistern barometer with fixed zero point.
(2) Cistern barometer with variable level or zero point.
(3) Siphon barometer without cistern.
(4) Siphon barometer with adjustable cistern.

51. CISTERN BAROMETER WITH FIXED ZERO POINT.

In this form a broad cistern is provided such that the effect of capillarity on the mercury in it is very much diminished. The surface of the mercury in the cistern is adjustable and the scale is fixed in its position. The observations are made either by adjusting the surface of the mercury in the cistern to coincidence with the zero point, and then reading the scale division at the top of the column, or by reading the scale divisions both at the bottom and top of the mercurial column without adjusting the mercury, and then taking the difference of the two readings. The former method is that used in the ordinary portable barometer first adopted by Fortin, and subsequently by Ernst, Green, Adie, Beck, and other makers; the latter method is followed in using the normal barometer at Kew.

52. CISTERN BAROMETER WITH VARIABLE LEVEL.

In this form, as first made by Kappeller, the mercury flowing in and out of the tube alters the level of the surface of the mercury in the cistern in the ratio of the sectional areas of the cistern and the tube. So long as the quantity of mercury remains the same and the cistern and tube are unchanged a given reading at the upper end must correspond to a definite position of the lower surface. In making an observation the scale reading at the top of the column only is taken and a correction for the level of the lower surface is applied. This correction may be computed and applied numerically, as in the Kappeller form, or the scale divisions may be contracted sufficiently to allow instrumentally for the change of level in the cistern, as in the modern Kew form. In this latter self-correcting form the quantity of mercury is adjusted so that the scale has its correct length for a height of one atmosphere; the readings need then only slight additional corrections, that are to be determined by comparison with a standard instrument. As a check against changes in capillarity or in the volume of mercury the comparisons should be frequently repeated.

The Kappeller form, with numerical correction, after having been for some time in use by seamen, was adopted by Kreil in 1851, for use at Austrian stations. The formula for its correction is given in Jelinek's Anleitung, etc., Vienna, 1884.

53. SIPHON BAROMETER WITHOUT CISTERN.

In its simplest form this consists of a U-shaped tube, one of whose legs is about 3 feet long and sealed at its upper end, while the other leg is short and open. The height of the mercury is read off on a fixed vertical scale, and the difference of the two readings gives the height of the column. In this form a falling pressure produces in the open leg a rising and very convex mercurial surface, but for rising pressures the reverse is the case. Consequently the capillary corrections are very uncertain.

54. SIPHON BAROMETER WITH ADJUSTABLE CISTERN.

In this form the long and short legs of the siphon are separate tubes fastened to the top of a cistern box full of mercury, whose volume can be increased or diminished by the turning of a screw. The advantages of this form are, (1) by diminishing the cistern the mercury in both legs may be made to rise and present a surface of maximum convexity; (2) by taking readings of the mercury when at different heights in the tube the effect of any air in the vacuum chamber may be determined, since the volume of this chamber is thus altered at will. This form is substantially that adopted in all the new normal barometers and in the portable-station barometers made by Fuess (see Fig. 18).

55. NORMAL BAROMETERS.

The elaborate investigations of the latter form of barometer that have been made at St. Petersburg, Paris, and Berlin have served to exhibit the errors of the past barometric work in physics and meteorology, and to show the possibility of attaining results more nearly in accord with the needs of modern science. The standard of barometry adopted by recent meteorological conventions is that of the International Bureau of Weights and Measures at Sevres, and extensive comparisons between the normal barometers at that bureau and the standards of the meteorological services of the world are in progress.

Institutions that have no normal barometer frequently adopt a high grade siphon barometer as a provisional standard. The accompanying engravings will illustrate the construction of the best normal and standard barometers. Fig. 14 shows the normal, devised by Wild for the Central Physical Observatory at St. Petersburg. Fig. 15 shows that constructed by Fuess for the Normal Standards Commission at Berlin, and Fig. 16 that constructed by Turrettini for the International Bureau of Weights and Measures at Sevres. These all embody the idea of separate vertical tubes for the vacuum and the open-air chambers, respectively, and the scale readings are made upon an independent vertical divided bar by means of reading telescopes; the volume of the vacuum chamber can be varied at will, as also can the portion of the divided scale, to be used in any given reading. Fig.

17 shows the standard devised by James Green in 1879 for the use of the Army Signal Office. In it no provision is made for determining the correction for imperfect vacuum. Fig. 18 shows the form of portable siphon barometer, known as the Wild-Fuess pattern; slight modifications of this pattern are due to Koeppen, and in one or the other of these forms it is now extensively used in the comparison of barometers used at the stations with a central standard. With six of this form Mr. Waldo made his extensive international comparisons in Europe and America in 1883 to 1885, the results of which are published in the Signal Service Monthly Weather Review, February, 1886.

[Since writing the above there has appeared Sundell's Barometric Comparisons during 1886 and 1887, using his own modification of the Wild-Fuess pattern in which Mendeleeff's capillary tube is utilized (see section 63)].

CHAPTER VIII.

THE CORRECTIONS OF THE NORMAL MERCURIAL BAROMETER; INTERNATIONAL COMPARISONS.

In most forms of both normal and ordinary mercurial barometers the same causes operate to affect their accuracy; consequently a classification and treatment of the individual sources of error necessary to be considered in the use of a normal barometer will include the corrections applicable to less accurate instruments. Any special forms in which the corrections enter will be indicated for each form of barometer in the following sections on barometer corrections.

56. CORRECTIONS FOR ERRORS IN GRADUATION.

The corrections for the errors of subdivision at every part of the scale are obtained by some method of calibration. Any errors of appreciable magnitude in the graduations of the vernier are so troublesome to correct for that only those scales should be used for verniers whose graduation is more accurate than the desired limit of accuracy in the resulting measures; thus, if the degree of accuracy desired for the resulting measures is one thousandth of an inch, then, whatever be the errors in the graduation of the main scale, there should be no error amounting to 0.0005 inch in the subdivision of the vernier.

Method of calibration.—The following method for determining the errors in the graduations of a rule or scale, first developed by Hansen, is given as arranged by Broch for use at the International Bureau of Weights and Measures (Travaux et Memoires, tome V, 1886).

Suppose that the scale to be examined is divided into n parts by graduations very nearly equidistant;

Let $\chi_1\ \chi_2\ \chi_3\ \ .\ .\ .\ .\ \ \chi_{n+1}$ be the desired corrections to be applied to the readings of the successive graduations $p, p+1, \ .\ .\ .\ .\ \ p+n$.

To find these corrections each unit interval of the scale, viz,

$$[\text{from } p \text{ to } p+1] \qquad [p+1 \text{ to } p+2] \quad .\ .\ .\ . \quad [p+n-1 \text{ to } p+n]$$

is compared successively with a similar unit interval, L_1, taken on the same rule, or on another rule, and the correction to which, on the ideal scale, is λ_n, so that $L_1=1+\lambda_n$.

Designating by $a_1\ a_2\ a_3\ \ldots\ a_n$, the differences found in comparing L_1 with the successive n intervals, we have

$$\begin{aligned}
L_1-[p \text{ to } (p+1)] &= a_1 \\
L_1-[(p+1) \text{ to } (p+2)] &= a_2 \\
L_1-[(p+2) \text{ to } (p+3)] &= a_3 \\
\ldots\ldots\ldots\ldots\ldots \\
L_1-[(p+n-1) \text{ to } (p+n)] &= a_n
\end{aligned}$$

whence

$$\begin{aligned}
(1+\lambda_n)-(1+\chi_2-\chi_1) &= a_1 \\
(1+\lambda_n)-(1+\chi_3-\chi_2) &= a_2 \\
(1+\lambda_n)-(1+\chi_4-\chi_3) &= a_3 \\
\ldots\ldots\ldots\ldots\ldots \\
(1+\lambda_n)-(1+\chi_{n+1}-\chi_n) &= a_n
\end{aligned}$$

whence

$$\begin{aligned}
\lambda_n+\chi_1-\chi_2 &= a_1 \\
\lambda_n+\chi_2-\chi_3 &= a_2 \\
\lambda_n+\chi_3-\chi_4 &= a_3 \\
\ldots\ldots \\
\lambda_n+\chi_n-\chi_{n+1} &= a_n
\end{aligned}$$

Similarly each of the $(n-1)$ intervals, whose length is equal to two units of the scale, namely,

$[p \text{ to } p+2]$; $[p+1 \text{ to } p+3]$; $[p+2 \text{ to } p+4]$ . . $[p+n-2 \text{ to } p+n]$

is compared with an interval, L_2, whose value is about two units of the scale and whose correction to reduce to the ideal scale is λ_{n-1}, whence

$$L_2=2+\lambda_{n-1}$$

Designating by $b_1\ b_2\ b_3\ \ldots\ b_n$, the differences found in comparing successively L_2 with the $n-1$ intervals, we have

$$\begin{aligned}
L_2-[p \text{ to } (p+2)] &= b_1 \\
L_2-[(p+1) \text{ to } (p+3)] &= b_2 \\
L_2-[(p+2) \text{ to } (p+4)] &= b_3 \\
\ldots\ldots\ldots\ldots\ldots \\
L_2-[(p+n-2) \text{ to } (p+n)] &= b_{n-1}
\end{aligned}$$

whence

$$\begin{aligned}
(2+\lambda_{n-1})-(2+\chi_3-\chi_1) &= b_1 \\
(2+\lambda_{n-1})-(2+\chi_4-\chi_2) &= b_2 \\
(2+\lambda_{n-1})-(2+\chi_5-\chi_3) &= b_3 \\
\ldots\ldots\ldots\ldots\ldots \\
(2+\lambda_{n-1})-(2+\chi_{n+1}-\chi_{n-1}) &= b_{n-1}
\end{aligned}$$

whence

$$\lambda_{n-1}+\chi-\chi_3 \qquad =b_1$$
$$\lambda_{n-1}+\chi_2-\chi_4 \qquad =b_2$$
$$\lambda_{n-1}+\chi_3-\chi_5 \qquad =b_3$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$\lambda_{n-1}+\chi_{n-1}-\chi_{n+1}=b_{n-1}$$

In the same way comparisons are continued with the intervals

$$L_3=3+\lambda_{n-2};\ L_4=4+\lambda_{n-3},\ \text{etc.},$$

whose values are very nearly 3, 4, etc., units of the scale.

The last comparison will be that of an interval of $n-1$ units

$$L_{n-1}=n-1+\lambda_2$$

with the two intervals on the scale

$$[p \text{ to } (p+n-1)] \text{ and } [(p+1) \text{ to } (p+n)]$$

Designating by k_1 and k_2, the two differences resulting from this last comparison, we have

$$L_{n-1}-[p \text{ to } (p+n-1)]=k_1 \qquad\qquad L_{n-1}-[(p+1) \text{ to } (p+n)]=k_2$$

whence

$$(n-1+\lambda_2)-(n-1+\chi_n-\chi_1) \ =k_1$$
$$(n-1+\lambda_2)-(n-1+\chi_{n+1}-\chi_2)=k_2$$

whence

$$\lambda_2+\chi_1-\chi_n=k_1 \qquad\qquad \lambda_2+\chi_2-\chi_{n+1}=k_2$$

If in any way the total length L_n of the scale under examination, between its extreme limits p and $p+n$, is known in terms of some standard unit, and if this length has thus been found to be subject to the correction λ_1 to reduce it to the ideal scale, we shall have the equation

$$L_n=n+\lambda_1$$

or

$$[p \text{ to } (p+n)]=n+\lambda_1$$

whence

$$(n+\lambda_1)=(p+n+\chi_{n+1})-(p+\chi_1)$$

or

$$\lambda_1+\chi_1-\chi_{n+1}=0$$

If the interval $[p \text{ to } (p+n)]$ is accepted as a fundamental interval equal to n ideal or mean units, we shall have

$$\lambda_1=0 \qquad\qquad \chi_1-\chi_{n+1}=0$$

in this way there results

$$n+(n-1)+(n-2)+ \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ +2+1=\frac{n(n-1)}{2}$$

equations of condition between the $n+1$ corrections χ to wit: $\chi_1, \chi_2 \ . \ . \ . \ . \ \chi_{n+1}$, and the n corrections λ to wit: $\lambda_1, \lambda_2 \ . \ . \ . \ . \ . \ \lambda_n$.

Of these $2n+1$ corrections λ is known in advance by a special study, or is supposed equal to zero; the same is also true of χ. There are then in reality only $2n-1$ unknowns to be found from $\frac{n(n+1)}{2}$ equations of condition. These are therefore to be solved by the method of least squares, the resulting formulæ and computations are similar to those already given for five subdivisions in the section on calibration (see section 17.)

57. FUNDAMENTAL LENGTH.

The length of the divided scale of the barometer is usually at least 800 millimeters, or 32 inches, but for normal barometers 1 meter, or 1 yard.

(*a*) The distance between the two extreme points of the scale between which the calibration has been effected is the fundamental length. This distance must be known accurately in terms of the standard measure, and for this purpose the scale is compared with an official standard, the result of which comparison is expressed as follows: "This scale of . . . . inches (or millimeters), when lying horizontally at the temperature of the freezing-point, is . . . . inches (or millimeters) too long (or short)."

(*b*) As the barometer scale must be used in a vertical position, and as in some forms of the instrument the scale supports a considerable weight, it is necessary to determine what additional correction to the fundamental length is due to elastic compression or extension.

The total correction for fundamental length is the sum of the corrections resulting from the horizontal comparison with the standard and the change due to elasticity and temperature.

If the vertical divided scale is supported at its center so that the upper half is compressed, but the lower half elongated, and by its own weight only, then the length of the whole scale is not sensibly changed; but as the lengths of separate portions may be changed, it is therefore no material advantage to support the scale at the center as suggested by some, and the computation is simpler if we consider that the whole scale is extended by the weight of the barometer, as in the ordinary portable instrument, or is compressed, by its own weight alone, as in most forms of construction of normal or standard barometers. The amount of this compression or extension can be determined experimentally for each instrument by attaching known weights to the top or bottom of the barometer and comparing the scale so altered with a standard parallel to it. The effect of a unit weight upon the length of the scale has then to be multiplied by the actual weight which the scale supports. If the area of the section of the scale and the weight that it supports and the modulus of elasticity proper for the material are known the elastic effect may be computed directly by the formula

$$\frac{\text{weight} \times \text{length}}{\text{section} \times \text{modulus.}}$$

For example, a brass scale 30 inches long and one-tenth of a square inch in section supports a weight of 10 pounds; Young's modulus for brass for 1 square inch section is 15,500,000; whence the extension will be 0.0002 of an inch. A direct determination of this extension should be made by measurement with the cathetometer if the scale is known to be supporting a weight at all approaching a limit such that a permanent set in the length of the scale is possible.

58. VERTICALITY.

According to the laws of hydrostatics the vertical distance between the free surfaces of the mercury in the vacuum chamber and in the cistern or free-air chamber is the desired measure of the pressure normal to the surface in the cistern; consequently the divided scale by which the measures are made must itself be vertical; if inclined to the horizon it represents the hypothenuse of a triangle whose vertical side is desired.

Let the length measured along the hypothenuse be h, and the angle of inclination to the vertical be i, then the true vertical length desired will be

$$h' = h \cos i$$

whence the correction for verticality of scale is

$$h - h' = h(1 - \cos i) = 2h \sin \frac{i}{2}$$

The following table gives the amount of this correction for deviations up to 5 degrees for lengths of column of 20 and 30 inches:

Inclination i.	$2 \sin \frac{i}{2}$	Correction for barometer 20 inches high.	Correction for barometer 30 inches high.
°		*Inches.*	*Inches.*
1	.00015	—0.003	—0.004
2	.00061	— .012	— .018
3	.00137	— .027	— .041
4	.00244	— .049	— .073

An inclination of 1 or 2 degrees is not readily detected by the eye; it is necessary therefore that a plumb-line be attached to the barometer, or some other means be adopted for determining the verticality of the scale.

In the cistern barometer, as ordinarily constructed, the instrument, if it hangs by a hook, should be so balanced that the scale can not deviate sensibly from the vertical; but when the cistern barometer is fastened at the upper and lower ends, or when, as in Green's standard, its verticality depends entirely upon the adjustment of the base plate, or when an ordinary free-hanging barometer is steadied by the observer when making an observation by holding the cistern in the left hand, then a deviation from the vertical is not a rare occurrence. Such a deviation introduces in these forms of instruments the two following errors additional to the above.

In the cistern barometer, as shown in Fig. 19, the graduated scale *SO* is attached to the tube and is parallel to the axis of its bore. The measurement that ought to be made in this axis from the top of the mercury at *a* to the point *b* vertically below it is really made along the scale by transferring to it the top of the mercury (a or a') and the ivory point (b') in the cistern. The upper transfer is made by the lower edge of the vernier, which is perpendicular to the scale and tangent to the mercurial meniscus; the lower transfer is made ordinarily by assuming that the ivory point b' and the zero of the scale *d* are on the same level as the central axis at *b*; but when the barometer is tipped out of the vertical the meniscus is deformed and the distance between the tangent points a and a', as measured parallel to the scale *SO*, introduces an error in the measured height of the barometer. Again, the tipping of the ivory point and the zero of the scale, raising one above and the other below the level of the bottom of the axis of the bore, thereby introduces an error equal to the difference of level. This error may be either positive or negative, and is a maximum when the tipping takes place in the vertical plane through the ivory point and the scale. The diagram shows the maximum effect on the assumption that the ivory point is in the same plane with the scale *SO* and the axis *ab*, and on the opposite side to the scale. Let the distance of the ivory point from the axis be *d* and its depression below the zero of scale be *p* when the axis is vertical; let the distance of the scale from the axis be d', then the two cistern corrections for verticality, when a maximum become

$$p \cos i \pm d' \sin i$$

In Green's portable barometer the distances d and d' are about one-half inch, and in Green's standard barometer about 2 inches; therefore the correction for an inclination of 1 degree in these instruments will be, respectively, 0.0088 and 0.035 inches. It is evident therefore that for ordinary station barometers, if an accuracy of 0.01 inch is to be attained, the scale must be within one-half of a degree of the true vertical.

The error at the meniscus is illustrated by the accompanying figure No. 20. The deformation of the shape of the meniscus when the tube is tipped at an angle *i* causes the horizontal point of the surface to be transferred from a to a''; but as the vernier plate remains perpendicular to the axis, the point of tangency moves higher up the scale and becomes a'. Therefore the plane tangent at a' cuts the scale *SO* at a point above that which would have been observed if the vernier plate could have been set upon the proper level a'', and the correction of the scale reading is $-a'\,a'' \sin i \cos i$ for wide tubes, the displacement $a'a''$, and the resulting correction becomes quite large.

59. TRANSFERRING THE LEVEL OF THE MERCURY TO THE SCALE.

In ordinary station and portable barometers a brass tube surrounds the glass tube, and two sight lines, which are the bottom edges of two plates of brass, define a plane perpendicular to the tube; to the front brass plate is fastened the vernier that slides along the scale. The observation consists in keeping the eye in this definite plane, which is slowly raised or lowered until it appears tangent to the mercurial meniscus. By this means the level portion of the meniscus surface is transferred to the scale and its height read off.

In general, for accurate work, it is necessary that the method of transference be the same for the top and bottom of the tube. In all forms of open-cistern barometers, however, it is customary to make the transfer, or its equivalent, in a different manner at the top and bottom, thereby introducing a variable form of error. In most forms the above tangent plane is used for the upper meniscus, but an ivory point, a plane edge, or a saw-tooth edge is provided for the cistern, and each observer usually has his own method of raising the mercury into contact with this point or edge. Observers also have individual peculiarities in making tangent contacts at the top of the meniscus. The readings will vary with the illumination, the cleanness of the mercury surface, the condition of the observer, so that the difference of the upper and lower readings thus made by two different methods of contact has an appreciable variability even with the best observers; but when both contacts are made in precisely the same style the resulting difference is almost entirely free from the observer's peculiarities. The importance of eliminating this source of error has led to the use of some form of siphon barometer and to making contacts or transfers by some one of the following methods, in all of which the fundamental condition is that the diameter of the bore and the size of the meniscus shall be as nearly as possible alike at the upper and lower contacts:

(*a*) *Tangent planes.*—If the tangent plane and vernier, ordinarily used for the upper contact, be also applied by a second vernier to the lower contact, the illumination being the same, personal errors are eliminated, but errors due to the want of exact coincidence between the tangent plane and the zero of each vernier still remain, and the sum of the two errors constitutes a constant correction for all measurements.

For barometers whose reduction to a standard is determined by direct comparison this constant vernier error is combined with others, which are given by the comparison; in this case the lower vernier is replaced by a single index line. But when independent accurate measures are made the sum of the two vernier errors is easily determined by making a number of readings of the same meniscus alternately with each vernier. Since the mean of the readings of the two verniers should agree, any difference between the two constitutes the correction.

If the same vernier be slid down to the bottom of the brass tube, and a similar sighting be made by it upon the mercury in the short leg, then any constant error made in both the sights becomes eliminated when the difference of the readings is taken. This latter method was originally devised for the Wild-Fuess pattern, and is applicable to all forms of the siphon barometer.

(*b*) *Duplicate points.*—In this method a fine cone is firmly attached within the chamber, so that the point of the cone marks the center of the bore, approximately opposite the 30-inch or 760^{mm} point of the scale. A similar point, moving up and down within the open tube, is attached to the lower vernier. In making an observation the mercury is first adjusted to contact with the upper point; then, by means of the lower vernier, the lower point is adjusted to contact in the open leg.

(*c*) *Electric method.*—In this method the points remain as in the preceding, but the contacts are made known by an electric signal running through the mercury and the two metallic points; a bell operated by the current gives the signal at the instant of contact. As the electric current alters the surface tension and capillary correction this method should only be applied to very wide tubes and cisterns, and a duplicate observation be made with reversed current.

(*d*) *Pernet's method.*—In this points are used, but actual contacts are not made. In place of contacts the point and its reflection in the mercury are viewed with the reading microscope of a cathetometer. When the distance between the point and its image is small the horizontal thread of the reading microscope can be made to bisect with great accuracy the distance between the two, and therefore represents the actual surface of the mercury, which is itself frequently invisible. The position of the level plane being thus defined, it is transferred to the vertical divided scale by either one of the processes used in cathetometry; the best of these is that in which the sighting telescope slides upon a vertical axis, about which it can also turn horizontally as shown in Figures 14 and 15. By this means the horizontal thread, when properly set for the top of the mercurial column, is moved horizontally until the observer sees the neighboring divided scale in the field of view, and can read the altitude corresponding to the thread.

(*e*) *Marek's method.*—In this the vertical scale is supposed to be some distance behind the barometer tube; in front of the scale, midway between it and the tube, is a lens, which forms near the axis of the bore an inverted image of the scale. The observer recognizes through his reading telescope the image of the scale division just above the mercurial surface, and below that sees an image of the same division reflected from the mercury. He brings the horizontal thread of the telescope to bisect the distance between the two images, and its apparant distance above the mercurial surface, as measured by the micrometer, is converted into scale divisions by a formula which is a function of the relative distances of the lenses, and of the angle of inclination of the axis of the microscope to the horizon.

(*f*) *Thiesen's method.*—This is similar to that of Marek's, except that the lens in front of the scale is omitted. Through his reading telescope the observer looks over the mercury directly through the glass, and in the ordinary inverted field he sees a given scale division, x, and above it the image x' of the same division reflected; the unseen mercurial surface is, of course, midway between them. The micrometer screw gives the apparent distance between x and x' in terms of the scale division, whence the true height of the mercury surface is $x+\frac{1}{2}(x'-x')$. If an accuracy of only one-tenth of a scale division is required the reading microscope may be replaced by a hand lens and eye estimates of tenths of divisions. This method has the great advantage of bringing the scale close to the glass tube, and both may be inclosed in an outer casing, so as to secure great control over the temperature of the mercury and scale.

(*g*) *Parallelism of the walls of the tube.*—The accuracy of all methods of sighting through glass tubes upon mercury depends in part upon the perfect parallelism of the walls of the tubes. The correction for any want of parallelism may be determined as follows for Thiesen's method:

If the two walls of the tube are so inclined as to change the direction of a ray of light by a small angle, θ, then the ray emergent at 0 (see Fig. 21) deviates by 2θ, and two errors are incurred in the measurement of the height of the surface, viz:

(*a*) The parallel walls m and n displace the original ray SA parallel to itself through a distance

$$d \sec r \sin (I-r)$$

where d is the thickness of the glass, I the angle of incidence, and r the angle of refraction. If the front and rear walls, or those at the upper and lower mercurial surfaces, are parallel, but not of the same thickness, the differential displacement remains as a constant error, and is equal to

$$(d'-d) \sec r \sin (I-r)$$

If the walls are not parallel the residual correction, which is the difference of the upper and lower corrections, is

$$d \sec r \sin (I-r+2\theta)-d' \sec r \sin (I-r+2\theta')$$

(*b*) Besides the preceding differential effects the following occurs: The observer sees a scale division and its reflection in the mercury through the same wall, *i. e.*, that nearest the eye; any difference between the two rays is, therefore, affected by it sensibly; but between the center C of the tube and the scale beyond it comes the further wall (m) with its slightly inclined side, and the angle θ has introduced a deviation such that the actual ray differs from that for parallel surfaces by an amount equal to $d \tan \theta$. The deviation by the wall m having been once produced affects both the direct and reflected ray alike so long as the angle θ does not change.

60. CORRECTION FOR TEMPERATURE OF THE SCALE.

The brass scale has its normal length, according to metric usage, at a temperature of 0° C.; at any other temperature its length is

$$L=L_0(1+\beta t)$$

where β is the coefficient of expansion for the metal of the scale. As the scale divisions, with their numbers, rise with increase of temperature, at any temperature above freezing the scale reading at the top of the column is too low and should be reduced to that which would be observed with the scale at 0°. If the true height of the column is B' in inches at the standard temperature, while the reading is B'' in elongated inches, then

$$B'=B''(1+\beta t)$$

whence the correction is

$$B'-B''=\beta t B''$$

This correction is itself in error in proportion to the uncertainty in the value of the coefficient β, and to that in the determination of the actual temperature of the scale.

Various kinds of brass show differences in their coefficients of expansion amounting to 10 per cent. of the whole. The values of B range from 0.00001744 to 0.00001898 for 1° C., or 0.0000097 to 0.0000105 for 1° F. The average value 0.00001878 (or 0.00001043 for 1° F.) has been adopted by Guyot in the Smithsonian Tables, but for any individual scale this value may easily be in error by 5 per cent. A change of a unit in the sixth place, or about 5 per cent. in the coefficient of expansion, produces a change in the temperature correction of 0.000001 tB', or $0.000001(t-62)B''$ for the Fahrenheit scale and English barometers, in which 62°, instead of freezing, is taken as the standard temperature of the scale. For any ordinary temperature the error is inappreciable.

An error in the determination of the temperature of the scale produces an error in the resulting correction given by the expression $B'\times 0.00001878dt$ (centigrade), or $B'\times 0.00001043dt$ (Fahrenheit); for $B'=$ 30 inches, and an error of 1° F., the error in the reading becomes 0.00031 inches.

The ordinary methods of determining the temperature of the bar are very crude; an accuracy of 1° is rarely attained, and errors of 5° are possible.

The only methods of attaining accuracy in this respect consist either in maintaining the whole room at a uniform temperature or in wrapping the whole barometer in some non-conducting substance, or inclosing it within a large glass tube, adopting Thiesen's method of readings, or best of all by inclosing all that is practicable in a liquid bath, whose temperature is determined by immersed thermometers.

For barometers as ordinarily exposed the errors introduced by erroneous temperatures of brass and mercury are likely to go through diurnal periods that obscure the correct diurnal periodicity of the atmospheric pressure.

61. CORRECTIONS FOR THE DENSITY AND WEIGHT OF THE MERCURY.

The atmospheric pressure on a unit of surface is equal to the weight that that pressure will support, and in the mercurial barometer this weight is that of the mercurial column. But by common usage pressures are expressed in terms of the *height* of the barometric column instead of by its *weight*. The observed height, however, is not a true measure of the pressure, because the height of the column supported by the atmospheric pressure changes with the temperature of the mercury and with the variations of gravity. Therefore, to obtain a height that shall be a true relative measure of the atmospheric pressure the observed height of the mercurial column is reduced to that which would be measured at a standard temperature and under a uniform standard force of gravity. The standard temperature is taken as 0° C., and the standard force of gravity that prevailing at sea level at latitude 45°.

Let P be the atmospheric pressure on a surface, s.

h the observed height of the mercurial column of section s.

Δ the density of mercury.

g the acceleration of gravity.

g_0 the value of g at latitude 45° and sea level.

Δ_0 the value of Δ at 0° C.

t the temperature of the mercury.

γ the coefficient of expansion of mercury.

$g=ng_0$, where n is a function of the latitude and altitude of the place.

Then,

$$P=sh\Delta g=s\Delta_0 g_0\times hn(1-\gamma t)$$

whence the barometric height to be used as a true relative measure of atmospheric pressure becomes

$$\frac{P}{s\Delta_0 g_0}=H_0=hn(1-\gamma t)$$

In correction tables for English barometers it is customary to combine the correction for the temperature of the mercury with that for the temperature of the scale, as given in the preceding article, and assuming that both temperatures are given by one attached thermometer, in this case the corrected reading is given by the formula

$$H_0 = \frac{h[1+\beta(t-62^\circ)]}{1+\gamma(t-32^\circ)}$$

or the total temperature correction to the observed reading is

$$H_0 - h = \frac{h[\beta(t-62^\circ)-\gamma(t-32^\circ)]}{1-\gamma(t-32^\circ)}$$

(*a*) *The density of mercury.*—The density of mercury depends upon its purity; hence, in order that the value of H_0 given by different barometers shall be comparable, the density Δ_0 must be the same in all, or, if not the same, an additional correction to reduce to a standard density must be added. Careful determinations of the density of the purest mercury, as purified in various reliable methods, have been made by Marek with the conclusion that the density may vary by nearly ± 0.0001 of its own value, according to the method of preparation, and that, for accurate barometry, the mercury actually employed must have its specific gravity determined by direct measures. As the result of a large number of observations on different specimens of mercury, purified by different processes, Marek gives the value

$$\Delta_0 = 13.5956$$

and finds that individual specimens differ from this by from 4 to 9 units in the fourth decimal.

The effect of an error of ± 0.0001 in Δ_0 is equivalent to an error of $\frac{.0001}{\Delta}H$ in the measured height of the mercurial column; for $h=30$ inches this error amounts to 0.0002 inch. An amalgam of 1 per cent. of lead alters the density of the mercury by two-thirds of 1 per cent. of its amount, and thereby alters the height of the mercurial column by 0.046 inch.

(*b*) *The effect of compression.*—An additional correction to the density of the mercury, not ordinarily of sufficient magnitude to be considered, has been introduced by Marek (Travaux et Mémoires du Bureau des Poids et Mésures, III, D. 44) for application to the normal barometer. This is based upon the fact that the column of mercury within the tube is compressed by its own superincumbent weight, which varies from 0 at the top surface within the vacuum chamber to the weight of the whole column at the bottom. For different heights of or pressures upon the column, therefore, the average density of the whole column of mercury will be different; but by definition the pressure is measured by the height of a column of uniform standard density, consequently the measured height b must be reduced to what it would be at a standard density corresponding to a standard hydrostatic pressure, b_0. Let the coefficient of compression be c for a unit of pressure, the correction will then be

$$\Delta_1 = -c(b_0 - \tfrac{1}{2}b)b$$

where $c = 0.0000000366$ for a pressure of 1 millimeter. This correction amounts to 0.0011^{mm} for a column 760 millimeters high and becomes appreciable for pressures of several atmospheres.

(*c*) *Effect of error in the coefficient of expansion.*—The expansion of mercury with temperature has been most carefully investigated by Regnault and Broch; the latter (see Travaux et Mémoires, Tome II) presents the results of Regnault's measurements in the formula

$$\Delta = \Delta_0(1 - .000181792t - .000000000175t^2 - .000000000035136t^3)$$

For temperatures at which observations are usually made, namely, about 10° C., the coefficients of t^2 and t^3 may be combined with that for t, and a constant coefficient of dilatation, $\lambda = -.00018153$, may be adopted. The error in the barometer correction introduced by an error of .0000001 in this coefficient is inappreciable in ordinary work.

(*d*) *Effect of error in the temperature of mercury.*—At the standard atmospheric pressure of 750 millimeters an error of 1° C. in determining the temperature of the column of mercury will produce an error of 0.13 millimeters in the barometric reading; an error of one-third of a degree Fahrenheit produces an error of .001 inch. The importance of getting an accurate determination of the temperature of the mercury is thus very evident. In the ordinary portable-station barometer with a single attached thermometer the temperature given by the latter may easily differ from that of the mercury by several degrees; the ordinary methods of obtaining the temperature with sufficient correctness consist in either wrapping the whole instrument in some non-conducting substance, so as to retard changes of temperature, or else keeping the barometer in a room of uniform temperature. An approximate good determination is sometimes attempted by establishing several thermometers in short tubes of mercury, similar to the barometer tube, and placed near it. Differences of temperature amounting to several degrees Fahrenheit between the top and bottom of the barometer have thus been detected. It is evidently more important that the attached thermometer should give the average temperature of the mercury than that of the scale. After a considerable investigation of the subject Broun states (see Nature, 1875) that scarcely any of the many series of hourly observations are reliable to within several thousandths of an inch, owing to the systematic errors introduced by the uncertainties in the correction for temperature.

(*e*) *Correction for changes in gravity.*—The ratio $n = \frac{g}{g_0}$ is known exactly when pendulum observations have been made to determine the value of g at the observing station, g_0 being assumed, as given from the mean results of all pendulum work, as 9.806056 meters per second. When observations of local gravity are not available the value of g is obtained by a formula deduced from geodetic operations and expressing the force of gravity as a function of the latitude (λ) and altitude (h) above sea level. For this formula the International Bureau of Weights and Measures has provisionally adopted

$$g=g_0(1-0.000259 \cos 2\lambda)(1-0.000000196h)$$

In this equation the coefficient of h is obtained on the hypothesis, originally made by Poisson, that the mass of earth lying between the observer and sea level is heaped upon the sphere and is so much attractive matter additional to that at sea level. But a more correct geodetic theory, as well as actual observation, shows that this is not the case. A mountain or plateau is merely a portion of the earth's crust raised by geological action above the present sea-level surface, and beneath such an elevation a deficiency in density apparently exists. Consequently the effect of such an elevated mass is not its whole attractive force, but only the difference in its attraction due to its displacement. This effect is small and diminishes but slightly the decrease in force of gravity with increase of elevation.

For vessels at sea and stations on islands in the ocean, the force of gravity is considered to be diminished by the effect of elevation above the bottom of the sea (considered as a removal from the earth's center), but to be increased by the attraction of the surrounding mass of water and by that of the solid conical elevation that forms the island.

The value 0.0000003 for the coefficient of h makes a slight allowance for displacement, and represents the true attraction much more nearly than the coefficient of Poisson, but the formula and tables of the International Bureau are provisionally adopted by meteorologists.

For accurate work the force of gravity must be determined by pendulum observations at the barometric station; an error of 0.00001 in the ratio $\frac{g}{g_0}$ affects the reduced barometer by $0.0076^{mm}+h$.

62. CAPILLARITY.

The hydrostatic equation of section 61, between the weight of the mercurial column and the atmospheric pressure, is based on the assumption that the pressure of the column is due to its weight only; but two other forces have to be considered, namely, pressures due to capillarity at the two surfaces of mercury, and the elastic pressure due to any gas or vapor in the vacuum chamber. Capillarity is a phenomenon of surface tension, and arises from the attraction or repulsion that exists between the particles of a liquid surface and a solid. The amount of such attraction or repulsion between the same liquid and solid depends on the condition of the surfaces of the solid and of the liquid and on the gas that is present, if there be any. For example, when mercury is boiled in barometer tubes the occluded gases are partially detached from the walls of the tube (they may even be replaced by an invisible film of mercury and its combinations), by virtue of which the capillary repulsion of mercury for ordinary glass becomes partially neutralized, and in extreme cases converted into an attraction.

The tension of the mercurial surface in a barometer tube varies with the purity and temperature of the mercury, with the cleanness of its surface, with the nature of the gas or vapor above it, and with the cleanness of the surface of the glass tube. Strict accuracy, therefore, would require the vertical pressure due to the tensions of the surfaces of the mercury in the tube and in the cistern to be determined for each instrument and at every observation.

(*a*) *The laws of capillarity.*—Let H be the superficial tension of the surface of the mercury when in contact with the gas above it; H_1 the superficial tension between the gas and the walls of the tube; H_2 the superficial tension of the surface of the mercury when in contact with the walls of the tube; θ the angle of capillarity or that at the junction of the meniscus of mercury and the tube.

The laws of capillarity furnish the relation

$$\cos\theta=\frac{H_1-H_2}{H}$$

This formula serves to show the dependence of θ on the three variable tensions, but its value is best obtained by direct measurement on the barometer.

Let h=the vertical capillary pressure in cylindrical tubes, and hence the barometric correction for capillarity.

r=the radius of the tube.

a^2=6.528, a constant that has been well determined for pure mercury, but is subject to variation with temperature and cleanness of surface.

ρ=radius of curvature of the meniscus$=r\sec\theta=\frac{r^2+i^2}{2i}$

i=the height of the meniscus.

Then, as in section 24,

$$h=\frac{a^2}{\rho}=\frac{a^2}{r}\cos\theta=\frac{a^2\,.\,2i}{r^2+i^2}$$

Values of the capillary correction h have been tabulated by Bravais with θ and r as arguments; and by others with the arguments i and r, such as given in the Smithsonian Meteorological Tables.

(*b*) *Neumann's method of observation.*—In order to avoid the error introduced by using an assumed constant value of a^2 Prof. F. E. Neumann proposed (Physik, page 156) a device by which h may be computed more rigorously for the surface of the mercury in the vacuum chamber. But the following extension of his suggestion allows of determining the still more troublesome capillarity correction for the open-air surface. The method consists simply in placing in the vacuum chamber and in the cistern or short leg of the siphon additional tubes whose diameters are considerably less than that of the main tube.

Let b' be the height of the column observed in the small tube.

b that observed in the large tube.

ρ' and ρ the corresponding radii of curvature of the small and large tubes, respectively.

Then for the small tube

$$h'=\frac{a^2}{\rho'} \text{ and } b_0=b'+h'$$

and for the large tube

$$h=\frac{a^2}{\rho} \text{ and } b_0=b+h$$

whence

$$h=\frac{b-b'}{\frac{\rho}{\rho'}-1}$$

This operation is repeated for both the lower and upper chambers of the barometric column.

A more elegant arrangement consists in simply contracting to a smaller bore the upper portions of both the vacuum and the open chambers by inserting appropriate glass tubes; the atmospheric pressure is then to be determined by four sets of measures, in which the upper and lower menisci are located successively in the larger and smaller bores, the position of the mercury being adjusted for this purpose by means of the screw of the cistern. This method is, however, slightly defective, in that it assumes that the glass surfaces in the two upper (or lower) tubes are of the same nature, so that the same value of a^2 obtains in the tubes of either pair.

(*c*) *Cleanliness of surface.*—The effect of the condition of the glass surface is to change the angle θ or the inclination of the mercury to the surface of the glass at the point of contact. This angle varies between 120° and 160° in ordinary barometric tubes. The normal angle is 148° for perfectly fresh clean surfaces. The angle may even become an acute angle in boiled tubes, in which case the repulsion between glass and mercury becomes an attraction, and the mercury may cling to the top of the vacuum chamber.

(*d*) *Measurement of the radius of curvature.*—Ordinarily it has been considered sufficient to compute the correction for capillarity at the upper and lower ends of the column by assuming an average value of a^2 and carefully measuring either the angle θ or the height of the meniscus or its radius of curvature. The radius of curvature has been determined by Wiebe, by Neumann's method, by sighting upon the meniscus with a microscope, in whose field is a ruled glass plate (see Fig. 22), on which may be read off the horizontal and vertical co-ordinates, h and v respectively, of the surface of mercury, from which the radius of curvature is easily computed by the formula

$$\rho=\frac{h^2-v^2}{2v}$$

It is thus found that the curvature of the meniscus in the vacuum chamber is much less and subject to less variation than that of the meniscus in the open cistern or siphon tube. Measures of the simple height of the meniscus are easily made by an index attached to an independent vernier. Thus, in the case of the normal barometer of the Berlin Standards Commission, the height of the upper meniscus varied in a few days between the limit, 1.12^{mm} and 1.37^{mm}, while for the lower meniscus it varied between 2.29^{mm} and 2.60^{mm}. The effect of altering the condition of the sides of the glass tube is illustrated by the following measurements made by Wiebe: Siphon barometers (f) (n) (g) were compared before and after cleaning the open end of the short leg of barometer (g).

Before cleaning, the heights of the menisci in barometer (g) were

Upper meniscus, 0.33^{mm}.

Lower meniscus, 1.52^{mm}.

After cleaning:

Upper meniscus, 0.35^{mm}.

Lower meniscus, 0.88^{mm}.

Showing that the lower meniscus in the open air was diminished to about one-half its height by simply cleaning the tube by washing it with dilute sulphuric acid.

(c) *Effect of atmospheric exposure on capillary correction.*—The capillary pressure ordinarily tends to lower the mercury in the vacuum chamber, and also to lower the mercury in the short leg or open-air cistern; therefore the total effect on the barometric column is the difference of the depressions, and the correction to the barometric reading becomes $h_u - h_o$. The total correction therefore will be negative when the depression due to capillarity at the lower end of the tube is greater than at the upper end. This is the ordinary case in siphon barometers, but in boiled tubes and in narrow tubes with wide cisterns the correction may become positive.

As the surfaces of mercury and glass change their condition with time and exposure to the atmosphere, a steady change in the capillary effect is usually observed, ordinarily producing a steady rise in the barometric reading or an increasing negative correction. In the case of the barometers f and g, above mentioned, the differential capillary correction was reduced from 0.54^{mm} to 0.28^{mm} as the effect of the steady change during a few months in the capillary condition of the surfaces.

Quincke (Wiedemann Annalen, XXVII, 1886, p. 219) has shown that these changes take place very rapidly during the first few hours of exposure of fresh surfaces, but after that more slowly. His measures gave him

$a^2=7.84$ for pure fresh mercury in vacuum.

$=7.07$ for pure fresh mercury in the air.

$=5.01$ for pure mercury, after standing several hours in the air.

Therefore the value adopted in Bravais' tables, as ordinarily used, is only a crude approximation to the value that may actually obtain in a given barometer at a given time.

(*f*) *Effect of temperature on capillarity.*—The effect of temperature on the surface tension is also appreciable. According to Frankenheim, for ordinary temperatures the expression for the capillary constant is

$$a^2=3.978(1+0.001329t)$$

where t is the temperature in centigrade degrees.

(*g*) *Effect of electric current on capillarity.*—The effect of passing a galvanic current through the mercurial column is to dull and flatten the meniscus at the positive pole, but brighten and give greater curvature to the meniscus at the negative pole, thus corresponding, respectively, to a decrease and increase of a^2. Therefore, in using the electric method of contacts, already described, an additional source of error is introduced that can, however, be partly eliminated if care is taken to make the measurements with both direct and reverse currents.

(*h*) *Desiderata.*—A simple method of determining the exact capillary effect for either meniscus in any position is still a desideratum, especially in the case of barometers of small bore, where the total uncertainty is large. In case mercurial contacts are made by the electric method the effect of the current in altering the capillary correction needs careful determination.

Pressures measured when the meniscus is rising differ from those measured when the meniscus is falling, and it is possible that the measurement of the fall of a meniscus from its maximum to its minimum convexity may give a convenient method of determining the surface tension and capillary depression at any point of the tube.

The effect of the unknown variations in capillarity enters directly into the determinations of the correction for imperfect vacuum, and into the comparisons for the reduction of barometers at various localities to a common central standard. The capillarity should therefore be investigated for the different temperatures that occur, and at the different points of the barometer tubes that are used.

(*i*) *Diameter of the tube.*—In computing the capillary correction the inner bore of the tube is required to be known with some precision. This may be ascertained if the tube is first weighed, then filled with mercury, and again weighed. Then we have

$$\text{radius}=\frac{1}{\pi}\sqrt{\frac{\text{total weight}}{\text{length}\times\text{density}}}$$

The following method of determining the radius has been given by Lépinay (see Fig. 23). The method is obviously applicable also to thermometer tubes:

Let the observer at C determine the apparent angles φ and φ', subtended, respectively, by the inner bore and the outer side of the tube

Let $CO=d$, the distance of the observer, be known.

α=the interior angle of refraction OJM of the ray CJM.

$OM=\rho$ =the radius of the bore.

$OJ=R$=the radius of the outer surface.

$CJO'=\beta$.

n=the coefficient of refraction from glass to air.

We have, then, the following relations:

$$\frac{\rho}{R}=\sin\alpha \qquad n\sin\alpha=\sin\beta \qquad R\sin\beta=d\sin\varphi$$

whence, if d be known,

$$\rho=d\frac{\sin\varphi}{n}$$

or, if R be known,

$$\rho=R\frac{\sin\varphi}{n\sin\varphi'}$$

This method fails when the apparent angle φ is greater than φ'; but for this case, which may occur in large thermometer or small barometer tubes, the measures may be made under water, when n becomes the coefficient of refraction from glass to water, which is much smaller than for air, or the value of R may be temporarily increased by surrounding the tube by a concentric cylinder, with water between.

The following method was used by Schumacher (Mém. Acad. Roy. Bruxelles, 1841, Tome XIV): When the pressure is falling rapidly measure the rise, h, of mercury in the cistern that corresponds to a given fall, H, at the top of the column, then the free area of the cistern A being known we find the area of the tube to be

$$a=A\frac{h}{H}$$

whence the radius is found.

The following was used by Bravais (Ann. de Chim. et de Physique, (3), 1842, V, p. 504). Let two marks, s_1 and s_2, be made on the outside surface of the tube whose distance a, parallel to the axis, can be accurately measured. The observer can so place his eye at θ that he will see s_1 reflected from the mercury at m, so that its image is covered by the mark s_2. Measuring the angle i between the line of vision and the normal, and assuming that the bore at m is parallel to the line $s_1 s_2$, and putting n=index of refraction, and R=external radius, we have

$$\sin r=\frac{1}{m}\sin i \qquad d=\frac{1}{2}\frac{a}{\tan r} \qquad \text{radius of bore} = R-d$$

Various modifications of the method of reflection are easily devised.

63. SLUGGISHNESS.

In portable-cistern barometers the construction is frequently such that the free air does not have free access to the cistern, and a change in outside pressure can only be communicated to the air in the cistern by slow percolation through the packings. This defect should be immediately remedied, either by loosening the packing, as in Green's barometers, or by making a special hole through the top of the cistern, as in the Adie barometers. A barometer should always be tested for this defect before being used. The test is easily made by adjusting the mercury in the cistern and setting the vernier plate, then lowering the mercury in the cistern, and, after giving the air time to enter, screwing rapidly up and adjusting to the zero point. If the air in the cistern was confined it will now be considerably compressed, and the mercury will be observed standing above, but slowly settling down to the position of the vernier.

It is customary to contract the lower end of the barometer tube or to introduce a contraction within the tube, in order to diminish the pumping when on board vessels, and also to diminish the chance of the introduction of air bubbles. In consequence of this the ascending and descending motions of the mercury are somewhat retarded. This retard, like the preceding mechanical effect, is so easily remedied that even in extreme cases of rapid changes of pressure it ought not to be allowed to exist. In general it may be said that a barometer will not show a given change of pressure until that change is sufficient to overcome the capillary resistance of the tube to the flow of the mercury. Therefore the time for which an observed air pressure holds good extends backward to the moment since which the changes in pressure have not exceeded the capillary resistance. In order to overcome this capillary sluggishness it is customary to move or tap the barometer, or lower and raise the mercury by the cistern screw, whereby the adhesions to the tube are broken up.

64. IMPERFECT VACUUM.

(*a*) *Arago's method.*—The assumption that there is a perfect vacuum above the mercurial column is tested by means of the method suggested by Arago, which consists in altering the capacity of the vacuum chamber and determining the resulting effect upon the measured pressures p_1 and p_2.

Let v be the volume, under one atmospheric pressure, of a small mass of air in the vacuum chamber, and v_1 the volume of the chamber; then will the pressure exerted by the expanded air, be

$$x_1=\frac{pv}{v_1}$$

p being the pressure of one atmosphere. For another size of the vacuum chamber we shall have

$$x_2=\frac{pv}{v_2}$$

If, now, the atmospheric pressure P has not changed while the volume of the chamber has been altered from v_1 to v_2, and the barometric height has changed from p_1 to p_2, we shall have for these two cases

$$P=p_1+x_1=p_1+\frac{pv}{v_1}$$

$$P=p_2+x_2=p_2+\frac{pv}{v_2}$$

therefore

$$pv=\frac{p_1-p_2}{\frac{1}{v_2}-\frac{1}{v_1}}=(p_1-p_2)\cdot\frac{v_1v_2}{v_1-v_2}$$

whence the correction

$$x_1=\frac{p_1-p_2}{v_1-v_2}\cdot v_2=(p_1-p_2)\cdot\left(\frac{1}{\frac{v_1}{v_2}-1}\right)$$

The change of volume of the vacuum chamber is effected by raising the level of the mercury in the cistern or open leg of the siphon; thus the volumes of the chamber and the corresponding corrections are considered to pertain to the readings of the upper end of the mercurial column, the head of the glass tube being fixed with regard to the scale.

If the correction is considerable then the pressure will vary appreciably with the temperature of the inclosed air. In fact, the operation of diminishing the volume compresses and warms the air, wherefore the observations must be so made as to give time for the chamber to cool.

Let v_1 x_1 t_1 be the volume, pressure and temperature prevailing in the vacuum chamber at the first observation, and v_2 x_2 t_2 the same data for the second observation; let D_1 D_2 be the true readings of the mercurial column, so that $A-D_1$ and $A-D_2$ are the corresponding distances in scale divisions to the top of the vacuum chamber; therefore $(A-D_1)S$ and $(A-D_2)S$ are the desired volumes of the chamber.

The pressures are therefore as before

$$P=p_1+x_1=p_1+p(1+0.00365t_1)\frac{v}{(A-D_1)S}$$

$$P=p_2+x_2=p_2+p\frac{(1+0.00365t_2)v}{(A-D_2)S}$$

whence

$$pv=\frac{p_1-p_2}{\frac{(1+\alpha t_2)}{v_2}-\frac{1+\alpha t_1}{v_1}}=\frac{(p_1-p_2)v_1v_2}{v_1-v_2+\alpha(t_2v_1-t_1v_2)}$$

With (pv) thus determined the correction for any other temperature or reading of the upper end of the column becomes

$$x=(pv)\frac{1+\alpha t}{v}$$

In the determination of this correction, which is itself quite small, it is sometimes necessary to take account of the fact that as the mercury is raised in the closed and open tubes, in order to diminish the volume of the chamber the last measure corresponds to the atmospheric pressure at the then level of the mercury in the open leg, and needs to have a reduction applied to it to obtain the pressure at the original level. This is best done by reducing all measures to a standard level in the atmosphere, *i. e.*, by correcting for the weight of the atmosphere between the zero point in the scale and the surface of the mercury in the open leg. This correction is given by

$$+0.0000951(1-0.0037t)\frac{p}{760}D_0$$

where D_0 is the scale division read off from the open mercury surface, p the approximate pressure and temperature of the air, and 0.0000951 the height in millimeters of mercury corresponding to the weight of a column of 1^{mm} of air at normal density.

The variation of capillarity with the temperature and the condition of the surface of the mercury appreciably affect the determination of the correction for defective vacuum. The necessity of measuring pressures for very different heights of the mercurial column introduces the possibility of extensive differences in the condition of the bore of the tube and in the resulting capillarity; it is at present the current tendency to attribute to these the perplexing anomalies that occur in the attempt to determine the vacuum correction. The following illustrations of such determinations show the anomalies that occur.

(*b*) *Anomalous experiences.*—In the normal barometer II, at the International Bureau of Weights and Measures, the correction $\frac{pv}{v'}$ for the standard reading of one atmosphere, when the upper column reads

811.0 on the scale of millimeters, was $+0.708^{mm}$ in June, 1882 (see Travaux et Mémoires, III, D, 47).

The auxiliary barometer No. 1 gave the following values:

	mm.
1879, July 11	+0.175
Aug. 1	+0.170
Sept. 1	+0.167
1880, Mar. 2	+0.088
Mar. 10	+0.097

(See Travaux et Mémoires, II, D, 11.)

Auxiliary barometer No. 3, after elaborate efforts to expel the air, gave the following corrections:

	mm.
1880, Aug. 12	+0.612
Sept. 1	+0.624
Dec. 13	+0.445
1881, Jan. 5	+0.408
Feb. 25	+0.473

(See Travaux et Mémoires, II, D, 13.)

The auxiliary barometer W. II, filled under a vacuum by Wild's method, had no correction for air in the chamber, so far as could be ascertained up to 1883 (see Travaux et Mémoires, IV, B. 59).

The Wild-Pernet normal barometer, as studied in 1884, gave 0 as the correction for vacuum.

The attempt to determine the vacuum correction during a journey with one of the Wild-Fuess barometers led Schreiber to conclude that the laws of gaseous pressure changed at these low pressures; but although this is true according to the kinetic theory of gases, and is appreciable at the lowest pressures, yet, probably, in Schreiber's observations the effect of this change is far smaller than the errors due to variable capillarity and uncertain temperatures. Krajewitsch finds that this effect on the low barometric pressure in the vacuum chamber is much less than the Boyle-Mariotte law would give.

The Arago method is limited in its application by our ignorance of the specific capillary effect at every measure that is taken, and the great desideratum at present is a convenient method of accurately observing this effect; several methods, refined and tedious, have already been described in the section on capillarity. In addition to those it is pos-

* The memoir of Krajewitsch (Exner, Repertorium, XXIII, p. 339), received since writing this, maintains that barometers filled with cold pure mercury under the Sprengel pump vacuum by himself and Mendeleeff, always show a perfect vacuum to less than 0.003^{mm}; after many weeks a small air bubble may accumulate, and this he removes to an air-trap and isolates from the barometer. Sundell, in a method just published, appears also to have virtually filled his tubes under a vacuum, but the vacuum correction remains appreciable.

sibly desirable to develop a method that consists in measuring the height of a meniscus when the convexity is a maximum; and again when the mercury has been flattened to its maximum extent by lowering the cistern screw, and just before the sides of the meniscus break the connection at any point with the sides of the tube.

(*b*) *Electric method of testing.*—An independent test of the perfection of the vacuum has been attempted, notably by Krajewitsch, Mendeleeff, and Grunmach. An electric discharge between electrodes varies its character with the perfection of the vacuum, the nature of the residual gases and vapors, and the nature of the electrodes. In the vacua ultimately obtained, when a barometer tube is being filled under the working of a Sprengel pump, only rarely do electric discharges take place through it. According to Grunmach all the spectrum lines disappear, except the *F* and *G* lines of hydrogen and those of mercury, and fluorescent light appears; after a time, however, this fluorescence disappears, owing to the general liberation of foreign matter by the tube and the mercury. Now, let the Arago method be executed while the pump is so working that the electric light still presents the appearance peculiar to a vacuum as perfect as can be obtained, and there will still be found as the result of the computation a slight correction for vacuum, which, for the barometer used by Grunmach, was -0.4^{mm}.

Experience seems to show that tubes filled under a Sprengel pump have more perfect and more constant vacua; but the determination of the vacuum correction even in the best work has not yet been made independent of the changes in density of mercury, due to its temperature, and the changes in capillarity, due to temperature, oxidation, and other influences, including the occasional electrified condition of the tubes and the mercury.

(*c*) *Correction for mercurial vapor.*—If both a liquid and its vapor, such as water, be present in the vacuum chamber, the Arago method does not suffice to detect the full effect of the vapor, since the latter is condensed upon its liquid as the volume diminishes. A barometer in this condition is therefore unserviceable. The only vapor that is present in a well-made barometer is that of mercury; this vapor adds its own pressure to any already prevailing in the vacuum chamber. The small correction thus indicated is adopted by Marek from the determinations of Hagen and Hertz, and is as follows (see Travaux et Mémoires, III, D, 43):

Correction for pressure of vapor of mercury.

Temperature.	Correction.
°C.	*mm.*
0	+0.0002
10	.0004
20	.0013

(*d*) *Surface adhesion.*—It is now known that air, aqueous vapor, and carbon dioxide are all strongly adherent to the surface of glass, thus Bottomley (Nature, March, 1885, XXXI, page 423) dislodged 0.45 and 0.41 cubic centimeters of gas from masses of spun-glass thread, whose total surface was 1,448 and 3,527 square centimeters, respectively, and chemical analysis showed that the gases thus dislodged were composed of the following mixtures:

Gas.	Analysis.	
	Weight.	Volume.
CO_2	78.6	8.24
O	10.5	22.76
N.	89.5	69.00
	168.6	100.00

Kayser and Bunsen have shown that a thin layer of water adheres to glass threads even up to a temperature of 500° C., and that this absorbs carbon dioxide from the air and retains it. From these layers of water, vapor, and gas may come the small amount of gas present in nearly every barometer, and their variable effect on surface tension becomes a part of the variability of the correction for capillarity.

The dislodgment from the mercury or the walls of the tube of any occluded gas is usually considered a reasonable explanation of a part of the slow change in the vacuum correction, and there seems no reason why sometimes the walls of the tube and the mercury should not absorb certain gaseous constituents from the vacuum chamber; but any changes produced in this manner must be slow, although subject to great variability if high temperatures or electrification occur.

(*e*) *Air-trap.*—The usual cause of deterioration in the perfection of the vacuum in portable barometers is the escape into it of the visible bubbles that are to be seen clinging between the mercury and the tube. This source of error is easily remedied by adopting the construction due to Bunten, and shown in Fig. 24, where *a b c* represents a conical glass funnel attached to the inside of a tube at *ab*. The hydrostatic pressure is freely transmitted from above through *c* down to the cistern; but the region marked *TT*, between the walls of the tube and the sides of the funnel, constitutes a trap in which any ascending bubbles are retained, so as not to affect the vacuum above. In filling the barometer, the top of the tube being held downwards, when a little mercury has passed through *c* into the vacuum chamber, it is warmed until the chamber is full of its vapor and the air thoroughly expelled (or if filled cold the Sprengel pump extracts the air), when, of course, mercury will flow in as the vapor is cooled down. This construction therefore simplifies the filling of the barometer tube, since one is relieved of the necessity of further boiling and of anxiety about air bubbles.

(*f*) *Mendeleeff's method.*—This is applicable only to special investigations, and consists in elevating the mercury until the top of the tube is filled, and any air is driven on into a capillary tube, where it is isolated by applying a blow-pipe flame and closing the tube.

65. INTERNATIONAL COMPARISON OF BAROMETERS.

As normal barometers can not be safely transported it has been customary for each country to establish its own standard barometer with which to compare those issued to its own observers. It therefore is important that the meteorologist should have some means of reducing indications of local standards to an acceptable international normal.

To this end the three normals established at the International Bureau of Weights and Measures have been accepted by meteorologists, but as yet only a few comparisons have actually been made.

It is evident from the preceding analysis of barometer errors that slight differences in the scales, the temperatures, the coefficients of expansion, the density of the mercury, or in the determination of the errors of capillarity and vacuum, will introduce a series of systematic discrepancies between any two barometers, which will vary with temperature and scale readings, and may be expressed by the general formula

$$\delta = A + B(b_1 - b_0) + C(t_1 - t_0) + \text{etc.}$$

Therefore the proper formula for reduction of pressure at several stations to a common standard involves the direct comparison of the barometers through at least as large a range of pressure and temperature as ordinarily occurs, and the determination of the constants A, B, and C. It is, however, common to avoid this great labor and determine merely one value of δ for an average temperature and pressure, namely:

$$\delta_1 = A + B(b_1 - b_0) + C(t_1 - t_0)$$

and it is assumed that this value of δ holds good for the whole range of observations. The error of this assumption may be very appreciable if the standards are at very different elevations above sea level.

In order to compare the normal barometers a portable barometer of the best class is used (those of Krajewitsch, Wild, and Koeppen are most desirable), but as no portable can be so accurate as the normals, the intercomparison is by no means decisive as to the exact relation of the barometers. Therefore there will still remain some uncertainty as to whether the concluded difference between two normals may not be at least partially due to errors introduced by the portable barometers and the manner of using them.

In order to diminish the uncertainty the following conditions of a good comparison should be adhered to:

(1) The portable barometer should be of the siphon form with adjustable cistern, very much as made by Fuess, and should be used with strict attention to all sources of error hitherto described, especially capillarity and temperature.

(2) The same observer should do all the work with the portable barometer.

(3) The personal difference between the observers should be determined at each station.

(4) The comparisons should be made at times when the atmospheric pressure, as shown by the readings themselves, is very steady.

(5) The comparison of the portable, first at *A* and then at *B*, should be followed by a second series, first at *B* and then at *A*.

During the past century a large number of international comparisons have been made, although not always according to the preceding outlined conditions.* The general result of these has been to show that large outstanding differences exist between so-called standard barometers throughout the world. The reductions to normal range between $+0.5^{mm}$ and -1.5^{mm}. The minus corrections, however, are most numerous and largest. This appears to be generally due to changes in the surface tension of the meniscus, especially at the open end, by which all barometers suffer a secular deterioration. But the personal differences or methods of making the upper and lower contacts in cistern barometers is a fruitful source of trouble.

The most important "normal barometers" at present existing are those at St. Petersburg, Berlin, and Sevres, near Paris, as represented in Figs. 14, 15, and 16. In these instruments every refinement in construction and observation has been introduced, but on account of the labor of using these normals local substandards are ordinarily employed, whose relation to the normals are known by frequent comparison.

Comparisons between the normals have been made by means of the Wild-Fuess portable barometers; not directly, however, but through the local substandards. These show the general result that even between the normals apparent differences of two-tenths of a millimeter still exist. These differences are far greater than any probable accidental error, and point to constant sources of error still uncorrected, but whose origin is not yet fully agreed upon; according to some, they are due to changes in the substandard, and such changes are shown to be appreciable (see Wild, Repertorium, VIII, Beilage, p. 36).

There must certainly be, even in the normals themselves, a possible accumulation of errors due to inevitable errors in the density of mercury, the length of scale, the attached thermometers, the methods of transference, unknown local variations of gravity, and of the gases in the vacuum chamber; but these will partly balance each other, and the resulting total constant error can hardly be so large as a tenth of a millimeter for such thoroughly investigated instruments. The most probable explanation of the larger differences, which are found also to be variable, is that they are mostly due to a variable capillarity, from whose influence normals and substandards are not wholly free, but to which the smaller portable barometers used in the comparisons are very much exposed. This source of error similarly affects all siphon

* See the publications of Hellmann, Waldo, Sundell, etc.

barometers, such as that of Fuess, and any unknown changes in such barometers therefore appear in the result as an apparent difference between the normals. The only remedy for this is a most thorough study of capillarity in connection with any international intercomparison. Until this is accomplished it is impossible to say what is the precise relation between the principal normals of the world; but the care with which these latter have been investigated would lead to the reasonable expectation that neither can be in error by one-tenth of a millimeter or four one-thousandths of an inch.

The most important and probably decisive study as to the origin and prevention of these differences between normals would consist in the direct comparison among themselves of the three normals by Pernet, Wild, and Marek, respectively, now established at the International Bureau of Weights and Measures. In this work no portable or sub-standard barometer would need to intervene, and the only source of discrepancy would be that due to currents of air in the room, and the different effect of the external wind upon the pressure in the three different rooms in which these instruments are placed. Even these effects can be prevented by connecting the three open siphons together by one long closed tube, so that hydrostatic equilibrium shall prevail at the three mercurial surfaces. The results of such comparison, which, it would seem, has already engaged the attention of the physicists at that institution (see Procès-verbaux, 1885, page 96), will be looked for with interest, since it is as important to meteorology as it is to metrology.

66. EXPOSURE OF BAROMETERS.

The preceding sections have enumerated the corrections peculiar to the barometer, by applying which the elastic pressure in the layer of air immediately above the open surface of the mercury is obtained.

The problem of the meteorologist is to determine the pressure in the free air; but this is generally in motion; it is necessary therefore to consider the question how to expose a stationary barometer so that it will give the pressure in the moving free air.

When a current of air strikes any obstacle there is produced a variable distribution of the pressure which was before uniform within the current, consequently a local current in the room blowing across the open end of the cistern of the barometer causes the pressure upon the mercury to differ from that in the still air of the room. The first condition of a proper exposure is, therefore, that the air in the room should be quiet.

Similarly during a wind an irregular distribution of pressure exists in the air on all sides of a building, and within the chimneys leading to the interior of the rooms within; consequently the general pressure prevailing within a room represents an equilibrium between the outside pressures that have access to it. Thus a room open only on the windward side has a greater pressure than one opening only on the leeward side, and rooms with windows and chimneys will show intermediate pressures depending on the relations of the openings to the wind. The maximum effect of the wind in condensing the air on the windward side of an obstacle is shown by the corresponding increase of pressure, as given in the following table (see also Chapters XV and XVI):

Velocity per hour.	Increase of barometric pressure.
Miles.	*Inches.*
20	0.015
40	0.061
60	0.138
80	0.245
100	0.383
120	0.548

(*a*) *Observations on Mount Washington.*—The above maximum effect is diminished by openings and leakages as illustrated by the following results, attained at the Signal Service station at Mount Washington, where the several rooms are so connected that the specific influence of any one opening could not be easily separated from that of the leakages over the whole building; but by opening certain leeward and windward windows the following differential effects were observed. The effect of opening a windward window (whose aspect is given in column 8) as diminished by the counteracting effect of all leakages from the room is given in the last column. The effect of opening a leeward window, as diminished by all leakages into the room, is given in column 7; column 5 shows that the effect of opening a flue is masked by the leakages, as might have been expected, on account of the small size of the flue relative to the volume of the room and the leakages:

1886.	Wind.			Effect on barometer of opening the—				
	Direction.	Average velocity. (Miles per hour.)	Gusts. (Miles per hour.)	Chimney flue.	Leeward window.		Windward window.	
					Aspect.	Effect.	Aspect.	Effect.
				Inch.		*Inch.*		*Inch.*
Sept. 29	W. NW.	65	85	—0.005	E. NE.	—0.014	W. SW.	+0.029
Sept. 29	W. NW.	65	85	+0.003	E. NE.	—0.008	W. SW.	+0.045
Oct. 15	W. SW.	70	85	+0.006	E. NE.	—0.011	W. SW.	+0.045
Oct. 21	W. SW.	65	80	+0.003	E. NE.	—0.018	W. SW.	+0.062
Oct. 22	N. NW.	70	90	—0.006	E. NE.	—0.025	W. SW.	+0.032

The above figures in each case give the difference between the pressure observed with the mercurial barometer with everything closed and with a window or flue open. Observations with an aneroid gave similar results (see S. S. Monthly Weather Review, March, 1887).

(*b*) *Standard exposure.*—The general theoretical relation between the pressure in the free air and that within a room where large openings are closed and only the smaller leakages control the interior pressure is easily shown. The above Mount Washington observations give an average effect of about two-thirds of the maximum effect due to the prevailing wind, and suffice to confirm the conclusion originally announced by Col. Sir Henry James, in 1854, that the correction for the dynamic effect of the wind must be applied before observations made during high winds can be used for obtaining the static pressure in or elasticity of the free air.

But the determination of this dynamic effect of the wind for every special building or room and every direction and velocity of wind is difficult and impracticable; neither is it sufficient to carry the barometer into the open air, since the effect of any obstacle in the vicinity as well as that of the barometer cistern still remains to be considered. The only satisfactory method consists in connecting the barometer cistern by a closed tube with some point in the open air, the pressure at which, after allowing for the effect of the wind, has a definite established relation to the static pressure in the free air. Several methods have been proposed of allowing for this conversion by obstacles of kinetic energy of wind into static or barometric pressure, but it is best, if possible, to so expose the barometer that such effects will be avoided altogether, or at least reduced to an inappreciable quantity. The method given in the section on pressure anemometers (see Fig. 55) suggests the simplest and best solution of this problem as yet offered. This consists in establishing in the open air two large circular horizontal plates a short distance above each other, as shown in Fig. 25; from the center of the lower plate a tube extends to a small chamber that incloses a barometer, or the tube may extend simply to the open leg of the siphon barometer. If the opening O of the tube is precisely flush with the disk AA, and the wind between the plates moves perfectly parallel to AA, then no dynamic effect is produced at O, and the pressure within the tube is the same as that between the plates and in the free air around them. By subtracting from the pressure P the weight of the vertical column of air in the tube between O and P there results the correct static air pressure at O.

Strictly speaking, the plane of the plates should be parallel to the wind, but the effect of a small inclination is inappreciable. The distance apart of the plates is indicated by the fact that in this method it is assumed that the air between them has a steady flow in lamina parallel to the plates. This condition is not realized whenever any outside disturbance introduces the turbulent flow discussed by Sir William Thomson in the Philosophical Magazine, September, 1887. The condition of stable flow, as verbally given by him at the Manchester meeting of the British Association; for water must also obtain approximately for air, and for the present problem may be expressed as follows:

Steady motion becomes easy and turbulent motion becomes difficult when the distance between the plates is equal to or less than the diameter of the plates multiplied by the ratio —coefficient of viscosity of the air divided by coefficient of skin friction of the air on the plate.

CHAPTER IX.

MISCELLANEOUS BAROMETERS.

The mercurial barometer is not to be transported without great care and much risk of accident. The effort to obtain a more portable and less expensive instrument has led to the invention of several forms, which are in general designated as differential or interpolation instruments, because they can give true pressures only in so far as they are checked at several points of their scales by comparison with standards. Among these modified forms the most important are the sympiesometers and aneroids, designed as portable instruments, and the Moreland and Howson patterns, which are not portable, but by their enlarged scales allow slight changes in pressure to be easily recorded.

67. ANEROID BAROMETERS.

The aneroid measures change in atmospheric pressure by their effects in bending some form of metallic elastic spring. The two principal types are the Vidi and the Bourdon. In the Vidi, which is the earliest invention,* an elastic spring is attached to the ends of a flat cylinder whose top and bottom are elastic corrugated surfaces, which are kept from touching each other by the spring. The air is in great part exhausted from within the box, and the spring therefore bears up the greater part of the atmospheric pressure, but by its elasticity yields slightly to every change in pressure. In order to observe the motions of the spring they are greatly magnified by levers, cog-wheels, wheel and axle, microscopes, reflecting prisms, or other means. In the Bourdon aneroid the elastic spring is a thin, hollow, metallic ring, nearly exhausted of air, either circular or spiral, and whose cross-section at any point is an ellipse, whose longer axis is perpendicular to the plane of the ring. In this instrument changes of external pressure produce a change of curvature in the circular axis of the ring, whose closed ends therefore approach or recede from each other, and their movements are measured as before by either mechanical or optical means. An elementary analysis of the relation between the motions of the cylinder and the elastic pressure is given by Rev. E. Hill, in the Quarterly

* Paris, Comptes Rendus, 1847, T. XXIV, p. 975.

Journal, London Meteorological Society, Vol. 1, 1871–'73, p. 51. This principle is applied especially in the construction of the Bourdon steam-gauge; by filling the ring with liquid the instrument becomes a thermometer.

Aneroids are not designed to give absolute and independent measures of pressure, but serve as very sensitive instruments of interpolation. The value of their scale divisions must therefore be determined by comparisons at different pressures and temperatures with a standard mercurial. In some forms of instruments a mechanical temperature compensation is introduced, but even in these the conversion of the aneroid readings into standard pressures demands an occasional study of the effect of the temperature of the instrument and of the pressure by means of comparisons with a standard barometer. A slight secular change in the reductions usually exists, depending on the gradual changes in the elasticity of the metal, and such changes are especially aggravated when the instrument is strained by subjection to a wide range of pressure. As the measurement depends on elastic forces, no correction for changes in the force of gravity is required when the observer changes his location.

The best aneroids are those made entirely of well balanced metallic parts, as in the "anéroïde holostérique" of Naudet, or those on the Bourdon pattern, but especially those of still simpler construction, where the small motions are measured by micrometer screws, as in the Goldschmid aneroid and those patterned after it.

68. SYMPIESOMETER.

This was designed as a more convenient portable form of barometer. The instrument owes its present form to the studies of August, but its earliest form was a modification by Hooke of the earliest air thermometer of Galileo. As constructed by Kopp, and Brunner it consists essentially of a cistern and short vertical tube. In the latter is a short column of mercury, above which a column of air is introduced in place of the vacuum (see Fig. 26). The measurement consists in determining the height of a column of mercury required at any time to keep the inclosed air compressed into its standard volume. One such measurement made when the atmospheric pressure is also simultaneously observed by means of a standard barometer, gives the coefficient (c), by which at any time the observed height of column (h) corrected for temperature and gravity may be converted into true barometric pressure (b) by the formula $b=ch$.

69. THE MORELAND BAROMETER.

In this form of the mercurial barometer the tube B (Fig. 27) is attached to one end of a balanced beam, $R\ F\ L\ W$, whose other end is bent at L and carries the heavy weight W. The moment of pressure due to this weight is Wl sine α, where l is the lever arm and α the inclination of the beam to the horizon, or of the index arm F to the vertical. Any change in atmospheric pressure causes the barometer tube B to sink or rise with reference to the mercury in the basin M. An equilibrium is maintained by the corresponding change in the moment of pressure of the weight W. The position of the index F is read off on a scale placed behind it, and these readings are converted into atmospheric pressures by means of a few comparative simultaneous readings of a standard barometer.

70. THE HOWSON BAROMETER.

This instrument is a modification of the balance barometer, the counterpoise being a part of the barometer itself. It was first described by R. Howson (Proc. Inst. Civil Engineers, 1861; Proc. Brit. Met. Soc., I, page 81). By it a very open scale may be secured, and the changes in the position of the index proceed with sufficient force to make a direct record upon the registering sheet. This form has therefore been adopted in the "King Barograph" at the Liverpool observatory. In the Howson barometer (see Fig. 28) the glass tube ABB is firmly attached to a frame, FF; the cistern bb and the long rod C move automatically, so as to preserve a balance between the upward atmospheric pressure at the bottom of the cistern and the weight of the mercury that it carries.

There is attached to the cistern the long vertical glass rod C that penetrates the bore of the barometer tube; the pressure upon the top of this rod is very small (being that of a small column of mercury) which, with the weight of the rod, represents the downward pressure at the point where it joins the cistern; the upward pressure at B is the full atmospheric effect, and the nearly constant excess suffices to keep the cistern and its mercury from falling away from the tube down to the ground. When the pressure rises the rod and cistern are lifted, and *vice versa*, so that the difference in level of the mercury in the tube and cistern is that corresponding to the new pressure. The amount of rise or fall of the cistern necessary to accomplish this result depends upon the ratio between the areas of the horizontal sections of the mercurial columns in the cistern and in the tube.

71. THE INTEGRATING BAROMETER.

By reversing the pressure compensation attached to astronomical clocks the clock rate can be made to depend largely on the atmospheric pressure, so that it becomes an index of the changes in pressure that have prevailed during any given interval. This idea appears to have been first embodied in an apparatus by G. F. Hall, exhibited at the London World's Exhibition of 1851; Rankine in 1853 described an improved form. About 1882 W. F. Stanley constructed his chrono-barometer, but his report in 1886 (see Quarterly Journal, Royal Met. Soc., XII, p. 116) of the first three years' work shows that the apparatus needs simplification before it can become widely used. Stanley's chrono-barometer consists essentially of a pendulum clock, whose pendulum is a glass or steel barometer tube whose cistern is an open-air chamber; every rise in atmospheric pressure causes the mercury to rise in the barometer tube and gives the clock an increased gaining rate. His own instrument gained at the rate of twenty-five minutes daily for a rise of 1 inch pressure.

NOTE.—For a general description and bibliography of barometers, see W. Ellis Brief Historical Account of the Barometer (Quarterly Journal of the Royal Meteorological Society, July, 1886, Vol. XII.

CHAPTER X.

BAROGRAPHS.

72. GENERAL CONDITIONS.

In the construction of self-registering barometers there has not been, until very lately, an appreciable direct progress towards perfection. This is because the automatic register is merely a mechanical device, and each new form of instrument is usually constructed on a different principle from the previous ones. Thus we have several totally distinct systems of self-registration; barographs constructed on the same mechanical principle will often differ somewhat in the details of construction, and thus each new style of instrument has introduced new sources of error without eradicating the old; the object has seemed to be to devise a new, a cheaper, or a more portable form without special endeavor to secure higher accuracy.

Under each of these systems the following points are to be considered in deciding upon the excellence of any barograph:

(1) The instrument must be automatic and require the presence of an observer as seldom as possible to make changes.

(2) It must have as nearly as possible the accuracy of direct observations.

(3) It must be self-compensating as regards changes of temperature.

(4) The curves should indicate directly the changes of pressure.

(5) The direct control observations should not be required oftener than tri-daily.

(6) The instrument should be stable and not liable to get out of order easily.

(7) The registration should be continuous and on a sufficiently large scale.

(8) The instrument should not be too expensive.

(9) Errors of capillarity should be avoided.

(10) The time scale should be open enough to be read easily within a half minute, and be accurately regulated by a standard clock whose error is known at any time.

(11) The scale division should have uniform values expressed in atmospheric pressures.

73. HISTORICAL DEVELOPMENT.

Is was early recognized that some method of preserving a record of the fluctuations in the readings of barometers was needed for the use of meteorological observers, as a sufficient number of direct observations can not be made without much labor and expense. Accordingly, as early as 1670 or 1680, Samuel Moreland, an Englishman, invented* his so-called steel-yard barometer (the modern balance-gravity barometer), and exhibited it to King Charles II. One of the same kind was made by Adams for George III in 1780,† and Luz, 1784, mentions the fact that there were two of the same kind in existence in 1780.‡ The laxity in regard to meteorological observations, however, caused the consideration and general adoption of self-registers to languish until towards the middle of the present century. The early barograph-records only indicate in a general way the curve of barometric pressure, because the nature of the errors and the proper method of reduction of the instruments were but little understood, and because the comparison barometers were quite inaccurate.

The various forms of barographs have been described in papers scattered through scientific periodicals and reports. The best collected accounts are by Radau,§ Études sur l'Exposition, Paris, 1867, and Carl's Repertorium, 1867, III; Wild, in his Memoir on Barometers,‖ and Ellis's History of the Barometer (Quar. Jour. Royal Met. Soc., 1887). Most of the various forms at present in use can only be briefly mentioned here. Barographs are to be classified as (I) mercurial and (II) aneroid, and these again are divided into the following four systems, according to the general principle of registration: (1) Photographic, (2) mechanical; (3) electrical; (4) balance (gravity).

74. MERCURIAL BAROGRAPHS.

(a) *Photographic method.*—This manner of self-registering was suggested by C. Brooke,¶ in 1847; and also in a paper of the same year by Ronalds.** The modification now used is known as the Kew system,

* See Pogg. Ann., Bd. 133, S. 430, 1868; Encyclopedia Britannica; Rees's Encyclopedia; Hutton's Math. Dictionary, Vol. I, p. 208; Gehler's Phys. Wörterb., Bd. I, S. 774; Marbach's Wörterb., Bd. I, S. 157 der ersten Auflage, or S. 673, der zweiten Auflage; August's Wörterbuch, Bd. 1, S. 245; Radau, Études sur l'Exposition de 1867, Paris, 1867, p. 18.

† Joh. Friedr. Luz, Obercaplan zu Gunzenhausen, 1784, Vollständige . . . Beschreibung von allen . . . Barometern, etc., nebst einem Anhang, seine Thermometer betreffend.

‡ J. G. von Magellan's Beschreibung u. s. w., Leipzig, 1782; Rozier's Journal de Physique (Paris, 1782).

§ Radau, Zur Geschichte und Theorie des Wagebarometers, Pogg. Ann., Bd. 133 (1868).

‖ Ueber die Bestimmung des Luftdrucks, von H. Wild, Repertorium für Meteorologie, III, 1874, herausgegeben von der Kaiserl. Acad. der Wissenschaften zu St. Petersburg.

¶ Ch. Brooke, On the Automatic Registration of Magnetometers and other Meteorological Instruments by Photography, Philos. Transact. for 1847, P. I, p. 59 to 69.

** Ronalds, On Photogr., Self-registr., Meteorol. and Magnet. Instruments, Phil. Trans., London, 1847, p. 111.

it has been described in many text books; illustrations are given in the catalogues of English meteorological instrument makers. For a very full description see the report of the meteorological committee of the Royal Society for 1867. The process consists essentially in getting a continuous photographic record of the varying distance between the top of the mercury column in a barometer with large cistern and a fixed line above the mercury column. Thus, if A is the distance between the mercury surface in the cistern and this fixed line, and B is the total height of the column, the photograph will show the distance $A-B$.

These barographs are in official use at Kew, Greenwich, Oxford, Aberdeen, Armagh, Stonyhurst, Glasgow, Valentia, Falmouth, Brussels, San Fernando (by Salleron), Lisbon, Coimbra, Toronto (one was active for several years at Washington). By allowing the "fixed" line above the mercury column to rise and fall with temperature the resulting records are approximately corrected for the temperature, but the capillary irregularities of the mercurial column must still be noticeable, since the meniscus varies in all ordinary size barometer tubes as the mercury rises or falls in the tube. The mean error of a comparison of this photographic curve with a barometer observed directly is given by Carpmael as about ±0.0026; the greatest discordance in three months' work was 0.006 inch. The photographic method is not wholly satisfactory for several reasons, in addition to the expense and the care necessary in attending it. The records lack the clearness of registration that is desirable; the advantage that no work is required of the mercurial column is counterbalanced by the want of definite knowledge as to the influence of hygrometric and thermal influences on the photographic paper, and the inability to photograph in a few seconds the sudden changes of pressure that frequently occur. Finally the time scale is so small that changes within a few minutes would not be recognizable even if the photographic process could record them, so that the simultaneity of other registered phenomena with those of the barometer can not be established with any degree of refinement. In general, however, it may be allowed that as constructed by the English makers the contracted scale for time and pressure is quite in accord with the sluggishness of the barometric and photographic processes.

(*b*) *Mechanical methods.*—Under this class comes the barographs of Kreil,* Lamont,† Buyson,‡ Hough§ (which combines mechanical and electrical), Redier,|| Negretti and Zambra, and others.

These instruments, by purely mechanical means, give the fluctuations of the mercurial column as represented by the motions of a float in the

* Kreil, Magnet. und meteorol. Beobachtungen zu Prag, Bd. 3, S. 131, Prag, 1843.

† Lamont, Beschreibung der Neuen Instrumente . . . München, 1851.

‡ R. Bryson, Description of a new self-registering Barometer. Transact. of the R. Society of Edinburgh, Vol. XV, 1844.

§ Hough, Description of an automatic registering and printing Barometer, Albany, 1865. Silliman's Journal, 1866.

|| Annuaire de l'Observatoire de Mont-Souris pour l'an 1878, p. 259.

short leg of a syphon barometer. This form of instrument is the simplest in construction, and gives good results with a proper arrangement of recording apparatus.

The Kreil barograph is among the best of this kind, and has been used extensively in Europe. At the Central Meteorological Office in Vienna one has been in use for many years, and the published barometric pressures are as taken from this instrument after proper corrections. Kreil* gave $\pm 0.21^{mm}$ (0.008 inch) as the probable error of a comparison of his Prague barograph with a Fortin barometer; the probable error of a single registration was $\pm 0.10^{mm}$. The barograph at Vienna gives a probable error of about $\pm 0.15^{mm}$. The objection to this form of instrument is that the irregular capillary changes enter into the results, as there is no appliance for raising the mercury in the legs, so that when registrations are continuous the marking pencil, by its fractional contact with the recording paper, diminishes the sensitiveness.

Examples of these instruments are in use as follows:

Kreil barograph, at Brussels, St. Petersburg, Dorpat, Vienna, Kremsmünster, Prague, Pesth, Munich (?).

Redier barograph, at Mont-Souris, Puy-de-Dôme, Pic du Midi (?).

(c) *The electric methods.*—Wheatstone† seems to have been the first to suggest the application of electricity to the automatic registering barometers. In general, one pole of a battery is in contact with the mercury of a syphon barometer, and at fixed intervals of time the platinum contact point at the other pole of the battery is made to descend the open leg of the barometer until it comes in contact with the exposed mercury surface or metallic float on the surface, when the circuit will be closed. The current caused by this closing of the circuit can be utilized in several ways to record the height of the mercury column in the open leg at the time of contact.

Jelinek‡ suggested improvements on the Wheatstone form, but Du Moncel (1856), Regnard (1857), Montigny (1857), and Hough§‖ (about 1862) were inventors of an electric automatic registering and printing barometer, which has also been independently reinvented by Theorell.¶

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*Kreil, Magnet. und meteorol. Beobachtungen zu Prag, Bd. 3, S. 131, Prag, 1843.

†Wheatstone, Enregistreur électro-magnétique pour les observations météorol., Archives de l'électricité, par A. de la Rive, 1844, Vol. IV, pp. 170–173; and Traité de télégraphie électrique, par A. Moigno.

‡C. Jelinek, Beiträge zur Construction selbstregistrirenden meteorologischer Apparate, Sitzungsber. der Wiener Academie, Bd. V, 1850.

§Hough, Description of an automatic registering and printing Barometer, Albany, 1865. Silliman's Journal, 1866.

‖Hough, Annals of the Dudley Observatory, Vol. II, Introduction, p. XXV, Albany, 1871.

¶Dr. A. G. Theorell, Description d'un météorographe enregistreur, Nova Acta Reg. Soc. Sc. Upsala, Ser. III, Vol. VII, Fasc. I, 1869; and "Description d'un météorographe imprimeur. Kongl. Svenska Vetenskap-Akademiens Handl., Bandet 10, No. 7, 1871; also Oesterreichische Zeit. für Meteor., 1875.

The Rysselbergh barograph (or meteorograph) is a device similar to that of Hough and others for electrical registration, by which the pressure or other curve is engraved directly on a metal plate, from which it can be printed without further reduction.

The form finally adopted by Hough has a float in the open leg of the barometer, and the actual electrical contact is made outside of the barometer tube; this is to avoid oxidation of the mercury by the spark. Hough claims for his instrument that the mean error of a single reading, when compared with a standard barometer, is only *plus* or *minus* .0035 inch. His instrument is compensated for temperature changes by supporting the tube on a brass rod. Rubenson, in Upsala, finds the deviation of the Theorell barograph to be ±.005 inch for a single reading. This accuracy, like that given for Kreil and other barographs, when new, would seem scarcely probable, however, when the exposed mercury surface has become impure. In the case of the Hough and Theorell barographs the inventors have added a mechanical counting device, by which the reading of the barometer at each registration is printed in figures on a strip of paper. One advantage of this form of instrument is that the record can be simultaneously printed at any distant place.

There are instruments of the Hough form at the Dudley Observatory, Albany; at the Signal Office, Washington, D. C. Theorell meteorographs are at the Meteorological Bureau at Vienna, at Stockholm, at Upsala, and at Copenhagen. The one at Vienna, however, is not used for obtaining published hourly readings. The Rysselbergh barographs (meteorographs) are used at Brussels, Triest, Batavia, and Utrecht.

(*d*) *The gravity methods or the balance barographs.*—The principle of the steel-yard barograph, invented by Moreland about 1670, has been applied with success by many inventors. The idea is merely to give the variation in weight due to variations in atmospheric pressure. If we fasten to one end of a balance the upper end of a free filled barometer tube, which has its lower end immersed in a vessel of mercury, and we counterpoise the other end of the balance by means of a weight, then the variation in atmospheric pressure will cause the ends of the balance to rise and fall. That is, the downward force exerted on the end supporting the barometer will vary with the pressure of the atmosphere and a motion will be given to the balance arm, which motion is to be recorded and measured and interpreted.

Of the forms at present in use Secchi† was the first to construct an instrument, about 1858, and he seems to have considered his meteoro-

* Van Rysselbergh, Notice sur un système météorologique universel, Bruxelles, 1873. On a Universal System of Meteorography, Quarterly Journal of the Meteorological Society, 1875. Zeitschrift der Oesterreichischen Gesellschaft für Meteorologie, Band, X.

† Memorie dell' Osservatorio dell' Collegio Romano, 1859, No. I, p. 1.

graph to be new in principle. Then followed Wild-Hassler,* Schreiber,† Greiner-Fuess,‡** Salleron,§ and Sprung.|| The relative accuracy attained by these instruments is shown by the following summary statement:

Schreiber finds for his instrument the probable error of the difference between a reduced reading of his barograph and a normal barometer to be ±.007 inch, whence the probable error of a single reading of the barograph results 0.005 inch. Wild,¶ after ten years' progressive experiments, obtains 0.003 inch as the mean error of a single reduced reading of his barograph at Pawlosk (this instrument has a ~~tapping~~ device to overcome the capillary adhesions). The Greiner-Fuess-Moreland instruments at the Hamburg Seewarte attain an accuracy of ±0.16^{mm} or 0.006 inch.** Sprung has reached an accuracy of ±0.003 inch almost at the outset with his form. Wolfers finds a mean error ±0.10^{mm} or probable error ±0.07^{mm} for the Wild-Hassler barograph at Berne. Both Wolfers and Wild find a systematic difference of 0.28^{mm} between the rising and falling pressures due to capillarity.

The balance barograph is in actual use as follows: (*a*) Professor Wild's form at St. Petersburg, Pawlosk, Nicolaijew, Tiflis, Moscow, Pola, Pekin, Berne, and Washington; (*b*) Professor Schreiber's form at the Seewarte, Hamburg; (*c*) Messrs. Greiner-Fuess's form at the Seewarte, Hamburg, and at the principal German stations; (*d*) Dr. Sprung's form at Magdeburg, Copenhagen, the German circumpolar stations, Seewarte, Hamburg, and Spandau; (*e*) Salleron's form at Mont-Souris; (*f*) Secchi's at Rome (the one at Washington is now dismounted); (*g*) King's barograph at Liverpool.

(*e*) *Miscellaneous forms.*—In addition to the barographs already mentioned there have been proposed also other forms; for instance, the Schreiber hydrostatic†† barograph, of which there was one on exhibition at the Deutsche Seewarte in 1883; Fuess* also proposes to insert a metal horse-shoe magnet float in the closed end of a barometer and let the

* H. Wild, Die selbstregistrirenden meteorologischen Instrumente der Sternwarte in Bern; Carl's Repert. der physik. Technik, Bd. II, S. 161, 1866; Mittheilungen der Berner naturf. Gesellschaft von 1862 bis 1864.

† P. Schreiber, Untersuchungen über die Theorie und Praxis des Wagbarometers; Carl's Repertorium für exper. Physik, Bd. VIII, S. 245, 1872; also the volumes for 1878 and 1879; Oester. Zeitschrift für Meteor., 1879 and 1881; Aus dem Archiv der Deutschen Seewarte, Vols. I and VII.

‡ Löwenherz, Bericht über die wissenschaftlichen Inst. auf der Berliner Gewerbe-Ausstellung, 1879, p. 231; Archiv der Deutschen Seewarte, Vol. I.

§ Annuaire de l'Observatoire de Mont-Souris pour l'an 1878, p. 265.

|| A. Sprung, Bericht über die wissenschaftlichen Instrumente auf der Berliner Gewerbe-Ausstellung im Jahre 1879; Oester. Zeitschrift für Meteorologie, 1877, 1881, and 1882; Zeitschrift für Instrumentenkunde, 1886, pp. 189 and 232.

¶ H. Wild, Annalen des phys. Cent.-Obs. St. Petersburg, Jahrgang 1878, Theil I.

** H. Eylert, Untersuchungen über den Moreland'schen Gewichts-Barographen von R. Fuess in Berlin, Zeitschrift für Instrumentenkunde, 1886, VI.

†† Oesterr. Zeits. für Meteor., 1881.

motions of this float be conveyed to an index by means of a movable magnet on the outside of a tube, this outside magnet following the motions of the float inside.

The Draper barograph in use at Central Park, New York; Providence, and Blue Hill, Mass., has a fixed barometer tube immersed in a cup of mercury; this cup being hung on springs, its variations of weight are registered by means of a pencil which follows the motions of the vessel.

75. ANEROID BAROGRAPHS.

The Hipp aneroid barograph, which is described with illustrations in the Oester. Zeit. für Meteor., 1871, registers at fixed intervals the position of the indicator of an aneroid by means of pressing a point against the registering paper. An improvement by Osnaghi was made afterwards (Oester. Zeit. für Meteor., 1873), and a still later description is given by Neumayer (Seewarte Instructions, 1879, and Archiv., Vol. I).

The wide diffusion of this apparatus is shown by the following list of places where it is found. Cracow, Vienna, Pola, Prague, Agram, Moncalieri, Turin, Milan, Naples, Vicenza, Padua, Venice, Hamburg, and Modena.

The Höttinger aneroid barograph, as described and investigated in Oester. Zeit. für Meteor., 1881, appears to give results within $\pm 0.2^{mm}$ of the true barometer reading when it is used as an interpolation instrument, but Wolfers finds $\pm 0.10^{mm}$ and $\pm 0.08^{mm}$ as mean errors of one corrected reading for the two specimens of these barographs at Zurich (see Schweiz. met. Beob., XIII, 1876).

The Naudet and Richard barographs give good results, and several forms are advertised in the current catalogues of English and French makers.

The reduction of the records of an aneroid barograph is made by means of formulæ similar to those employed for ordinary barographs. It is particularly necessary, however, to have an even temperature for these instruments, and they can in no case be used as other than interpolation instruments. In most of the first-class observatories where these instruments are found they are merely used to interpolate any breaks that may occur in the mercurial barograph record. Their portability and cheapness fit them for use at all stations where the large instruments of Wild or Sprung are not practicable.

76. REDUCTION OF BAROGRAPH RECORDS.

(*a*) *Methods of registration.*—The curves by which the pressure is actually registered may be divided into two classes, continuous or intermittent. In the latter case the record may be at every minute or ten minutes, or any chosen fixed period.

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* Zeits. für Instr.-Kunde, 1883.

For most cases of continuous registration the marking-point must be continually in direct mechanical connection with the barometer, and must be very sensitive to barometric changes. The friction of this point (it may be a pencil, pen, or metallic point) against the recording paper can be avoided by using Thomson's electric pen. However, the photographic method and Hough's or Sprung's method are exceptions to the generality of instruments, and are the best form of continuous registration yet invented.

There are two general methods of discontinuous registration. In the one a hammer or its equivalent is made to descend on the marking-point, which is thereby caused to puncture the registration paper at the point over which it has been brought by reason of the communicated barometric variations; after the puncture is made the marking-point frees itself from the paper. The armature and attached hammer are operated by clock-work, which causes the circuit to close for an instant at stated intervals. Such an arrangement is found in the Wild barograph.

In the second arrangement, which is used in connection with the electrical method, a rod (usually with a platinum wire point) is made to descend at regular intervals whenever a circuit is closed by action of a clock-work. This descent can be so regulated that each 0.001 of an inch of motion causes a toothed wheel, geared into connection, to turn one tooth. This wheel communicates the motion to others, and they are so arranged that for each 0.001 of an inch of barometric motion type-wheels or counters with tenths, hundredths, and thousandths in the correct relation are brought over the registering paper, and by the automatic closing of a circuit the printing wheels are pressed against the paper. Thus we have the barometer reading printed automatically.

Such apparatus is used for the Hough and Theorell barographs. In all the forms of registration (with the exception of the self-printing meteorographs) there must be a reference point or line marked on the paper at the same time that the height of the barometer is recorded, so that the ordinate of the barometric curve can be read off from the sheet. These reference points are made by a marking-point, which has a permanent fixed position, so that the records of the point made on the paper will form a straight line, and along this straight line are to be laid off the abscissæ of the barometer readings corresponding to the time.

The recording sheet may be mounted either on a flat plate or on a cylinder, but in any case this support for the paper is moved by clock-work. In most cases a flat surface is used, as in the case of the barographs of Wild, Kreil, Sprung, Hough, Theorell, etc. In the Kew (photographic), Schreiber, Redier, etc., barographs a cylinder is used. It is not necessary to give a clear account of the various forms of registration, as almost every one utilizes a different mechanical device, and similar methods of reduction apply to all.

(*b*) *The reduction of the observations.*—The conversion of barograph records into correct pressures involves the following considerations: The record sheet shows a straight line at the bottom, which represents the position of the fixed reference point of the barograph. Above this reference line is the curve drawn by the moving point, which shows the fluctuations in the height of the mercurial column. On the fixed reference line the times are marked off, and are given by the distances or abscissæ from the beginning of the day to any point on the reference line. A point on the barometer curve, which lies on a perpendicular to a point on the reference line, shows the barometric registration for that special time. The length of the perpendicular between the fixed line of reference and the barometric curve is called the ordinate. It is with these ordinates that we have to deal in order to find the true pressures corresponding to the different points of the barograph curve. This curve in itself shows only that the barometer rose and fell. The pressure or change in pressure at any time can only be told when the values of its scales of ordinates and abscissæ are determined. In order to know these values and make the reduction to absolute measures a series of "control observations" is instituted, *i. e.*, a reading is made of a standard barometer at least three times a day, and the position of the marking point of the barograph at that time is marked on the barograph curve. The ordinate of that point corresponds to the height of the barometer, as observed by the eye, and from several of these ordinates on each sheet and the known actual height of the barometer the absolute height of the barometer at intermediate points can be found by a method of interpolation. Labor is saved and better results attained if, instead of dealing with each barograph sheet separately, the 90 or 100 control readings for a whole month (taken at very diverse pressures and temperatures) are used to determine the instrumental reduction constants once for all. These may then be used until a trial shows them to have changed. By means of these constants the ordinates can be reduced to true barometer readings with little trouble. Wild has given a theoretical discussion of a method of getting these constants in his paper on barometers (page 135); but different instruments require different formulæ, and a given formulæ is adopted only when it is found by actual trial to be satisfactory.

In practice Wild* arranges for each month the ordinates x according to their lengths in groups of ten, and the corresponding direct readings of the barometer reduced to 0° C., which we call h, are placed opposite. Thus nine groups of ten each of x's and of h's are obtained. These must satisfy nine empirical equations of the form

$$h=a+bx+cx^2$$

using the mean x and h of each group of ten; the a b c are the constants to be determined. From the nine equations find by the method

* H. Wild, Annalen des phys. Cent.-Obs. St. Petersburg, Jahrgang, 1878, Theil I.

of least squares the values of a, b, and c. The values of these constants will probably vary somewhat with the seasons. Assume values of x, commencing with the least observed values and taking them at intervals up to the greatest observed, and with the constants already known, a b c, compute the corresponding values of h. Plot the results, using x as abscissa and the h's as ordinates, on cross section paper and draw a curve through the points. From this curve the barometer height (h) can be read off corresponding to any ordinate (x) as read from the barograph sheet. A simpler and less accurate process is used at the observatory at San Fernando, Spain, where the photographic registrations on each sheet are reduced separately. The ordinates and abscissæ are read by means of a scale, and the reduction formula is

$$b-B=cx$$

where b is the barometric height observed simultaneously with the registration of the ordinate x, B is the distance to the fixed line, and c a constant. Control observations are made four times a day; the extreme ones are used in determining the c, and all four in getting the B.

The experience of Mr. Eylert in finding a method of reduction for the Geisler-Fuess "bent-lever barograph" used at the German stations is interesting and instructive.* He at first tried the formula

$$B=C+ay+bt$$

where B is the true pressure, C the pressure which corresponds to a registration on the abscissæ axis at 0° temperature, y the ordinate of the barogram, and t the temperature of the instrument. From a large number of comparative readings he determined C, a, and b. The results were very unsatisfactory. Eylert next introduced a term, ct^2, but that gave no better results. Finally, in November and December, 1885, a number of comparative readings were made, both with rising and falling barometer, with a variation of pressure of 32^{mm} and a variation of temperature 12° C., by cooling and heating the room. He now adopted as the formula for reduction

$$B=C+ay+bt+cy^2+dyt$$

which he had deduced from a study of the mechanism of this form of barographs. The terms cy^2 and dyt were due to inconstant action of the weight, and bt to the irregular effects of temperature. The new system of equations being solved by least squares for C, a, b, c, d gives excellent accordance between the standard pressure and the reduced records of the barograph sheets:

In 61% the deviation was under 0.2^{mm};
In 96% the deviation was under 0.3^{mm};
The mean deviation was only $\pm 0.16^{mm}$.

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* H. Eylert, Untersuchungen über den Moreland'schen Gewichts-Barographen von R. Fuess in Berlin. Zeitschrift für Instrumentenkunde, 1886, VI, p. 269.

This illustrates the important general principle that analysis must dictate the form of the equation or the manner in which the known and unknown quantities are to be connected.

77. SPRUNG'S FIRST BALANCE BAROGRAPH WITH SLIDING WEIGHT.

In the Austrian Zeitschrift für Meteorologie for 1877 Dr. A. Sprung, then of the "Deutsche Seewarte," published a new principle of construction for self-registering barometers. Fuess, of Berlin, constructed an instrument according to Sprung's designs, which was found to work satisfactorily, and with an accuracy equal to that of any other form of instrument. This instrument was first described in the "Bericht über die wissenschaftlichen Instrumente auf der Berliner Gewerbeausstellung, Berlin, 1879," where Sprung gives the mathematical analysis of its action.

It is important to consider its construction and error in detail, as the reliability of its records is apparently not inferior to the best eye and hand observations with ordinary station barometers.

The accompanying figure, No. 29, shows the plan of its construction, and the following analysis is as given by the inventor:

Instead of moving the whole barometer tube, as in other cases of balance barographs, in the present case the tube remains practically stationary, and a balance weight with the recording pencil is kept in motion instead; thus no friction affects the accuracy of the registration. The changes in statical moment caused by changes in weight of the barometer B are counterpoised by the automatic sliding of a weight on the other end of the balance beam, and the position of the sliding weight is shown (by a recording pencil, S) on a brass tablet covered with paper, which is moved by means of clock-work. The clock moves a vertical connecting rod, t, whose upper end, by means of a two-armed lever beam turning in h, which at the same time bears an iron anchor, a, can be shoved a little from left to right. This motion causes a beveled wheel with rough edges, fastened on the upper end of the connecting rod t, to work first to the left then to the right on two similar beveled wheels, which are both firmly fixed to a horizontal steel screw, $c\,c'$, running under the long beam on the left, and let this screw rotate first in one direction then in the other. These rotations of the screw produce corresponding alternating displacements of an arrangement, V, which, partly by riding on the steel screw, partly by being driven by a beam arranged behind the latter, transmits these motions to the pencil S and to the rolling wheel R. The transmission of the motion to R takes place by means of a small completely balanced lever arm, so that R always rests with its full weight on the long balance beam. The figure represents the connecting rod t in the position in which, by means of a spring, f, working on the right of both beveled wheels, it drives the rolling wheel R towards the left, and thereby increases the statical moment of the left balance arm; for a very little shoving of the rolling wheel produces a slight descending motion limited by the two screws s and s', and closes at c (mercury contact) an electric circuit, in which the electro-magnet E is inserted. By overcoming the force of the spring f this gives, then, to the rod t the opposite position, so that, now working on the left beveled wheel, it sets the rolling wheel R in motion in the opposite direction towards the right. In this way the statical moment of the left balance beam l' is soon so diminished that it lies again on the upper screw s', and therefore breaks the current at c, which then again causes the contrary motion of the running wheel, etc.

For a constant weight of the barometer B the pencil S will draw a zigzag line, whose mean line is exactly parallel to that straight line which is made by another pencil S' fastened on the frame-work of the instrument.

A slightly greater weight of the barometer will have the effect of making S draw a zigzag line a little further to the left. If the weight of the barometer is increased gradually the pencil S is so moved from right to left that the right one of the two beveled wheels, which are situated on the steel screw $c\,c'$, works longer each time than the left one.

Let L denote the length of the right balance beam;

- l the distance of the running wheel R from the axis of revolution of the beam;
- P the weight of the barometer;
- p that of the running wheel R;

and we have, neglecting the influences of temperature, the following relation, where Δ indicates a change in the value of the quantity following it.

$$p\Delta l = L\Delta P$$

or

$$\Delta l = \Delta P \frac{L}{p} \quad \ldots\ldots\ldots\ldots \quad (1)$$

that is, the movements of the running wheel along the beam are proportional to the changes in weight of the barometer. As, now, the sensitiveness of the apparatus is not at all connected with the enlargement of the tube in the neighborhood of the upper mercurial surface (but can be influenced to a high degree by the change of weight p), it is possible, by leaving out this enlargement, to avoid the chief causes of errors of the barograph due to temperature. One can, therefore, consider ΔP as proportional to the change of pressure ΔB, whence also the motion Δl of the running wheel is proportional to the change in pressure ΔB. The curve drawn by the pencil S gives, therefore, without any reduction, a fairly true representation of the changes in the air pressure. By the help of an evenly divided glass scale, whose spaces are determined by the dimensions of the instrument, the barometric height can be read off at any point on the curve.

The first adjustment on setting up the apparatus can best be done by the help of a standard barometer; but in order that a special barometer may not be needed for this and for the control of the registering apparatus a scale, *m*, divided into millimeters (whose steel zero point can be pushed by a screw into the lower mercury surface), is placed on the barometer tube *B*. As the diameter of the column amounts to 2 centimeters this arrangement must represent a good normal barometer.

The specimens of this barograph hitherto constructed show the changes in height of the mercurial barometer multiplied fivefold. It can easily be seen that this factor can be changed by applying another running wheel or weight *p* in equation (1) to the completed apparatus.

The scale weight *g*, placed on the left arm of the beam, is for the purpose of setting the pencil *S* at any desired place on the paper sheet.

The influences on the instrument of its own changes in temperature must now be considered before discussing the further problem of how the weight *P* of the barometer, as ascertained by the direct weighing, is influenced by its temperature. As the glass tube in this apparatus is made smaller at the lower end, for the sake of greater stability and less friction, we assume the general case (including also the Moreland or bent-lever or angle counterpoise barograph previously mentioned) of a barometer tube of the form represented in Fig. 29, and suppose this to be hung on an iron frame-work holding the cistern *F*.

Let P = the weight with which the barometer acts on the point of suspension.

G the absolute weight of the glass tube with the metallic scale fastened to it.

Q', Q, q, and r the cross sections of the cistern, the wide and narrow mercury column, and the wall of the narrow portion of glass tube, respectively.

H and h the height of the upper and lower mercury surface, counted from the bottom of the cistern.

i the height of the lower end of the glass tube above the bottom of the cistern.

K the height of the point where the tube narrows above the bottom of the cistern.

V_0 the whole volume of mercury in use at 0° C.

S and S_0 the specific gravity of mercury at $t°$ and 0°.

B the barometer reading reduced to 0°.

α, g, and ε the coefficients of expansion of the mercury, the glass, and the iron, respectively.

E the vertical distance of the lower end of the tube from the point of suspension.

The subscript index $_0$ in general indicates the value of any symbol at 0° C.

The weight P on the point of suspension consists of the weight of the glass tube diminished by the buoyancy of the parts immersed, and of the weight of the whole mass of mercury which is found above the under surface; therefore

$$P=G+(H-K)Qs+(K-h)qs-(h-i)rs \quad . \quad . \quad . \quad . \quad (2)$$

The condition that a rise of the mercury in the glass tube must correspond to a fall in the cistern is expressed by

$$(H-K)Qs+(K-h)qs+hQ's-(h-i)rs=V_0s_0 \quad . \quad . \quad . \quad (3)$$

Finally, the condition that the difference of level in tube and cistern is inversely proportional to the specific gravity of the mercury gives

$$(H-h)s=Bs_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If we eliminate H from (2) and (3) and from (4) and (3), and eliminate h from the two resulting equations, replace Q by $Q_0(1+2gt)$, etc., and s by $\frac{s_0}{1+\alpha t}$, and, for abbreviation, put

$$C=\frac{Q_0'}{Q_0+Q_0-q_0-r_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

there results

$$P=G+V_0s_0(1-C)+Cs_0Q_0B(1+2gt)-Cs_0\left\{K(Q_0-q_0)-ir_0\right\}\frac{1+2gt}{1+\alpha' t} \quad . \quad (6)$$

The partial differentiation of this expression with respect to t gives the increase of weight that corresponds to an increase of temperature of 1° C., namely:

$$\frac{dP}{dt}=Cs_0Q_0B2g+\frac{Cs_0}{1+\alpha t}[(Q_0-q_0)K-ir_0]\left(\alpha \cdot \frac{1+2gt}{1+\alpha t}-2g\right) \quad (7)$$
$$-Cs_0\frac{1+2gt}{1+\alpha t}\left((Q_0-q_0)\frac{dk}{dt}-r_0\frac{di}{dt}\right)$$

The partial differentiation with respect to B leads to the expression for the change in weight that corresponds to an increase in the height of the barometer by a unit of length, or

$$\frac{dP}{dB}=Cs_0Q(1+2gt) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Denoting by x the apparent increase in the barometric height which would correspond to an increase in weight occasioned by an increase in temperature of 1°, we have

$$\frac{dP}{dt}:\frac{dP}{dB}=x:1$$

from which, by substitution from equations (7) and (8), we find

$$\frac{\frac{dP}{dt}}{\frac{dP}{dB}}=\frac{2Bg}{1+2gt}+\frac{1}{Q_0(1+\alpha t)(1+2gt)}[(Q_0-q_0)K-ir_0]\left(\alpha\frac{1+2gt}{1+\alpha t}-2g\right) \quad (9)$$
$$-\frac{1}{Q_0(1+\alpha t)}\left((Q_0-q_0)\frac{dk}{dt}-r_0\frac{di}{dt}\right)$$

In a less degree x is dependent on t. For $t=0$ we find, by omitting all terms which contain the very small r_0 as factor,

$$x_0=2Bg+\frac{Q_0-q_0}{Q_0}(E_0-K)(\alpha-2g)-\frac{Q_0-q_0}{Q_0}(E_0e-K_0g) \qquad (10)$$

Wherein k is replaced by

$$E-K=E_0(1+et)-K_0(1+gt)$$

and i by

$$E-J=E_0\,(1+et)-J_0(1+gt)$$

For the bent-lever barograph of the ordinary dimensions we substitute the following numerical values:

$Q_0=7.0$ sq. cm. $\quad q_0=0.5$ sq. cm. $\quad E_0=83$ cm. $\quad K_0=16$ cm.

$$k=E_0-K_0=67^{mm}$$

Further

$\alpha=0.0001815 \qquad g=0.0000085 \qquad e=0.0000120$

Therefore from (10) there results

$$x_0=0.0129+0.1022-0.0080=0.1071^{mm}$$

The error due to changes in temperature, causing an apparent change of weight of the barometer, amounts, therefore, to little more than 1^{mm} for each 10° C. for the common barograph with enlarged tubes.

For the running weight barograph with perfectly cylindrical tube, $Q-q=0$; so that the expression (10) is reduced to

$$x_0=2Bg(=0.0129^{mm} \text{ for } 760^{mm} \text{ pressure}) \quad . \quad . \quad . \quad . \quad (11)$$

The latter holds good with considerable closeness if only a very short portion of the tube is narrowed at the lower end, because then the small amount of the second and third members in the value of x_0 nearly balance each other.

For 10° C. change in temperature an error amounting to about 0.13^{mm} is brought about by enlarging the cross-section Q of the glass tube; it is independent of the width of the glass tube.

Thus an enlargement of the tube, limited to a short distance, modifies to a high degree the influence of temperature in the usual form of the balance barograph. One can also be easily convinced of this without computation, by thinking of the mass of mercury in the enlarged part as divided into two parts by a thin-walled extension of the narrow tube, continued up to above the level of the upper mercurial surface. As long as the dividing wall exists a rise of temperature (independent of the enlargement of the cross-section) can cause no change in weight of the barometer. However, as the inner column expands in a rise of temperature much more than the shorter annular column that surrounds the inner cylinder, and as in the absence of the dividing wall the whole upper cross-section must reach the level of the inner narrow column, therefore as much mercury must enter the tube from the cistern as will be needed to fill the annular space to the height of the inner column. The increase in the barometric record occasioned by the rise in temperature corresponds to the weight of this ascending mass of mercury.

It is easier to exclude these causes of error by a proper construction at the outset than to compensate for them afterward by any other arrangement.

The influence of the change in temperature on the motion of the running wheel R is shown as follows: The following symbols are adopted in addition to those already defined:

Let W be the weight of the balance beam (including the scale weight g).

M the distance of the center of gravity of this system from the point of rotation.

P' the weight of the barometer for $l=0$, consequently at a very low barometric height.

γ the coefficient of expansion of the metal of which the balance beam is made.

The equality of the statical moments on both sides is expressed by

$$pl=PL-Wm \quad \quad (12)$$

or, in another form,

$$l=P\frac{L_0}{p}(1+\gamma t)-\frac{W}{p}m_0(1+\gamma t) \quad \quad (13)$$

From this there results

$$\frac{dl}{dt}=\frac{L_0}{p}(1+\gamma t)\frac{dP}{dt}+\frac{\gamma}{p}(PL_0-Wm_0) \quad \quad (14)$$

$$\frac{dl}{dB}=\frac{L_0}{p}(1+\gamma t)\frac{dP}{dB} \quad \quad (15)$$

Denoting by y the apparent increase of the recorded height, expressed in height of the mercury, which corresponds to an increase in temperature of 1°, we have

$$y=\frac{\frac{dl}{dt}}{\frac{dl}{dB}}=\frac{\frac{dP}{dt}}{\frac{dP}{dB}}+\frac{(P-P')\gamma}{(1+\gamma t)\frac{dP}{dB}}+\frac{(P'L_0-Wm_0)\gamma}{L_0(1+\gamma t)\frac{dP}{dB}}$$

According to the definition of P' we get it when in (13) we put $l=0$; so that

$$P'L_0-Wm_0=0 \quad \quad (16)$$

with which, consequently, the last member of y vanishes.

The difference, $P-P'$, in the second member of y, is to be found from equation (6), as follows:

As a result of the special construction of the apparatus h and i remain independent of the barometric height; therefore, only the third term in (6) is to be taken into account. It becomes

$$P-P'=Cs_0Q_0(B-B')(1+2gt)$$

and the expression for y, by considering the values of (9) and (8), changes into

$$y=x+\frac{(B-B')\gamma}{1+\gamma t}.$$

It was explained above that for this form of barometer tube x is reduced to

$$x=\frac{2Bg}{1+2gt}$$

Accordingly, the whole influence of temperature on the running wheel is.

$$y=\frac{2Bg}{1+2gt}+\frac{(B-B')\gamma}{1+\gamma t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

For temperatures near the freezing-point, if for brass γ is assumed equal to 0.000019, $B=76^{cm}$, and $B-B'=8^{cm}$ there results as the extreme value

$$y=0.0129+0.0015=0.0144^{mm}.$$

Therefore a temperature of 10° causes an error, such that the results of the apparatus come out too high by 0.144^{mm}.

The smaller term in the second number of (17) is based on the expansion of the balance arm L, and is entirely dependent on the barometric height; therefore, this lesser part can be considered as constant, and the correction is easily made, or, if it is considered desirable to be so accurate, an app aratus according to the principle of the metal thermometer can be applied.

The exact motion of the running wheel or its pencil for a change in pressure of l^{mm} is determined as follows:

Substitute $\frac{dP}{dB}$ from (8) in (15) and there results

$$\frac{dl}{dB}=Cs_0Q_0\frac{L_0}{p}(1+2gt)(1+\gamma t)$$

or, if the value for C from (5) is substituted,

$$\frac{dl}{dB}=\frac{Q_0Q_0'L_0s_0}{Q_0'+Q_0-q_0-r_0}\cdot\frac{1}{p}\cdot(1+2gt)(1+\gamma t) \quad . \quad . \quad . \quad . \quad (18)$$

In this equation the enlargement of the tube needs to be considered more carefully. A sudden narrowing of the glass tube at the lower end, no matter how short it is, has the effect of making the running wheel perform a little less motion than if the narrowing were not there. If the tube be cylindrical then $q=Q$, and

$$\frac{dl}{dB}=\frac{Q_0'}{Q_0-r_0}Q_0s_0\frac{L_0}{p}(1+2gt)(1+\gamma t) \quad . \quad . \quad . \quad . \quad . \quad (19)$$

This value is greater than (18). The cause of this is that for a narrowed tube the increase ΔB of the difference in level, corresponding to the increase in pressure, consists partly in an elongation of the narrow lighter column, so that the corresponding increase of weight ΔP must become smaller in proportion as the lower column is narrower; this elongation is as great as the sinking of the surface in the cistern. By the narrowing of the tube the enlargement of the scale becomes a function of the width of the cistern. Without the narrowing it is independent of it. However, as is evident from (18), the proportionality between the motions of the running wheel and the increase of pressure from the narrowing is not interfered with at all.

From these considerations it is evident that stress need not be laid on the importance of the greatest possible width of the cistern. One should choose this of only such a width that by making the narrow part as short as possible the point of narrowing will not reach the surface of mercury in the cistern.

The barograph under consideration is very well fitted for registration at a distance. The apparatus of the receiving station consists of the clock-work, the writing tablet, the steel screw with rider and pencil, the electro-magnet, etc., while the balance beam, the running wheel, and the barometer can be at a distance. If, now, the electro-magnet at the receiving station is inserted in the circuit of the barograph of the observing station both steel screws, and consequently the pencils, also, make identical motions if both beveled wheels on each screw are of the same size. For the purpose of registration at a distance a little different construction of wheels would be better. Evidently also this registration can take place at the same time at several places.

The result of several years' experience in the use of the Sprung barograph showed that the mercurial electrical contacts did not work as well as was desirable, and the later instruments have been provided with platinum contacts, according to the Rung-Lauritzen method, as described in the Zeitschrift für Instrumentenkunde, 1884, p. 318.

In the "Aus dem Archiv der Deutschen Seewarte," Vol. VII, 1886, is a description of a specimen of a slightly modified form of the Sprung barograph.

78. SPRUNG'S NEW THERMO-BAROGRAPH WITH SLIDING COUNTERPOISE.

Sprung has lately succeeded in combining the barograph with a self-registering air thermometer and has given the theory of this construction in the Zeit. f. Inst.-Kunde, 1886. In this new instrument he finds a mean error of $\pm 0.04^{mm}$ for a single registration, which would imply that his barograph gives results equaling in accuracy the best direct observations. This modified form apparently offers several advantages over the above first form, and the following analysis of its errors can easily be understood after what precedes. The thermograph as well as the barograph are here described and analyzed together, as they are intimately interdependent.

This apparatus was described first in 1881, but constructed in 1883, and examples of it are in good working order at Magdeburg and Spandau. It apparently ranks as the best form of this instrument that has hitherto been devised. The general arrangement of the parts is shown in Fig. 30. An air thermometer, preferably filled with pure nitrogen, has its large bulb placed in an appropriate shelter, and communicates by a fine tube, *T*, with the top of a cylindrical manometer tube, P', fixed permanently to the solid iron frame-work of the apparatus. The lower open end of P' dips into the mercury contained in a vessel, P_1', which latter hangs freely from the knife-edge shown above it, and which is at the right-hand extremity of a horizontal balance beam, whose left end carries the weight *W*. When the elastic pressure of the gas in the thermometer bulb is increased by temperature the mercury in P' is pushed downwards, and the vessel P_1', with its mercury, descends as it becomes heavier, but immediately the small counterpoise on the wheel u' is automatically pushed to the left by the automatic rotation of the horizontal screw below it. Thus maintaining the balance beam in its horizontal position, the motion of this counterpoise is recorded by the pencil on the register sheet, and is a measure of the change of pressure and temperature of the air in the thermometer bulb.

Now, a diminution of external atmospheric pressure also allows the mercury to flow from P' into P_1', thus simulating the effect of an increase of temperature, but if any fall in pressure has occurred it will have diminished the weight of a barometric column to the same extent. Therefore a barometric tube, *P*, is hung directly upon the same balance arm by the knife-edge A', and this counteracts the effect of pressure upon the manometer tube P', the result being that the small counterpoise u' is affected only by the changes in the temperature of the thermometer bulb.

The flow of mercury from the barometer *P* into the cistern P_1 causes the weight of the latter to vary inversely as the changes of pressure; therefore the vessel P_1 is hung upon another balance beam, and is balanced by the weight W' and the sliding counterpoise *u*; the record of the position of *u* is therefore the scale for barometric pressure.

A light bar, *K*, behind the two mercurial columns, is made to gently strike these every two minutes, so that the capillary adhesion of the mercury to the glass is broken up. Each balance beam carries at its left end a suspended disk, *DD*, which moves vertically in a bath of oil and dampens any slight vibration of the beam. Each vibration of the beam closes a galvanic circuit and sets in motion a wheel and screw that quickly opens the circuit, as in Hough's printing barometer. This is the motion that controls the screws that move the wheels u and u'.

The accuracy of the records and the beauty of the theory of the action of this instrument justifies the following detailed presentation of Sprung's analysis, which will indicate the method by which to secure automatic records as accurate as the best of ordinary personal observations.

(a) *The register of barometric pressure.*—Let the barometric tube have its section contracted at a point above the surface of the mercury in the cistern, so that the area of the inner section of the upper portion is Q, that of the inner section of the lower part of the tube is q, that of the outer section of the lower portion $q+r$, and the area of the inner section of the cistern Q'.

Adopt, also, the following notation:

e the vertical distance from the bottom of the cistern up to the lower end of the tube.

l the distance from the lower end of the tube up to the point of constriction.

h the distance from the bottom of the cistern up to the surface of the mercury in the cistern.

$H=e+l$ the distance from the bottom of the cistern up to the constriction in the interior of the tube.

L the distance from the constriction up to the surface of the mercury in the vacuum chamber.

$B=L+H-h$ the atmospheric pressure after allowing for the density of the mercury.

The total weight of mercury which is constant will be given by the product of its volume and density, whence

$$\text{constant}=Q'h-r(h-e)+(H-h)q+QL$$

Assuming the pressure $B=L+H-h$ to be constant, then in this equation e and h alone are variable; therefore by differentiation we obtain

$$\frac{dh}{de}=\frac{Q-(q+r)}{Q'+Q-(q+r)}$$

This ratio becomes 0, and therefore h becomes independent of e when $Q=q+r$, or when the area of the outside section of the submerged lower end of the tube is the same as the area of the inside section of the upper principal part of the tube. In this case, therefore, the surface of mercury in the cistern remains at a constant height with reference to the bottom of the cistern for any movement of cistern and tube not caused by change of pressure.

Let this condition be fulfilled in the construction of the tube P, and there will then result a further simplification as to the weight of the mercury in the cistern and tube; for if the tube is raised or lowered without altering the height of the mercury h, then the weight of the barometric tube and its contained mercury remains the same. Now, let s_0 be the density of the mercury at 0° C., then the previous expression gives in general the following relation between any change in the external air pressure P and the corresponding change in height of the mercurial column:

$$\frac{dP}{dB}=\frac{s_0QQ'}{Q'+Q-(q+r)}$$

But if the condition that $Q=q+r$ be fulfilled then this expression becomes $\frac{dP}{dB}=s_0Q$; consequently the change in weight that such a barometric tube experiences for a unit change in atmospheric pressure becomes quite independent of the area Q' of the cistern. Its actual value is the same as would be given by a cylindrical barometric tube of infinitely thin walls and an inside area, Q.

While, therefore, the cistern with its mercury varies in weight with the changes of mercury proportional to the changes of pressure, the barometer tube and its effect, as it hangs with the manometer on the beam of the temperature balance, are independent of pressure changes. The change in weight of the cistern is therefore the reverse of that of the barometer tube P, and is a measure of the pressure effect only, and it now remains only to determine its relation to the rectilinear movement u of its sliding counterpoise. Let a and A be the lever arms of the weights u and P_1, respectively, the latter being almost wholly counterpoised by the weight W' in its fixed position. The expression for static equilibrium

$$fW'+au=AP_1$$

by differentiation gives

$$uda=AdP_1$$

which, by introducing the preceding condition, becomes

$$u=As_0Q\div\frac{da}{dB}$$

The scale of enlargement decided upon in the instruments constructed by Fuess is for the barometer 5 to 1; therefore, substituting this for $\frac{da}{dB}$, we have for the weight of the sliding counterpoise under these conditions

$$u=\frac{1}{5}As_0Q$$

(*b*) *The thermometric register.*—Of the physical conditions which enter into the construction of this part of the apparatus the first to be considered is the law of Mariotte and Gay Lussac, connecting the temperature, pressure, and volume of the gas:

Let $B=$ the barometric pressure in centimeters.

V the volume of air bulb in cubic centimeters.

v the total volume of air inclosed in the bulb and the tube and the air space in the upper part of manometer P'.

q' the inner section of the manometer tube P'.

m the total length of the manometer tube.

n the elevation of surface of mercury in manometer above that in its cistern.

$\delta=\alpha-\gamma$ the difference of the coefficients of the cubic expansion of the thermometer bulb and its gas.

v_0, p_0, τ_0 the initial volume, pressure, and temperature of the gas in the reservoir.

v, p, τ the corresponding data for any other time.

Q' the inner section of the cistern P_1' of the manometer.

q'' the outer section of the manometer tube.

q' the inner section of the manometer tube.

η be the constant ratio $\frac{Q'-q''}{Q'-(q''-q')}$.

The initial volume $v_0 p_0$ then becomes for any other temperature and pressure

$$v_0 p_0(1+\delta\tau)=B(V+q'm)-(q'\eta B+V+q'm)n+q'\eta n^2$$

The ratio between the changes of τ and those of n, as depending on changes in B, is given by differentiating this equation, whence

$$\frac{dn}{d\tau}=-\frac{v_0 p_0 \delta}{V+q'(\eta B+m-2\eta n)}$$

(c) *Adjustment for temperature.*—The effect of a change of 1° C. in temperature of the thermometer bulb is to force mercury down its manometer tube and increase the weight of the cistern P_1' by

$$\frac{dP_1'}{d\tau}=-s_0 q'\frac{dn}{d\tau}\cdot\frac{Q'}{Q'+q'-q''}$$

Let A_1, a_1, and a' be the lever arms, respectively, of the manometer cistern P_1', the barometer tube P, and the thermometric sliding counterpoise u', the greater part of the weight of P_1' and P being counterpoised by W. The static equilibrium of the lever gives

$$\text{constant}+u'a'=Pa_1+P_1'A'$$

which, by differentiation (considering that, by the conditions of construction, as given in a preceding section, P does not vary with the temperature), becomes

$$A'dP_1'=u'da'$$

If, now, as in the construction by Fuess, the further condition is imposed that the scale of temperature shall be 5mm for 1° C., then

$$\frac{da'}{d\tau}=0.5^{cm}$$

whence

$$u'=2A'\frac{dP_1'}{d\tau}$$

which is, approximately, $=2A's_0q'p_0\delta F$, where

$$F=\frac{V_0}{V+q'(\eta B+m-2\eta n)}\times\frac{Q'}{Q'+q'-q''}$$

These equations, therefore, define the value of the weight u', in order that the scale of the temperature record may be exactly as agreed upon.

(*d*) *Adjustment for pressure.*—The effect of a change of atmospheric pressure upon the height of the mercury in the manometer tube is given by differentiating the first equation in paragraph (*b*) for the initial volume, with reference to n and B considered as variables, whence we obtain

$$\frac{dn}{dB}=\frac{V+q'(m-\eta n)}{V+q'(\eta B+m-2n\eta)}$$

Now, a change of 1 centimeter in the height of the mercury in the manometer implies a change of weight in the cistern such that

$$\frac{dP_1'}{dn}=-s_0q'\frac{Q'}{Q'+q'-q''}$$

and by combining these we obtain

$$\frac{dP_1'}{dB}=\frac{dP_1'}{dn}\cdot\frac{dn}{dB}=-s_0q'\frac{V+q'(m-\eta n)}{V+q'(\eta B+m-2\eta n)}\cdot\frac{Q'}{Q'+q'-q''}$$

This is very approximately equal to $-s_0q'F$, where F has the same significance as in paragraph (*c*).

If the effect upon the manometer of a change in atmospheric pressure is to be compensated for as in this instrument by attaching the barometer tube to the lever arm of the manometer, then the moments of pressure of these two changes must destroy each other and the principle of the lever must give

$$A'\cdot\frac{dP_1'}{dB}=-a_1\frac{dP}{dB}$$

or, by substitution,

$$a_1 s_0 Q = + A' s_0 q' F$$

whence

$$\frac{a_1}{A'} = \frac{q'}{Q} F$$

This shows that the relative lengths of the lever arms a_1 and A' are nearly constant, since F is subject to but slight variations, depending on the height n and diminishing as V is enlarged. For a value of $V=3{,}000$ cubic centimeters, and $\frac{1}{2}q'm=21$; the value of u' varies between 218.7 grams for the low temperatures and high pressures and 214.0 grams for the high temperatures and low pressures ordinarily experienced near sea level. The mean value of 216.6 is adopted in the instruments constructed by Fuess, by which construction the extreme error introduced into a recorded temperature of $\pm 25°$ C. is 0.167° C. at least for the pressures that occur in Europe. This is within the uncertainty of the temperatures ordinarily given by thermometers in shelters at meteorological stations.

The gas within the thermometer bulb should be very well dried, since, as shown by Sprung in his studies on the instrument at Spandau, if moisture be present, then the scale for temperatures below the dew-point of the inclosed air will differ from that for higher temperatures. Sprung concludes that the Spandau thermograph is more sensitive to changes of air temperature and follows them more quickly than does the ordinary mercurial thermometer, and that the mean deviation of the record from true air temperature is $\pm 0.15°$ C.

With regard to the Spandau barograph Sprung concludes that it is quite as sensitive and probably even more so than the barometer as observed by eye and hand; the mean difference between them is $\pm 0.041^{mm}$, if we treat the barograph as an independent instrument, but much less if we treat it as a differential instrument.

From the above it would appear that this instrument renders unnecessary the great labor of making frequent control observations and computing a formula for reducing its readings to standard.

79. WILD'S BALANCE BAROGRAPH.

The Wild balance barograph, as made by Hassler and Escher in Berne, is shown in the accompanying diagrams (Figs. 31 and 32), both front and side view. It is mounted by screwing it to a wall or pillar, and should be covered with a glass case to keep it clean and prevent the draughts of air from acting on it. This mounting is accomplished in general by firmly fixing the large cast plate GG by means of screws. The separate parts of the instrument, the balance, the registration apparatus, the paper reel, and mercury cistern, are fastened to the back plate GG. The brass balance W is hung on the cast-iron support T by means of the steel springs DD. On the right end of the balance there is a stirrup, *BB*, which supports the barometer tube, the lower end of this tube being immersed in the mercury cistern *Q*. On the left end of the balance beam is placed the temperature compensator (see Fig. 31), which consists of a stirrup which supports a glass tube, *V*, containing mercury, into which the lower end of a short barometer tube, *A*, projects. *A* is supported from the frame *G* by means of the arms *N* and *N*. This short barometer tube *A* has the upper part, *u*, filled with alcohol, below which is mercury having free connection with the mercury in *V*. *L* is a counterpoise or bent lever, and can be regulated by means of a sliding weight. The index *Z* is firmly screwed to the balance beam *W*, and has at its end the marking point *S*. On the roller *P* is wound a long strip of paper (a month's supply), and the free end of this paper is brought between the rollers *CC*; after passing between these it is wound up on the reel *R*. The registration takes place as follows:

Every ten minutes a current, for an instant only, is passed automatically by clock-work through the coils *E*, which, by acting on an armature through *H* and *F*, causes the frame *F* to strike on the index arm *Z*, and causes *S* to puncture a hole in the registration sheet. *P* is a counterpoise weight which causes *F* to move back to its place, freeing *F* from the paper as it does so, and at the same time turning the roller *C* slightly by means of a rachet. This turning of *C* causes the recording sheet to move forward a little and be ready to receive the next registration from *S*. Each time that the point *S* is made to puncture the paper there is a corresponding puncture on the edge of the sheet made by a fixed point being pressed against it. This point is used as a reference point for measuring the puncture made by *S*, and this row of reference punctures should form a straight line.

In practice the compensating tube *A* is filled four-fifths with mercury and one-fifth with alcohol, but the proper ratios can only be determined exactly by experiment. The scale of registration of the instrument gives a variation of about three times the actual change in the barometric height. A battery of six zinc-carbon elements is used for working this barograph.

This form of instrument has a considerable use in Europe, and several examples have already been introduced into America. The perfect simultaneity of the records on the complete set of Wild-Hassler instruments for recording pressure, temperature, wind, and rain is a marked advantage. In order that the records may be carefully reduced for publication the analysis of the errors of this instrument, as given by Wild and Schreiber, must be considered. The method of Wild is here given as he has applied it in the hourly records published by him:

(*a*) *Analysis of Wild's balance barograph for effect of temperature and pressure.*—The following analysis* shows the formula used by Wild in correcting the records of his barograph for errors due to temperature and pressure.

* See Wild's Repertorium für Meteorologie, Vol. III, art. 1.

Suppose the balance beam to be motionless in its average or horizontal position, then we have the following relations:

G the total pressure on the brass stirrup (B), which, in its turn, is carried by the balance beam.

R the weight of the empty glass tube.

S the specific gravity of the mercury.

Q the inner cross-section of the upper wide part of the barometer tube.

q the inner cross-section of the lower narrow part of the tube.

q'' the cross-section of the glass walls of the narrow tube.

Q'' the inner cross-section of the glass cistern C.

α the coefficient of cubic expansion of mercury for 1° C.

β the coefficient of linear expansion of the glass of the tube and the cistern.

γ the coefficient of linear expansion of the iron-supporting framework.

δ the linear coefficient of expansion of the brass of the stirrup.

Let the linear dimensions of the barometer tube, the stirrup in which it rests, and the cistern into which its lower end opens be as follows:

a the distance from fixed bottom of cistern up to lowest end of movable barometer tube.

e the distance from fixed bottom of cistern up to the constriction in tube where it rests in the stirrup.

g the distance from fixed bottom of cistern up to knife-edge on the beam supporting the stirrup.

c the distance from fixed bottom of cistern up to mercury in the cistern.

f the distance from fixed bottom of cistern up to the surface of the mercury in the enlarged portion of tube.

$b=f-c=$ the distance from mercury in the cistern to that in the enlarged portion of tube.

$h=f-e=$ the distance from constriction in tube to the mercury in the enlarged portion of tube.

$d=g-e=$ the distance from constriction in tube to the knife-edge on the beam supporting the stirrup.

Let all quantities which are for the temperature 0° C. have the subscript o attached, and those which are for the temperature $t°$ have the subscript t, then the laws of hydrostatics give for 0° C. the equation

$$G_0=R+S_0[Q_0(f_0-E_0)+q_0(e_0-c_0)-q_0''(c_0-a_0)] \quad . \quad . \quad . \quad (1)$$

For $t°$ the equation is

$$G_t=R+S_0\frac{1+2\beta t}{1+\alpha t}[Q_0(f_t-e_t)+q_0(e_t-c_t)-q_0''(c_t-a_t)] \quad . \quad . \quad (2)$$

The expansions among the parts give the following relations additional:

$$a_t=a_0+g_0\gamma t-(g_0-e_0)\delta t-(e_0-a_0)\beta t$$
$$e_t=e_0+g_0\gamma t-(g_0-e_0)\delta t$$

Introducing these values there results for the desired difference in the weights of the tube at the two temperatures 0°, and t° by neglecting quantities of the second order,

$$G_t - G_0 = S_0 t \Big\{ Q_0(f_t - f_0)\frac{1}{t} - (q_0 + q_0'')(c_t - c_0)\frac{1}{t} \qquad (3)$$
$$- (Q_0 - q_0)\,[g_0\gamma - (g_0 - e_0)\delta] + q_0''[g_0\gamma - (g_0 - e_0)\delta - (e_0 - a_0)\beta]$$
$$- (\alpha - 2\beta)\,[Q_0(f_0 - e_0) + q_0(e_0 - c_0) - q_0''(c_0 - a_0)] \Big\}$$

But for the determination of the unknown quantities $(f_t - f_0)$ and $(c_t - c_0)$ there are the two following equations of condition. In the first place, since the beam is motionless or the pressure is assumed to be constant, we must have

$$(f_0 - c_0)S_0 = (f_t - c_t)\frac{S_0}{1 + \alpha t}$$

or, by neglecting quantities of the second order,

$$\frac{f_t - f_0}{t} - \frac{c_t - c_0}{t} = \alpha(f_0 - c_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

In the second place, as no mercury is added or removed, the algebraic sum of all changes in volume due to the expansion of the cistern and its support, as well as to the rising of the mercury in the tube and the cistern, must equal the true expansion of the whole mass of mercury, and thus we obtain the equation

$$\alpha t[Q_0(f_0 - e_0) + q_0(e_0 - c_0) + Q_0''c_0 - q_0''(c_0 - a_0)]$$
$$= Q_0''[(1 + 2\beta t)c_t - c_0] - q_0''[(1 + 2\beta t)(c_t - a_t) - (c_0 - a_0)]$$
$$+ q_0[(1 + 2\beta t)(e_t - c_t) - (e_0 - c_0)] + Q_0[(1 + 2\beta t)(f_t - e_t) - (f_0 - e_0)]$$

From this, by neglecting quantities of the second order, there results

$$\frac{Q_0(f_t - f_0)}{t} + (Q_0'' - q_0'' - q_0)\,.\,\frac{c_t - c_0}{t} = (Q_0 - q_0)\,[g_0\gamma - (g_0 - e_0)\delta] \qquad (5)$$
$$- q_0''[g_0\gamma - (g_0 - e_0)\delta - (e_0 - a_0)\beta]$$
$$+ (\alpha - 2\beta)\,[Q_0''c_0 - q_0''(c_0 - a_0) + q_0(e_0 - c_0) + Q_0(f_0 - e_0)]$$

If we introduce in equation (3) the values computed from (4) and (5) of $(f_t - f_0)$ and $(c_t - c_0)$, we have finally

$$G_t - G_0 = \frac{S_0 Q_0''\,.\,t}{Q_0'' + Q_0 - q_0 - q_0''}\Big\{ 2\beta Q_0 b_0 + (\alpha - 2\beta)\,[Q_0 - q_0)c_0 - q_0''a_0] \qquad (I)$$
$$- (Q_0 - q_0 - q_0'')\,(g_0\gamma - d_0\delta) - q_0''(c_0 - a_0)\beta \Big\}$$

in which, for brevity, we have put

$f_0 - c_0 = b_0$, *i. e.*, the barometer height reduced to 0° C;

$g_0 - e_0 = d_0$, *i. e.*, the length of the brass stirrup *B*.

The numerical data are as follows:

$$S_0=13.56 \qquad \alpha=0.0001815 \qquad \beta=0.0000082$$
$$\gamma=0.0000126 \qquad \delta=0.0000188$$

In the apparatus at St. Petersburg the dimensions are

$Q_0''=2827$ square millimeters.	$g_0=1200$ millimeters.
$Q_0=1257$ square millimeters.	$d_0=250$ millimeters.
$q_0=28$ square millimeters.	$e_0=910$ millimeters.
$q_0''=20$ square millimeters.	$a_0=120$ millimeters.

Inserting these numerical values in equation (I) there results, for $b_0=760^{mm}$

$$G_t-G_0=13.56\times0.7004\times t(15.67+184.25-12.60-0.13)$$
$$=13.56\times131.12\times t=1778\times t \text{ milligrams}$$

For $b_0=790^{mm}$ there results

$$G_t-G_0=13.56\times131.55\times t=1784\times t \text{ milligrams}$$

Consequently, for the same change, from 0 to t, in temperature, and for a barometer reading 30^{mm} higher, the difference in weight at t° C. is only $6\times t$ milligrams.

On the contrary, under the supposition that the temperature remains constant, and that the pressure has changed by p millimeters, the difference in weight of the tube, as we shall see further on, would become

$$G_{p+p}-G_p=\frac{S_0Q_0''Q_0p}{Q_0''+Q_0-q_0-q_0''} \quad . \quad . \quad . \quad . \quad . \quad . \quad (II)$$

According to this the difference in weight of the tube for two temperatures differing by t° is equal to that for two pressures differing by p millimeters, where p is given by the relation

$$p=\frac{t}{Q_0}\Big\{2\beta Q_0b_0+(\alpha-2\beta)\,[(Q_0-q_0)e_0-q_0''a_0] \qquad (III)$$
$$-(Q_0-q_0-q_0'')\times(g_0\gamma-d_0\delta)-q_0''(e_0-a_0)\beta\Big\}$$

In the St. Petersburg apparatus Wild finds

for $b_0=760^{mm}$ . . . . $p=t\times0.1489^{mm}$
for $b_0=790^{mm}$ . . . . $p=t\times0.1494^{mm}$

According to which a change in temperature of 10° C. causes a change of weight of the tube corresponding to a change of 1.49^{mm} in barometric height; and for an absolute height greater by 30^{mm} the weight would increase only by an amount corresponding to 0.005^{mm} of the barometer.

The preceding assumes that the balance beam is fixed, and the computations for the reading 790^{mm} therefore hold good with this restriction. This gives us therefore an approximate idea only as to how much the readings would be affected for places having different mean barometric heights.

If we wish to know the amount of this change at any one place for the usual range of fluctuations of pressure we must take into consideration the actual motions of the balance beam. When a barometer reads higher than the mean the balance arm and tube sink, and in consequence of this not only does b_0 become greater in equation (I), but at the same time e_0 and a_0 become smaller (the last two quantities change by the same amount).

In the apparatus at St. Petersburg, for example, for an increase in pressure of 30^{mm} the stirrup and tube sink 45^{mm}; consequently for $b_0=790^{mm}$ we have $e_0=865^{mm}$ and $a_0=75^{mm}$.

If we insert these values in equation (I) we have

$$G_t-G_0=13.56\times 0.7004\times t(16.29+175.27-12.60-0.13)$$
$$=13.56\times 125.26\times t=1699\times t \text{ milligrams}$$

that is to say, for a barometric height higher or lower by 30^{mm} the same change in temperature causes a decrease in weight of 79 milligrams for each 1° C. Again, if we replace this influence of the temperature on the weight by its equivalent in barometric height, according to equation (III), we now find for $b_0=760^{mm}, p=t\ 0.1489^{mm}$, and for $b_0=790^{mm}$, $p=t\ 0.1423^{mm}$. Therefore for a mean barometric height of 760^{mm} a rise in temperature of 40° C. has the same effect on our balance barometer as if the barometric height had increased 1.439^{mm}; but the change of this temperature influence, caused by a variation in the barometer height of $\pm 30^{mm}$, reaches a value equivalent to only $\pm 0.066^{mm}$ in barometric height. The influence of temperature on the results given by this form of balance barometer, as computed for the mean barometric height of St. Petersburg, is consequently not affected by the ordinary fluctuations of pressure beyond the adopted limiting error of $\pm 0.1^{mm}$ if the range of temperature of the apparatus does not exceed 10° C. As the latter condition is usually fulfilled, therefore in this instrument the increase in weight for each 1° C. is to be regarded as constant; consequently the value

$$G_t-G_0=1784\times t \text{ milligrams}$$

holds good for this instrument without restriction. But these errors introduced by its own temperature may be nearly annulled mechanically by several devices, thus avoiding the labor of large numerical corrections when once the mechanical temperature compensator has been properly adjusted. Wild's mechanical method of doing this has been already explained in a general way, and the analytical formulæ are given in the following section.

(*b*) *Temperature compensation.*—Referring back to the first part of this section (79) for a general description of Wild's method, if we call V_0' the volume of the mercury and V_0'' that of the alcohol at 0° C. in the compensating thermometer, and also put ρ for the true cubical coefficient of expansion of alcohol for 1° C. and l and l' the lengths of the arms of the balance beam, then the equation for temperature compensation must be

$$t \,.\, l' \,.\, S_0[V_0'(\alpha-3\beta)+V_0''(\rho-3\beta)]=l\,(G_t-G_0)$$

Suppose the thermometer cistern to be represented by a cylindrical tube with cross-section Q_0 and height z_0' and z_0'' where z_0' denotes the height of the mercury and z_0'' that of the alcohol in it, and suppose the volume of the overflow tube so small as to be negligible, then will

$$z_0'\alpha+z_0''\rho-(z_0'+z_0'')3\beta=-\frac{l(G_t-G_0)}{l' \,.\, Q_0 \,.\, S_0 \,.\, t}$$

If to this we add the equation

$$z_0'+z_0''=Z_0$$

that is, if we assume the whole length of the thermometer cistern as given, we can compute from these two equations the relation of z_0' and z_0'', consequently the quantity of the mercury and alcohol which is necessary for the temperature compensation.

In the apparatus at St. Petersburg

Z_0=250 millimeters.	Q_0=1257 square millimeters.
l=200 millimeters.	l'=250 millimeters.

From the determination of Pierre and Kopp on the expansion of pure alcohol between 10° and 20° C.

$$\rho=0.0010899$$

Substituting these values and those for α, β, S_0, and G_t-G_0, there results

$$z_0''=48.68^{mm}$$

that is to say, the temperature compensation is accomplished by filling an outflow thermometer of the dimensions given with about one-fifth pure alcohol and four-fifths mercury.

(*c*) *Errors of capillarity.*—As at first made the "Wild" barograph was affected by a relatively large error, due to the varying capillary action of the rising and falling mercury. With rising barometer the barograph lagged behind the barometer readings by 0.17^{mm}, and by falling barometer the lagging was 0.12^{mm}. In order to overcome this difficulty Wild arranged that a weight should be temporarily applied* to the left

* Beseitigung des Capillaritätsfehlers beim Wag-Barograph. Bulletin Acad. Sc., T. XXIII, March 8, 1877, St. Petersburg.

arm of the balance between each registration. This causes the right arm to rise, and consequently the mercury falls in the tube to rise again when the weight has been removed. This weight is made to descend, and after acting to ascend again by automatic action of a clock-work arrangement at the top of the barograph. This has been found to give very satisfactory results, as the records are always made with a rising and maximum meniscus.

(*d*) *Value of a scale division.*—Wild finds that the change of true pressure of 1^{mm} corresponds to a movement of 3.2^{mm} of the registering point of his barograph, and is very nearly proportional to the sine of the angle through which the balance arm turns in its endeavor to maintain the counterpoise, and that the moment of the moving force is very large compared with any frictional resistances, so that the instrument is abundantly sensitive.

SECTION C.

THE MEASUREMENT OF THE MOTION OF THE AIR.

CHAPTER XI.

GENERAL REMARKS.

80. STATEMENT OF THE PROBLEM.

A knowledge of the motions of the atmosphere constitutes an essential element in all investigations of dynamic meteorology. These motions are conventionally classified into upper and lower currents, the upper currents being those in and above the region of the cirrus clouds, and the lower currents those near the surface of the earth; but between them no rigid distinction can be maintained. The motion of the upper currents is generally very different from that of those near the surface, but both, so far as we at present know, are of equal importance in the theory of atmospheric circulation and the development of storms.

Lower atmospheric currents, *i. e.*, currents near the earth's surface, are commonly called winds, while the motions of the upper atmosphere are termed currents. The direction and velocity of the wind are much influenced by the general character of the surface as to land and water, mountains and plains; our observations are further influenced locally by the buildings upon which meteorological instruments are exposed, and the hills and trees that surround them. Those peculiarities of exposure cause minor disturbances in the general movement of the wind, which latter is the datum important to dynamic meteorology. In general a proper exposure is obtained by securing as great freedom as possible from surrounding obstructions, and the observer must learn by experience to distinguish local disturbances from the general motion of the wind.

Although the observation of the winds has received far more attention than that of the upper currents, because the former constitutes an essential element of climate and have a direct practical relation to the business of the navigator, the engineer, and the architect, yet the upper currents are of such importance in meteorology that we shall give such methods of observing them as have been devised hitherto. The whole subject, therefore, is embraced in the seven following sections, embracing methods of observing the direction and strength of winds and upper currents.

As regards the strength of the wind, although the personal estimation of the velocity of the wind by wind scales still obtains as the general method of observation, yet instruments for accurate measurement also have been in use for over two centuries. The air in motion produces a variety of mechanical and physical effects, and it is evident that such of these effects as are proportional in their degree or intensity to the wind velocity may afford by their measurement an indirect means of its determination. A current of air sets in motion a system susceptible of rotation about an axis; produces an augmentation or diminution of pressure on different portions of fixed or constrained bodies; cools surfaces of higher temperature by convection; accelerates evaporation; evokes musical sounds of different pitch or intensity from properly constructed apparatus susceptible of vibration. Anemometers may therefore be classified with respect to the physical effect of which they afford a measurement. The cooling, evaporative, and musical effects above enumerated do not bear a sufficiently definite and measurable relation to the wind velocity to form the basis of accurate anemometers and have never come into extensive use; but the first three effects, rotation, increase of pressure, and diminution of pressure or suction, have all been successfully applied to the construction of anemometers, and all important anemometers may be classified under the three divisions, (*a*) pressure, (*b*) suction, and (*c*) rotation.

CHAPTER XII.

MEASUREMENT OF THE DIRECTION OF THE WIND.

81. THE PROBLEM IN GENERAL.

The movement of the air may have a vertical as well as a horizontal component. The direction of the horizontal component, which is generally the principal part of the wind, may be determined with considerable accuracy without the aid of special instruments. Observers in an open space, where the wind is free from deflections occasioned by buildings or other obstructions, usually estimate its direction to the nearest eight or sixteen points of the compass. On shipboard, where the motion of the vessel itself affects the indications of wind-vanes, flags, or smoke, the true direction is estimated (allowance being made for the effect of the ship's motion), together with the necessary correction from magnetic to true direction. After taking account of these influences an accuracy of one thirty-second of a circle is generally considered to be attainable by experienced seamen. On land it is only in comparatively isolated locations that an observer on the ground, who has nothing but his own feelings or the motion of surface objects for his guide, can feel assured that he is observing the true and not a deflected direction of the wind. Therefore a vane placed upon a mast or tower, or at some distance above an ordinary roof, is employed to indicate the undisturbed direction of the general wind, which at lower elevations along the city streets is whirled into innumerable eddies and cross currents that do not constitute the wind of the meteorologist.

The anemoscopes in common use may be treated of in two classes, (*a*) arrow vanes, (*b*) wind-mill vanes.

82. THE ARROW VANE.

The common weather-cock or wind-vane is no doubt the oldest as well as one of the simplest of meteorological instruments. The record of its use in systematic observations begins with 1650 in Italy and 16[illegible]7 in England. As a popular indicator of the wind the vane was given the shape of a cock, a fish, a trumpeter, dragon, diabolus, or other ornamental device. But in the arrow vanes used for accurate meteorological observations these forms have been replaced by simple plane plates, disks, or arrows, designed solely with regard to the mechanical action of the wind upon them. The vane may either turn about a fixed spindle, or it may be fixed to the spindle, which latter then turns on fixed bearings of friction rollers. The advantages of the latter method in affording protection from the weather render it a much preferable construction, and the best vanes designed for meteorological purposes are of this pattern. When desirable the spindle, or a rod connected therewith, is extended through the roof, and the wind direction is indicated in the room below by a small arrow rigidly fixed to the rod and moving under a compass circle painted upon the ceiling. The adjustments of the vane are as follows:

(1) The axis or spindle should be vertical. Lack of verticality of itself alone impairs the sensitiveness and increases the friction and wear of the bearings.

(2) The center of gravity of the vane should pass through the axis of rotation. This is attained by a counterpoise movable along the pointer. To make this adjustment the axis of the vane is placed in a horizontal position and the counterpoise is set to a position in which the vane will have no tendency to turn, but will remain at rest in any position in which it is placed.

(3) If the vane be not thus counterpoised, and the center of gravity lie at some point not over the axis, the following conditions will obtain:

First. If the spindle be not vertical the vane will tend to assume a position in which its center of gravity will be at the lowest point. It will, therefore, not be in neutral equilibrium.

Second. If the spindle be vertical the direction of the vane will not be directly affected in the same manner as in the preceding case by the lack of a counterpoise, but the weight of the vane will press upon the side bearings, thereby increasing the friction and causing an unequal wear at certain points, depending somewhat upon the prevalent wind. This unequal wear tends eventually to destroy the verticality of the axis, and the additional friction diminishes the sensitiveness of the instrument.

Sensitiveness.—The sensitiveness of a vane adjusted as above depends primarily upon the friction of its bearings. The friction varies chiefly with the weight of the vane and the character of the bearings, so that by reducing its weight and by the use of oiled friction rollers, hardened steel points, and polished glass or agate bearings a vane may be made as sensitive as desired. For ordinary purposes, however, where the simple direction of the wind to sixteen points is all that interests the observer, extreme sensitiveness is not requisite and becomes an objection rather than an advantage, inasmuch as in strong or even moderate winds their continual small veerings keep the vane in oscillation and render the main direction difficult of determination.

Adjustment to the meridian.—For indicating the direction of the vane when the rod does not pass through the roof, either, first, points of reference, properly adjusted to the meridian, are employed, such as fixed cross bars placed beneath the vane and bearing the cardinal letters N., E., S., W.; or, second, a drum marked with the eight compass points or simply eight arms may be fixed to the rotating rod beneath the vane and the direction be indicated by a fixed index. Having established the index in any position convenient for observation the adjustment is made as follows: Turn the vane until the arrow points to some distant object whose bearing is known. Turn the drum about the spindle until the direction on the drum which corresponds to the direction of the vane is coincident with the fixed index. Clamp the drum to the spindle in that position and the adjustment is complete.

At stations where the vane rod passes through the roof and in the room below indicates the direction of the wind by an arrow near the ceiling moving under a fixed vane circle, an observer should proceed as follows:

Draw a vane circle upon the ceiling around the rod as the center and a little greater in diameter than the length of the arrow, paint thereon the eight points of the compass, with the north and south line approximately in the meridian. Let the wind vane be turned by an assistant on the roof, to a distant object whose bearing is known, and while in that position let the arrow be fastened to the vane rod with a set-screw so that it shall exactly coincide with the direction on the vane circle corresponding to that of the wind vane. The instrument will then be in adjustment. The above instructions involve turning the wind vane so that it shall point to some object whose bearing is known. For this purpose the pole star affords the most accurate point of reference. Strictly, the pole star bears north only at its upper and lower transit, but for latitudes under 45° the greatest departure from the meridian is less than 2°. In carrying out this portion of the operation of adjustment the following method is recommended: Suspend a plumb-line from a movable support at a considerable distance from the vane, and by successive trials bring it into perfect alignment with both the pole star and the rod of the vane. The meridian is thus defined by the plane passing through these vertical lines. Then one person should turn the vane until its axis, as defined by the pointer and middle of the tail, appears to be in coincidence with the vertical plane passing through the plumb-line and the vane rod. With the vane in this position the drum or arrow of the vane circle should be set according to the above instructions. If there be a wall or other fixed object at a sufficient distance from the vane the observer should make a permanent mark thereon in agreement with the vane, the star, and the plummet, by reference to which the vane can be set or tested at any time. The position of the mark must, of course, be obtained at night; but having established the mark the operation for setting or testing the vane should be performed by daylight in calm weather. As the adjustment is liable to be disturbed by the loosening of clamps it is desirable to test its accuracy at occasional intervals, or at any time when there is occasion to suspect that a disagreement exists between the vane and its recording index.

83. FORMS OF WIND VANE.

Among the forms of arrow vanes the following types are presented:

(*a*) *Kreil's wind vane.*—The above-named conditions of sensitiveness were embodied by Kreil in 1849 in the construction of a self-recording anemoscope for the Austrian meteorological service. The vane (Fig. 33) consists of a disk, *A*, of thin sheet-iron at one end of a horizontal iron rod, which carries a lead ring at its other end as a counterpoise. The rod is screwed fast at its center of gravity to a sheath, which turns upon the vertical spindle *E*, having a hard-pointed tip to reduce the friction of the apparatus. The lower end of the sheath (see Fig. 34) is provided with a plate of friction rollers, *F*, upon which it turns about the spindle. This apparatus was found to be exceedingly sensitive, and, in fact, presented a difficulty thereby. Often in the same day, especially during light winds, the vane made five or six complete rotations in the same direction, carrying the pencil off the registering sheet.

(*b*) *Osborne's experimental vane.*—A careful study of the action of wind vanes has been made by Mr. J. W. Osborne, of Washington, D. C. (Am. Assoc. Adv. Sci., 1878), the results of which illustrate and confirm several general principles necessary to be considered in the construction of a good wind vane. Starting out with the definition of a perfect vane as one which instantly responds to the slightest change in the wind direction and remains stationary when it has made the necessary angular movement, he made the following construction in order to attain as nearly as possible these conditions:

The frame of the vane (see Figs. 35 and 36) was 7 feet long, while its weight was only 3 ounces; the material was the thinnest sheet-brass rolled hard and made into tubes five-sixteenths of an inch in diameter. By his method of suspension a counterpoise was rendered unnecessary, thus contributing to the lightness of the vane. At the end of the arm, constructed as in the drawing (Fig. 36), a flat circular disk 7 inches in diameter was attached to receive the impulse of the wind. With several cross wires to give rigidity to the vane the whole was found to possess remarkable elasticity and strength. The spindle, three-eighths of an inch in diameter, ran between two groups of friction rollers, and at its lower end terminated in a hardened steel point, turning in a polished cavity in a piece of glass, on which rested the whole weight of the instrument. In this construction it will be seen that every effort was made to reduce friction and inertia. Preliminary experiments showed that 1¼ grains of pressure perpendicular to the center of the flat disk was sufficient to start it and keep it in motion against quiescent air. The behavior of this wind vane was most admirable, as it exhibited all the oscillations of both light and heavy winds and yet was free from any oscillations due to its own momentum. For practical purposes, however, when the main direction of the wind is all that interests the observer, it is often difficult to determine it from watching the unceasing oscillations of a highly sensitive vane. It is obviously unphilosophical to overcome this difficulty by adding to the inertia of the vane and by retarding it with an overwhelming amount of friction, which varies from day to day, and which is greater at the beginning of motion than when motion is established. The true plan is to allow the vane to represent all the actual deviations of the wind, and then so to deal with the record as to obtain a mean direction. The solution of this problem was attempted mechanically by a method of damping, as follows:

A vertical shaft connected with the vane rod by a belt oscillates synchronously with the vane; a long delicate spiral spring encircles the shaft and is fastened at its upper end by a set-screw. At its lower end it carries a free collar, to which a pointer is attached, and also two or more horizontal arms, from which blades hang in a vessel containing oil. When the wind vane oscillates the little vertical shaft oscillates synchronously with it, but the lower collar, with its pointer and the blades immersed in the oil, receive the impulses through the spring as a medium, whereby the quick spasmodic movements of the wind vane are in part eliminated and those of a more permanent character proportionately represented. This damping can be made to any degree desired, either slight or considerable, by altering the stiffness of the spring, the viscosity of the fluid, or the size of the containing vessel.

In a second paper Mr. Osborne gives the results of comparisons between the above-described delicate vane and a somewhat heavier and shorter one previously constructed. The following are his results:

In light winds the lighter and longer vane made the larger number of complete rotations, but in strong winds this was true of the heavier vane, a result obviously due to the greater sensitiveness of the one and the greater inertia of the other, and therefore not a characteristic of the wind; the modifying effect of the force of the wind is, therefore, an important element in the comparison. As the wind increases in velocity the number of total rotations of the lighter vane generally decrease in number, while those of the heavier vane, especially in gusty weather, are often relatively increased, a result evidently due to the momentum of the heavy vane. The lighter vane gives the true state of things for strong as well as gentle winds; its oscillations are generally smaller and its rotations fewer in windy weather, because the onward motion is so quick that an impulse from one side is immediately succeeded by one from the other, thus causing short quick movements of the vane. But the characteristics of the various strong winds differ among themselves just as widely as those of feeble currents, and in such cases the origin of differences in the action of the wind vane is referable essentially to the motions of the wind itself.

(c) *The tablet vane.*—In this form the vane is a thin rectangular plate supported near one end, to which also a pointer is attached; it differs therefore from the arrow, disk, and other forms in that the shape is strictly rectangular. Owing to the action of the currents that flow around the edge of the plate such a rectangle comes to rest in a position inclined to that of the wind at an angle depending on the proportions of the plate and the relative distance of the supporting axis from the ends of the rectangle. For certain locations of the axis the angle becomes quite indeterminate.

In general the flow of air produces a more uniform distribution of pressure over the windward than over the leeward side of a plane plate, especially when it is not normal. The air that flows around the anterior edge of an inclined plate reaches the leeward side as a stream that impinges on it most forcibly at a point some distance behind the anterior edge, so that the difference between the front and rear pressures at any point of the plate is least at the posterior edge and greatest in the anterior portion of the plate; if, therefore, the plate is supported by an axis parallel to the anterior edge it is possible to place this axis at such a position, always between the anterior edge and the center of the plate, that the moment of the pressures on the anterior portion will just balance the moment of pressures on the posterior portion, and this position will vary with the inclination of the plate to the wind. The exact values of the angle for given positions of the axis in the case of a plate moving through the air on a whirling machine is given by Kummer's experiments (Berlin Abhandlungen, etc., 1876), and suffices to show the inappropriateness of this form of anemoscope. The wings of a single or double vane should therefore be supported by an axis that is either in or to the windward of the anterior edge of the wing.

(*d*) *Double tail vane.*—In order to diminish the continual oscillations to which the vane is subject during variable winds, the elder Parrot, in 1797, constructed a vane of two thin plates, joined at one end, and making an angle with each other of about 45°. This form was adopted by the younger Parrot on his journey in the Caucasus in 1811. In 1840 vanes with spread tails were in common use in England, and with regard to their advantages Jelinek makes the following statement:

> The English seek to obviate the disadvantage of the complete rotation by making the wind vane of two plane surfaces set at an angle to each other of generally about 22½°. They say that thereby the oscillations become smaller and the complete rotations less frequent (Sitzungsberichte Kaiser. Akad. Wissenschaften, Wien, 1850).

Stating that this had been shown to be true in his own experience, Jelinek adopted the same form for an anemoscope, which he proposed as an improvement on that constructed by Kreil. The use of similar spread tails has also become general throughout the United States through their adoption by the Signal Service.

The accompanying drawings (Figs. 37 and 38) show the details of the construction of the Jelinek's double-tail wind vane. As now in general use at the stations of the Signal Service, the vane proper consists of two wings of light wood at the leeward end of a rod whose windward end carries an arrow-tipped pointer and the counterpoise. The wings and the pointer are approximately 3 feet long, so that the vane rests symmetrically upon the axis. The total weight is a little less than 12 pounds.

(*e*) The most advantageous construction for a wind vane is attained by attaching to the lower end of the rod a buoyant drum with damping vanes, the whole immersed in a bath of oil. The buoyancy of the drum reduces the friction to a minimum or gives the vane a maximum sensitiveness, while the dampers increase the stability.

84. THEORY OF THE WIND VANE DURING STEADY WINDS.

To the experimental results already quoted relative to the action of arrow vanes should be added the results of a deductive treatment by Curtis (Am. Journ. Science, XXXIV, p. 45) of the mechanical principles involved in the action of the vane during steady winds.

(*a*) *Straight vanes.*—Let the pressure on the windward side of a plane plate, due to a change in the direction of the wind, be P, and the defect of pressure on the leeward side be nP, where n has been found by experiment to have values ranging from 0.2 to 0.36 for normal incidence on small planes, and may be assumed as 0.5 or less for such sizes as occur in wind vanes. P and nP vary approximately as the sine of the angle between the plane of the plate and the wind direction; therefore the pressure due to a wind inclined at an angle, θ, will be $(1+n)P \sin \theta$. This will move the vane if the moment of pressure overcomes the moment of friction (this latter is due, first, to the weight of the vane and resulting friction on its vertical supports; second, to the pressure against its side bearings, due to the pressure of the wind against the vane). The smaller the moment of friction and the larger the moment of the pressure the more quickly will the vane respond to a change in the direction of the wind or the more sensitive will the vane be.

Once started into motion the inertia of the vane carries it beyond the new wind direction, and it performs a series of oscillations, which, unless the wind direction again changes, will rapidly diminish, owing to the friction of the bearings and the resistance of the air. Assuming the wind to continue unchanged in direction and force, and omitting friction, but considering only the resistance of the air, Curtis shows that the angle θ, by which the oscillating vane successively departs from its mean position, constitutes a series in geometric progression of which the common factor is $e^{-\frac{k}{h}\pi}$ in which the ratio $\frac{k}{h}$ depends upon the dimensions of the vane and the resistance of the air, so that the angular oscillations of the vane are smaller as the area of the vane is larger, and the rates at which the oscillations diminish increases as $\frac{k}{h}$ increases. The latter is therefore an index of stability.

The final position of rest may be anywhere between exact coincidence with the wind direction and a small inclination, θ_0, such that the moment of pressure is just equal to the moment of friction, or

$$\sin \theta_0 = \frac{\text{moment of friction}}{\text{moment of pressure for normal wind.}}$$

This equation, therefore, expresses the outside limit of error of the vane when it has come to rest in a steady wind.

A liquid damper attached to the vane is equivalent to diminishing the inertia without affecting the friction or the angle of final rest. Therefore it increases the stability without diminishing the accuracy.

(*b*) *Spread vane.*—Let the angle between either wing and the axis of the vane be ε; that between the wind and the axis be θ.

The influence of the wind in producing gyration is to be considered for two cases:

(1) When $\theta<\varepsilon$ the pressure perpendicular to the axis is

$$P \sin(\theta+\varepsilon)+P \sin(\theta-\varepsilon)=2P \sin\theta \cos\varepsilon$$

For an angle, ε, such as most frequently occurs, namely, not exceeding 25°, $\cos\varepsilon$ may be considered constant and equal 0.95.

(2) When $\theta=\varepsilon$ or $\theta>\varepsilon$ the pressure perpendicular to the axis is

$$P \sin(\theta+\varepsilon)+nP \sin(\theta-\varepsilon)$$

if we neglect the effect of the wind that flows in between the wings of the vane, or rather if we assume that the included space is boxed in.

The above-quoted treatise by Curtis shows that in both these cases the expression for this gyratory force is larger than for straight vanes for all small values of ε, and the value of ε, which gives a maximum excess, depends on the value of n, and is 41.4° when n equals 0.5. Therefore for all moderate values of ε and θ, whether θ be greater or less than ε, the pressure producing gyration is larger for the spread than for the straight vane; therefore, other things being equal, the spread vane will be the more sensitive. But in a spread vane the lateral friction in general is larger, owing to the larger surface presented to the wind, and varies nearly as $A \sin^2\varepsilon$ (where A is the surface of either wing), which indicates that ε should be as small as consistent with other conditions. The best value of ε is given by the condition that the gyratory force minus the moment of the lateral friction shall be a minimum; whence Mr. Curtis deduces that for $\theta<30°$ the maximum sensitiveness is attained when the half angle of the wings is about the average angle through which the wind suddenly changes its direction, and for $\theta>30°$ the angle ε should be less than 30°. Consequently for all ranges of θ the angle ε is less than 30°, and if ε be made equal to the average θ the condition of maximum sensitiveness will generally be fulfilled.

The oscillations that the spread vane makes after being started by a change of wind direction diminish rapidly in a geometrical series whose common multiple is $e^{\frac{-2k}{h_1}\pi}$. The exponent of this factor is always somewhat larger than the corresponding exponent for the straight vane; therefore the amplitude of the oscillations will diminish more rapidly and the spread vane will be more stable than the straight vane.

The accuracy is decided by the fact that the vane comes to rest finally at a slight inclination to the wind direction; for the straight vane the limiting angle of rest was θ_0, determined above by the condition that the moment of pressure equals the moment of friction. Other things being equal in the spread vane the moment of pressure is larger than for the straight vane; therefore the limiting angle is less and the accuracy greater.

(*c*) *Gusts.*—All the preceding paragraphs assume that the wind velocity is steady and that the direction only changes; but the change in direction also frequently comes with a gust, whose duration is so short that it has traveled the entire length of the vane and ceased to act before the latter has wholly adjusted itself to the new direction. For such cases the force acting on the vane is to be considered simply as an impulse, after receiving which the vane moves by its inertia until the momentum of its initial motion is used up in work done by friction and resistance of the air. The angle through which the vane will turn will therefore be determined by the ratio

$$\frac{\text{moment of initial momentum of vane}}{\text{sum of moments of friction and resistance.}}$$

For very sensitive vanes, therefore, when the denominator is small and the numerator large, the vane may make a complete rotation, and even if it describes only a small angle its position of rest gives no indication of the direction of the gust. If the resultant pressure of the gust on the vane does not pass through the center of percussion there will be a strain on the vertical supporting rod and on the vane and an increased friction, all of which should be avoided as much as possible.

The true direction of momentary gusts and winds of great variability can be best obtained by observing a light string or penant, where the inertia is a minimum, the normal resistance is zero, and the principal force acting on the string is the so-called skin friction of the wind, as it blows along the string and temporarily stretches it in the direction of its motion. The direction of gusts can also be obtained by pendulous spheres or cylinders, whose initial motion will be in the direction of the gust. The oscillations of the ordinary wind vane, in so far as they are due to gusts, show nothing of the actual variations in the direction of the wind.

85. THE WINDMILL VANE.

A distinct class of vanes, differing in the method of their action from all forms of arrow vanes, is that in which a pointer is brought into the wind direction by the action of fan-wheels (see Figs. 39 and 40), similar to the governor attributed to Sir William Curbitt, and long ago applied to European windmills to keep them facing the wind.

This governor was first used as a meteorological instrument by Osler, in 1836, in the construction of his self-recording anemograph; only one fan-wheel was employed by him. In Beckley's anemograph the additional wheel is added for symmetry, and their common horizontal axis is placed at right angles to the pointer which indicates the wind direction.

The fan-blades, generally eight in number, are adjusted at such an angle that when the pointer is in line with the wind the wind pressures on the opposite arms of each pair of fans are exactly balanced and the fan remains at rest; but if the direction of the wind forms an angle with the pointer the fan turns so as to bring back the pointer into the wind direction. If the wind suddenly shifts through a large angle the fans will begin to turn, at first rapidly, then more and more slowly, until the pointer is again toward the wind, when their motion will cease.

In this form of vane there is no oscillation about the position of equilibrium, as in the case of the arrow vane; but on the other hand it is far less sensitive.

86. VERTICAL VANES OR INCLINOMETERS.

The wind vanes described in the preceding sections serve to indicate only the direction of the horizontal component of the wind's motion; but the wind usually has also a vertical component by virtue of its forming part of an ascending or descending current. Attention has occasionally been given by individual meteorologists or observatories to the measurement of inclination of the wind or to the amount of the vertical component, but it has not become a part of the regular observations of any meteorological service. As yet the proper interpretation to be placed upon the records of vertical-component-anemometers or inclination-vanes is by no means well determined, nor is it clear that such data for the air next the surface of the earth would form a valuable addition to the present data of meteorological observations; for higher altitudes, however, it is very much desired. Benzenberg first proposed a double vane for this class of observations, and Montigny, in 1853, Hennessey, in 1856, and Dechevrens, in 1881, have claimed that their observations exhibit the existence of general vertical currents in the atmosphere. If this were assured the data would unquestionably be of importance; but the vertical component of the air current indicated by vanes established on the galleries of towers, the roofs of buildings, the sides of hills, or under any uneven condition of topography is likely to be largely due to eddies and deflections produced by these environments, and is not a general current such as the meteorologist desires to observe. Only in a level country or at sea, with a vane established upon a very high tower, can we feel assured that the results of vertical measurements will be of meteorological importance, and that general currents, vertical or inclined, are really the subject of observation.

(a) *Montigny's Inclinometer.*—The inclination of the wind to the horizon was investigated by Ch. Montigny, from 1851 to 1875, by means of a portable tipping vane (see Fig. 41). The apparatus consists essentially of a horizontal vane, LL, which keeps the tipping vane facing the wind. The latter is a balanced spread or double tail vane, made of a pair of thin plates, BB′, each 120^{mm} long by 60^{mm} broad, inclined to each other at an angle of about 20°. To these are attached the balancing rod *D* and the pointer *C*, extending down to the graduated arc *G*, on which two indices slide, so as to record the extreme oscillations of the pointer. In the course of long-continued observations on the tower of the cathedral of Anvers Montigny found that the influence of the tower in deflecting the wind was perceptible, but in his regular observations he was only able to place the vane 2 meters away from it. He states that when in action "the vane often tipped and remained at a fixed position for some moments, but sometimes also it oscillated continually between rather extensive limits without settling in any definite position, which indicated frequent variations in the inclination of the wind. In this last case I have taken for the inclination the average of the extreme limits of oscillation of the pointer."

As before shown, a perfectly balanced vane of this kind, whose moment of friction is small, is quite at the mercy of gusts of wind, and its extreme oscillations simply show the arc described by it in order that the work of friction and resistance may use up the energy of its initial motion. If when the vane comes to rest at one extremity of its oscillations the sliding index indicates a plunging or descending wind of 20° inclination, while at the other extremity the other sliding index indicates an ascending wind of 15° inclination, this simply means that some one downward gust was stronger than the strongest ascending one. Montigny, however, concludes from his observations that there is an average downward inclination of the wind increasing as we rise above the ground, and that the average inclination at 64, 89, and 104 meters is respectively 2.17°, 8.36°, and 12.28°. But as these figures are simply the means of the extreme oscillations they are open to the criticism above made. These results agree with statements made by Langsdorf and Poncelet to the effect that in Europe the planes of rotation of windmill sails are inclined to the vertical at an angle of 10° or 20°, because the wind has that average downward inclination; but it does not seem likely that this can be true of the total movement of the air, although it may be true of the strongest gusts, such as become dangerous to the safety of the structure.

It is evident that if the inclination of the wind is to be given by the tipping vane the vane must be perfectly counterpoised and symmetrical about its axis of rotation. Moreover the proper method of using the tipping vane necessitates obtaining a continuous and simultaneous register of both direction and velocity. This is accomplished in the anemographs of Dechevrens and Casella, as described in the following paragraph.

87. ANEMOGRAPHS FOR DIRECTION.

Very early in the history of anemometry attention was given to constructing apparatus for recording the direction of the wind and thereby preserving a continuous register of its variations. The earliest of such inventions is that described and illustrated by Leupold, in 1724, which, except for the bulkiness of its mechanism, is a simple and adequate apparatus, containing the essential features of our present instruments. In the present century anemographs have frequently combined on the same sheet traces showing both the direction of the wind and its pressure or velocity. In the pressure anemometers, which are necessarily attached to the vane in order to keep them face to the wind, the combination of the two records is especially convenient. The anemographs of Kreil, Jelinek, and Osler are of this class. Beckley's anemograph, in use at the Kew Observatory and very extensively throughout England, registers on the same sheet the velocity and direction; in Howlett's anemometer the wind pressure and direction are recorded by the same trace. In these forms the motion of the vane is conveyed mechanically to the registering sheet. But when, by reason of the location of the vane, the vane shaft can not be connected with the self-register an electrical connection is used in which wires making contacts with the respective quadrants of the vane shaft are led to a registering apparatus in the observer's room. This may easily be made self-recording. It is usually not a continuous register, but records at every 5 or 10 miles of wind or every five or fifteen minutes. With this electrical connection a wind vane or an anemometer may be established in any high or distant location at which the wind direction and velocity are desired. But in cases which allow of its use the mechanical continuous registration gives the more satisfactory record.

The operating of a mechanical self-register necessitates that an additional amount of work be performed by the wind in turning the vane, and thus contributes to diminish the sensitiveness of the latter. But if the mechanism of the self register be light and delicately constructed, as in the Signal Service pattern, the additional work is inappreciable.

(a) *Signal Service self-recording anemoscope.*—This instrument (see Fig. 42) needs only a general description. A vertical cylinder, carrying the anemograph sheet, is by clock-work caused to make a complete rotation about its vertical axis every twenty-four hours. The anemograph sheet is ruled both horizontally and vertically; the vertical lines represent hours of the day; the horizontal lines designate the cardinal points of the compass, and recur in successive bands about 1 inch broad from the top to the bottom of the sheet when the latter is in place, their distance apart being such that a pencil driven by a screw at the lower end of the vane staff moves over the space between the lines when the vane turns through one quadrant. A sheet contains thirty-two horizontal spaces for eight complete rotations of the vane, and is therefore sufficient to hold the traces for at least three or four days. At noon every day the nut carrying the pencil is screwed down to a lower space for the formation of a new trace, or if necessary a new sheet is put on. The instrument requires two adjustments; one provides that the pencil shall so rest upon the lines as to accurately indicate the direction of the wind vane; the other provides for a similar accuracy in the time scale. The latter is obtained first by an approximate adjustment when the pencil is originally put in place, and is further secured by marking (at specific hours and minutes twice a day) the position of the pencil on the sheet. With this care the instrument is susceptible of high accuracy.

The continual oscillations of a sensitive vane, which give rise to uncertainty in determining the true wind direction by direct observation, are a matter of no difficulty when recorded on an anemograph sheet. The examination of any anemograph sheet shows that where the oscillations are most frequent the true wind direction can *generally* be best determined. The existence of oscillations is an evidence in light winds that the vane is keeping to the true direction, for a straight line on the sheet denotes either a calm or a wind too light to turn the vane, in which latter case the true direction may differ greatly from that indicated by the trace. The oscillations shown by the anemoscope sheet suggest a method of testing the sensitiveness of the vane. By comparison with simultaneous velocity sheets we may obtain the *least* velocities under which the vane will turn when the wind changes its direction by 5°, 10°, 15°, and 20°, respectively. This is the criterion of sensitiveness.

(*b*) *Signal Service electrical anemograph.*—This gives an electrical registration of both velocity and direction on a horizontal cylinder revolving by clock-work and bearing the register paper. The record of the direction is made by means of an electrical arrangement, similar to those of Secchi and Du Moncel, of four electro-magnets, each of which acts on an armature corresponding to one of the four quadrants, and is furnished with a marker, so placed that the position of the mark made on the paper by each marker indicates the direction of the wind to which that mark corresponds. In one form of the instrument the direction markers consist of disks, which print on the paper the letters *N E S W*, and thus the position of the marks is not essential. Each of the four electro-magnets is connected by wire with its proper one of the four contact springs at the foot of the vane shaft, and a return wire, passing through the battery and the driving clock and thence to the magnets, completes the circuit. A contrivance within the clock closes the circuit, which is ordinarily left open, every five minutes; the current passes through the magnets, whose wires are in contact at the vane shaft, and the corresponding disks print the direction.

The system of contacts, by means of which the current is made to pass through the proper wires corresponding to the wind direction, is shown in Figs. 43 and 44. Upon the vane shaft are fitted four rubber cams, *cccc*, each extending over five-eighths of the circumference of the shaft and arranged symmetrically around it. The contact springs *AA* (Fig. 43) are raised by the cams, and the circuit which is made by a platinum point on their under surface is opened. During five-eighths of a revolution of the vane therefore any given contact spring will rest upon its cam and so be out of circuit, and during three-eighths of the revolution, or 67½°, will be in circuit. Hence, by the symmetrical arrangement of the cams the successive octants of the vane shaft will be in circuit with one or two wires alternately. Thus, for instance, when the cams are properly adjusted and the vane points between N. 22½° W. and N. 67½° W., both the north and west wires are in circuit; when the vane is between N. 67½° W. and N. 112½° W., the west wire only is in circuit. The wind direction is therefore given to the nearest one of the eight points of the compass.

The construction and operation will be understood by observing the following directions for establishing and adjusting the whole instrument:

Fig. 43 shows the position of the contact when the vane rod passes through the roof to the ceiling of the office.

Fig. 44 shows the position of the contact and of the friction rollers when inclosed in a cast-iron base on the roof of a building.

(1) To place the contact in position, as shown in Fig. 43, the vane rod must extend from 3½ to 4 inches below the ceiling and the following directions observed:

(2) Fasten the circuit closer *cccc* to the vane rod with the set-screws close to the ceiling, and then pass the end of the rod through the aperture in the lower side of brass support *BB*, which must be securely fastened to the ceiling.

(3) Screw the brass plate *AA*, with which the four contact springs *N E S W*, are connected, to the support *BB* in such a manner that the contact springs will be parallel with and opposite to their respective cams on the circuit closer *cccc*.

(4) Adjust the circuit closer *cccc* by having an assistant hold the wind vane in a due north direction; loosen the set-screws of the circuit closer *cccc* and turn it around until the point of the spring *N* corresponds with the notch filed in the uppermost cam on the circuit closer opposite to the spring *N*. When this has been done firmly refasten the circuit closer to the vane rod by means of the set-screws.

(5) Before connecting the five-wire cable test it and mark the wires *N E S W* and *R*, respectively, at both ends. Remove the covering from the cable for at least 6 inches at the end to be connected with the contact, clean the ends of the wires, connect them with the five binding posts at the ends of the springs *N E S W*, then fasten the cable to the ceiling and conduct it to the instrument. The fifth wire, *R*, must be connected with one pole of the battery.

(6) When all the wires have been properly placed the arrow or indicator must be screwed to the vane rod immediately under the lower portion of the brass support *BB*.

Where an office is so located that the vane rod can not be passed through the roof to the ceiling a cast-iron base *BB*, (Fig. 44), is used. This is set up as follows:

A suitable wooden block is to be firmly fastened to the roof upon which the cast-iron base will be securely attached. The lower end of the pipe *P*, through which the vane rod passes, will be placed in the socket at the top of the cast-iron base, and the vane rod itself, *DD*, passed through the two apertures in the top and bottom of the base *BB* and through the wooden block underneath the base. A hole should be bored in the wooden block of greater diameter than the vane rod in order to prevent friction.

In erecting the vane care must be taken to place it at such an elevation that the vane rod will not extend too far below the iron base. After the large pipe *P* supporting the vane is firmly braced to the roof, and before the vane is attached to the rod, the contact friction rollers *c* must be placed in position. This will be done by raising the vane rod sufficiently to enable the circuit closer *cccc* and friction rollers *G* to be slipped on the lower end of the rod. Lower the rod to its former position, so that the upper friction rollers (which are immediately below the vane) will rest upon the cap of the pipe. Then fasten the lower friction rollers, *G*, to the vane rod *DD*, so that the weight of the rod will be as equally distributed as possible between the upper and lower friction rollers; place the circuit closer *cccc* and plate *AA* in position.

(*c*) *Dechevren's anemograph.*—A vane with both vertical and horizontal balanced plane plates is mounted on a horizontal axis, supported by a copper stirrup soldered to the extremity of a tubular shaft, which descends into the observatory. At the lower end of this shaft is a zinc cylinder floating in a vessel of water and glycerine. This cylinder supports the shaft, vane, driving clock, and drum for recording the inclination of the air currents, and being unattached to the vessel in which it floats it turns freely with every change in direction of the vane. Attached to the axis on which the vane swings is a grooved pulley, over which passes a light chain, one end of which is fixed to the pulley and the other end passes down the vertical shaft to the top of the zinc float, where it gives motion to a light metallic lever, which traverses the shaft by two longitudinal slits. This lever carries a pencil, which traces an arc of a circle on a cylinder driven by a clock. The whole of this apparatus being, as before mentioned, carried by the float turns with it as the wind shifts. The horizontal direction of the wind is recorded on a separate sheet. On the vertical shaft carrying the vane there is a fixed toothed wheel, which engages with a similar toothed wheel fixed to a paper-covered cylinder. This cylinder therefore turns to the same extent as the vane, but in an opposite direction. A pencil is by clock-work drawn parallel to the axis of this cylinder and the position of the mark indicates the direction of the wind.

(*d*) *Casella's alt-azimuth anemometer.*—This ingenious instrument (see Fig. 45), devised by Louis M. Casella, records the horizontal and vertical component of direction, and also the normal pressure of the wind, and is thus described by the inventor: The apparatus for indicating the direction of the wind consists of a vane constructed of a pair of diverging blades fixed to a cap mounted so as to rotate about a vertical axis, the motion of this vane being transmitted by a vertical tubular shaft passing downwards through the usual fixed column to the registering mechanism. The apparatus for indicating the inclination of the wind to a horizontal plane consists of a similar vane (composed of a pair of diverging blades) mounted on a horizontal axis within the direction vane, so balanced as to assume normally, when no wind is blowing, a position in which its longitudinal axis is horizontal. To insure this the vane is brought to a condition of stable but very delicate and sensitive equilibrium.

The oscillating motion of this inclination vane is transmitted to its registering mechanism by a tubular connecting rod, jointed to the vane by a pair of links, which move up and down inside the direction tube. Thus the inclination tube is so connected with the carriage of a stylus that its longitudinal motion affects the latter only, so that the record on the scale represents oscillations of the vane due to the varying inclination of the wind.

The pressure plate is fixed to a guide rod sliding between pairs of rollers in the frame of the inclination vane and moving with it. In order to prevent the varying positions of the pressure plate from affecting the balance of the vane its motion is constantly and exactly compensated by a movable weight running on rollers, so arranged that the weight moves to a proportionate extent in the opposite direction to the pressure plate, so as to maintain the balance of the vane in all positions of the pressure plate. The motion of the pressure plate is transmitted to the apparatus for measuring the force by means of a chain attached to the guide rod of the plate and passing down over a pulley through the tubular shaft of the inclination vane. To prevent the weight of this chain from affecting the accuracy of the records it is exactly balanced by a counterpoise hanging in a casing carried by the cap. In this way the pressure plate is kept perpendicular to the direction of the air current, not only for all azimuths, but also for all altitudes.

CHAPTER XIII.

DIRECT MEASURES OF WIND VELOCITY.

88. VELOCITY OF FLOATING OBJECTS.

The velocity of the wind was first directly measured by allowing light substances to be borne along in the air while the observer timed their velocity. Thus Derham, about 1700, determined the velocity of a storm wind to be 53 miles per hour; thus, in 1751, Lomonosoff determined the constants for reduction of his anemometer records; thus Coulomb, in 1820, determined the velocity of a hurricane to be 100 miles per hour; thus Stamkart, about 1850, in Holland, by a large number of accurate observations of smoke, closely determined the velocity of wind for each of the terms commonly used by Dutch seamen.

A modified method of observation was proposed by Forbes, in which a rather heavy body is allowed to fall freely from a point vertically above a horizontal circle; the deflection of the body from the vertical by the wind is given by observing the point at which it strikes the circle, from which the velocity is deduced when we know the law of resistance to the falling body. As a method of systematic observation this proposal could be recommended only in extreme cases.

89. EFFECT OF WIND ON SOUND.

A method of obtaining directly the velocity of wind was suggested in the last century by the determination of the velocity of sound.

If sounds start simultaneously from two points, *A* and *B*, while the wind is blowing from *A* towards *B*, the velocity of the sound from *A* to *B* is increased by that of the wind, while the velocity from *B* to *A* is decreased by that of the wind. If at each place an observer determines the velocity of the sound that comes to him, the half difference of the results will be the velocity of the wind. By this method wind velocities were determined by the Academicians Cassini and Lacaille, in 1737 and 1738, acting as a commission for the Academy of Sciences at Paris; they occupied stations from 3 to 20 miles apart. Similar determinations have subsequently been made by others, as the French Bureau of Longitudes in 1822, and by Moll and Vanbeck in Holland, 1823. By this method we get a determination of the average movement of the air over a large horizontal space.

The earlier observations were made with the object of determining the velocity of sound, but now that its velocity has been independently determined by many other methods we may assume it to be known, and by a proper arrangement one observer may at any time determine the general wind velocity by this method. In order to carry out this method of observation two men are stationed at considerable distances from a principal observer and in directions from him preferably at right angles to each other. Some loud sounds are produced by these two men, as from a gun or cannon, and the observer records the time elapsing between his observation of the flash and the sound. He has then for each case the velocity of the sound plus the effect of the respective wind components in the two directions, whence the wind velocity is easily computed. The adopted velocity of sound is, however, dependent upon the temperature of the air, and the uncertainty of the latter contributes to make this method unreliable. The relation between the velocity of sound and the air temperature is given by the following formula:

$$\text{vel. (feet per sec.)} = 1089.42\sqrt{1+0.00208\times(t-32)}$$

The slight effect of moisture in changing the elasticity of the air is disregarded in this formula. Four guns and one observer can be so located that the velocities of both the wind and sound and the direction of the wind are directly deducible from the observations; in which case this formula gives us the means of determining from the velocity of sound the average temperature of the air.

90. VELOCITY AT MODERATE ELEVATIONS.

The preceding methods give velocities and directions quite near the surface of the ground, but by several other devices the velocity and direction of the wind can be observed at moderately high elevations.

(*a*) Historically the kite is the oldest method of demonstrating not only the electric condition of the upper air, but also the strength and direction of the upper winds; it has been used up to elevations exceeding 1,500 feet and higher elevations have been obtained by attaching several kites successively one above the other. Owing to the complexity of the forces acting on the string and the tail, as well as on the body of the kite, it is best to consider the whole as simply a means of elevating into the upper regions some other apparatus proper for observing the force and direction of the wind. In this way E. D. Archibald, having elevated Biram's form of Woltmann's air meter, has determined the relative velocity of the wind, and finds that it follows very closely the law expressed by the formula

$$\frac{V}{v}=\left(\frac{H}{h}\right)^{0.25}$$

where v and V are the velocities at the altitudes h and H above sea level. Other apparatus, such as Casella's air meter, Robinson's anemometer, or some form of pendulum anemometer, can be substituted for Biram's air meter for special studies.

In a similar way the direction of the wind at any altitude is shown by thus carrying up with the kite or by a messenger on the string, one or more long streamers, whose direction can be accurately observed from below, when the wind becomes known with any required degree of accuracy.

Both direction and velocity are easily given by setting free at various altitudes on the kite string some light substance, as a ball of down or a cloud of smoke, whose movement can be accurately observed from two stations on the ground. Such objects are easily sent up by a messenger and discharged by automatic apparatus.

The balloon kite invented by Archibald (see "Nature," 1886) seems to answer all requirements for elevations less than 3,000 feet.

(*b*) Observations upon clouds of smoke produced by the explosion of bomb-shells have, it is reported, been systematically made by the British meteorological office, but the results have not yet been published.

CHAPTER XIV.

PERSONAL ESTIMATES AND ARBITRARY SCALES OF WIND FORCE.

91. VARIETY OF METHODS.

The personal observation of the estimated force of the wind is a method that belongs to the simplest and earliest meteorological records, and continues to be practiced in its elementary form by all observers, including even those who, having special apparatus, can supplement personal estimates by instrumental measurements.

Although devices for measuring the wind have frequently been used for the past two hundred years the majority of observers are still, and probably always will be, necessarily dependent upon estimates based largely upon their own judgment, and so reliable can such estimates be made that for many purposes they abundantly answer the needs of meteorology as well as of climatology. Unfortunately, however, different observers have adopted different scales or words for expressing their estimated force of wind, and it is a difficult matter to reduce their methods to a uniform system. A fairly complete collection of the terms or numbers used to express the strength of the wind, as made by me in 1875, shows that there are about two hundred systems already adopted or suggested as practicable in the absence of actual measurements. The experience of those who have daily used the Robinson anemometer shows that direct estimate of velocity in miles hourly can easily be made.

The general tendency of meteorological conventions has been in the direction of adopting some arbitrary decimal scale, according to which the strongest winds are represented by the number 10, and calms by 0; but it is generally acknowledged that the wide diffusion of the duodecimal scale of Admiral Beaufort, in which the strongest winds are represented by the number 12, makes it impracticable entirely to displace that scale, and perhaps undesirable, inasmuch as the navigators of all nations have generally acquired a very considerable degree of expertness in its use.

As recommended by the International Congress at Leipsic, Vienna, and Rome, the meteorological observers of the world should be divided into three classes: (1) Those using instruments giving either velocity or force, and who publish their instrumental records; (2) those who use personal estimates, recording by the 0 to 10 scale, and generally adopt the metric system of measures; and, (3), those who estimate and record by the 0 to 12 scale, and generally adopt the English system of measures.*

92. WIND SCALES IN ORDINARY USE.

As preliminary to this desirable simplification the various arbitrary scales actually in use may be classified as follows:

(1) *The 12 scale.*—This was introduced into the British navy by Admiral Beaufort about 1800, and has been adopted, with slightly varying interpretations as to the meanings to be attached to the numbers, by the Brussels Conference, by the British meteorological office, by the Austrian service, and by the British ordnance bureau. Attempts to determine whether any satisfactory values of wind velocity could be assigned to the respective numbers as used by observers were made first by Schott in his discussion of McClintock's observations, and by Jelinek in his instructions for meteorological observations.

The more recent determinations of this kind are by Scott, Sprung, and Köppen.

(2) *The 10 scale.*—"Ten" scales were disseminated among the United States observers by Espy in 1837, by Nicollet in 1838; about the same time they were adopted in Europe, especially in the Netherlands. Similar scales were adopted for the Smithsonian observers about 1850, by Jelinek in his instructions to Austrian observers, and by Wild in his instructions of 1871 to Russian observers. The latest 10 scale is that adopted in the Signal Service Bulletin of Simultaneous International Observations. In each of these and other cases a slightly different interpretation of the values of the scale numbers is supposed to obtain; the values adopted by the Smithsonian were first assigned by Schott, in 1862, in his discussions of McClintock's observations; the scale adopted in Holland was interpreted first by Stamkart, about 1850, by means of observations on the velocity of smoke.

(3) *The 9 scale.*—This was used in European telegraphic dispatches, where the words indicating force of wind were necessarily limited to the simple numerals; a similar scale was used for a short time by the United States Signal Office in 1870–'71, before the adoption of its printed cipher code.

(4) *The 8 scale.*—Adopted in 1871 by the Signal Office, but only for temporary special use to supplement the telegraphic cipher code when the station anemometer is out of order.

(5) *The 7 scale.*—Introduced early in this century into France and adopted by the Meteorological Society of France, and subsequently by Leverrier for use in his International Bulletin; this is still used very commonly in France, Austria, Italy, and Turkey.

* Russian observers use Wild's tablet anemometer for velocities up to 20 meters per second and personal estimates for stronger winds, but record all observations in meters per second.

(6) *The 6 scale.*—This is more frequently known as the English land scale, and was intended to be one-half of the Beaufort nautical scale, so that 6 on the former corresponds to 12 on the latter. Introduced apparently for use in England and Scotland, this scale has also been widely adopted in other countries, and is used in the meteorological publications of Belgium, Norway, Sweden, and Denmark, and more recently by the central bureau at Paris. Although sometimes known as the Glaisher and the Fitzroy land scale, yet I do not know that it was originated by either one of these; it is by some called the continental scale, a term that has, however, also been applied to the 7 and the 4 scales.

(7) *The 4 scale.*—This is generally known as the Mannheim or old continental or old land scale; it was first introduced by the observers at Upsala, and was adopted by the Mannheim Meteorological Association about 1780. For a time this scale was very widely used, but now only by an occasional observer in Saxony, Netherlands, Italy, Switzerland and Russia.

93. TEN SCALE OF THE SIGNAL SERVICE BULLETIN OF INTERNATIONAL OBSERVATIONS.

The International Congress at Leipsic and Vienna recommended that there be a general agreement on a uniform 10 scale, and appointed a committee to devise a normal scale of numbers and velocities; but the only one that has come into actual use is that devised by me in 1875, when it became necessary for me to edit the daily publication by the Signal Office of its International Bulletin, based upon a large mass of observations, representing nearly all the arbitrary scales then in use.

At that time I decided to attempt the formation of a 10 scale that might have some claim to international recognition on the ground that it did not differ too much from those already widely established; the following was adopted for the use of the bulletin, and through it has become widely disseminated. Absolute velocities corresponding to the scale numbers were adopted, as in the following table:

Scale numbers.	Limiting velocities.		Average velocities.	
	Miles per hour.	Meters per second.	Miles per hour.	Meters per second.
0	Calm.	Calm.	0	0
1	0 to 4.5	0 to 2.0	2.2	1.0
2	4.6 to 9.0	2.1 to 4.0	6.8	3.3
3	9.1 to 13.5	4.1 to 6.0	11.3	5.0
4	13.6 to 22.5	6.1 to 10.1	18.0	8.0
5	22.6 to 31.5	10.1 to 14.1	27.0	12.1
6	31.6 to 40.5	14.2 to 18.1	36.0	16.0
7	40.6 to 49.5	18.2 to 22.1	45.0	20.1
8	49.6 to 67.5	22.2 to 30.2	58.5	26.1
9	67.6 to 85.5	30.3 to 38.2	76.5	34.2
10	85.6 upwards.	38.3 upwards.	95.0	42.5

The above velocities, when converted into pressures by the Smeaton-Rouse formula, give the following numbers, which may therefore be adopted for directly converting pressures observed by pressure-plate anemometers into the 10 scale of the International Bulletin:

Scale numbers.	Limiting pressure.		Average pressure.	
	Pounds per square foot.	Kilograms per square meter.	Pounds per square foot.	Kilograms per square meter.
0	0	0	0	0
1	0.00 to 0.10	0.0 to 0.5	0.03	0.15
2	0.11 to 0.40	0.6 to 2.0	0.23	1.13
3	0.41 to 0.90	2.1 to 4.5	0.64	3.15
4	0.91 to 2.53	4.6 to 12.5	1.62	7.97
5	2.54 to 4.96	12.6 to 24.5	3.64	17.9
6	4.97 to 8.20	24.6 to 40.5	6.48	31.9
7	8.21 to 12.25	40.6 to 60.0	10.12	49.8
8	12.26 to 22.90	60.1 to 112.7	17.12	84.2
9	22.91 to 36.50	112.8 to 170.6	29.26	143.9
10	36.51 upwards.	179.7 upwards.	45.12	222.0

No special formal regularity will be found in the relation of these velocities and pressures to the respective scale numbers, except that the differences in the velocity table proceed by steps of 4.5 miles per hour or some multiple thereof. All attempts to force some arbitrary relation, such as, for example, that the pressure of the wind should double for each increase in the scale number, or that the velocity of the wind should be some simple multiple or power of the scale number, gave results differing so much from the scales in actual use that it did not seem desirable in introducing a new arbitrary scale for the use of an International Bulletin, to depart unnecessarily from one that would nearly represent the average of all others, and would therefore imply the least possible change from scales in current use. The absolute value of the new scale numbers being thus decided on, the next step was the conversion of the Beaufort scale numbers into velocities, which was done by means of Scott's table of equivalents, and the corresponding pressures were then deduced by the Smeaton-Rouse formula. This, then, afforded the means of converting the observations of pressure on normal plates into the Beaufort, and thence into the 10 scale. As a result the following tables have been adopted for the direct conversion of other scales into the 10 scale of the International Bulletin. These tables also show approximately the mutual relations of the arbitrary scales that are now in use:

Tables for the conversion of various wind scales into the 10 scale of the Bulletin of International Simultaneous Observations.

Beaufort scale as now used by British seamen.		Corresponding International Bulletin scale.
Terms.	Numbers.	
Calm	0	Nos. 0
Light air	0.5 and 1	1
Light breeze	1.5 and 2	2
Gentle breeze	2.5 and 3	3
Moderate breeze	3.5 and 4	4
Fresh breeze	4.5 and 5	5
Strong breeze	5.5 and 6	6
Moderate gale	7	7
Fresh gale	7.5 and 8 and 8.5	8
Strong gale	9	8
Whole gale	10	9
Storm	11	10
Hurricane	12	10

NOTE.—Intermediate numbers, *e. g.*, 2.5, frequently occur in the observer's reports, and are therefore included in the second column.

Jelinek's Austrian scale.		Corresponding International Bulletin scale.
Terms.	Numbers.	
Calm	0	Nos. 0
Scarcely sensible	1	2
Feeble wind	2	4
Moderate wind	3	5
Stronger wind	4	6
Rather strong wind	5	7
Strong wind	6	8
Very strong wind	7	8
Stormy wind	8	9
Storm	9	9
Hurricane	10	10

Smithsonian scale and original scale of 1874 for voluntary observers of the Signal Service.		International Bulletin scale.
Terms.	Numbers,	
Calm	0	Nos. 0
Very light breeze	1	1
Gentle breeze	2	1 or 2
Fresh breeze	3	3
Strong wind	4	4 or 5
High wind	5	6
Gale	6	7
Strong gale	7	8
Violent gale	8	9
Hurricane	9	9
Most violent hurricane	10	10

Signal Service scale for telegraphic cipher.			International Bulletin scale.
Measured velocity per hour.	Terms.	Numbers.	
Miles.			
0	Calm	0	Nos. 0
1 to 2	Light	1	1
3 to 5	Gentle	2	3
6 to 14	Fresh	3	5
15 to 29	Brisk	4	6
30 to 39	High	5	7
40 to 59	Gale	6	8
60 to 79	Storm	7	9
80 upwards.	Hurricane ...	8	10

Wild's new Russian scale.		The old Russian scale. Numbers.	International Bulletin scale.
Terms.	Numbers.		
Calm	0	0	Nos. 0
Very feeble	1	0	1
Feeble	2	1	2
Light	3	1	3
Moderate	4	2	4
Fresh	5	2	5
Strong	6	3	6
Very strong	7	3	7
Storm	8	4	8
Heavy storm	9	4½	9
Hurricane	10	....	10

Scale of the French Meteórological Association.			International Bulletin scale.
Old terms.	Modern terms.	Numbers.	
Calm	Calm	0	Nos. 0
Feeble	Very Feeble	1	1
Moderate	Feeble	2	2
Rather strong ..	Moderate	3	3
Strong	Strong	4	4
Very strong	Very strong	5	6
Violent	Violent	6	8
Hurricane	Hurricane	7	10

Scale of the central meteorological bureau of Paris.	International Bulletin scale.
Nos. 0	Nos. 0
1	1
2	3
3	4
4	6
5	8
6	10

The Mannheim scale.		International Bulletin scale.
Terms.	Numbers.	
Calm or very weak	0	Nos. 0
Moderate or light	1	2
Strong, or the wind moves the branches	2	4
Very strong, or the wind moves the trees and hinders walking	3	6
Storm, or the wind breaks the branches	4	8
"From 1 to 2"	....	8

English land scale.		International Bulletin scale.
Terms.	Numbers.	
Calm	0	Nos. 0 or 1
Light	1	2
Moderate	2	4
Strong	3	6
Fresh gale	4	7
Whole gale	5	8
Hurricane	6	9

CHAPTER XV.

PRESSURE ANEMOMETERS.

94. THE ULTIMATE OBJECT OF ANEMOMETRY.

The pressure exerted by the wind against an obstacle in its path is a function of the kinetic energy of the wind, and of the size and shape of the object against which the pressure is exerted. The determination of this function, and thereby the amount of wind pressure to which structures are liable, is of great importance to architects and engineers, while the velocity of the wind is the element most important to meteorology. It is often urged that observations with pressure anemometers furnish directly the data needed by practical engineers, and are therefore more immediately useful than the records of velocity anemometers, which must be converted into pressure in order to be of utility. But this is a mistake, since the records of a given form of pressure anemometer can not in strictness be generalized and assumed to give the same pressure per square foot that the same wind would exert upon surfaces of different shape and size; thus the wind that produces a pressure of 10 pounds on an anemometer plate 1 foot square will not produce 1,000 pounds on a surface 10 feet square. Even for engineers, therefore, a knowledge of the velocity of the wind is of greater use than any observed pressure, for, knowing the velocity, the pressure upon the special surface under consideration may be determined by such formulæ as theory and experiment may indicate to be appropriate for that surface.

The real point of advantage in a pressure anemometer, as compared with a rotation anemometer, consists in the fact that it registers momentary gusts, thereby furnishing data for obtaining maximum velocities and pressures, while the velocity anemometer is essentially an integrating machine, giving total movements and average velocities for definite intervals of time. Thus each class of instruments fills its own place, and is needed to supply the deficiencies of the others. Pressure anemometers may be properly subdivided into several varieties, each of which will be specially considered in the following paragraphs of this chapter.

95. PENDULUM ANEMOMETERS.

The history of accurate anemometry begins with the use of one form of pressure anemometer, of which a first description was published in 1667 in Instructions for Seamen, by Robert Hooke; in consequence this has been called Hooke's anemometer, although it is by no means clear whether he or Rooke, who died in 1662, or Sir Christopher Wren was its inventor, and it ought rather to be known as the anemometer of the first meteorological committee of the Royal Society (see Sprat, History of the Royal Society, 1667). This instrument (see Fig. 46) consists essentially of a plane tablet, *CD*, suspended from a horizontal axis, and swinging upwards along a graduated quadrant, *AB*. The quadrant turning freely as a vane on a vertical shaft keeps the plate facing to the wind, and also serves to measure its angular deflection. This type, in which the wind force is balanced by the force of gravity, has been designated "pendulum," and has been the subject of numerous structural modifications. Among these may be mentioned the form devised by the Rev. Roger Pickering in 1744, in which the rim of the graduated quadrant was notched to each degree, and the tablet held up by a spring. catching at every notch, so as to register the maximum deflection.

Another modification is by Dr. Schmidt, of Giessen, in 1828, who used a circular plate and registered on a quadrant the maximum deflection by an index pushed forward by the deviation of the plate.

All forms of pressure plates require to be kept face to the wind by being attached to a wind vane; but, especially during high winds, the wind vane itself is subject to continual veering, and it is impossible to maintain the exact perpendicularity of the plate. This complication is obviated by the use of a sphere, upon which the wind acts with the same force from whatever direction it may come. When suspended by a rod and universal joint the deflection of the sphere from the vertical will measure the force of the wind and show its direction. This suggestion was first made by Parrot (Voigt's Magazine, Vol. L, 1797, p. 153), who studied also the significance of the indications of the instrument, and concluded that the pressure upon the front face of the sphere is very approximately equal to one-half the pressure upon the flat surface of a section through its center.

An anemometer embodying Parrot's idea was constructed by Howlett in 1868, wherein the sphere is placed at the upper end of a rod pivoted near the middle, and bearing at its lower end a counterpoise. Thus the sphere and upper half of the rod are exposed to the wind, while its lower portion below the pivot is protected in an inclosure. A pencil attached to the rod traces the direction and force, so that the instrument becomes a self-recording anemometer for both direction and intensity. This is the simplest apparatus for obtaining both the *direction* and *force* of momentary gusts.

The same object of obviating the use of the wind vane for the proper exposure of the pressure surface has been attained by Mr. Daniel Draper in the use of a cylinder suspended by a chain, and recording by means of a pencil attached by a string to the lower end of the cylinder. This more complex instrument has been in use at the meteorological observatory, Central Park, New York, since 1869. The proper interpretation of its indications has not yet been made evident. The theory of its action requires careful analysis, the chain suspension being an important additional element in the determination of its position of equilibrium.

Of these various forms of the pendulum anemometer only the first form, as given by Hooke and Wren, has come into any extended use. In 1861, nearly two centuries after Hooke's publication, Prof. H. Wild introduced this simple instrument at the meteorological stations of the canton Berne, and later throughout Switzerland and Baden. Since 1870, as director of the Central Physical Observatory at St. Petersburg, he has made it a part of the instrumental outfit of meteorological stations in Russia. It is now known as Wild's tablet anemometer. The accompanying Fig. 47 shows the details of its construction. A slightly modified form was introduced into Germany by Prestel, 1865, but was not widely used.

96. NORMAL PRESSURE-PLATE ANEMOMETERS.

The principal objections to the swinging plate as an anemometer of precision, and especially the theoretical difficulties with respect to the proper function of the angle to be used in reducing the indications of the former, are obviated in the perpendicular pressure plate. The earliest reference to this type of pressure anemometer, and which represents a distinct class, is contained in Leupold's Theatrum Machinarum Generale, Leipzig, 1724.

A perpendicular pressure plate, 6 by 12 inches, is set into a base movable in horizontal grooves and placed face to the wind. The wind pressure on the plate is counterbalanced by a weight, acting with a variable leverage, attached to the plate by a cord passing over a pulley. In any position of equilibrium the pressure upon the plate is proportional to the moment of the weight supported, if the friction of the parts be negligible.

As to the supposed advantage which this instrument possesses Leupold says:

> Because the plate always stands straight against the wind, and as it has a certain definite surface, we can make a computation (of pressure) for any other surface, greater or smaller, which is not practicable with other instruments.

In 1746 M. Bouguer used a card-board pressing against a spring, and urged its advantages in that "the pressure on each square foot of surface is at once found without having to take into consideration the obliquity of the impact." Numerous forms of construction and registration were suggested during the next hundred years, among which the most important improvements were in the application of gravity resistances in place of springs. Kirwan in 1810, and Beaufoy in 1821, are quoted as contributing to the advancement of anemometry in this way, but they made no advance in the method of measurement beyond the construction described by Leupold in 1724.

These various modifications attained nothing more than an individual use until the stimulus given to meteorological observation by the British Association and the Royal Meteorological Society from 1830 to 1840 led to the general adoption in Great Britain of uniformly constructed instruments.

A self-registering pressure-plate anemometer, presented to the British Association in 1837 by Osler, became generally accepted in Great Britain and Europe as the standard pressure anemometer, and is still in use as such. An Osler anemometer is in use at Brussels, and instruments of the same form are in general use at meteorological stations in the Netherlands. In this instrument (Fig. 48) the plate, 1 foot square, is kept face to the wind by a vane, and the pressure is received against springs placed behind it. To indicate the amount of compression, and so the force of the wind, a light chain connected with the pressure plate is conveyed through the hollow vertical shaft of the wind vane, and then, turning over a pulley, draws a pencil up or down, as the intensity increases or diminishes; a tracing is thus made on a sheet of paper moved by clock-work. The pressure plate is supported by light bars running horizontally on friction rollers and communicating with flattened springs (1) (2) (3), so that the plate, when affected by the pressure of the wind, acts upon them, and they transfer such action to the chain passing down the interior of the shaft. A second pencil connected with the shaft of the vane registers the direction on the same sheet, and thus a continuous registration of the wind force and direction is maintained.

The diagram (Fig. 49) gives a representation of an improved construction of Osler's anemometer, as used at the royal observatory, Greenwich. The acting portion of this apparatus consists of a circular disk of iron having 2 square feet of area. It is very light, but is strengthened by stays placed behind it. When the wind blows against the circular plate the springs *DDDD* are brought successively into action, the stronger being so arranged as to take up the pressure before the weaker are subjected to any strain. A rod, *G*, which moves with the pressure plate, carries a fine flexible wire cord, *H*, which, not readily seen in the drawing, passes over a small pulley in the side of the rod carrying the direction vane and runs down through the rod. This wire continues on into the building, where it governs the motion of a pencil on an anemometer sheet.

This instrument is so sensitive that it will record pressures as light as a fraction of an ounce on the square foot, while in gales it will support a force as great as 40 pounds in the same area.

In 1850 Jelinek devised a modification of Osler's anemometer, in which the springs behind the pressure plate were inclosed in a cylindrical case. This case served the purpose both of protecting the springs from exposure to the weather and of eliminating the action of the wind at the back of the plate, which action forms an important factor in the total pressure experienced by a simple plane lamina immersed transversely in a moving fluid.

In Cator's anemometer (see Fig. 50), designed in 1864, the pressure in the rear of the plane was sought to be avoided by making the pressure plate the base of a cone. Another and the principal feature of Mr. Cator's anemometer was the substitution of a gravity resistance in place of the sets of weather-exposed springs, which constitute the essential constructional defect of Osler's anemometer. The diagram shows the only portion of the instrument exposed to the weather. The plane base *a* of the cone has an area of 1 square foot, and is kept face to the wind by the vane. This base is pushed by the wind towards *b*, and in its motion draws up the chain, which passes downward through the axis *cc* into the observing-room below, where it winds around a spiral drum that is rigidly connected with a second spiral that supports a weight not shown in the figure. By the change in the point of application of the force the weight exerts a pull *continuously* increasing with the wind force. A cord fastened to the drum conveys its motion to a registering apparatus, and the instrument becomes self-recording. The use of a gradually varying gravity resistance in place of a spring is a constructional improvement, because the strength and elasticity of a spring can not be depended on to remain constant under variations of temperature, exposure to weather, and continual use.

Professor Wilke, of Stockholm, in 1785, suggested a method for measuring the pressure, whereby the plate presses against a leather bag containing mercury, which is thus pressed up into a glass tube and pushes up colored spirits in a finer tube above, thus magnifying the scale. The varying height of the column affords a good indication of the variations in wind pressure. Pujoux adopted a similar idea, the pressure being received against a bag filled with air, which, by compression, forced a column of liquid to rise in a double siphon tube.

All the above modifications of the pressure-plate anemometer (except that of Jelinek, which eliminates, and that of Cator, which reduces the diminution of pressure behind the plate) pertain only to the measurement of the pressure experienced, but do not touch the anemometrical principle of the instrument, namely, the relation of the pressure produced on the plate to the velocity of the wind.

97. TUBULAR PRESSURE ANEMOMETERS.

A third class of pressure anemometers is that in which the wind pressure is received by a tube and conveyed to the surface of the liquid in the manometer. Leupold, in his Theatrum Staticum, Leipzig, 1726, gives among the diagrams of anemometers a representation of an instrument of this kind (see Fig. 51). His figure shows a funnel-shaped receiver leading to a jar of water or other liquid, upon which the wind, blowing into the funnel, exerts its pressure. The height of liquid rising in a vertical tube, whose lower end is immersed in the liquid, measures the pressure on the surface.

In 1775 Dr. Lind, making use of the same principle, brought out an instrument known as Lind's wind-gauge or anemometer, which in different forms has since been widely used. In its original form this consisted of a glass tube (see Fig. 52) bent in the form of a letter U and partly filled with water; the two limbs were connected by a smaller tube, whose bore was one-tenth of an inch in diameter, to check the undulations of the water, and in time of gusts to prevent it from being thrown up by inertia to a greater altitude than the wind at that time is able to sustain. Both limbs were open above, and the receiving end was bent at right angles, so as to directly face the wind. In later modifications this open end is sometimes made bell-shaped, with a view to enlarging the area of the opening on which the wind acts, thus returning to Leupold's arrangement. The tube is attached to a broad vane and poised on a pivot, so that the mouth is constantly presented to the wind. The pressure of the wind on the air which fills the upper part of the windward tube is transmitted to the water, which is thereby depressed in the first limb and raised in the other limb; the difference in the height of the two columns, as read off on an attached scale, indicates the pressure of the wind in the windward tube increased by the suction in the lee tube, which latter was not taken into account by Dr. Lind. Sir David Brewster made the lee tube with a closed bulb at the top. In this case the pressure of the wind is balanced by the column of liquid supported, and by the increased pressure of the confined air; the uncertain amount of suction was thereby eliminated. A new form of Lind's wind-gauge was made by Sir Snow Harris, in which the leeward tube is made of a smaller bore than the other, in order to increase the sensitiveness, and a plumb-line is hung within the frame of the instrument to show when it is exactly vertical. These forms of Lind's anemometers have received very extensive use in England for many years. The pressure indicated by the measured difference of height in the two limbs depends, so far as the instrument is concerned, upon the area of the receiving orifice, the sections of the tubes, and the density of the liquid.

In an instrument devised by Adie in 1836, one of which was in use for several years at the Royal Observatory in Edinburgh, the wind blows into a bell-mouthed tube, and the compressed air within is led into a cylinder inverted over water, which acts as a gasometer or manometer and rises as the pressure of the wind increases. This method of measuring has been adopted by Hagemann for suction tubes at fixed stations; his most recent portable anemometer, for use at sea, employs the V-tube, as in Lind's arrangement. A third method of measuring the pressure of the wind upon the air in a tube has been suggested (see Ann. Rep. C. S. O., 1882, page 99) by the present writer, in which a delicate aneroid barometer, specially adapted in its construction to the object in view, is inclosed at the bottom of each single vertical tube. The excess of pressure upon this barometer over the general static pressure of the air represents the pressure due to dynamic effect of the wind, and the accuracy attainable by this means is greater than that possessed by most of the above-described methods of measurement applied to Lind's gauge. The principle of Lind's gauge is essentially the same as that of the Pitot tube as ordinarily applied to measurements of the velocity of water.

98. BRIDLED ANEMOMETERS.

The term "bridled" has been given by Laughton to a fourth class of pressure anemometers in which windmill sails, blades, paddle-wheels, or cups fixed to an axis receive the wind, but whose rotation is prevented by interposed resistances. The earliest of these was by Christian Wolf in 1708.* A set of small windmill sails on a horizontal axis communicates motion to a toothed wheel. A counterbalancing weight, acting with a variable leverage upon the axle of this wheel prevents the sails from rotating, and they are brought to rest in positions of equilibrium corresponding to the variable force of the wind. The horizontal axis, about which the windmill sails revolve, is kept permanently directed toward the wind by being attached to an ordinary wind vane; an index on a dial face shows the angle of rest of the wheel, and thus the relative pressure of the wind against the sails.

An improvement on Wolf's anemometer was made by Gärthner, an ingenious and celebrated mechanician of Dresden, by attaching a set of curved sails to a vertical spindle, thus obviating the necessity of a wind vane to keep the sails properly facing the wind.†

The diagram (Fig. 53) from Leupold's Theatrum Staticum, plate XXI, shows a horizontal section of this instrument; the six curved radial arms

* The Edinburgh Encyclopædia describes an instrument, apparently very much like Wolf's, invented by Dr. Croune in 1667, the same year of the description of the pendulum anemometer by Hooke, but Croune's instrument never came into actual use.

† In his Historical Sketch of Anemometry, Quarterly Journal Meteorological Society, VIII, 1882, pp. 161–189, Mr. J. K. Laughton gives the credit of the invention of this anemometer to Leupold. This statement is made on the basis of its description in Leupold's Theatrum Machinarum Generale, Leipzig, 1724, where it is described without reference to its inventor; but in his Theatrum Staticum, Leipzig, 1726, it is called Gaerthner's anemometer, and one is said to have been erected on the house of Dinglinger in Dresden. This latter fact suggests a note on Laughton's further statement that "the earliest rotation anemometer, measuring the velocity of the wind, was that of Dinglinger," of which a scant mention is made by Leupold in 1724. The whole of Leupold's note is as follows:

"Herr Dinglinger in Dressden hat nebst vielen andern Kunst-Stücken an seinem wohl-angelegten curieusen Hause, solches zu einem Wind-Weiser wie starck der Wind gehet, appliciren lassen, wobey es gute Dienste thut. Die ganze Machine ist von Blech, unten aber in einem Zimmer mit grossen saubern Tafeln und Abtheilungen, davon auch eine Tafel die Gegend des Windes, und eine die Stunden und Minuten weiset, gezieret."

The plate accompanying this reference shows an exterior instrument similar to the bridled anemometer of Gärthner, which Leupold states was established on Dinglinger's house. This above passage gives no indication as to whether the instrument was used as a rotation or bridled anemometer. The clause "davon auch eine Tafel die Gegend" refers to a wind vane whose rotation was carried to the room below and registered by a dial or perhaps made self-recording; the Tafel, which showed hours and minutes, seems to belong to the anemescope, or perhaps was used in connection with both the anemometer and anemescope. It seems more probable, therefore, that the anemometer here referred to was the same instrument (Gärthner's bridled anemometer) described in the Theatrum Staticum, and not a rotation anemometer, for which the text gives no authority.

respond to the wind pressure; an exterior system of fixed inclined plates leads the wind to the concave surface of the arms and screens the convex surface therefrom. The rotation of the arms is prevented by a self-adjusting gravity resistance. Leupold's description of this instrument in 1724 is its first publication; in the following year Leutmann describes a similar instrument, which he apparently claims as his own invention.

Sir Francis Galton has recently made a bridled anemometer with Robinson's cups, preventing rotation by a resistance spring, and Prof. G. G. Stokes has used five such cups, whose rotation is prevented by attached weights.

99. CONVERSION OF ANEMOMETRIC PRESSURE INTO WIND VELOCITY.

The velocity of the wind can be deduced from records of the various forms of pressure anemometers described in sections 95–98 only by calculations founded on experimental data, and on formulæ deduced from the laws of mechanics, the whole of which constitutes the theory of anemometers.

As regards the deductive treatment of this subject it need only be said that the present state of ærodynamics does not furnish any complete and general relation between velocity and pressure available for all pressure anemometers, so that independent formulæ and experiments are necessary for each instrument, according to the form and size of the surface or the object upon which the wind acts.

As regards the experimental treatment investigations have been limited to measuring pressures produced by two methods, namely, (1) currents of air pressing against a stationary object; (2) still air resisting a moving object. The latter method of obtaining definite relations between velocities and pressures is again subdivided into (1) resistances to bodies moved by whirling-machines and by pendulums, both of which give circular motions, and (2) resistances to bodies having rectilinear motions.

These three modes of experimentation apparently give different results, and extensive studies have been instituted as to the reasons therefor. In general, for the same linear velocities, rectilinear motions in still air give resistances less than the pressures of rectilinear winds against the stationary bodies. Again, circular motions in still air give greater resistances than rectilinear motions in still air.

100. MECHANICAL CONSIDERATIONS.

The mechanical principles involved in fluid motion may be classified as follows: *

(*a*) *Inertia of impact, Newtonian theorem.*—The deductive mathematical study of the laws of resistance of fluids begins with Galileo, and its

* While reading the proof of this page I received through the kindness of M. Boussinesq an early copy of the valuable posthumous work of Saint Venant, "Résistance des Fluids," Mém. Inst. de France, 1887, in which our subject is treated in eminently practical and brilliant style. August 1, 1888.

first century ends with the publication of Sir Isaac Newton's Principia, according to whom, and with an approximation sufficient to represent the observations of earlier days at low velocities, the resistance between the solid and fluid is equal to the pressure due to the weight of a column of the fluid of the same density D, whose base is the cross-section A of the body, and whose height H is the height required for gravity g to give to a freely falling body the same velocity v, or $H=\frac{v^2}{2g}$. This resistance is therefore expressed by the equation

$$R=DA\frac{v^2}{2g}$$

The pressure calculated by this formula is in general different from that observed, and this is therefore frequently called the theoretical pressure or resistance due to impact, etc., but as no really complete theory of the motions of natural fluids leads to this formula it is more proper to call it "the Newtonian theorem," although he himself modified it afterwards.

(*b*) *Viscosity.*—In the further explanation of the phenomena of fluid resistance the idea of viscosity was early resorted to. In 1720 'sGravesande, of Leyden, in his Introduction to the Newtonian Philosophy, concludes that the resistance to the motion of a body through a fluid is due to two causes: First, the cohesion of the particles of the fluid which introduces a term proportional to the velocity; second, the inertia of the particles which introduces a term proportional to the square of the velocity. In this view Daniel Bernoulli, in his Hydro-dynamica, 1738, coincided, but Borda, in several memoirs, between 1763 and 1770, concludes that the viscosity and the term proportional to the velocity are not sustained by actual observation; he therefore returned to the Newtonian theorem. The accurate observations of the oscillations of the pendulum made first by Dubuat in 1780, but afterwards more thoroughly still by Bessel in 1828, Sabine in 1829, and Baily in 1832, afforded Stokes the data for memoirs, wherein he was able to show the law and amount of the action of the internal friction or viscosity of gases and liquids. Viscosity is a resisting force, and like gravity is measured or expressed as an acceleration per unit of time. The coefficient of viscosity is the force required to slide a layer of fluid of a unit area at a unit's velocity over a layer of the same fluid at a unit's distance from it. This coefficient in English measures for air is, according to Maxwell, 0.000-000 0256 ($461°+t$), which expresses the pressure in pounds required to slide 1 square foot of air at the rate of 1 foot per second parallel to a layer 1 foot distant, and where t is the temperature in Fahrenheit. Similarly, according to Helmholtz and Pietrowski, for pure water at a temperature 77° F. the coefficient is 0.000 00091 in English measures. For all gases the coefficient increases nearly in proportion to the absolute temperature, but for liquids it diminishes. In C. G. S. units the above coefficients are 0.0141 for water at 24.5° C., but 0.00018 $(1+0.0027\theta)$ for air at temperature θ° C. Therefore air is more viscous than water, after allowing for its density $\frac{1}{784}$; moist air is more viscous than dry air; hot air more viscous than cold air. The conclusion has generally been entertained that the viscosity proper, otherwise called internal friction or shearing stress, however important in many other physical phenomena, may be neglected in anemometry, and that the study of the laws of resistance may be restricted to the consideration of an ideal non-viscous or perfect fluid. This, however, may lead to appreciable defects in our results.

(*c*) *Waves.*—Waves of compression are propagated rapidly through a gas or a liquid; therefore in an unlimited mass of fluid every obstacle or moving body is preceded by the great primary wave of translation, so called by J. Scott Russell, and a slight dissipation of energy is due thereto. This is generally negligible in anemometry. A wave of sound is such a primary wave of compression; a cannon-ball that moves faster than sound is perpetually striking quiescent air, but if it moves with the velocity of sound or very slightly less it is perpetually striking air that has already been set in motion by the primary wave of translation that started from the front of the obstacle a moment before; therefore the ball experiences a proportionate diminution of its resistance, precisely similar to the diminution experienced by canal-boats that keep up with the wave of translation. Obstacles that move with a velocity much less than that of sound are perpetually meeting air that has nearly come to rest after the passage of the primary wave. In the preceding the body is assumed to be wholly immersed in a fluid or current of indefinite extent; if the liquid is limited, as by the surface of a river, or the sides of channels, or of pipes, an additional amount of energy is dissipated in the production of small surface waves and the reactions of waves reflected from the boundaries. For the moderate velocities that occur in anemometry the resistance corresponding to the energy dissipated in the production of waves is neglected, but for higher velocities it is very appreciable.

(*d*) *Skin friction.*—When a smooth well-shaped body (a "fair body") passes slowly through a fluid of indefinite extent it produces very little disturbance or permanent wake; in proportion as its shape becomes that of a body of least resistance the relative importance of the resistance produced by the eddies between the liquid and the surface of the body increases. This, which is a principal factor in the resistance of a well-shaped, smooth, long ship, moving through the water, is called skin friction, and has been accurately measured by Froude (see his Report to the Lords Commissioners of the Admiralty on Experiments for the Determination of the Frictional Resistance of Water on a Surface under Various Conditions, performed at Chelston Cross, Torquay, under the Authority of their Lordships. By W. Froude. London: Taylor & Francis. 1874). If the liquid does not adhere to or wet the solid then the liquid and solid slip past each other with a sliding friction similar to that of smooth solids separated by unguents, but this rarely happens, and the smoothest plane surface carries a layer of small regular whirls with it; if a slight roughness of the surface causes still more liquid to be carried along in innumerable small whirls this is equivalent to an increase of cross-section and of inertia of the solid. In the air for moderate velocities simple viscous resistance becomes less important than skin friction; for still higher velocities skin friction is negligible as compared with the fluid friction attending the spheres and plates and other "unfair bodies" that are used in anemometry, as is shown by the relative measures of resistances of rough and smooth surfaces made by Hagen, Recknagel, Haughton, and others.

(*e*) *Fluid friction.*—When a normal pressure plate or other "unfair" body moves through a perfect liquid, *i. e.*, one without viscosity, still discontinuous motions, surfaces, and spaces in the liquid may attend the movement; large eddies are produced near and especially behind the body; a small quantity of dead fluid is carried in front and a larger quantity in the rear; the particles, separating from each other in front in order to be deflected around the solid, acquire eddying motions and give up a part of their energies to the adjacent portions of the liquid, and the total effect is as though the energy of the moving solid is dissipated throughout a large portion of the liquid, or as though the mass of the solid were steadily increasing. The total amount of energy thus communicated per unit of time is said to be used up in overcoming "fluid friction," as distinguished from the skin friction due to lateral eddies and from the shearing effect that is resisted by viscosity. Similar eddies in discontinuous spaces within natural viscous fluids drag the neighboring liquid with them into still larger eddies, and the theoretical part of anemometry is very largely to be considered as a study of the laws of fluid friction or the consumption of the energy of the moving body or moving current by the numerous eddies produced within both perfect and non-viscous fluids. Experimental anemometry takes also into account approximately the properties of the natural viscous gases and liquids and the effects of the shapes and roughness of the solids that enter into the apparatus.

(*f*) *Discontinuity.*—If the particles successively flowing past an obstacle have at any point always the same velocity and motion then the motion is said to be steady; the paths pursued by the particles passing any point are the stream lines, and in steady motion these will be similar and permanent. In steady motion the stream line is the same as the line of flow, which is a line whose tangent at any point of the fluid has a direction the same as the resultant of all pressures at that point. But if at any point in the fluid the internal pressure becomes negative and exceeds a certain limit (depending on the density, pressure, and cohesion of the fluid particles), then a rupture will take place and a surface of discontinuity will separate the fluid from a vacuous space; into this space, such as the dead space behind a body, other fluid may rush under the hydrostatic pressure of the liquid and the atmosphere and eddies be formed. Such a rupture takes place when water under atmospheric pressure flows past an obstacle at a velocity of about 45 feet per second; for air, under the same circumstances, a partial diminution only takes place, but eddying motion still exists. For ellipsoidal bodies the stream lines are now known for ideal non-viscous fluids, whence those for ordinary fluids for spheres and plates may be determined as special cases, at least approximately. The discontinuous space behind a sphere is investigated by Sir William Thomson in the L. E. D., Phil. Mag., 1886 and 1887.

(*g*) *Surface tension.*—The assumptions that the pressure and density within a mass of liquid are uniform, and controlled by the exterior atmospheric pressure, fail when surfaces of great curvature are considered. For such cases the surface tension or capillary forces become the controlling feature, as shown in the phenomena of drops, jets, etc. Vacuous regions of discontinuity, when once established by forces that overcome the cohesion of the liquid, are then at once bounded by a surface layer of the liquid, and this, by its high tension, strongly tends to contract to its smallest dimensions the discontinuous space within it; thus the surface tension becomes a new and very important force. For example, a small bubble of air within a mass of water is bounded by a surface layer of water, whose compression of the air within is such that the barometric pressure within the bubble is expressed by the formula

$$P=\frac{4\gamma}{R}$$

where

γ=tension due to a unit of water surface=8.4 milligrams for 1 square millimeter of water surface.

R=the radius of the sphere in millimeters.

Again, a falling jet of water, A, striking a disk, B, as in the accompanying diagram (Fig. 54), divides into the hollow egg-shaped figure BDC, the enlargement to D and the contraction to C being controlled by the surface tension of the thin layer of liquid.

The viscosity of the surface layers of a liquid are also (at ordinary temperatures) far greater than that within the mass of the fluid.

(*h*) *Compressibility.*—Fluids are divided into gases and liquids or compressible and incompressible, air and water being the respective types. For small velocities (*i. e.*, considerably less than the limit for discontinuity) the stream lines are sensibly the same for both classes of fluids, and the resistances also, after allowance, is made for the density. The compression of the air in front of an obstacle is very slight at ordinary velocities. Therefore as a first approximation, sufficiently correct for velocities under 30 miles an hour, the results of observations in air and water can be combined, the most important proviso being that in both cases the resisting obstacle shall be surrounded by the fluid to a distance many times the diameter of the obstacle itself, or practically to infinity.

(*i*) *Motion of solid versus motion of fluid.*—With regard to the question as to the difference in the resistances experienced by moving a body through a motionless liquid, and by moving the liquid past the stationary body, Langsdorf, in 1794, states in a general way that the pressure of a current must be greater than the resistance to a moving body, and that the problem of anemometry is to ascertain in these two cases the law of the pressure in the rear as depending on the inflow of the displaced fluid. But thus far, although the impression seems to prevail that there may be some reasons why pressures and resistances should differ, as observations seem to indicate, yet mathematical mechanicians have not expressed these impressions in definite form, and have generally discussed the problems of hydro-mechanics under the assumption that it matters not which moves, the fluid or the body.

(*k*) *Coefficient of resistance.*—Poncelet and Unwin give the following explanation of the fact that whether the liquid or the body moves in either case the resistance is greater than that given by the Newtonian theorem. They assume the body to be in the center of a cylinder of liquid whose diameter is the same as that of the stream whose surface is not affected by the presence of the obstacle. Thus let A be the area of the section of the current represented by this cylinder, a the section of the obstacle; then the resistance will be a function of both a and $\frac{A}{a}=\rho$. Let c be the coefficient of contraction, namely, the ratio of the area of the contracted section behind a to the full section a itself, then will the true resistance be expressed by the product of the Newtonian resistance into the factor K, recognized by the earlier mechanicians and called the coefficient of resistance, where K will have values depending on the shape of the obstacle, namely, for plane circular plates

$$K=\rho\left(\frac{\rho}{c(\rho-1)}-1\right)^2$$

for circular cylinders

$$K=\rho\left[\left(\frac{\rho}{\rho-1}\right)^2\left(\frac{1}{c}-1\right)^2\left(\frac{\rho}{\rho-1}-1\right)\right]$$

C is approximately 0.85 or $\frac{1}{c}=1.176$. In accordance with experience this gives the pressure on a cylinder less than on a thin plate, but it is best to determine K experimentally for each solid rather than to compute it by Poncelet's formulæ, since it is found that the formula does not fully represent the observed resistances.

The above formula, based on the principle of living force, is an extension of the ideas developed by Dubuat, who considered the total resistance of a plate surrounded by a moving fluid as due to the fact that it experiences in the rear an average hydrostatic pressure less than that existing when the fluid is not in motion, while in front it experiences a greater pressure. The general hydrostatic pressure of the fluid pushes the outside quiet water into the region of low pressure in the rear, and partly compensates but still leaves a deficit of hydrostatic pressure. By actual measures Dubuat determined the pressure prevailing in the front, and the deficiency or minus pressure existing in the rear. The last he expressed by the fraction 0.433 of the Newtonian pressure, while the front pressure was unity. The sum of these gave him for the total pressure of a plate 1 foot square moving in water the coefficient of resistance $K=1.433$; similarly Morin found for water 2.18 and for air 1.36, from which Poncelet and Unwin conclude that 1.3 is a plausible value of K. When the plate is fixed and the water flows past the value seems to be larger, thus Mariotte in water finds $K=1.25$, Dubuat in water finds $K=1.856$, and Thibault in air $K=1.834$, from which 1.8 is adopted as plausible by Poncelet and Unwin. It is at present impossible to decide whether the differences in these numbers for moving plate and moving fluid, respectively, are errors of observations or represent laws of nature, but as the observations on rectilinear motions are liable to considerable error, it may be fair to ascribe equal weight to each of the preceding six determinations of K, in which case the average becomes for the moving body 1.66, but for the moving fluid 1.65, whence $K=1.66$, or $1\frac{2}{3}$ may be adopted as an approximate common factor for both cases.

The six experimental values of this constant just quoted are based on rectilinear motions of the fluid and the obstacle. The observations made with circular motions that are frequently quoted in this connection apparently need a special correction for circular motion, such as makes it improper to use these in the preceding determination of the value of K.

(*l*) *Reduction of the resistance for circular motion to that of rectilinear.*—As the rectilinear motion, whether of the air or the obstacle, is more difficult to obtain and regulate than the circular motion of the solid therefore by far the most attention has been given to the experimental testing of all forms of anemometers by mounting them upon a whirling-machine and giving them a rapid circular motion in still air. Similarly it has been attempted to determine the pressure of the wind against an obstacle by mounting the latter upon a whirling-machine; but the results of such experiments can not be rightly reduced until the theory of the action of the whirled body on the fluid is understood.

The whirling-machine consists essentially of a long horizontal arm attached to a vertical axis; the plate or other resisting solid is carried by this arm and describes a circle of known radius at a uniform measured rate produced by the pressure of a known force upon the arm. The moment of this force must therefore be equal to the sum of the moments of the friction of the machine and the resistance of the air. After subtracting the moment of friction* there remains the moment of the resistance of the air, and the remaining problem is to determine the lever arm and thence the force of resistance, or the total pressure normal to the radial arm of the machine. As this normal pressure may, for the same resisting body, vary with the length of the radius of the circle described by it, it is necessary to repeat the determination with radii of various lengths, and derive thence experimentally some indication of the value proper for an infinite radius, *i. e.*, equivalent to rectilinear motion. Unfortunately, although experimenters have varied considerably the sizes, shapes, and velocities of the solids experimented upon (Recknagel gives results with radii of 1 and 2 meters and with rectilinear motions), yet no one person has varied the radii of the whirling-machines used in the same course of experiments in the air sufficiently to thoroughly elucidate this subject experimentally. Nor, on the other hand, is it possible to arrive at any conclusion by the comparison of the results obtained with diverse pieces of apparatus and methods of observations, as will be seen by examining the discrepant results quoted hereafter.

The oscillations of a pendulum offer a case of circular but not uniform motion; the accurate observations of Dubuat afforded Duchemin a basis for the conclusion that the resistance to circular motion is a little greater than for straight motion, and that the excess increased the larger the surface and the smaller the radius. Hence Duchemin conceived that for a rotating body in uniform rotation there must also be a similar increase. His idea was that a rotating or oscillating body can not be accompanied in its motion by the same system of stream lines that would surround it if the body moved in a straight line since the centrifugal force due to the rotation acts on the molecules composing those streams; the fluid at rest beyond the stream lines must then resist this action as communicated to them, and there results an additional pressure on the body and a greater resistance to circular than to a straight line motion. A more rigorous discussion by Stokes of the mutual reaction between water and spheres oscillating therein apparently gives a different explanation of the increased pressure observed by Dubuat and Duchemin, but as the latter states that he tested his formula by application to whirling experiments in the air, I give it here as an empirical and first attempt to reduce circular to rectilinear resistances. Duchemin finds that for a given linear velocity and a given form of body the resistance for circular motion is expressed by the following formula:

$$R' = R\left(1 + 1.624\frac{\beta}{\rho(f-s)}\right)$$

where

R = the resistance for rectilinear motion at the velocity V.

R' = the resistance for circular motion when the center of figure of the largest section of the body has a linear velocity, V.

* Wied. Ann., 1878, IV, p. 171.

ω=the area of the greatest section of the body.

Δ=the density of the liquid.

ρ=the coefficient of the resistance to straight-line motion of the body in still water, variously assumed at 1.3, 1.6, 1.8, and for which the preceding paragraph where this is designated by K has given 1.66 as the best available.

V=the linear velocity of the body in its circular path.

f=the distance of the center of figure of ω from the axis of rotation.

s=the distance of the center of figure of ω from the center of gravity of that half of the section ω that is on the side nearer the axis of rotation.

β=a linear quantity representing the thickness of the region in front of the body, within which the stream lines are so deviated as to be sensibly parallel to the front surface of the body.

In general we may assume

$$\beta=\frac{1}{2}\sqrt{\omega}\sin\alpha$$

where α=the angle of incidence of the air striking the front surface of the body at the front point of the axis. For a solid of revolution whose axis is in the direction of the motion

$$\beta=\frac{1}{2}C\sin\alpha$$

where C is the diameter of the maximum section ω.

For a plane surface $R=\rho\omega\Delta v^2$ by the Newtonian theorem, where ρ=1.3 was assumed by Duchemin.

101. RESULTS OF EXPERIMENT AND OBSERVATION.

If, now, we pass from deductive considerations in a field of study that is confessedly the most difficult branch of mathematical mechanics, and collect together the results of actual observations on the resistance of the air, we may summarize them as follows:

(A) *Pressure of moving fluid on a stationary solid.*—(1) Mariotte was the first to directly measure the pressure due to the wind; his determination of the velocity was probably by means of light objects floating in the air, but the details of his methods are not accessible to me. The ratio of the theoretical pressure to that given by his experiments is as 1 to 1.73, as quoted by Muncke and Weisbach, but as 1 to 1.75 as quoted by Langsdorf. According to Poncelet two observations by Mariotte on plates 6 inches square in a straight current of water, at velocities of from 1 to 4 feet per second, gave K=1.25. The accuracy of these resulting coefficients can not be very great, as the pressures and temperatures of the air are unknown and apparently were assumed to have their normal values.

(2) Experiments made by Woltmann between 1785 and 1790 at Hamburg, in which the wind velocity was apparently measured by his newly-invented "hydrometrische flügel" are quoted by Langsdorf and others as the most reliable of any results obtained up to that time. The best of these observations are selected by Langsdorf and reduced with an assumed density of the air equal to $\frac{1}{800}$ of the density of water with the result that the coefficient K is 1.19; but the density of the air seems to have been a matter of assumption, and with a density of $\frac{1}{900}$ Langsdorf subsequently got K=1.49, while Crelle in his reduction deduces 1.34.

(3) Dubuat, with a plate 1 Paris foot square, in a current of water at a velocity of 3 feet per second, deduces K=1.856; but this result is not from direct measures of the total pressure, but is attained by the subtraction of the measured pressures positive on the front and negative on the rear of the plate. It is therefore not directly comparable with the results of other observers. But Dubuat did also make direct measures of the pressure on this plate and found the same coefficient, thereby apparently confirming the principles of the method by which his former value was obtained.

(4) Thibault, 1826: The work of this observer is very highly praised for its accuracy by Morin, Poncelet, and others, but is known to me only by their quotations. Thin square plates 0.33 and 0.48 meters on a side were exposed to the wind; the wind velocity varied from 1.8 to 8.2 meters per second; the pressure was measured by a spring; the resulting value of K is 1.834.

(5) Experiments were made by Paris, of which I can learn but little, but which are said to agree well with the preceding mean values.

(6) Result: The discrepancies in the above figures are so great that it can not be affirmed that their evidence is strongly in favor of a value of K as large as 1.85, but the adoption of this value is recommended by Poncelet and indorsed by Unwin, with the proviso that more accurate experiments must be made; the value 1.86 is generally adopted by engineers in accordance with the usage of Weisbach in his Mechanics of Engineering and Construction.

(*B*) *Resistance of stationary fluid to a solid moving rectilinearly through it.*—The resistance of the air to falling bodies and the resistance of water to objects drawn through it are the two ordinary and most convenient methods of experimentation with straight-line motions. The following are the results of many investigations:

(1) The resistance of the air to falling bodies was first experimented upon by Galileo. Since his time the most prominent investigators of falling bodies have been Mariotte, Newton, and Benzenberg (1802), but the accuracy of all such work, even on small falling spheres, scarcely suffices for anemometric problems. [See below (3) *j*.]

(2) Observations with the ballistic pendulum give the momentum of balls shot against it from different distances, whence the energy lost by

the resistance of the intervening air may be computed. Thus Robins concludes that the resistance to an iron cannon-ball about $4\frac{1}{4}$ inches in diameter, at a velocity of 25 feet per second, is not less than 0.5 ounce. From similar observations Hutton concludes that for velocities under 300 feet per second the resistance to a sphere is about 1.4 times that given by the Newtonian theorem.

Similar observations have been repeated frequently in connection with gunnery, notably by Bashforth and Mayevski, but the results are not sufficiently accurate for use in anemometry.

(3) Measurements of objects moving through water afford larger resistances, and the stillness of the water can be assured by observation of floating particles. Among the results of this class of work are the following:

(*a*) Bouguer, for a plate 1 (Paris) foot square at the rate of 1 (Paris) foot per second, finds the resistance in sea-water to be 23 ounces.

(*b*) Da Borda, for a cube 1 foot on a side at 1 foot per second, finds 21 ounces.

(*c*) Ulloa, for a plate 1 English square foot in area at the rate of 2 feet per second, finds the resistance 244 ounces.

(*d*) Buchanan, experimenting for the English Society for the Improvement of Naval Architecture, finds for a thin plate 1 foot square, moving with a velocity of 5 nautical miles per hour or 8.35 feet per second, the resistance 80.76 pounds avoirdupois.

(*e*) The commission of the French Academy found that a plate 1 Paris foot square, drawn through water at 2.56 Paris feet per second, has a resistance 7.625 French livres. The same observers, for a rectangle 2 feet by one-half a foot, at the rate of 3 Paris feet per second, found a resistance 14.45 French livres.

(*f*) The Swedish commission of 1810–'15, for small spheres falling through water at small velocities, found the resistance as given by the following formula:

$$R=0.5\ ms\frac{v^2}{4g}$$

where

R=the resistance in Swedish pounds.

m=the weight of a cubic foot of water=61.46 Swedish pounds.

g =the descent of a falling body in 1 second=16.534 Swedish feet or one-half the acceleration due to gravity.

s =the area of section in square feet (Swedish).

v =the velocity per second in Swedish feet.

(*g*) Nordmark: Observations on horizontal motions of bodies were made for this Swedish commission by Nordmark, who found for bodies sharp pointed at the front and rear the resistance equals 0.67 times the Newtonian resistance for the sectional area; for cylindrical bodies moving perpendicularly to the axis R=0.907 times the Newtonian resistance; for cubical bodies moving normal to front face R=0.83 times the Newtonian resistance.

(*h*) Dubuat, from observations on a plate 1 French foot square in currents of water at velocities of 1 and 2 French feet per second, found a resistance equal to 1.433 times the Newtonian, but this, as in his previously quoted work, is the result of separate measures on the front and rear of the plate, which, added together, gave the above coefficient. The difference between this and the coefficient 1.856, as given by Dubuat for this same plate when stationary in a current of water, has been the basis for extensive studies by several French physicists (see the works of Thibault, Duchemin, Poncelet, and Morin).

(*i*) Piobert, Morin, and Didion, in their joint work, found for thin vertical square plates from 0.03 to 0.25 meters on a side, and for velocities from 0 to 5 meters per second, moving horizontally in shallow water, resistances that were closely proportional to the surfaces, and expressed by the formula

$$R=0.934\,A+2.81\,pAH$$

where A is the surface, $H=\frac{v^2}{2g}$ or the falling height, namely, the descent under the action of gravity necessary to attain the velocity V; $p=$the weight of a unit volume of water. For any velocity over 0.5 meters per second this gives the coefficient $A=2.81$. Resistances in shallow water are very different from those within an indefinite extent of fluid owing to the work done in producing waves.

(*j*) The same experimenters for thin square horizontal plates 0.5 to 1.0 meters on a side, moving vertically in the air at velocities from 0 to 9 meters per second, found the resistance

$$R=(0.03+1.3574H)pA$$

where H and A have the same significance as before, but p is the weight of a unit volume of air; for a standard density of air at 760^{mm} and 10° C. 1 cubic meter of air weighs 1.214 kilograms. This formula gives for the ratio of the observed to the Newtonian resistance $K=1.3574$.

(*k*) General result: Notwithstanding the above discrepancies the evidence is in favor of the values of K as found by Dubuat (1.43), by Duchemin (1.254), and Didion (1.3574); Poncelet recommends the adoption of $K=1.30$, with a proviso that a small correction for the size of the surface of the plate may still be found. In this recommendation Unwin coincides (see Encyc. Britannica, 9 ed., art. Hydro-mechanics, p. 517).

(*C*) *The resistance of motionless fluids to solids moving circularly.*—Circular motion is obtained by the whirling-machine and by the pendulum. The former was first used by Robins, Schober, Rouse, and Smeaton, who have been followed by numerous others. The general results, as far as we need quote them, are as follows:

(1) Robins: The dimensions of his apparatus were; radius of arm, 49.5 English inches; radius of circle described by the center of press-

ure, 51.75 English inches; the resisting object, a very smooth hollow 12-pound cannon-ball covered with paper, making a smooth sphere, 4.5 inches in diameter. At a linear velocity of 25 feet per second in the air this ball experienced a resistance of 36 ounces; the resistance increased as the square of the velocity.

(2) Vince: A plane circular disk, 0.927 square inch area, whose center describes a circle of 7.57 inches radius, experiences in water a resistance 1.4520 multiplied by the Newtonian resistance.

A hemisphere whose diameter is 1.1 and whose center describes a circle of radius 6.22, the flat side being forward, gave in water $K=1.5172$. The same, flat side to the rear, gave $K=0.6186$.

(3) Coulomb: A sphere 59 lines in diameter described a circle of (unknown) small radius in water, and gave

$$K=\frac{1240}{1127}=1.10$$

A circular plate gave in water $K=2.75$.

(4) Da Borda (1763), as quoted by Poncelet: Three thin plates describing a circle whose radius is 1.208 meters, with a velocity of from 3 to 4 meters per second, gave the following (in which, however, the friction of the machine and the density of the air are not allowed for):

area of plate	value of K
0.102 square meter	1.39
0.059 square meter	1.64
0.026 square meter	1.39

(5) Hutton (1786): Using the same machine experimented with by Robins: a circular plate whose diameter is 6⅔ inches and area two-ninths of a square foot or 0.011 square meter, describing a circle in the air whose radius is 52.7 inches for velocities up to 20 feet per second, gives

$$K=\frac{1.312}{1.057}=1.24$$

For a larger plate, whose area equals 0.021 square meter, $K=1.43$.

(6) Rouse, as quoted by Smeaton, who gives no details except that a whirling-machine was used, found the well-known formula for resistance of the air: $R=0.00492v^2$ in pounds per square foot, v being given in miles per hour. This gives

$$K=\frac{0.005}{0.0027}=1.85$$

Smeaton himself gives only relative values for different forms of sails, which hardly apply to anemometry.

(7) Edgeworth (1783): Using a whirling-machine similar to that of Robins; the thin plates described circles whose radius was 6.4 feet. He concluded that the shape and size of the plane plates both affected the resistance by reason of the stagnation of the air near the middle of the forward surface. The meager numerical results given by him are all

for the velocity 10 feet per second or about 7 miles per hour, and for a nearly uniform radius of circle. He states that for a square plate 16 inches area $R=2.50$ ounces per square inch, and for a square plate of 64 inches area $R=3.47$ ounces per square inch.

(8) Beaufoy, both for himself and subsequently for the Society for the Improvement of Naval Architecture, used a whirling-machine, and for a plate whirled in air of 1 square foot area and at a velocity of 6 feet per second, found

$$K=\frac{40.382}{34.975}=1.154$$

For a larger plate, whose size is not given, he found $K=1.225$.

(9) Rennie (1831) used Smeaton's form of whirling-machine revolving in air, but with very short arms, viz, about 8 inches, and always with two similar objects at the extremities of the radii, so that each was to a certain extent in the wake of the other. For his highest velocities he obtained the following:

Object.	Area of section.	Velocity per second.
		Feet.
Circular disk ...	10¾ inches in diameter ..	27.5
Square disk.....	9×9 inches each side....	22.9
Wooden ball....	10¾ inches in diameter ..	11.4

From this he concludes that the mean resistances of circular plates, square plates, and globes in the air are to each other as the numbers 25.180, 22.010, and 10.627.

(10) Prechtel: For a flat square plate of paper rotating around one edge as an axis Prechtel finds $R=3.7931\ HAq$ for velocities from 2 to 10 feet per second, where

$$H=\frac{v^2}{2g}.$$

A=area of the plate.

q=weight of a cubic foot of air.

This formula gives K=the observed divided by the theoretical pressure=1.896.

(11) Hagen, Recknagel, Schellbach: The exact measures published by these physicists will be discussed in the next sections (102 and 103).

(*D*) *Resistance to pendulous oscillations in air and water.*—Pendulums allowed to vibrate in the air move more slowly than in a vacuum. Bodies of the same size but different densities also show different effects. These phenomena would afford a very accurate means of determining the resistance of the air were it not for analytical difficulties in determining the effect of the varying action of the pendulum during each oscillation. Borda concluded, from the observations accessible to him, that the resistance of the air was approximately $R=1.86\ Av^2$ or $K=1.86$.

Stokes (1851) showed that the pendulum and vibration experiments of Baily, Bessel, Coulomb, Dubuat, and Sabine agree with his analysis, in which the old impact theory is replaced by a more natural one involving the mutual reaction of the solid and the fluid. He showed that the resistance to a moving body immersed in a fluid, whether of definite or indefinite extent, is composed of two parts: (*a*) That due to viscosity, and which varies with the velocity; (*b*) that due to whirling motions, and varying with the square of the velocity and the boundaries of the fluid; (*c*) he also showed that for motions so slow that no whirls or discontinuous motions or vacuous spaces are produced. The second form of resistance is zero, but the first remains appreciable. For such slow rectilinear motions of a sphere in an indefinite fluid that does not slip on the sphere Stokes finds the resistance to be

$$-F=6\pi\mu'\rho a V=6\pi\mu a V$$

(See On the Effect of Internal Friction, etc., equation 126, Cambridge Phil. Trans., Vol. X); the notation is $\pi=3.1416$;

$\mu'=\frac{\mu}{\rho}$=coefficient of viscosity of fluid divided by its density, or that which Stokes calls "index of friction" and which Maxwell calls "kinematic measure of viscosity;" a=radius of sphere; V=linear velocity of relative motion of sphere and fluid.

Stokes deduces the value of μ' from the above-mentioned pendulum experiments and finds it to be $(0.0564)^2$ for water and $(0.116)^2$ for air where the units are seconds and English inches, and these results he converts into $(0.05291)^2$ and $(0.1088)^2$, respectively, when the units are seconds and Paris inches. The better methods of determining the viscosity that have been used by Maxwell, Meyer, Helmholtz, Holman, Barus, and others have given values about one-half the above.

Stokes's formula shows that for the slow motion of a sphere the resistance increases directly as the product of the diameter by the velocity. This formula for resistance applies to vibrations only when the angle of oscillation is small and the path nearly a straight line; it does not apply rigorously to spheres revolving throughout a whole circle even at low velocities (see Stokes, section 9), even if the sphere be so mounted as to have no rotation around its own axis. The problem rigorously solved by him was the periodic oscillation of the sphere along a right line.

Stokes also showed from the above formula that when a sphere falling in the air under its own weight attains its maximum and ultimately permanent velocity, where the resistance must equal the weight, and provided the velocity be so small as to comply with the above conditions, it can then be computed by the formula

$$V_m=\frac{2g}{9\mu'}\left(\frac{\sigma}{\rho}-1\right)a^2=\frac{2ga^2}{9\mu}(\sigma-\rho)$$

where g=gravitation, or 386 inches per second.

σ=the density of the sphere.

ρ=the density of the fluid.

Computation thus shows that for small globules of water at small velocities the viscous resistance of the air is far larger than the impact resistance as computed by the old Newtonian theorem. Thus, for globules of water 0.001 inch diameter in air and Stokes's value of μ', adopting $\sigma=1$ and $\rho=0.001$, at the height of ordinary clouds we have viscous resistance=weight when $V=1.593$ inches per second. ~~Whence~~ The Newtonian impact resistance=$\frac{1}{400}$ of weight when $V=1.593$ inches per second; impact resistance=whole weight when $V=32.07$ inches per second (see Stokes, etc., section 43).

[The resistance of air to slow vibrations and motions of fair bodies has been of late years discussed mostly with a view to determine the viscosity coefficient (see the memoirs by Stokes, Meyer, Lambe, Helmholtz, Pietrowski, Maxwell, Braun, Kurz, Töpler, Boedeker, Kirchhoff, Freiburg, Auerbach, Froude, etc.]

102. OBSERVATIONS BY THIBAULT, HAGEN, AND RECKNAGEL.

The best observations of the normal pressure experienced by the total surface of flat plates in uniform circular motion, and moving so rapidly that viscous and skin friction are small compared with fluid friction as above defined, are those made by Thibault in 1828, by Recknagel in 1880, and by Hagen in 1874, and these merit being given in detail.

A careful study of the pressure at the center of the front of a plane plate and at the apex of a solid of revolution whirled in still air is published by Recknagel (see Wiedemann Annalen, 1880, X, p. 677, etc.). He shows that P_1, the static pressure in the air at the central spot, is

$$P_1=P\left(1+\frac{\beta-1}{\beta}\cdot\frac{s_0v_0^2}{2gP}\right)^{\frac{\beta}{\beta-1}}$$

or for low velocities

$$P_1=P+\frac{1}{2}mv_0^2$$

where

P=the pressure in the surrounding still air in kilograms per square meter.

$m=\frac{s_0}{g}$=mass of a cubic meter of air at density $s_0=1.293$ at 0° C.

v_0=velocity of air relative to that of the central point.

$\beta=1.41$=ratio of specific heats for constant pressure and volume whence

$$\frac{\beta-1}{\beta}=\frac{2}{7}$$

In the experimental confirmation of this formula Recknagel adopts the amount of motion or "mit-wind" communicated to the surrounding air by the whirling-plate to be 5 per cent. of the absolute velocity of the plate v, so that $v_0=0.95v$ when the radius of whirl is 2 meters. This is determined directly for a delicate Robinson anemometer, whence it is inferred for his experimental plates. The pressure evidently diminishes as one proceeds from the center towards the edge of the plate, and, although the law of diminution is not determined by Recknagel, yet the above confirmation of an obscure law of motion of fluids is very acceptable. The pressure in front of the plate and the mit-wind do not sensibly vary with plates of zinc, brass, and wood, as in all cases a thin layer of air evidently adheres to the plate over which the remaining air slides. The convex side of a hemispherical cup, moving forward at a true velocity of $v=9.42$ meters per second, gave $v_0=9.19$ or a mit wind of 0.23, while the concave side forward for $v=9.42$ gave $v_0=9.09$ or a mit wind of 0.33.

Thibault experimented with thin plane plates moving in a direction normal to the surface by a circular motion. The radius of the circle described by the center of figure was 1.37 meters; the velocities varied between 0.5 and 11.0 meters per second. The coefficients deduced by him are quoted as follows from Poncelet:

Square plate: Area, 0.026 square meter	$K=1.525$
Square plate: Area, 0.10304 square meter	$K=1.784$
Rectangular plate: Area, 0.10304, long side radial . . .	$K=1.900$
Rectangular plate: Area, 0.10304, short side radial . . .	$K=1.677$
Square plate: Area, 0.323×0.323; radius of the center, 1.370	$K=1.784$
Square plate: Area, 0.227×0.227; radius of the center, 0.966	$K=1.784$
Square plate: Area, 0.161×0.161; radius of the center, 0.685	$K=1.784$

The preceding observations indicate that for plates moving circularly the resistance per unit area may increase with the extent of the surface; but the following more extended observations by Hagen give the fullest information at present available.

Hagen, in the Berlin Academy Mathematische Abhandlungen, 1874, has published in full his determination of the pressures against thin plates moving circularly. The following table gives the shape and dimensions of the disks, the velocities and resistances reduced to a standard density of the air. The corrections for the friction of the apparatus and the standard density were applied by Hagen. The rearrangement for the present study is, however, new. The measures are all expressed in the units actually employed by Hagen, namely: Rhenish inches, of which 1 equals 1.0297217 English inches=26.15446 millimeters; old Prussian loths, of which 32=1 Prussian pound=1.031236 pounds avoirdupois=467.7 grams. The standard density of the air is that for a pressure of 28 Paris inches of mercury at 0° R. under gravity at Berlin, and for a temperature of 12° Reaumur. These correspond to 758.6^{mm} mercury at standard gravity and to 12.75° C., respectively.

TABLE I.—*Original results of observations by Hagen of resistance of the air to circular motion.*

Shape.	Dimensions of apparatus.			
	Dimensions.	Areas of disks, F	Radii of rotation.	
			For center of surface, A	For center of pressure, R
	Inches.	*Sq. inches.*	*Inches.*	*Inches.*
Radii of circular disks	1.25	4.909	97.25	97.252
	1.745	9.566	97.745	97.754
	2.245	15.834	98.245	98.256
	2.75	23.758	98.75	98.760
	3.25	33.182	99.25	99.260
Sides of square disks	2	4	97	97.002
	3	9	97.5	97.505
	4	16	98	98.008
	5	25	98.5	98.512
	6	36	99	99.015
Equilateral triangle	7.6	25	98.2	98.204
Square disk	5	25	98.5	98.512
Long rectangle	1×16	16	96.5	96.5

Shape.	First determination.			Second determination.			
	Time for driving weight to fall 1 inch, t	Corresponding resistance, "reduced r."	Resistance for unit area at unit velocity, K	t	"Reduced r."	K	Approximating linear velocities.
	Seconds.	*Loths.*	*Loths.*	*Sec'ds.*	*Loths.*	*Loths.*	*Inches per second.*
Radii of circular disks	1.994	18.791	0.0000022760	1.91	18.952	0.0000022894	65
	1.990	38.463	23476	1.87	38.576	23549	61
	2.035	66.165	24028	2.045	66.575	24176	58
	2.240	104.695	24810	2.235	103.221	24609	54
	2.435	149.942	25199	2.44	149.683	25154	50
Sides of square disks	1.835	15.607	0.0000023317	1.835	15.704	0.0000023461	65
	1.965	35.810	23472	1.965	35.998	23595	56
	2.080	67.053	24281	2.070	67.524	24452	57
	2.290	106.455	24338	2.290	107.493	24574	54
	2.545	160.522	25055	2.530	169.376	25032	50
Equilateral triangle	2.12	....	0.0000025026	......	....	....	58
Square disk	....	....	....	2.10	....	0.0000024491	58
Long rectangle	2.68	66.65	0.0000025286	1.87	66.373	0.0000025178	60

NOTE.—The linear velocities C, with which the center of pressure moved through the air, can be exactly found, if desired, from the expression

$$C=\frac{R}{0.81705\times t}$$

They were approximately as given in the last column.

Hagen finds that these 20 values of K can be represented, with a probable error of $\pm 0.000\,000\,252$, by the expression

$$K=0.000\,002\,263\,9+0.000\,000\,009\,416p$$

where p is the circumference of the disk in inches; hence he concludes that for the conditions of his experiments and for his standard density of air the pressure against a plane disk moving circularly at low velocities, keeping its surface normal to the quiet air, is represented by the following equations, which are given in various systems of units on account of the frequent reference that is made to them:

For air of Hagen's standard density the resistance in Prussian loths per square Rhenish inch for velocities in Rhenish inches per second is

$$\frac{2.264+0.00942p}{1\,000\,000}Fc^2$$

For air of Hagen's standard density the resistance in grams per square decimeter for velocities in decimeters per second is

$$(0.00707+0.0001125p)Fc^2$$

For air of Hagen's standard density the resistance in pounds avoirdupois per square English foot for velocities in English miles per hour is

$$(0.0028934+0.0001403p)Fc^2$$

The Newtonian theorem would give for a disk whose area is 1 square Rhenish inch the resistance $=0.000\,001\,992$ loths; the ratio of this to the coefficient $\frac{2.264}{1\,000\,000}$ in the above equation therefore gives for a unit velocity $K=1.13654$; but if we allow for the perimeter of the plate this is enlarged by the ratio of the second term in the numerator of the above equation; for a square plate of 1 inch the ratio becomes $K=1.16$. Therefore for circular motion the factor K is smaller than those for rectilinear motion instead of being larger, as Duchemin expected.

The following are the values of K thus computed for square plates:

Sides of square plates.	$K=\frac{\text{Hagen}}{\text{Newton}}$
Rhenish inches.	
1	1.155
2	.174
3	.193
4	.212
5	.231
6	.250
7	.268
8	.287
9	.306
10	1.325

Assuming Duchemin's value to be $K=1.3$, although this is too large for very small plates, and introducing it into Duchemin's formula* as already given, we thereby reduce Hagen's results for circular motion on the

* See § 225, paragraph (*l*), page 225.

whirling-machine to an approximate value for the resistances to rectilinear motion. The numerical processes are indicated in full in the following table where Duchemin's notation is adopted from pages 225–226:

TABLE II.

Resistance to rectilinear motion deduced by Duchemin's formula from Hagen's observations on circular motion.

Disk. Shape.	Disk. Size.	f	s	$\beta=\frac{1}{2}\sqrt{\omega}$	$f-s$	$\frac{\beta}{f-s}$	$x=\frac{1.624}{\rho}\cdot\frac{\beta}{f-s}$
	Inches.	*Inches.*	*Ins.*	*Inches.*	*Ins.*		
Circles:							
Radius ..	1.25	97.25	0.42	1.25	96.8	0.018	.016
	1.745	97.745	0.58	1.745	97.1	0.018	.022
	2.245	98.245	0.74	2.245	97.5	.024	.030
	2.75	98.75	0.92	2.75	97.8	.029	.036
	3.25	99.4	1.08	3.25	98.3	.033	.041
Squares:							
Sides	2	97.	0.5	1	96.8	0.012	.015
	3	97.5	0.75	1.5	96.8	.016	.020
	4	98.	1.0	2	97.0	.020	.027
	5	98.5	1.25	2.5	97.2	.026	.033
	6	99.	1.5	3	97.5	.031	.040
Triangle:							
Side	7.6	98.2	2.53	2.5	95.7	.039	.049
Square:							
Side	4	98.5	1.2	2	97.3	.021	.027
Rectangle:							
Side	1×16	96.5	0.25	0.5	96.2	.005	.006

Disk. Shape	Disk. Size.	Hagen's observed K or R'. First series.	Second series.	Mean value.	Duchemin $R=\frac{R'}{1+x}$
	Inches.	*Loths.*	*Loths.*	*Loths.*	*Loths.*
Circles:					
Radius ..	1.25	0.000002270	0.000002289	0.000002280	0.00000225
	1.745	2348	2355	2352	233
	2.245	2403	2418	2410	234
	2.75	2481	2460	2470	239
	3.25	2520	2515	2518	242
Squares:					
Sides	2	2332	2346	2339	231
	3	2347	2360	2354	231
	4	2428	2445	2436	237
	5	2434	2457	2446	236
	6	2506	2503	2504	240
Triangle:					
Side	7.6	2503	....	2503	238
Square:					
Side	4	....	2440	2440	230
Rectangle:					
Side	1×16	2529	2518	2524	251

These figures illustrate the small amount of reduction, according to Duchemin's formula

See p 225

$$R'=R\left(1+1.624\frac{\beta}{\rho(f-s)}\right)$$

needed in order to pass from R' (or the resistance per unit area per unit velocity for a plate revolving on a whirling table in still air) to R (or the resistance that would have been observed had the plate moved in a straight line). The further reduction from a plate moving in a straight line to a rectilinear wind moving against a still plate is approximately given according to the previous section (k§100) by multiplying R by the factor $\frac{1.8}{1.3}=1.4$, if we follow Poncelet and Unwin, but which factor is more nearly 1.00 if our general view of the subject in that section is correct.

The comparison of the computed coefficients R for rectilinear motion in the last column shows that for the circular and square plates, respectively, the rate of increase of the pressure per unit of area still varies with the size and the perimeter of the plate, as in the case of the R' for circular motion, although the amount of its increase is reduced by nearly one-half. Hagen's observations seem to be sufficiently accurate to show that the empirical formula deduced by him for the purpose of representing his observed R' really represents in its form some law of nature, and may be due, as he seems to suggest in other writings, to the fact that the viscosity, whose effect is large in the motion of water in pipes and channels, has a similar effect in the flow of the air around obstacles; but the reduction to rectilinear motion reduces this coefficient for the effect of viscosity *on these plates* to about 0.6 of the value given by Hagen. The numerical accuracy of this coefficient depends also upon the value adopted for the coefficient ρ, for which 1.3 would appear to be the maximum allowable. The steady increase of R' is I think due to fluid friction quite as much as to viscous resistance.

103. OBSERVATIONS BY SCHELLBACH AND THIESEN.

The most recent attempts at accurate measures of resistance by means of a whirling-machine are those of Schellbach and his pupils. The preliminary portion of this work has been published by Thiesen (Wiedemann, Annalen, 1885, Vol. XXVI), who therein confines his attention to observations made on the long rods used to carry the spheres and globes and plane disks. The result given by Thiesen is that for a long thin cylindrical rod whose axis lies radially in the plane of the circle described by it, the resistance is expressed by the following formula:

$$\Re=37.4\eta vL+0.3972\varepsilon v^2dL+0.000\,009\,04\frac{\varepsilon^2}{\eta}v^3d^2L$$

where the units are centimeters, grams, and seconds, and the notation is

d=diameter of the cylindrical rod.

L=the length of the cylindrical rod.

v=linear velocity of the center of the rod.

ε=standard density of the air or the weight of 1 cubic centimeter, which is assumed at 0.001200 grams.

η=the coefficient of viscosity of the air=0.000190.

In regard to these results, and, indeed, as bearing on the whole subject of the resistance of the air, the following excellent remarks are made by Thiesen.

The prevalent views as to the laws, according to which the hydro-dynamic resistance depends upon the velocity and the dimensions of the moving body, can be summarized as follows: For very small velocities and very small dimensions of the body the frictional [*i. e.*, internal friction or viscosity] forces alone play an important part; therefore one can omit the terms in the hydro-dynamic equations that are proportional to the square of the velocity, and the resistance thus becomes directly proportional to the velocity and to the linear dimensions of the body. On the other hand, if for a given incompressible fluid the velocities and dimensions exceed certain limits, then the influence of friction becomes negligible, and the resistance is proportional to the square of the velocities and of the dimensions. In this case the terms that are multiplied by the coefficient of friction can be omitted from the hydro-dynamic equations. Therefore, for ordinary velocities one would expect to assume that the resistance is made up approximately of terms that depend upon the fricton alone and of those that are independent of it.

But, examining this matter more closely, these views rest upon erroneous reasoning. Of course, for great velocities the terms in the equations of motion that arise from friction are of less importance, but only in so far as the change in the conditions of motion in a given time depending upon them is slight. In the determination of the hydro-dynamic resistance in its ordinary narrower meaning the changes in the conditions of motion are not at all considered, but only the character of the motion, since it is assumed that the motion has become steady and stationary. But the character can depend very materially upon any existing friction, no matter how slight that may be; the difference between fluids of small and large friction will consist only in that the stationary conditions occur sooner for the latter than for the former. Therefore the conclusions that have been drawn from the hydro-dynamic equations after simultaneously omitting the terms depending on the time and the friction are not conformable with experience.

The foregoing considerations were suggested by the results of the experiments by Schellbach and his student, Heyne, according to which for large velocities the resistance of cylindrical rods increased faster than the square of the velocity. This could not be attributed to the compressibility of the air, since the discussion of older experiments showed that for water also the resistance increases sensibly faster than the square of the velocities and dimensions. On the other hand, the experiments and their reductions appeared not sufficiently free from criticism to suffice of themselves to overturn a law that had apparently stood firm since Newton's time, and which is confirmed by the rigid theory based on the hydro-dynamic equations since Helmholtz, by the introduction of the idea of discontinuous fluid motion, had removed the previous paradoxical laws for the hydro-dynamic resistance. The knowledge of the theoretical law is desirable for the more accurate deduction of the results of the experiments, since my preliminary approximate development by powers does not converge sufficiently. I have, therefore, for a long time endeavored to find a theoretical solution of the problem of the hydro-dynamic resistance, but have arrived at no valuable results, and have concluded to communicate the results of the experiments on the resistance of cylindrical rods substantially in the same form as they were in my hands four years ago (see Wiedemann's Annalen, 1885, Vol. XXVI, p. 309).

The general conclusion is, therefore, that no general hydro-dynamic equations are at present available for the desired conversion of direct observations of wind pressure into wind velocities, and that a special study and experiment must be applied to each of the four types of instruments described in the first four sections of this chapter. Their consideration from this point of view is now to be again taken up.

104. REDUCTIONS FOR PENDULUM ANEMOMETERS.

(*a*) *The pendulous sphere of Parrot.*—This was first studied by Parrot himself (see Voigt's Magazine, Vol. I, 1797) in order to derive the velocity of the wind from the indications of his instrument. As quoted by Laughton and by Muncke it would seem that Parrot concluded that the pressure on the front face of the sphere is approximately equal to one-half that given by the Newtonian theorem for the area of a central section of a sphere.

On the other hand the experiments of Newton, Borda, and Hutton show that the resistance of a sphere that is not too small, moving in still air, is two-fifths the measured resistance of a circumscribed cylinder whose axis is parallel to the motion, or

$$R=\frac{32}{125}\omega\delta v^2$$

where δ=the density of the air, v=the velocity, and ω=the area of the section. With this we have to combine the equation that expresses the fact that the vertical component of the resistance balances the weight of the pendulous sphere, or $R=w\sec\theta$, where θ is the angle between the vertical and the rod that supports the sphere. Therefore the final equation for velocity becomes

$$v=\sqrt{\frac{w\sec\theta}{\frac{32}{125}\omega\delta}}$$

(*b*) *The pendulous circular plate of Schmidt.*—For this form of pressure anemometer Schmidt computed a table of wind velocities corresponding to different angles of deviation from the vertical by assuming $p\sin\varphi=s\cos^2\varphi$, where p is the weight of the plate, φ the angle of its deviation from the vertical, and s the pressure thereon, when $\varphi=0$, that is, when normal to the wind. As is stated in the theory of the wind vane, section **84**, the pressure of a fluid on an inclined plane is not $s\cos^2\varphi$, but is much more nearly equal to the first power of the cosine of the angle of deviation from the normal than to its square, so that Schmidt's theory and his resulting velocities were appreciably in error.

(*c*) *Wild's pendulous rectangle—Theory of Wild's tablet anemometer.*—This anemometer has been studied experimentally and theoretically by

Dohrandt and Thiesen (Wild, Repertorium, Vols. IV, Nos. 5 and 9), whose results may be presented as follows:

Let e=the distance of the center of pressure from the middle of the plate.

c=distance of the axis from the middle of the plate.

s=distance of the center of gravity from the middle of the plate.

$c-s$=distance of center of gravity from the axis.

G=the weight of the plate.

N=the resultant wind pressure normal to the plate.

φ=the angle of the plate from the perpendicular.

For a condition of equilibrium the moments of the forces about the axis must vanish; whence we have the equation

$$N(c-e)=G(c-s)\sin\varphi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The quantities N and e are dependent upon the velocity V of the wind, the density of the air δ, upon the size and form of the plate, and upon the angle φ, and may be represented by

$$N=\delta\,.\,V^2ab\,.\,F\left(\frac{a}{b},\varphi\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$e=a\,.\,f_1\left(\frac{a}{b},\varphi\right)$$

where a and b are the length and breadth of the plate, and F and f_1 functions depending on the shape of the plate and on the angle φ.

We may place $F\left(\frac{a}{b},\varphi\right)=F(0)\cos\varphi\, f(\varphi)$, where $F(0)$ is the value of F when $\varphi=0$, and must be found by experiment.

From (1) and (2)

$$\delta\,.\,V^2=\frac{G\,.\,(c-s)\sin\varphi}{a\,.\,b\,.\,F(0)\cos\varphi\,.\,f(\varphi)\,.\,\left[c-af_1\left(\frac{a}{b},\varphi\right)\right]}=A^2\tan\varphi\, f_2(\varphi)$$

where

$$A^2=\frac{G(c-s)}{abcF(0)} \text{ and } \frac{1}{f_2(\varphi)}=f(\varphi)\,.\,\left[1-\frac{a}{c}f_1\left(\frac{a}{b},\varphi\right)\right]$$

In order to determine $f_2(\varphi)$ experiments with tablets placed on a whirling-machine were made in 1873 by M. Dohrandt, at the central physical observatory in St. Petersburg, with the result that $f_2(\varphi)$ was found to be constant for the first approximation and equal to unity.

In 1875 these experiments were continued with greater accuracy in order to obtain a more precise determination.

For the case in which b is very large with respect to a, as in the blade of an oar, Thiesen states that he finds that the hydro-dynamic equations developed by Kirchhoff [see also Rayleigh; Phil. Mag., Dec., 1876 make the resultant pressure due to purely fluid friction in non-viscous fluids become

$$N=\delta V^2 a\, b\, F(0) \cos\varphi f(\varphi)=P\frac{(\pi) \cos \varphi}{4+\pi \cos \varphi}$$

and the distance of the center of pressure from the center of the plate is

$$e=af_1\left(\frac{a}{b}, \varphi\right)=a\tfrac{3}{4}\frac{\sin \varphi}{4+\pi \cos \varphi}$$

Dohrandt's observations were then reduced by Dr. Thiesen under the assumption that an equation of the same form as this, with generalized coefficients, will also satisfy the observations in the case of square plates or rectangles of other proportions. Whence

$$\frac{1}{f_2(\varphi)}=\frac{1}{1+m \cos \varphi}\left(1-\frac{a}{c}\cdot\frac{p \sin \varphi}{1+n \cos \varphi}\right)$$

and the normal pressure on the plate is

$$N=P\frac{2 \cos \varphi}{1+m \cos \varphi}\left(1-\frac{a}{c}\,\frac{p \sin \varphi}{1+n \cos \varphi}\right)$$

For the undetermined coefficients Dohrandt's observations gave the values

$$m=1 \qquad n=1 \qquad p=0.31$$

and the resulting formula for the wind velocity for any angle φ of the plate is

$$V=A\frac{1}{\sqrt{\delta}}\sqrt{\tan \varphi}\sqrt{\frac{1+\cos \varphi}{1-\left(\frac{a}{c}\cdot\frac{\sin \varphi}{1+\cos \varphi}\times 0.31\right)}}$$

If the axis is very close to the upper edge of the plate then we may place $c=\frac{a}{2}$, and the equation may be written

$$V=\frac{A}{\sqrt{\delta}}\sqrt{\tan \varphi}\,.\sqrt{\frac{\cos^2 \frac{\varphi}{2}}{1-0.62 \tan \frac{\varphi}{2}}}$$

where

$$A=\sqrt{123.7\frac{G}{ab}}$$

for Wild's anemometer.

These results, obtained from the experiments of Dohrandt, can now be compared directly with similar experiments made by Hutton, Thibault, and others on the pressure upon planes moving obliquely in air. Duchemin summarizes the work done by these experimenters and finds that according to them the normal pressure upon a plane moving obliquely and circularly in a fluid may be well represented by the empirical formula

$$N=P\frac{2\cos\varphi}{1+\cos^2\varphi}$$

where P is the pressure on a plate normal to the direction of motion.

The following table gives the values for each 10° of the three formulæ given by Thiesen, Duchemin, Rayleigh, and Kirchhoff:

Authority.	Formula.	80	70	60	50	40	30	20	0
Thiesen ...	$\left\{\frac{2\cos\phi}{1+\cos\phi}\left(1-\frac{0.62\sin\phi}{1+\cos\phi}\right)\right\}$ ...	.142	.288	.428	.557	.672	.774	.863	1
Duchemin ..	$\left\{\frac{2\cos\phi}{1+\cos^2\phi}\right\}$	.337	.612	.800	.910	.965	.990	......	1
Rayleigh ...	$\left\{\frac{(4+\pi)\cos\phi}{4+\pi\cos\phi}\right\}$	.273	.481	.641	.762	.854	.918	.965	1

These results show wide discordances.

The experiments of Hutton and Thibault, which are fairly represented by Duchemin's formula, give for $\varphi=60°$ nearly twice, and for $\varphi=70°$ over twice the pressure given by the formula of Thiesen. It would seem desirable, therefore, that the results of experiments upon oblique plates by different methods should show a closer agreement before any one of these formulæ can be accepted. The formula of Rayleigh agrees closely, as he shows, with Vince's experiments on plates whirled in water, although it was deduced for the case of rectilinear motion of long blades in non-viscous fluid.

In addition to this disagreement in experimental work the application of Thiesen's formula to Wild's anemometer involves the assumption that the pressure upon a fixed plate in a fluid current is the same as that upon a plate moving circularly with the same velocity in the fluid at rest. The coefficients of Thiesen are derived from experiments in the latter case, but the resulting table is applied to pressures that obtain for the former case. Experiments by Dubuat and Duchemin indicate that for the two cases there is a difference in the amount of the diminution of pressure behind the plate, and the latter gives the following formula for the resistance R' of a fixed oblique plane to a fluid current, R being the resistance to a normal plane (see page 225, and Duchemin, section 84):

$$R'=R\cdot\frac{2\cos\varphi}{1+\cos^2\varphi}\left(1-\frac{\sin\varphi\cos\varphi}{6.48}+\frac{\sin^2\varphi}{3.52}\right)$$

This formula presents a very satisfactory agreement with the observations of Thibault on the normal resistance of a plane surface exposed at different angles of incidence to the pressure of the wind. A maximum pressure is exhibited at an angle of about 45°, in which position the pressure is slightly greater than when the plate is normal to the current. This apparently anomalous result is explained and supported by a consideration of the diminution of pressure behind the plate, thus the experiments on the diminution of pressure in tubes (see Chapter XVI, section 111) made in 1848 at Boston and in 1842 at Philadelphia, show that when the tubes are at angles of 30°, 45°, and 60° to the current, the suction within them is greater than when normal or parallel and is a maximum between 30° and 45°.

[NOTE.—Since writing the above I have obtained the measures published in the Annual Report for 1871 of the London Aeronautical Society, as made by Messrs. Wenham and Browning on the horizontal and vertical components of the resistance of plane plates to a horizontal blast. The blast was from a horizontal pipe 18 inches square, and the plates were so large that the conditions are not those of a small plate in an indefinite fluid; the observations were too few to give very valuable results, and the following extract gives whatever is most nearly comparable with Theisen's work.

A thin rectangular plate, 4.25 inches wide and 18 inches long, was supported at the centers of the long sides so that the axis of rotation and support passing through these centers should be horizontal, while the longer central axis of the plate was inclined θ° to the blast and lay in a vertical plane through the horizontal axis of the air blast, which latter was driven under a pressure of 0.8 inch of water.

Observed pressure in pounds.

Component.	$\theta = 15°$	20°	45°	60°	90°
Vertical	0.8	1.3	1.65	1.3	0.0
Horizontal....	0.24	0.43	1.65	2.1	2.57

The plate was evidently not supported at the center of pressure, since there was always a tendency to lift the front edge.

The location of the center of pressure has been elaborately investigated by E. Kümmer (Memoirs of the Berlin Academy, 1875-'77), who shows that it is always in front of the geometrical center of figure, owing to the manner in which the air flowing around strikes on the back of the plate.—*August*, 1888.]

Thiesen's formula for the action of the tablet anemometer contains the factor A, which depends on $F(0)$, or on the pressure per square unit of area on a plate normal to the wind. For the value of $F(0)$ Thiesen adopts the result obtained by Hagen in experiments with normal plates on a whirling-machine (see page 236)

$$F(0)=\frac{7.07+0.0225(a+b)}{1000}$$

a and b are in centimeters and $F(0)$ is the pressure in grams on a square centimeter for a velocity of 1 meter per second. The second term of this formula, involving the perimeter of the plate, is in part accounted for, as above shown by Duchemin's theory of the centrifugal action developed in circular motion. As above stated, French experiments indicate also that the total resistance to a thin plate in motion in a fluid is less than the pressure on a fixed plate in a fluid current, owing to a difference in the two cases in the amount of the diminution of pressure (Duchemin section 71) behind the plate. Unless these experiments be shown to be untrustworthy, and additional observations prove an identity of pressure in the two cases, it is prudent to apply the corrections already given in this chapter, section 102, to Hagen's results, so as to obtain approximately the coefficients for a fixed pressure plate. It is most desirable that the value of $F(0)$ should be obtained, if practicable, from experiments whose conditions are similar to those obtaining in the actual use of the tablet anemometer. Extreme accuracy in the theory of the instrument is not so necessary at present, for no great precision can be obtained in observation. The continual oscillations of the plate, and the consequent roughness in the estimation of its angular inclination to the vertical, form a source of uncertainty inherent in the tablet anemometer, except when used, like Pickering's, simply as a register of maximum velocities. The accuracy attainable is stated by Professor Wild to be ± 10 per cent. of the wind velocity; but for high winds and consequently large values of φ the positions of the plate vary but little for large changes in velocity, and over this portion of the scale the range of uncertainty must be materially increased.

105. REDUCTION OF THE NORMAL PRESSURE PLATE ANEMOMETER.

(a) The kinetic energy of a unit volume of moving mass is represented by the expression $\frac{1}{2}\rho v^2$, where ρ is the density and v the velocity of the stream. In steady motion of an incompressible liquid mass the increment of pressure due to the loss of velocity $(v-v_1)$ is $\frac{1}{2}\rho v^2-\frac{1}{2}\rho v_1^2$ and can never exceed $\frac{1}{2}\rho v^2$, which value corresponds to a state of rest where all of the kinetic energy has become potential. If the velocity of the stream were destroyed over the whole of the anterior face of a surface exposed normally it would follow that the resistance would amount to $\frac{1}{2}\rho v^2$ for each unit of area exposed. Newton, therefore, in his Principia, 1687, assumed that the resistance is equal to the weight of a column of fluid whose height is equal to that required to produce this velocity in a freely falling body, namely, $h=\frac{v^2}{2g}$; but it is only near the middle of the anterior face that the fluid is approximately at rest or without lateral motion; towards the edge the fluid moves outwards with increasing velocity, and at or just beyond the edge itself retains the full velocity of the stream.

The pressure by the Newtonian theorem $\frac{1}{2}\rho v^2$ is therefore an overestimate; nevertheless the amount of error is frequently small, only amounting to 0.01 for velocity of 20 meters per second, as shown by Recknagel's observations on circular plates. For the case of a blade-shaped surface Kirchhoff's solution of the problem makes the resistance for frictionless blades and fluids per unit of area $\frac{\pi}{4+\pi}\rho v^2$ instead of $\frac{1}{2}\rho v^2$. The distribution of pressure over the front surfaces of plane-plates in a current of water was experimentally determined by Dubuat.

(*b*) When the fluid is compressible there is an increase of density in front of the plate, produced by the crowding together of the molecules of the gas, which must be taken account of in a rigid formula, as has been done in that of Recknagel (see page 233.) Assuming that this change of density is adiabatic, the compression being supposed to take place without loss of heat, the resistance per unit of area is given by the expression

$$p_1 - p = p\left\{\left(1+\frac{\gamma-1}{2}\frac{v^2}{c^2}\right)^{\frac{\gamma}{\gamma-1}}-1\right\}$$

p is the normal elastic pressure in the current; $c=\sqrt{\gamma p \frac{g}{s_0}}$ the velocity of sound in the free fluid; γ, the ratio of the two specific heats, equal to 1.408 for air. When v is small in comparison with c the resistance follows the same law as if the fluid were incompressible.

The distribution of pressure over plane plates exposed to wind in the open air has been studied by Curtis and Burton (see Quart. Journal Met. Soc., London, 1882, Vol. VIII), whose results accord herewith.

Duchemin attempted to take account of the increase of density in front of the plate by adopting the formula $p=\frac{1}{2}\rho v^2$ and assuming for the density

$$\rho=\rho_0\left(1+\frac{v}{\eta}\right)$$

where η is the velocity at which air under the density ρ_0 rushes into a vacuum.

(*c*) The elementary Newtonian formula for pressure

$$p=\frac{1}{2}\rho v^2$$

when expressed numerically in English measures becomes

$$p=\frac{0.0027}{1+.003665t}\frac{P}{P'}v^2$$

in which p is the pressure on a square foot in pounds avoirdupois, v the velocity of the wind in miles per hour, t the temperature in centigrade degrees, $P'=760^{mm}$, P the barometric pressure of the air under consideration. This formula, subject to the small negative correction due to incomplete stoppage of the stream over the whole exposed surface, and subject to the small positive correction due to the compression of the gas in front of the obstacle, represents the pressure on the anterior surface of a plane exposed normally in a very large fluid current, and therefore applies without further correction to Jelinek's improved anemometer, *i. e.*, with the cylinder at the rear to protect the pressure plate.

The above small negative correction, applicable to thin pressure-plates in the open air, is obviated when the plate is the central portion of the bottom of a box-shaped vessel whose sides project in the direction from which the stream is flowing, and are sufficiently extended to produce approximate quiescence over the whole of the bottom. If the end of the box be inclosed, as in Jelinek's arrangement (see section 96), and the central portion of the bottom be used as the pressure plate, the force required to keep it in place must be very exactly equal to $\frac{1}{2}\rho v^2$. This form, therefore, constitutes a very simple and reliable pressure-plate anemometer. (See also Rayleigh, L. E. D. Phil. Mag., 1876, II, p. 430.)

(*d*) When the diminution of pressure behind the obstacle enters as a portion of the measured pressure the total effect is the sum of two terms; but since this second term is of the same order as the first, namely, proportional to ρv^2, and assuming the pressure uniform over the front surface, the total effective pressure may be obtained from the anterior pressure by multiplying by a factor determined by experiment. This factor differs for bodies of different forms, being greatest for thin plates. The table of wind pressures given by Smeaton and Rouse, 1750, apparently assumed the factor to be 1.8. Duchemin has given the following table, representing the value of this factor obtained by different experimenters for bodies of various shapes and proportions, as indicated by the ratio $\frac{b}{c}$ or the ratio of the length of the body to the diameter of the exposed cross-section, where the length is measured in the direction of the current:

Ratio $\frac{b}{c}$	0.00	0.50	1.00	1.50	2.00	2.50	3.00	5.00
Coefficients ...	1.86	1.85	1.48	1.39	1.34	1.32	1.33	1.36

This table exhibits a minimum ratio and pressure for $\frac{b}{c}=2\frac{1}{2}$, for which fact no very satisfactory explanation presents itself. Evidently these factors are a complex result of viscous, skin, and fluid resistances, and the results are in need of further experimental confirmation.

[This subject is greatly elucidated in the memoirs of Saint Venant, Paris, 1887, which I have received through M. Boussinesq while reading these proof-sheets.—August, 1888].

(*e*) For thin normal plates the factor 1.86 is in very satisfactory accord with the formula given by Smeaton, with the experiments of Mariotte, Thibault, and Paris in air, and with those of Dubuat in water. It differs from the results of Woltmann's observations, to which Langsdorff attaches much weight, and is open to further investigation. Since the diminution of pressure behind the plate is due, not directly to viscosity or to skin friction but to fluid friction, as above defined, being maintained by eddies and whirls at the surfaces of discontinuity of the continuous stream lines, therefore this coefficient should depend to some extent upon the form and size of the plate and the velocity of the fluid.

The amount of fluid to be carried away from behind the plate each second, in order to maintain there a static pressure less than the normal elastic pressure within the undisturbed current, increases with the area of the plate, while the bounding surface of discontinuity through which the fluid must pass, and where the eddies are formed, increases with the perimeter; therefore the diminution of pressure per unit area of the plate will be greater the greater the ratio of the perimeter to the area.

Consequently the above factor, giving total effective pressure, will diminish as the surface increases, and for the same area will be least for a circular plate and greatest for a long slender rod, assuming that the velocities are such as to produce a preponderance of fluid over viscous frictions. Within what limits or to what extent such a variation takes place needs to be determined by additional experiments. Thibault's observations on square plates 33 and 48 centimeters on a side give a mean coefficient 1.86, but his groups of observations show internal variations from this of over 10 per cent. in the measured resistances, discrepancies too large to allow his data to be used in detecting the influence of the size or shape of the plate. Hagen's experiments give a large positive coefficient similar to what would be directly produced by viscosity proper, but his figures are diminished somewhat by the correction for Duchemin's theoretical term depending on the radius of the whirling-machine. This term has its origin in the fact that the air impinging on the whirling-plate is forced outwardly by centrifugal action, and the energy thus dissipated has to be made up by increasing the driving force or the apparent resistance.

The viscosity term introduced by Hagen (section 102) is in accord with his previous works on the steady flow of liquids in tubes and channels, which show that his coefficient should increase with the ratio of the perimeter to the area; but the fluid friction also increases when sharp angles, such as those of squares and triangles, introduce discontinuous motions. Therefore Hagen's coefficients for the resistance of the air, even when reduced by Duchemin's correction for centrifugal action, may still be too large, as they are derived from experiments with squares, rectangles, and triangles, and must therefore include at least a part of that effect of discontinuity, to which I restrict the expression "fluid friction."

106. REDUCTION OF THE TUBULAR PRESSURE ANEMOMETERS.

The theory of the action of the wind on this class of anemometers is the same as that of the action of water on the Pitot tube, as applied by that engineer in 1760 in measuring the velocity of streams. For a straight-mouthed tube with sharp clean edges facing up stream $h=\frac{v^2}{2g}$, where h is the height of water in the tube above the surface level of the stream and v the velocity of the stream at the center of the mouth of tube. For funnel-shaped mouths Pitot found the height h to be $1\frac{1}{2}$ times greater than the value just given. In this latter case, however, we can not be certain that the velocity in front of the orifice is the same as that of the unobstructed stream, so that this modification introduces an element of uncertainty into a remarkably precise instrument. For a straight-mouthed tube with short edges, directed towards a current of air, the increase of pressure within is $\frac{1}{2}\rho v^2$. This is the same expression as above found for the wind pressure per unit area on the bottom of a box open to the wind, which is a parallel case. If the pressure within the tube is conveyed to the surface of a liquid it will support a liquid column whose height furnished the measure of the pressure per unit of area; the equation

$$p=\frac{1}{2}\rho v^2$$

will give the changes of the barometer due to the whole pressure of the wind blowing into a Pitot tube if p be expressed in terms of the height of a mercurial column instead of pounds per square foot, and this is effected as follows: Putting b for the barometric pressure in inches of mercury corresponding to p we have

$$b=\frac{0.00003827v^2}{1+.003665t}\frac{P}{P'}$$

in which $P'=760^{mm}$ and P is the barometric pressure prevailing in the free air; t is the temperature in centigrade degrees and v the velocity of the wind in miles per hour. With a velocity of 50 miles per hour, temperature 0° C., and pressure 760^{mm}; $b=.0957$ inch. When, instead of the mercury, the wind supports a column of water the equation becomes for an atmospheric temperature of 0° C. and pressure of 760^{mm}

$$v=\frac{h}{0.0228}$$

h being the height of the water column in inches. The best form of tubular pressure anemometer is that in which this equation is utilized to obtain the velocity of the wind from the difference between the readings of an aneroid inclosed in a tube whose orifice is face to the wind, and one so exposed as to give the elastic pressure in the moving air free from all dynamic effects. This latter may be attained by inclosing an aneroid in a tube, whose mouth opens flush with a large flat surface, across which the wind blows in continuous parallel straight stream lines. Fig. 55 shows such a tube, T, and surface, a, surmounted by a second parallel flat surface, a'. In order to accomplish a perfect result the surfaces should be parallel to the wind, but if fixed horizontally they will give erroneous results only when the wind is inclined considerably to the horizon.

This method of connection of the barometer with the outer air, so as to obtain the elastic pressure uninfluenced by the dynamic effect of the wind, combined with a similar barometer at the bottom of a Pitot tube, constitutes the improvement devised by Mr. Curtis upon the anemo-barometer, described by me in the Annual Report of the Chief Signal Officer for 1882, page 99. Apparently this gives an unexceptional complete solution of the problem to determine the air pressure, the wind pressure, and the wind velocity, and avoids the complicated and uncertain relations introduced by Hagemann's use of the Magius tube, as well as those attending the measurement of pressure in the rear of any obstacle.

107. REDUCTION OF THE BRIDLED ANEMOMETERS.

These anemometers have thus far been used only to give relative indications of the force of the wind, and the conversion of their arbitrary scales into wind velocity based upon a precise hydro-dynamic theory of their action has not yet been attempted. On such bridled windmill sails Smeaton made his studies of the best form of sails and the resistances in windmill construction, but his results are not sufficiently accurate for use in anemometry. The first condition to be satisfied by this anemometer, in order to get correct absolute or even relative indications of wind force, is that the radial arms or cups shall be sufficient in number and so placed as in any position to present the same pressure surface to the wind.

In the four cups of Robinson's anemometer this condition is not even approximately fulfilled, so that a given increment in wind velocity will act with a different force upon the instrument, according to the relative positions of the cups. The five cups used by Professor Stokes come nearer to satisfying this condition, and for the six curved radial arms of Gärthner's anemometer the pressure surface is very nearly constant in all positions of exposure. For any form of bridled anemometer special experiments are needed to determine the accuracy with which this condition is realized. When the number of vanes needed to satisfy any given limit of precision in this respect has been determined on, the formula for its action will be similar to that of the pressure plate, namely

$$v^2 = K_0 + KP\frac{1}{\rho}$$

Here K_0 and K are instrumental constants to be determined by comparison with a normal or standard anemometer of any kind, ρ is the density of the air, and P the observed pressure indicated by the counterbalancing weights or springs.

The bridled anemometer with vertical axis is exposed to the wind from all directions, and therefore need not be attached to a wind vane. It thus has a great advantage over the pressure plate, and if its indications can be evaluated with as much precision as those of the plate, is to be highly recommended as a pressure anemometer.

108. MEASUREMENT OF GUSTS.

In the preceding portion of this chapter the pressure anemometers have been supposed to be exposed only to a steady wind that brings the instruments to rest in equilibrium with its steady pressure. The ordinary intermixture of temporary gusts makes it necessary to properly interpret the readings of the instruments for such cases. A gust produces a momentary impact upon the anemometer and ceases to act long before the instrument has fully attained the condition that indicates the maximum force of the gust. The wind's action is therefore that of an impulsive force.

For the pendulum anemometer the problem is similar to that of the ballistic pendulum. The resultant of the gust passes through the center of pressure of the surface exposed to the wind, and in the ideal pendulum anemometer is assumed to pass also through the center of percussion with reference to the axis of suspension which thus becomes an axis of spontaneous rotation. For a pendulum thus constituted none of the impulsive force of a horizontal gust will be lost in producing strain on the axis of suspension. The initial angular velocity about the axis of suspension will be the sum of the moments of the impressed forces divided by the moment of inertia with reference to this axis. The impressed force is in general some fraction μ of the momentum of the air that strikes the pendulum. Let m and v be the mass and average velocity of the striking air, V_1 the resulting initial velocity of the pendulum, M the mass of the pendulum, l the lever arm of the center of inertia, K the lever arm of the center of gyration, then will $\mu mvl = MV_1K$.

(*a*) If the mass of the pendulum is considerable in comparison with the area that it exposes to the resistance of the air, then μ will be small and V_1 will be small, and the arc of vibration described before coming to rest may be approximately determined without considering the resistance of the air or the friction of the axis of suspension. In this case the pendulum moves from the vertical to an inclination, ψ_1, and is temporarily brought to its first rest there by the work done against gravity; therefore the ordinary ballistic formula gives the maximum velocity V_1 for $\psi=0$ in terms of the maximum value ψ_1 of the arc of vibration, as

$$V_1=\sqrt{\frac{4g}{l_1}}\sin\frac{1}{2}\psi_1$$

If the time T of vibration of the pendulum has been observed for small arcs of observation then we may substitute

$$\frac{\pi}{T}=\sqrt{\frac{g}{l}}$$

whence

$$V_1=2\frac{\pi}{T}\sin\frac{1}{2}\psi_1$$

by which the initial velocity of the center of inertia of the pendulum plate or sphere may be computed from the observed ψ_1.

(*b*) If the pendulum is very light in comparison with the surface exposed to the wind μ will be nearly unity and its initial velocity will be more nearly that of the incident wind, but the angle ψ_1, by which it is driven from the vertical, will depend upon the action of the resistance of the air behind the plate and the friction of the axis as well as on gravity. Assuming the moment of resistance of the air that now resists the plate to be lR, which must approximately equal l_1Kv^2, and the moment of friction to be l_2f, and the moment of inertia to be Ml^2, then will the solution of the following equation for each special form of pendulum plate or sphere give the relation between the limiting angle ψ_1 at which the pendulum comes to rest and its maximum velocity V_1:

$$Ml^2\frac{d^2\psi}{dt^2}=Mlg\sin\psi+l_1K\frac{d\psi^2}{dt^2}+l_2f\frac{d\psi}{dt}$$

(c) Having thus found the initial maximum velocity V_1 of the pendulum, the next step in the problem is to derive therefrom the maximum velocity of the air particles constituting the gusts. A gust may be considered as a rapidly advancing wave of compression, whose elementary masses communicate their own velocity to the particles in front and then subside to rest; or at other times it consists of a vortical motion, the front surface of the vortex being the front of the gust. In either case the pressure plate receives a pressure increasing very rapidly up to a maximum, and then as rapidly followed by a diminution.

The forward component of the velocity of any particles within the advancing thin shell of the gust of air may be expressed by the formula

$$V=V_0\sin m\left(\frac{x}{n}\cdot\theta\right)$$

where x expresses the distance of the particle from the front edge, m and n are constant coefficients, and θ is a function of the time. The introduction of this variable force into the equations of motion for the pendulum shows that the maximum velocity of the plate will be about $\frac{a}{2}\cdot V_0$, where a depends on the relative density and dimensions of the plate, and V_0 is the maximum velocity of the air particles.

CHAPTER XVI.

SUCTION ANEMOMETERS.

109. GENERAL PRINCIPLE.

A second general class of anemometers is that in which the object of measurement is the so-called suction effect due to the diminution of pressure produced by the wind in the neighborhood of some obstacle. When a fluid flows past an obstacle the deviations of the stream lines are accompanied by variations in the pressure and by regions of so-called discontinuity in which in gases the pressure is especially low, and in liquids becomes zero. If the air flows into such a region it is said to be sucked in, although the true impelling force is the excess of pressure on the outside.

Suction anemometers may be divided into classes, according to the nature of the obstacle that opposes the flow of the wind. Of these two classes only need be considered. First, the wind blows through a horizontal tube, and the obstacle consists of a contraction or diaphragm within the tube. Second, the obstacle consists of a straight vertical tube, open at the top, erected in the free air, so that the wind blows freely against and around the tube and across the top.

The case in which the obstacle is a sphere or other surface, at several points of whose surface measurements of the relative suction and pressure effects can be made, has been worked out both experimentally and theoretically by Sir William Thomson (Math. and Phys. Papers, Vol. I, Parts III and IV), and by Recknagel (Wied. Annalen, 1880, X, p. 677), but its application to anemometry, as suggested by me a few years since, has no material advantage over the two preceding simpler cases.

Although the suction anemometers have not as yet been very widely adopted by meteorologists, yet it will be seen that some forms are in no respect inferior in accuracy and convenience to the pressure and the rotation instruments. The formulæ for the computation of wind velocity in all three classes involve one or more coefficients that must be determined by experiment, and the formulæ resulting from the mechanical theory of the flow of fluids are simpler for this class of anemometers than for either of the other two.

110. HORIZONTAL TUBES.

Wind blowing through a smooth horizontal tube of uniform bore is under the same elastic pressure within as without the tube, if we neglect the very slight effects of skin friction and viscosity, and if the walls of the tube be so thin that we may neglect the disturbance produced by the impact of the wind on the anterior edge of the tube. Therefore the elastic pressure against the walls of the tube is everywhere the same, inside and outside. If, however, the tube be contracted and again expanded somewhere in the course of its length there will be a corresponding variation in the velocity of the flow of liquid through it, and a corresponding change in the density of the fluid and in the pressure on the interior wall of the tube, the pressure diminishing as the square of the velocity increases.

(*a*) The first current meter based on this principle is that of Overduyn, in 1854. He adopted the contracted tube known as Venturi's tube, which consists of a small frustrum of a cone, through which the current flows towards the smaller section, where it enters a much larger conical tube. The shape and proportions of Venturi's tube are as shown in Fig. 56, where

$$AB=18 \qquad CD=15.5 \qquad EF=23 \qquad GH=11 \qquad HI=78$$

These are the proportions that were found by Venturi to give the maximum discharge when his tube is applied to an orifice. Equivalent curved surfaces may be substituted for these cones. At the contracted section *CD*, where the fluid would have the greatest velocity and the pressure on the walls of the tube would be the least, Overduyn fitted a small tube leading to a hermetically sealed case containing an aneroid barometer, which latter indicated therefore a pressure the same as that against the section *CD*. A similar arrangement for the free portion of the stream gives the full pressure such as it must exist at the section *AB* or *EF*, and the difference between the two aneroids gives the basis for computing the velocity of the liquid.

The theory of the flow of liquids, as applied to suction anemometers, is deduced from the general methods of hydrodynamics.*

For the present we need only recall the following general relations. For any two sections such as *CD* and *EF* of a stream in steady flow the conditions of continuity as to mass give

$$\rho_1 \omega_1 V_1 = \rho_2 \omega_2 V_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The condition of adiabatic flow, *i. e.*, continuity as to heat, give

$$\frac{\rho_1}{\rho_2} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The Eulerian equations of motion for incompressible non-viscous fluid give

$$\frac{p_1}{\rho} + \tfrac{1}{2} V_1^2 = C \qquad \frac{p_2}{\rho} + \tfrac{1}{2} V_2^2 = C \quad . \quad . \quad . \quad . \quad (3)$$

* See Besant, Hydromechanics; Lamb, Motions of Fluids; Unwin, Hydromechanics; a special memoir by G. E. Curtis is also promised.

The corresponding equations for adiabatic flow of non-viscous gas give

$$\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1}+\tfrac{1}{2}V_1^2=C' \qquad \frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2}+\tfrac{1}{2}V_2^2=C' \qquad (4)$$

Elimination between (1) and (3) gives V_2 for water, and elimination between (1), (2), and (4) gives V_2 for air. If the density is expressed as the weight of a unit volume we have $\rho_2=G$, and if the acceleration p_2 is expressed as a barometric pressure we have $p_2=p_a g$. The result of this elimination and substitution shows that the velocity at EF or AB of the stream V_0 is computed by the following formulæ:

(1) For water non-viscous, incompressible, and flowing steadily without discontinuity.

$$V_0^2=\frac{2g}{G}\frac{p_a-p_1}{\frac{\omega_2^2}{\omega_1^2}-1}$$

(2) For air the expression for the velocity must be developed from the principles of the adiabatic flow of gases, from which for non-viscous, steady, non-discontinuous flow there results the equation

$$\left[\left(\frac{\omega_2}{\omega_1}\right)^2\left(\frac{p_a}{p_1}\right)^{\frac{2}{\gamma}}-1\right]\frac{V_0^2}{2g}=\frac{\gamma}{\gamma-1}\cdot\frac{p_a}{G}\left[1-\left(\frac{p_1}{p_a}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

where ω_1 and ω_2 are the areas of the sections CD and EF, p_1 and p_a are the pressures at the sections CD and EF, g is the acceleration of gravity, G is the weight of the unit volume of liquid, and $\gamma=1.408$ is the ratio of specific heats under constant pressure and constant volume.

The difference between the equations for compressible and incompressible fluid is small, amounting to between 3 and 4 per cent. for a diminution of barometric pressure of one-half inch. The simpler formula for incompressible fluids is therefore sufficiently accurate for ordinary winds.

(*b*) Professor Overduyn showed that by placing a corresponding small Venturi tube, as in Fig. 57, so that its mouth $A'B'$ would lie in the plane of the section CD, there would be an exaggeration of the suction effect, and the pressures measured at the sections $C'D'$ could be utilized for obtaining the velocity of the stream. This idea has been experimentally realized by E. Bourdon (see Paris, Comptes-Rendu, 1882, tome 94, p. 229), who, however, altered Overduyn's arrangement (see Fig. 58) by placing the mouth $A''B''$ in the plane of AB and the opening $E''F''$ in the plane of CD.

Bourdon found that the additional tubes introduced into the larger one act as obstructions to diminish the full effect. Nevertheless the suction effect at $C''D''$ was to that at CD as 20 to 6.

(*c*) The anemometer proposed by Arson (Assoc. Francais, Nantes, 1875), is on the same principle as the Venturi tube (see Fig. 59), and consists of a cylinder whose bore is constricted as shown in the figure, from L to M, where it suddenly widens to its original dimensions.

Arson makes the area of the contracted section at M equal to $\frac{1}{\sqrt{2}}$ of the area at L, whence he deduces the following formula on the assumption that there is no discontinuity in the flow of liquid:

$$V=46.9\sqrt{h\cdot\frac{P}{29.92}\cdot\frac{519}{459^\circ+t}}$$

where V is the velocity in feet per second, h is the observed deficiency of pressure at the contracted section MN, as measured by the height of a column of water raised by the suction in a U-tube, one of whose ends is open to the air while the other is directly connected with the inner surface of the contracted tube, P is the atmospheric pressure in inches of mercury, and t is the temperature of the air in Fahrenheit degrees.

If, however, discontinuity exists, and in general it must be so, then, as shown by Curtis, the computed velocities are too small, and need to be increased by the factor 1.082.

It would seem, therefore, that it is best to use a gradual enlargement from M towards N rather than the sudden enlargement adopted by Arson, or, in other words, to adhere to the Venturi tube.

In the use of the horizontal tubes the tube must be attached to a wind vane in order that it may be parallel to the wind direction. It is therefore subject to extensive oscillations with the vane, and any want of parallelism to the wind introduces an error in its indications, as in a similar case of pressure plates. This difficulty is obviated in the vertical tube described in the following section.

III. VERTICAL TUBES.

A vertical tube exposed to the wind constitutes an obstacle. At the sharp horizontal edge of the upper end of the tube the wind deflected by the surface immediately below bounds over and leaves a small region of discontinuity; into this the air flows from some surrounding point, but a definite deficiency of pressure still prevails in this region.

If the tube be opened at the lower end a free flow of air takes place through it upwards into this discontinuous space and is immediately carried thence by the wind. This is the ordinary phenomenon of the suction of a chimney, the laws of which were experimentally investigated for many forms of cowls by a committee of the American Academy of Arts and Sciences, Boston, 1847. It will be perceived that the phenomenon is not one of "suction due to the viscosity of the air drawing the air from the vertical tube," but is one of fluid friction proper, as defined in the section on the theory of the wind vane. If the lower end of the tube be closed the pressure within the tube becomes constant and the same as in the discontinuous space above it, except for the slight periodic variations attending the sonorous waves that travel up and down the tube. The law of diminution of pressure in this case was experimentally investigated by Ewbank and Mott, Philadelphia, 1842, and by Magius, of Copenhagen (see Hagemann in Met. Aarbok, 1876; translation in Journal Franklin Institute, 1887).

(*a*) The vertical suction tube was first applied to anemometry by Fletcher, in 1867. It was reinvented by Hagemann, Copenhagen, 1876, and has been used to some extent in Denmark, Norway, Sweden, and Russia, as also on the ocean. The original Fletcher-Hagemann anemometer consists essentially of a vertical tube, whose upper end is open and beveled, the plane of the section being perfectly horizontal; the lower end connects with a **U**-tube manometer or other form of measuring pressures. A modification of this, suggested both by Fletcher and Hagemann, consists in directly combining the vertical and **U**-tube, and is a modified Lind anemometer, in which the wind presses into one branch of the **U**, but produces a suction in the other branch, so that both effects combine to raise the column of water in one leg and depress it in the other. This is the form now adopted in the portable Hagemann anemometer used in Denmark. But as all such arrangements require to be oriented by attachment to a wind vane they lose the advantage of the simplicity of the single fixed vertical tube.

The pressure within the vertical tube is that within the discontinuous space at its top, and is a measure of the velocity of the wind; but the exact relation between the two can not be stated satisfactorily from a deductive point of view, owing to the mathematical difficulties in the problem. It can be seen, however, that the relation is, approximately, $P_a - P_1 = K(V_o)^2$, where the factor K must be determined by experiment for the special form of end or nozzle that is adopted for the upper end of the tube. The value of K was assumed equal to $\frac{1}{2}g$ by Fletcher; Hagemann and Magius came to the same conclusion from their experiments with the beveled nozzles. The velocities in their experiments were, however, quite moderate, and further investigations have been promised.

The simplicity of the vertical suction-tube anemometer makes it peculiarly adapted for transportation and use at sea; the trouble caused by the stoppage of the aperture by frost-work and snow can be partly obviated by bending the top of the tube, causing the mouth to open downward instead of upward, and by occasionally forcing a current of warm air up and out of the tube, whereby to melt the snow and frost.

If the section of the mouth or the motion of the wind be not horizontal a source of error is introduced. The results given by two tubes opening upward and downward, respectively, afford data for approximating to the effect of an inclination of the wind or the tube.

As stated in the section 109, tubes ending in the surface of a sphere have been experimented with, but none of these forms excel in simplicity and directness the combination of barometer and pressure anemometer described in sections 66 and 106.

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CHAPTER XVII.

ROTATION ANEMOMETERS.

112. GENERAL CONSIDERATIONS.

The third general class of anemometers is that in which the wind sets in motion a system susceptible of rotation about an axis, and the velocity of its rotation is taken as the object of measurement. Instruments of this kind are often and properly termed "velocity" anemometers, since we observe the velocity of rotation of a movable mass of matter, but in the use of this term its application is often improperly transferred from the velocity of rotation to the velocity of the wind, and the erroneous impression is fostered that velocity anemometers are able to give the wind's velocity directly, while other instruments furnish other data. It is now known, however, that the relation between the velocity of rotation of the instrument and the linear velocity of the wind is, in general, far from being a simple one. The preceding chapters have shown that for pressure and suction anemometers the relation of their indications to the wind velocity is, in some cases, fairly well established. This will also be found to be approximately true of rotation anemometers; therefore all anemometers may be considered as velocity anemometers in the sense that they lead to a knowledge of the velocity of the wind by measuring some of its effects, some by means of resistance, some by suction, and those of the present class by their velocity of rotation. An important difference, however, in the action of the rotation anemometer, as distinguished from the other classes lies in the fact that it gives a continuous integral result instead of a momentary differential one, and its general acceptability results from its character as an integrating machine, giving total movements and average velocities.

113. EARLY FORMS OF ROTATION ANEMOMETERS.

(*a*) The first rotation anemometer of which we have record is described by d'Ons-en-Bray, in 1734. In this form the windmill with vertical axis (moulin à la Polonaise), which formed the outer exposed portion of the bridled anemometers of Gärthner and Leutmann (see Fig. 53), was allowed by d'Ons-en-Bray to rotate continuously, and to record by a self-registering attachment. For every 400 revolutions of the windmill a hammer was let fall, thereby making a perforation in a recording sheet moved by clock-work. The quantity of gearing by which this was accomplished rendered its records of doubtful value, but the essential idea aimed at in this anemometer is the same as that in Robinson's anemometer, brought out more than one hundred years afterward.

Observations to determine the relation between the velocity of the wind and the record of this anemometer were promised by d'Ons-en-Bray, but never carried out, so far as I can discover.

(*b*) The next rotation anemometer was described by Lomonosoff in 1751. It consisted of a wheel (see Fig. 60) something like an overshot water-wheel, whose horizontal axis was kept perpendicular to the wind by being attached to a wind vane. The wind vane, being made in the form of a narrow box, served also the purpose of covering the lower half of the wheel, while the wind, continuously striking the upper half, set it in rotation. On the axle is a pinion that works a spur-wheel, on whose axis another pinion works another wheel, and so, by means of a thread passing down the hollow axis of the vane, transmits the motion to a wheel below. The velocity of this lower wheel is the object of measurement; Lomonosoff determined the relation between its rotations and the wind velocity by observing the latter directly by means of feathers.

114. SCHOBER-WOLTMANN WIND METER.

(*a*) With the exception of Lomonosoff's instrument, which never received further development, the progress of rotation anemometers during the century following d'Ons-en-Bray lay in the application of small wind-mill sails, and under this head come the various patterns of current meters used for measuring the velocity of natural or artificial currents of all kinds. In 1752 Schober described the first anemometer of this class of which we have record. It was an adaptation of Wolf's bridled anemometer, and consisted of four thin rectangular brass sails, 1.25 by 2.5 inches in size, placed at the ends of two light cross arms. The shorter dimensions of the rectangles lay in the direction of the arms; the distance from the axis to the middle of the sails was 4 inches. To determine the angle at which the plane of the sails should be set the anemometer was placed on a whirling-machine, and with the sails placed at different angles, was whirled with a linear velocity of 25 feet per second.

When the sails were set at an angle of 50° with their axis of rotation the velocity of the middle of each sail in its rotation on its axis was 23.7 feet per second, and when set at 60° was 30.5. By interpolation 52° was taken as the angle which the sails should make with their axis in order that their middle points should rotate with the same velocity as the wind. For recording the velocity of the sails a bell was so attached to the axle of the anemometer as to be struck at every sixth rotation, thus permitting the number of rotations in a given time to be counted, and thereby the velocity of the wind to be determined.

In 1783 Edgeworth made an anemometer of a similar type, consisting of four light windmill sails delicately mounted on a horizontal axis, in the arbor of which was cut an endless screw that recorded its revolutions by means of the well-known elegant device now called the cotton counter and adopted in the anemometers manufactured by J. & H. J. Green, of New York.

No extended application appears to have been made of this form of rotation anemometers until 1790, when Woltmann published his "Theorie und Gebrauch des hydrometrischen Flügels," wherein he showed the value of Schober's anemometer as a meter for both air and water currents, he having spent the year 1786 in experiments for its improvement. The resulting instrument is shown in the accompanying diagram (Fig. 61). The radial arms of Woltmann's Flügel Anemometer were of polished steel, and their length from the axis to the middle of the little sails was 19.1 inches. The sails or "Flügel" were of polished hard wood, 2½ by 5 inches in size, and set at an angle with their axis of rotation of 48½°. On the axis, which was 14 inches long, was an endless screw, into which fitted a toothed wheel containing 100 teeth. The following explanation of the action of this anemometer is due to Woltmann.

Let a plane, $CBLM$ (Fig. 62), be inclined to the direction of the current PQ, in which it is immersed, and be so constrained as to be movable only in a plane perpendicular to the current, *i. e.*, parallel to BA; let the plane make an angle, α, with the current. The pressure of the current will set the plane in motion. At first the current will strike the plane with its whole force, but as the plane begins to move and increases its velocity the pressure of the current diminishes and vanishes, when the plane moves with such a velocity that a particle of air is not deviated by the plate, that is, when a point of the plane moves from B to A while the wind moves from C to A. Then, if V be the velocity of the wind and v the linear velocity of the plane, we have

$$V : v = CA : AB$$

$$V = v \cdot \frac{CA}{AB} = v \cot \alpha$$

For circular motion, as in windmill sails, different points of the plane move with different linear velocities, so that v, the velocity of the plane, refers to some line within the plane, and for $\alpha = 45°$ the equation states that the velocity of the wind is just equal to the velocity of this line. Woltmann states that this line is a little outside of the center of the plane reckoned from the axis, but that for small planes without sensible error we may assume it to pass through the center. The portion of the sail inside of this line moves slower than the wind, and so the wind tends to accelerate its rotation, while the portion of the sail outside of the line moves faster than the wind, which latter therefore acts to retard the rotation. The arms offer slight resistances to the motion of the sails that must be taken account of, whereby their velocity is less than that given by the above equation. This resistance is proportional to the square of the velocity, and in like manner the force which overcomes the resistance increases in the same ratio. Therefore the retardation always remains a constant part of the actual velocity whether it be great or small, and the corrected equation becomes

$$V=\left(\cot\alpha+\frac{1}{m}\right)v=v\cot(\alpha-\beta)$$

in which $\frac{1}{m}$ is the diminution of the velocity due to the resistance of the arms.

By experiments in water Woltmann found that in order to have $(\alpha-\beta)=45^\circ$ he must make $\alpha=48\frac{1}{2}^\circ$. As this correction depends on the size and form of the instrument, and is independent of the density of the medium, the results of the experiments in water hold good for motion in air, consequently the angle $48\frac{1}{2}^\circ$ was adopted by Woltmann for his anemometer.

The effect of friction must yet be introduced into the equation of the instrument. Assuming that the apparatus has perfect symmetry, so that its center of gravity and pressure lie in the axis of rotation, then the fundamental condition for the correction for friction is that the total energy of the wind expended on the instrument must equal the energy produced in the moving parts plus the amount used up by friction. The condition is expressed by the equation $V^2=v^2+F$. The friction term F is really composed of two parts: First, a constant term which represents the friction due to the weight of the apparatus and the friction of the gearing, its amount being determined by the condition that its moment is equal to the moment of the wind pressure on the sails when they just begin to start; second, a term due to the hitherto neglected component of the wind pressure parallel to the axis of rotation and producing a friction at the bearing of the end of the axis, which is also proportional to V^2. Woltmann took account only of the first of these friction terms and found for his special instrument

$$V^2=v^2+(2.11)^2$$

in which V and v are given in feet per second.

The second friction term, being proportional to V^2, may be combined with that term in the equation. The general equation of the air meter may therefore be expressed in the form $V^2=a^2n^2+V_0^2$ where V_0 is the wind velocity at which the instrument just begins to turn, n the number of rotations in a unit of time, and a a factor, constant for each instrument, to be determined by experiment. In this form the equation holds true, whatever the angle at which the sails may be set, and in the experimental determination of a and V_0 the friction and resistances of the apparatus are allowed for. The term V_0^2 varies with the delicacy of construction and the square of the density of the medium in which the instrument is submerged; for water the term becomes insensible, and the velocity of the stream becomes directly proportional to the velocity of the instrument. For observations of wind velocity and considering the changes of air density the equation becomes

$$V^2=a^2n^2+\frac{\rho_0 V_0^2}{\rho}$$

in which V_0 is the wind velocity at which the instrument begins to turn, as determined under a standard air density ρ_0, and ρ is the density of the air at the time of any observation expressed in terms of the standard density ρ_0.

(*b*) Kallstenius in 1820 made an instrument of this type with twelve sails instead of four, and with their planes set at an angle of 37° to their axis of rotation. He omitted the question of the friction of the apparatus.

(*c*) In 1837 Combes, in France, and Whewell, in England, brought out modifications of Woltmann's instrument, which have had an extensive use. Combes's anemometer was designed as a delicate current meter for use in the ventilation of buildings and mines, and for any purposes of physics, engineering, or the arts, while Whewell's anemometer was intended to be a meteorological instrument.

Combes's anemometer had four square sails, 0.0225 meter on a side. The distance from the axis to the center of the sails was 0.02625 meter; the angle between the sails and the axis of rotation was about 61°. By assuming the gyratory force upon the sails proportional to the square of the angle between the current and the plane of the sails, Combes deduced an equation of the form $V=an+b$ where a and b are constants and n the number of turns of the instrument in a unit of time. As already stated, in considering the wind vane, section 84, the pressure on an oblique surface is more nearly proportional to the sine of the angle than to its square; consequently the form of the equation given by Combes would seem to be subject to the error of this assumption and inferior to the equation

$$V^2=a^2n^2+\frac{\rho_0 V_0^2}{\rho}$$

given above. The equation given by Combes is, however, simpler in form and has been very largely used in graduating air meters. Indeed for large velocities it can lead to no appreciable error.

(*d*) The conditions of accuracy of the Woltmann anemometer in the various forms manufactured by French, German, and English makers (which are all slight modifications of the general pattern adopted by Combes) have been several times investigated, but perhaps most thoroughly by Recknagel (see Wiedemann Annalen, 1878, IV, p. 149).

The results of the latter confirm the accuracy, at least for small velocities, of the linear formula given by Combes, which is

$$V=a+bn$$

where

n=the number of rotations of the axis per second.

$b=2\pi\rho \tan \varphi$.

φ=the angle between the wind and the normal to the vanes.

a=the friction constant or least velocity at which the vanes just begin to turn.

According to Combes's theory

$$a=\sqrt{\frac{2Krg}{\delta f\rho \sin \varphi \cos^2 \varphi}}$$

Kr=the moment of friction of the axis in kilogram-meters.

f=the total area of the vanes in square meters.

$\frac{\delta}{g}$=mass of a cubic meter of air.

ρ=the radius of the circle described by the center of the vane in meters.

Recknagel finds that for linear velocities of between one-third and 4 meters per second, which he obtains by moving the anemometer along a small railway, the above formula well represents the observations, and that it also equally well applies to circular motions obtained by whirling the anemometer in circles of 1 and 2 meters radius. For different values of the angle φ he determines the sensitiveness or least velocity a that will start the vanes into rotation, and the coefficient b for velocities up to 12 meters per second. The following table gives the values of a and b for the different angles of inclination:

ϕ	a	b	Range of experimental velocities.
	Meters per second.		*Meters per second.*
° ′			
11 00	0.40	(?)	....
20 00	0.30	(?)	....
25 40	0.25	0.04764	0.2 to 4
32 50	0.18	.06222	0.2 to 5
46 30	0.16	.09892	0.2 to 6
60 30	0.16	.15784	0.2 to 12
62 30	0.175	.18576	0.2 to 12
75 0	0.20?	....	....

From these results he concludes that the sensitiveness is a maximum when the inclination is >25° and <62°, and that the linear feature of the equation for v holds good throughout the whole range of velocities above given, but that for higher velocities these values of a and b give velocities too large by 1 per cent. or more.

The preceding figures hold good for winds of uniform velocity. When Recknagel varied the velocity with rapid regularity the computed velocities were all a little higher than the true average, evidently due to the greater inertia of the moving mass when forced to whirl rapidly. When the wind struck his experimental anemometer from behind the recorded velocities were 1 or 2 per cent. smaller than when it struck the front. The formula deduced for the anemometer whirled at low velocities upon a whirling machine gave results sensibly correct when the instrument was moved rectilinearly through still air at low velocities.

The constants a and b deduced from these experiments agree very closely with those computed by Combes's formula for apparatus having the dimensions employed by Recknagel, who used four vanes, which were plates of mica about five-eighths inch long by three-eighths broad, and whose centers described circles of about three-fourths inch radius.

Recknagel concludes by recommending the adoption of the angle of inclination $\varphi=60°$, for which he estimates that anemometers so constructed will probably not be in error by more than +0.03 meter for velocities between 0.25 and 8 meters per second, but which may be in error by +0.02 for velocities of 12 meters per second. The angle 52°, apparently adopted by Dr. Parkes in 1860 for "Casella's air meter," accords very nearly with Recknagel's recommendation.

(*e*) Whewell's self-recording anemometer (Fig. 63) consists essentially of a small windmill fly formed by eight brass planes, each set at an angle of 45° to the common axis of rotation, which latter is kept in the wind direction by the vane *V*. The fly revolves by the action of the wind and gives motion to an endless screw, *a*. This screw, operating on a vertical wheel, *b*, gives motion to a horizontal wheel, *d*. This latter wheel is attached to the long vertical screw *S*, by which motion is given to the pencil of the registering apparatus.

The length of each sail, 1.92 inches; the distance from the axis to the center of each sail, 1.34 inches. This size of sails is much too small for the work that the instrument has to perform. The endless screws working in toothed wheels, so as to convert the rapid motion of the fly into a slow descending vertical motion, which is again carried out by a thread turning in a movable nut, all involve the greatest amount of friction incidental to any machine. Notwithstanding this feature, no provision was made by Whewell to allow for the effect of friction in evaluating the wind velocities of the record, the trace being considered by him to be directly proportional to the wind velocity.

In 1844 Sir W. Snow Harris made a report to the British Association on the working of Whewell's anemometer during the preceding three years' trials. He obtained an evaluation of the pencil trace in terms of the wind velocity by comparing the records with simultaneous observations of Lind's gauge, and found, as was to be anticipated from the amount of friction, that the ratio of the velocity of the wind to that of the pencil increased with decreasing wind velocities, as shown in the following table:

Wind velocities by Lind's gauge (feet per second).	Whewell's anemometer (space described by pencil in one hour).	Ratio (velocity of wind divided by velocity of pencil).
10.68	1.0	10.6
11.68	1.5	7.8
13.50	2.0	6.7
15.30	2.5	6.1
17.50	3.0	5.8
19.10	3.5	5.5
21.00	4.0	5.2
22.60	4.5	5.0
24.60	5.0	4.9
26.14	5.5	4.7
27.50	6.0	4.5
28.26	6.5	4.3
30.17	7.0	4.3

In 1843 a Whewell anemometer was erected at the Royal Observatory, Greenwich, and its records published annually until 1862. During 1860 and 1861 continuous observations were also made with a Robinson anemometer, the latter being reduced with the factor 3. The comparisons between the two published records showed that, as the observations had been reduced, the total movement of the air given by the Whewell anemometer was less than one-half of that shown by the Robinson anemometer in strong winds, while in light winds this difference was very much greater. By a table of factors for different velocities the back records of the Whewell anemometer were corrected to agree with the Robinson.

(*f*) Numerous modifications have been made in the construction of air meters, both in the sails and in the enregistering devices. Biram in 1843 made the sails curved instead of straight, as previously constructed. By this means a greater efficiency is attained, understanding by this term simply the ratio of the amount of useful work performed to the energy expended. This is important in the case of water-wheels and other similar mechanical appliances, but in the case of an anemometer the additional efficiency given by a curved sail is not of very much advantage, as sufficient power to move the sails even in light winds can be obtained with straight surfaces. The small air meter (see Fig. 64) first made by Casella, of London, in 1860, for Dr. Parkes, is a form of Woltmann's current meter. It consists of eight small plane windmill-sails set at an angle with their axis, and therefore with the wind, a little greater than 45°. It is now very widely used for all purposes for which small air meters are applicable. These air meters are tested by the maker and correction cards furnished with each instrument, but nothing is published or known to me as to the accuracy of the method of determining these corrections.

115. THE ROBINSON ANEMOMETER.

In 1846 Dr. Robinson brought out an instrument, suggested to him by Edgeworth many years previously, that has almost superseded all other forms of velocity anemometers, and has come into general use throughout the meteorological world as "the Robinson hemispherical cup anemometer" (see Fig. 65).

(*a*) The general description of this instrument is as follows: A vertical spindle has at its upper end four horizontal arms at right angles to each other, bearing at their extremities hollow hemispherical cups of thin sheet metal, whose circular rims are in the vertical planes, passing through the respective arms and the common axis of rotation. The convex side of each cup faces the direction of rotation, therefore two opposite cups have their convex surfaces facing in opposite azimuths. The pressure of the wind against the cup whose concave side receives the wind is greater than against the opposite cup whose convex side simultaneously receives the wind, and consequently, whatever the direction of the wind, the system is always caused to rotate in the same direction, each cup moving with its convex side forward. The spindle has at its lower end an endless screw, which is geared to movable dials, the number of whose revolutions count the rotations of the cups, or it is connected with a shaft and registering apparatus, whereby motion is given to a movable pencil and a trace described upon an anemograph sheet.

Electrical registration is obtained by attaching to one of the dials one or more metallic pins that at a given point in each revolution press upon a metal spring, whereby an electric current is completed and an automatic registration effected on a registering sheet.

(*b*) In devising the cup anemometer Dr. Robinson was guided by the following principles:

(1) The moving power should be so great in comparison with the friction that the correction due to the latter may be inconsiderable.

(2) The instrument should be acted upon by a section of the current sufficiently large to give an average result.

(3) The movement of the exposed surfaces should be as slow relatively to that of the wind as may be consistent with a sufficiency of moving power. This diminishes the wear of the machine and lessens the train of wheels required to bring down the speed of the recording point when mechanical self-registers are used.

(4) The instrument should act without requiring special provision for bringing it to face the wind direction.

Experiments with an anemometer (diameter of cups 3 inches, length of arms from axle to center of cups about 6 inches) placed with its axis horizontal on a whirling-machine gave the ratio $m=\frac{V}{v}$ of the velocity of the wind V to the velocity of the centers of the cups v, which ratio Robinson concluded to be almost exactly equal to 3.

From this and a slight comparison with larger instruments Robinson concluded that for any windmill of that description the centers of the hemispheres move with one-third of the wind's velocity, except in so far as they are retarded by friction, and this latter he considered negligible. This ratio was adopted on Robinson's authority by all observers using the cup anemometer and by instrument makers in the graduation of the dials, by which the wind movement is read directly in miles. Having made the above experiment, Robinson adopted the following dimensions for his standard anemometer: Diameter of cups, 12 inches; distance from axis to center of cups, 23 inches. The friction was made very small by causing the weight of the instrument and the lateral pressure to be borne by friction rollers or ball bearings.

Dr. Lloyd introduced this instrument in 1844 into his observatory, and Prof. C. P. Smyth in 1846, after testing one with 4-inch cups and 6-inch arms, adopted one of that size for the observatory at Edinburgh, where it is still carefully preserved.

(*c*) In 1856 a self-recording Robinson anemometer devised by Mr. Beckley was established at the Kew Observatory. The arms and cups of this instrument, which has since been known as the Kew standard, had the following dimensions: Diameter of cups, 9 inches; distance from axle to center of cups, 24 inches. In 1867 self-registering instruments of these proportions were supplied by the meteorological committee of the Royal Society to the observatories at Falmouth, Stonyhurst, Glasgow, Aberdeen, and Valencia.

(*d*) A cup anemometer made by Negretti and Zambra was established at the Greenwich Observatory in 1859, of which the diameter of cups was 3.75 inches, and the distance from axis to center of cups was 6.72 inches. On three days in July, 1860, this anemometer was tested by Glaisher by means of an improved whirling-machine of 17⅜ feet radius; 1,000 revolutions were made backwards and forwards. When the machine revolved in the direction opposite to the direction of rotation of the cups the record of the cups, reduced by the usual factor, gave 1.15 of the velocity of the whirling-machine; when the machine was revolved in the same direction as the cups 0.97 was registered. The mean of these results, although evidently discrepant by 6 per cent., was considered as sufficiently confirming Robinson's theory. These experiments were made during calms in the open air in Greenwich Park, and it was not considered necessary to eliminate the effect of any wind which might be blowing during the experiment.

(*e*) In October, 1866, the anemometer just described was taken down and a larger anemometer substituted at the observatory, whose registration was made by means of a mechanical anemograph. The cups of this anemometer, which is still in use, are 5 inches in diameter, and the distance of the center of the cups from the axis is 15 inches.

The accompanying Fig. 66 shows the action of this anemograph. The foot of the axis of the cups is a hollow flat cone bearing upon a sharp cone, which rises up from a cup of oil. A pinion, *C*, connected with the wheel-work acts in a rack, *J*, drawing it upwards by the motion of the cups. The rack is connected at the bottom with a sliding rod, *D*, which passes down into the chamber below where it draws up the pencil carrier *E*. The pencil *F* traces its record on a cylinder revolved by clock-work once every twenty-four hours. A motion of the pencil of 1 inch represents 100 miles of wind, the velocity of the wind being computed by the factor 3.

(*f*) In respect to the dimensions of the arms and cups a wide diversity at present exists among meteorologists. The dimensions adopted by the London meteorological office are those of the above-mentioned Kew standard, as first designed by Beckley, and made in 1856 by Adie; the radius of the hemispheres is, as before stated, 4.5 inches, and the radial arms, with reference to the vertical axis of rotation, 24 inches. These are the largest dimensions that have come into extended use, and are recognized as the standard for all who use this form of anemometer. The radial-dimensions adopted by the Signal Service in 1870 were those then in use by the manufacturer, James Green, of New York City, namely, 2 inches or 50.8 millimeters for the cups, and 6.6 inches or 168.3 millimeters for the circle described by their centers. These were essentially the same as had been adopted previously for portable instruments, made by London makers, and have been substantially retained in the usage of the Signal Service, only the cups and arms have been made a little heavier by the more recent manufacturers for the Service. These are among the smallest dimensions as yet adopted for station use by any Government weather bureau, the next largest being those of the Deutsche Seewarte, for which the cups have a radius of 2.5 inches or 64 millimeters, and the circle described by their centers has a radius of 9.5 inches or 239 millimeters.* In these, and in all Robinson anemometers hitherto made for meteorological use, the factor 3 is introduced into the wheel-work of the apparatus, so that the readings of the dial are presumed to give the movements of the winddirectly in miles or meters.

(*g*) Instead of the hemispherical cups a few trials have been made with conical and shallow cups. The simplest form seems to me to consist in applying the horizontal >-shaped trough described by Mariotte in his "Traité des Eaux Courantes." †The relative efficiency of conical and hemispherical cups may be inferred from the measures of Schellbach (Pogg. Ann., 1871, CLXIII), who found the relative times of rotation of different cups under a constant driving force to be—

Normal circular plate	0.322
Conical cup, base forwards	0.306
Conical cup, apex forwards	0.282
Hemispherical cup, concave forwards	0.317
Hemispherical cup, convex forwards	0.223
Closed hemisphere, base forwards	0.296

* A much smaller and very delicate portable size is made by R. Fuess & Co., of Berlin, for the use of travellers.

† See also Leupold Theatrum Machinarum Generale 1724, page 136, and plate XLIX.

116. DEDUCTION OF THE VELOCITY OF THE WIND FROM THE INDICATIONS OF ROBINSON'S ANEMOMETER—ENGLISH INVESTIGATIONS.

Owing to the complex nature of the currents of air that flow through the region within which the arms of any rotation anemometer are rapidly revolving, it will be readily understood that mathematical analysis has not yet been able to satisfactorily state the mechanical relations that must exist between the linear velocity of the wind and the circular motion of the vanes or cups. As, however, this relation must be known before the instrument can be used to get the velocity of the wind there have been made several comprehensive investigations into this matter. The present state of our knowledge of the subject can best be summarized after the reader has carefully considered the following analysis of the more important memoirs relating thereto, and the fullness with which certain discrepant results are here presented is fully justified by the mechanical difficulties and the meteorological importance of the subject.

(*a*) *Open air comparisons by Rev. Fenwick Stow.*—One of the earliest meteorologists to question the uniformity of the results given by different patterns of cup anemometers was Rev. Fenwick Stow. Instruments much smaller than the Kew pattern having come into use, he decided upon a direct comparison of these in the open air. With several styles of these small anemometers and a Kew pattern erected on tall poles in a large open field Stow made a series of simultaneous comparisons during 1870 and 1872.* The description of the instruments and the results of the comparisons are given in the following tables, in which the Kew pattern is taken as the standard for comparison. The figures given for anemometer *C* are the mean of the readings of two instruments, which agree closely with each other. All the readings are reduced by the factor 3, no account being taken of friction.

Comparative observations by Rev. Fenwick Stow with large and small anemometers on high poles in the open air.

DIMENSIONS OF ANEMOMETERS.

Designation.	Pattern.	Diameter of cups.	Length of arm to center of cup.
		Inches.	*Inches.*
A (standard).	Kew	9.0	24.0
B	Casella	3.0	6.7
$C=\frac{C_1+C_2}{2}$	Negretti and Zambra	3.7	6.7
D	Adie	4.0	5.6
E	Adie, with shortened arms	4.0	4.0
F	Adie, with lengthened arms	4.0	9.0
G	Adie, with longer arms and larger cups	4.6	11.2

* Quart. Journ. Met. Soc., London, Vol. I, 1872.

Comparative observations by Rev. Fenwick Stowe with large and small anemometers on high poles in the open air—Continued.

TABLE I.

Mean velocity per hour by *A* (standard).	Total movement of air according to—		Ratio.	Mean velocity by *A* (standard).	Total movement.		Ratio.
	A	*B*	$\frac{B}{A}$		*A*	*C*	$\frac{C}{A}$
3.0	24	20	0.833	3.0	24	25	1.041
7.4	280	223	0.796	4.1	58	63	1.086
11.0	320	255	0.797	5.8	75	70	0.933
12.9	490	371	0.757	7.4	311	270	0.868
16.2	406	303	0.759	11.5	1,077	874	0.811
20.2	829	652	0.786	14.0	1,136	921	0.810
24.0	336	260	0.774	16.7	502	394	0.785
26.2	420	327	0.778	20.2	829	641	0.773
28.0	126	97	0.770	24.3	877	689	0.785
30.3	364	287	0.788	26.0	780	591	0.758
32.7	180	139	0.772	28.1	352	266	0.755
34.8	139	106	0.762	30.3	364	280	0.760
				32.7	180	135	0.750
				34.8	139	104	0.748

TABLE II.

Mean velocity by *A* (standard).	Total movement.		Ratio.	Mean velocity by *A* (standard).	Total movement.		Ratio.
	A	*D*	$\frac{D}{A}$		*A*	*E*	$\frac{E}{A}$
4.1	58	65	1.129				
6.4	246	210	0.850				
7.1	186	160	0.860	11.0	320	260	0.810
7.9	189	162	0.857	13.5	81	66	0.815
9.4	478	401	0.837	16.0	272	210	0.772
11.2	118	99	0.838	20.2	829	600	0.727
13.3	241	200	0.830	24.0	336	241	0.719
14.2	1,024	832	0.812	30.9	216	149	0.692
15.4	592	482	0.814	34.8	139	86	0.619
17.8	1,091	905	0.829				
23.2	1,043	849	0.814				
24.6	541	432	0.799				
27.5	165	126	0.764				
29.6	148	111	0.750				

Comparative observations by Rev. Fenwick Stowe with large and small anemometers on high poles in the open air—Continued.

TABLE III.

Mean velocity by A (standard).	Total movement.		Ratio.	Mean velocity by A (standard).	Total movement.		Ratio.
	A	F	$\frac{F}{A}$		A	G	$\frac{G}{A}$
7.2	118	101	0.856	9.0	664	608	0.915
8.6	66	53	0.803	13.2	385	344	0.893
10.1	163	131	0.804	15.7	1,054	894	0.848
12.7	369	301	0.815	18.4	1,694	1,440	0.850
16.3	802	644	0.803				
19.0	1,123	899	0.800				
21.8	262	210	0.801				
23.0	268	216	0.806				
24.7	371	302	0.815				
31.0	96	77	0.802				
40.3	141	116	0.822				

These comparisons showed that anemometers of relatively small size as reduced by the factor 3 do not agree even approximately with the Kew standard, except at low velocities.

The cups of most of the small instruments registered from 15 per cent. to 25 per cent. less wind than the standard.

A peculiarity of the observations is that the anemometers that have the smallest cups relatively to the length of arms maintain at all velocities a tolerably uniform percentage of the motion of the standard, while those whose cups are large relatively to the length of arms have a variable ratio, registering more than the standard in low velocities, and falling much below the standard in high velocities. As an explanation of this peculiarity Mr. Stow suggested the following:

The effect of friction is most easily overcome by the instrument that at a certain velocity of wind has (1) the greatest angular velocity, and (2) that acquires from the wind the greatest motive power relatively to the amount of friction.

The first qualification is possessed by all anemometers with short arms, the second pre-eminently by those with the largest cups. Small anemometers ought, therefore, relatively to large ones, to move faster at low velocities of the wind than at high, and those which have the largest cups should have the fastest relative motion.

Again, at velocities of 26 miles per hour and over, the Casella *B* moves faster than the Negretti and Zambra *C*, both instruments having the same length of arms, and the Casella the smaller cups. Mr. Stow suggests an explanation by supposing that the cups shelter one another at certain points in each revolution, and the larger the cups relatively to the length of arms the greater the arc through which the interference will take place. For low velocities where much friction is to be overcome the larger cup has a greater motive power, and consequently a higher speed, but when power is no longer necessary, interference by sheltering begins to make itself felt; anemometers *D* and *E* illustrate this feature.

(*b*) Although the Kew was, with the factor 3, taken as the standard for comparison it is not to be assumed that its readings give the true velocities, so that these comparisons by themselves are able to give only relative results. Later experiments, to be hereafter described, agree in indicating that the factor for the Kew anemometer at high velocities, where the effect of friction is a negligible part of the recorded velocity, is not greater than 2.4, and does not differ much from that value.

Assuming, then, that the action of a Robinson anemometer may be approximately represented by the formula $V=a+bv$, in which V, v are the linear velocities of the wind and of the centers of the cups, respectively, and a and b instrumental constants, let the approximate values be assumed for the Kew standard, viz, $b=2.4$ and $a=1.50$ miles per hour, which latter is a fair average value, and equivalent to 0.75 meter per second. The true velocity of the wind may then be obtained from the preceding tables by the equation $V=1.50+0.8\times A$, where A is the observed Kew velocity computed by the factor 3, as above given by Stow in the first column of each series of comparisons. With the values of V thus obtained the observations of the remaining anemometers may be compared, and the constants a and b of the equation given above be determined for each anemometer, so as to convert its records as well as possible into standard wind velocities.

The computations by this method give the following results:

Anemometer.	Diameter of cup.	Length of arm to center of cup.	*a*	*b*
	Inches.	*Inches.*	*Miles hourly.*	*Miles hourly.*
A (Kew)	9.0	24.0	1.50	2.40
B	3.0	6.7	1.65	3.07
C	3.7	6.7	0.14	3.33
D	4.0	5.6	0.30	3.25
E	4.0	4.0	0.00	3.53
F	4.0	9.0	0.58	3.10
G	4.6	11.2	0.18	3.08

By this method of reduction, if the action can be approximately represented by the formula, the effect of friction is taken account of and the factor *b* becomes the desired anemometer factor. The resulting values show unmistakably the great difference between the Kew anemometer with factor 2.4 and the six anemometers of smaller patterns, all of which give factors greater than 3. The latter divide themselves into two groups; the first, *B*, *F*, and *G* (*G* was the second pattern adopted in 1866 at Greenwich Observatory), have arms relatively long with respect to the diameter of the cups; and the second, *C*, *D*, and *E* (*C* was the first pattern adopted in 1859 by the Greenwich Observatory), which have arms relatively short with respect to the diameter of the cups. In the first group the factors are a little greater than 3, and have so small a range that they may be considered as identical results. In the second group the factors are considerably larger and have a greater range. In this group the smallest factor is 3.25, and the largest belongs to the anemometer *E*, whose arms are very short relatively to the diameter of the cups. This result bears out the suggestion made by Mr. Stow that, for the latter group, the cups shelter one another in a large part of their rotation and so lose a portion of the wind force. A practical rule derived from these observations would seem to be that the diameter of the cups should never be greater than one-half the distance from the axis to the center of the cup.

(*c*) *Experiments of Jeffery and Whipple—Whirling-machine in open air.*—In 1872 the meteorological committee of the Royal Society allowed to Mr. Scott a grant for defraying the expenses of a series of whirling-machine experiments on anemometers of different patterns. These experiments were made by Mr. Jeffery and Mr. Whipple, respectively the director and first assistant of the Kew Observatory, by means of a steam "merry-go-round" at the Crystal Palace.

The anemometers were placed a little beyond and above the outer edge of the machine, so as to be as far as practicable out of the way of the disturbance in the air caused by its own rotation. As the machine would go round only one way the cups had to be taken off their spindle and replaced in a reverse position in order to reverse the direction of revolution of the anemometers. The motion of the wind during the experiment was determined by observations of a dial anemometer with 3-inch cups on 8-inch arms, placed about 30 feet from the outside of the whirling-machine. Let V be the linear velocity of translation of the axis of the anemometer, W that of the wind, and θ the angle between the direction of motion of the anemometer and that of the wind; then the resultant velocity of the anemometer at any point in its whirl relatively to the wind will be

$$\sqrt{V^2-2VW\cos\theta+W^2}$$

and a correction must be applied to the observed velocity v to take account of the attendant wind W. A negative correction should also be applied to V to take account of any induced air current or vortex motion set up by the rotation of the machine itself, but in these experiments this effect was assumed to be small and so not investigated.

Three anemometers were tested, namely: One of the old Kew standard pattern, one by Adie, and Kraft's portable anemometer. The dimensions are as follows:

Pattern.	Diameter of cups.	Length of arms from axis to center of cup.	Distance from axis of machine to anemometer.
	Inches.	*Inches.*	*Feet.*
Kew	9.0	24.0	22.3
Adie	2.5	6.7	20.7
Kraft	3.3	4.15	19.1

With each anemometer the experiments were made in three groups, with high, moderate, and low velocities, respectively, averaging about 28 miles per hour for the high, 14 for the moderate, and 7 for the low. Each group was divided into sets according as the cups rotated in the direct or reverse directions. In the former case the rotation of the anemometer and the "merry" were opposite, and in the latter case the same. The object of the experiment was, of course, to compare the mean velocity of the centers of the cups with the mean velocity of the air relatively to the anemometer. In the reduction of these experiments by Professor Stokes (Proc. Roy. Soc., Vol. XXXII, 1881) the velocity of the anemometer, *i. e.*, of its spindle, is increased by the effect of the wind velocity W in order to get the resultant relative velocity of the wind and the anemometer, which is the total impelling force. The mean effect of the wind will be different according as we suppose the moment of inertia of the anemometer very small or very great. If the moment of inertia be supposed to be small, and if W, as is practically the case, be small as compared with V, the expression

$$\sqrt{V^2-2VW\cos\theta+W^2}$$

may be expanded in a series of ascending powers of $\frac{W}{V}$; odd powers will disappear in taking the mean and fourth, and higher powers may be neglected.

Professor Stokes finds that the resultant velocity V_1 will be approximately

$$V_1=V+\frac{W^2}{4V} \text{ or } V_1=V+\frac{3}{4}\frac{W^2}{V}$$

according as the moment of inertia of the anemometer is small or large. The registered velocity is that obtained from the velocity of the cups by employing the usual factor 3, noting in this process that the true number of rotations of the cups is given by adding the number of revolutions of the "merry" to the number of apparent revolutions of the cups when the rotations of the anemometer and the machine are in the same direction and subtracting when in opposite directions.

The results of the experiments at moderate and high velocities are given in the following table, where the percentages indicate the ratio $\frac{V_1}{V}$. Thus the Kew anemometer cups rotating in a direction opposite to that of the whirling "merry" gave velocities that are 122.6 per cent. of the true linear velocity of the axis of the anemometer.

The experiments with low velocities were rejected owing to their irregularity, due to the fact that the machine could not be regulated to a uniform low velocity.

Crystal Palace experiments.

Anemometer.	Direction of rotation.	High velocities.		Moderate velocities.	
		Moment of inertia small.	Moment of inertia large.	Moment of inertia small.	Moment of inertia large.
Kew	Opposite	122.6	121.9	115.1	113.2
	Alike	118.4	117.5	109.7	108.5
	Mean	120.5	119.7	112.4	110.8
Adie	Opposite	95.1	94.2	88.5	86.8
	Alike	98.0	97.3	82.6	81.0
	Mean	96.5	95.7	85.5	83.9
Kraft	Opposite	101.5	100.8	89.1	86.9
	Alike	100.8	99.4	87.8	86.0
	Mean	101.1	100.1	88.4	86.4

From the internal agreement of the individual numbers whose averages are given in this table the probable errors of these mean percentages are deduced as follows:

Anemometer.	For high velocities.	For low velocities.
Kew	1.0	2.7
Adie	1.5	2.0
Kraft	0.9	1.8

As the registered velocity is computed by using a simple factor with no friction term the difference between the results for moderate and high velocities may be attributed to this omission, and the discrepancy between the two cases may fairly serve to indicate the amount of error arising from the neglect of a friction term for velocities like those of these experiments.

In whirling-machine experiments it is to be noted, moreover, that there is a lateral pressure on the anemometer axis, arising from centrifugal force and gyroscopic action, which does not exist in the normal action of the stationary anemometer; if possible this ought to be allowed for, since the resulting friction, which is not constant but proportional to V^2, must sensibly reduce the registered velocity, and consequently the experiments will tend to give too large a value for the anemometer factor represented by b in the preceding article. With this exception the results of the experiments at high velocities should give a good indication of the approximate value of b. Therefore for the Kew pattern, which is the one adopted by the meteorological office at London, the factor to be recommended is 2.5 less the effect of the various frictions at the velocities of the experiments. For the two smaller instruments the experiments indicate a factor very nearly 3, but for the coefficient of the Kraft anemometer the result of Scott's observations does not correspond with the larger values obtained for the somewhat similar experimental anemometers Nos. *C*, *D*, and *E* in the comparisons by Mr. Stow.

(*d*) *Robinson's theory*.—The experiments of Dohrandt (see next section) led Dr. Robinson to re-examine the theory of the cup anemometer, and as a result he published in 1873 a new investigation, the substance of which is as follows:

A correct theory of the anemometer must equate the impelling force of the wind to the sum of the various resistances which oppose it. These resistances can arise only from the action of the anemometer itself on the air and the friction of its parts; and therefore V, the velocity of the wind, is a function of v, the velocity of the cups, and R, the sum total of the resistances.

The nature of this function can not in the present state of hydro-dynamics be completely determined, but a conception of its form may be obtained. Suppose the wind at velocity V to be incident on an anemometer cup at an angle, θ, with its arm.

If a cup has the velocity v the resultant relative velocity of the wind and the cup is

$$U=\sqrt{V^2+v^2\mp 2Vv\sin\theta}$$

the negative sign belonging to the case when the cup is moving with the wind, and the positive sign to the case when the cup is moving against the wind.

This resultant makes with the radial arm bearing the cup an angle, φ, such that

$$\tan\varphi=\tan\theta\mp\frac{v}{V}\sec\theta$$

The first approximation to the resultant impelling pressure of the wind upon a cup will therefore be

$$a_1 U^2 \sin\varphi$$

in which the coefficient a depends on the size of the cup, the length of arm, and the density of the medium.

As an equal arm carries on the other side of the axis an equal cup, but in a reversed position, the action of the wind on its convex surface will oppose the pressure on the first cup, and the resultant moment, acting on the axis, will be the difference of the two pressures acting with the leverage of the radial arm. The difference of the pressures will be

$$a_1 \sin \varphi(V^2+v^2-2Vv \sin \theta)-a_2 \sin \varphi(V^2+v^2+2Vv \sin \theta)$$
$$=\sin \varphi[(a_1-a_2)(V^2+v^2)-(a_1+a_2)\times 2Vv \sin \theta]$$

This force is opposed, first, by the moment of friction at the centers of the cups; secondly, by the moment of resistances depending on v^2, of which the chief are (1) the resistance of the air to the arms, and (2) the reaction on the cups of the air thrown outwards by centrifugal force.

When these opposing forces balance each other on the average of a complete rotation the angular velocity of the instrument will be constant, except for small periodical fluctuations, whose effects are lessened by increasing the moment of inertia of the anemometer and by increasing the number of pairs of cups, *e. g.*, two pairs instead of one. If we could find the mean values of $a_1 \sin \varphi$, $a_2 \sin \varphi$, $a_1 \sin \theta \sin \varphi$, and $a_2 \sin \theta \sin \varphi$ through a revolution we could express this state of permanent motion by the equation

$$\alpha V^2-2\beta Vv-\gamma v^2-R=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (I)$$

in which the resistances R can be obtained by experiment.

As the resistances which are proportional to v^2 may be combined in the term γv^2, R may be approximately replaced by F or the total friction of the moving parts. Then, solving the equation with respect to V, there results

$$V=v\left(\sqrt{\frac{\beta^2}{\alpha^2}+\frac{\gamma}{\alpha}+\frac{F}{\alpha v^2}}+\frac{\beta}{\alpha}\right)$$

The coefficients α, β, and γ are functions of the areas of the cups, of the length of arms, of the density of the air, and of the angle φ. As functions of φ it is possible that α, β, and γ may vary with V, but the form of the expressions indicates that any such variation would be small and may be neglected in the first approximation. Therefore, for the same instrument, α, β, and γ may be considered to be constant coefficients.

If we have a series of observations with whirled anemometers in which V (which now becomes the velocity of the axis less the current of air in the room), v, and F are accurately known, we may determine by least squares the values of α, β, and γ. If, then, the residuals be arranged according to the ascending values of v, they will indicate whether the assumption that the coefficients are sensibly constant is true within the limits of the errors of observation. This was done by Robinson in his discussion of his elaborate experiments with a whirled anemometer made by him at Dublin, and no dependence on v was indicated. He adopts the notation

$$\eta=\frac{F}{\rho V^2} \qquad \xi=\frac{v}{V}$$

where ρ is the factor for reducing to a standard density of the air, and writes the above expression (I) in the following form:

$$\eta=\alpha-2\beta\xi-\gamma\xi^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (II)$$

Dr. Robinson plotted his observations, using η and ξ as variables, and found that the best smooth curve to represent the observations is either accurately or very approximately a straight line, whence $\eta=\alpha'-2\beta'\xi$, and the term $\gamma\xi^2$ or the coefficient γ is inappreciable. If this latter form of the equation is true it leads to a very simple equation for the action of these cup anemometers when stationary in the open air, *i. e.*,

$$v=aV-\frac{b}{V}$$

in which b depends on the friction of the instrument and a varies with the length of arms, diameter of cups, and density of the air.

(*e*). *Robinson's experiments.*—In order to test his theory and to evaluate the constants of his formula, especially for the standard Kew pattern anemometer, Dr. Robinson made some careful whirling-machine experiments at Dublin. The whirling apparatus was erected in the large dome of Mr. Howard Grub's optical establishment at Rathmines, the least diameter of which dome is 42 feet, and the height to the summit of the dome also 42 feet.

The vertical shaft of the whirling-machine was placed in the center of the dome with its horizontal bar at a height of 14 feet from the floor. The anemometers were attached to the end of this bar with their axes pointing outwards horizontally instead of vertically, so that the centers of the cups revolved in vertical planes tangent to a vertical cylinder 9.077 feet in radius. The cups were therefore 12 feet from the walls and, in the lowest case, 12 feet from the floor.

The anemometers tested were the following:

Designation.	Diameter of cup.	Length of arm from axis of revolution to center of cup.
	Inches.	*Inches.*
I (Kew)	9	24
II	4	24
III	9	12
IV	4	12

(e_1) The mean velocity of the current of air set in motion around the room by the whirling of the machine itself and the revolutions of the cups must be subtracted from the computed velocity of the anemometer center in order to get the true relative velocity with which the wind strikes the air. It was therefore measured in the following way: A rod of deal one-fourth of an inch square and 23 feet long was suspended horizontally by a fine thread from the summit of the dome. In order to prevent bending it was braced by other threads fastened 4 feet above it to the vertical thread; a light balloon, about 8 inches in diameter, was suspended at each end, so as to hang about 4 feet below the rod, with their centers on a level with that of the anemometer and 14 inches outside the plane of revolution of the cups. The suspension thread of the rod was 23 feet long, and its force of torsion was insensible in these experiments. The balloons therefore revolved with sensibly the same velocity as the horizontal component of the air current that prevailed at their level and location, and the observation to be made consisted in timing their velocity for given velocities of the whirling-machine and attached anemometer. The velocity of the air current at the balloons was not, however, the same as that in the exact plane of the anemometer cups, which was the velocity desired. Dr. Robinson assumed that this latter can be obtained by multiplying the observed velocity of the balloons by a factor obtained on the assumption that the velocity in any part of the vortex created within the dome is inversely as the square of the distance from the center of the whirling-machine, an assumption which was confirmed by the velocities of the balloons at two distances from the anemometer of 14 and 30 inches, respectively.

The observations gave values for W, the velocity of the induced current, ranging from 7½ to 10 per cent. of the value of the velocity V of the anemometer center, but Dr. Robinson states that this determination is perhaps the weakest portion of the whole investigation.

Besides the great uncertainty of the reduction just described there is still an additional source of error in the assumption as to this data given by the balloons. The quantity desired is the velocity of that component of the induced current that lies in the plane of revolution of the cups at the moment when they overtake air set in motion by the whirled anemometer at some point in its preceding circular path. During the time of one whirl, however, the impulse given to the air at any point has to some extent been used up against resistances, and a diminution of velocity has resulted approximately proportional to the time occupied by the whirl. When the whirling is slow the air may come to rest and the anemometer may meet still air at the beginning of its successive whirlings, but with greater velocities of the whirling-machine the residual velocity of the induced current at the end of each whirl increases. For high velocities any periodic variation in the velocity of the induced current at the end of the whirls probably becomes inappreciable. For a radius of whirl of 9 feet, as used by Robinson, at a velocity of 30 miles per hour, the anemometer center makes one rotation in 1.3 seconds. For this velocity any periodicity in the horizontal component of the induced current must be inappreciable. But for velocities less than 20 miles per hour a sensible periodic variation may exist, and the residual or minimum velocity is the one required, because it is the velocity of the air when it is met by the anemometer. The method of observation, however, does not give the minimum value, but one much larger, larger even than a mean value; for the inertia of the solid rod and other parts of the measuring apparatus being greater than that of the air itself, their velocity is less quickly overcome by resistance, so that at the end of a whirl the measuring apparatus is moving with a velocity greater than that of the average induced current.

For these reasons, therefore, the computation of the velocity of the induced current as made by Robinson tends to give too large results at any but high velocities. That such was actually the case is well illustrated by the following table, giving the measured velocity *W* of the revolving balloons (assumed to be the same as that of the induced current) as a percentage of the velocity *V* of the center of the whirling anemometer:

V	W	$\frac{W}{V}$	Number of observations.
Miles per hour.	*Miles per hour.*	*Per cent.*	
29.53	2.21	7.5	8
27.15	2.13	7.8	8
25.40	2.02	7.9	8
21.66	1.82	8.4	7
19.71	1.69	8.6	7
15.19	1.37	9.0	7
12.57	1.30	10.3	5
10.40	1.04	10.0	4
8.70	0.92	10.6	4

The relative velocity of the induced current, as measured by the balloons and rod, is seen to increase steadily with decreasing velocities of the whirling arm, a result that is consistent with the preceding analysis of the nature of the error introduced by using so heavy a moving apparatus. The current that actually affects the anemometer is probably less than that shown by the balloons, so that a correction for an induced current of 7.5 per cent. for high velocities of the whirling-machine, with a diminishing percentage for lower velocities, would probably give values nearer to the truth.

Such a diminishing correction would be given by the formula

$$W^2 = b^2 V^2 - a^2$$

in which $\frac{a}{b}$ is the value of V, at which W begins to be appreciable, and b is the ratio $\frac{W}{V}$ at high velocities.

In the equation

$$\alpha V'^2 - 2\beta V'v - \gamma v^2 = F \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(III)}$$

V' represents the relative velocity with which the wind strikes the anemometer, and in whirling-machine experiments in a room protected from the wind, is equal to V, the velocity of whirl of the center of the anemometer less W, the velocity of the induced current.

(e_2) Determination of α: Making $v=0$ in the above equation we have

$$\alpha = \frac{F}{V_0'^2}$$

where V_0' is the wind velocity at which the cups just begin to turn, and when F is known the velocity may be made the subject of direct experiment.

(e_3) Determination of β and γ: Examining the anemometer equation (II) it will be seen that β and γ can not be determined accurately in a series of ordinary experiments because of the very small variation of ξ. To overcome this difficulty Dr. Robinson applied to the anemometer a brake apparatus by which the friction could be increased to any desired amount, and thus ξ increased to any extent. This friction brake consisted of two rubbers made of stout cloth, which were pressed with adjustable force against the opposite edges of a friction disk attached to the axis of the anemometer. The friction thus applied caused neither vertical nor lateral pressure on the supports of the shaft. When, therefore, the brake was made to act it merely added another friction F_B without altering that acting from normal causes.

The normal friction F, when the axis of the anemometer is held horizontally while being whirled, is threefold, and may be designated as follows: f', or that due to the weight of the axis and cups; $f_1''+f_2''$, or that caused by the lateral pressure of the axle on its bearings, produced partly by the resistance of the air and partly by the force which is required to change the plane of the anemometer's motion as it turns with the whirling apparatus; f''', or that due to centrifugal force, pushing the axis outward against its bearings.

f' and f_1'' exist in the regular meteorological use of the anemometer, when its axis is vertical and has no motion of translation; f_2'' and f''' are incident to Robinson's method of mounting the anemometer on the whirling-machine.

So far as possible these various frictions were directly measured by Dr. Robinson and expressed in grains weight.

The brake friction F_B, and the friction f', due to the weight of the anemometer, were obtained by the process used by Atwood in determining the constants of his well-known machine. The results of the observations of the brake friction are given in the following table, in which B is the load on the brake in ounces (minus 200 grains, the force required to bring the rubbers into contact):

B	F
Ounces.	*Grains.*
3	284.7
6	574.3
9	841.7
12	1,098.1
15	1,347.6
18	1,593.5
24	2,026.9
30	2,556.0
36	3,021.1

On account of the long axis, its horizontal position, and the weight of the brake apparatus f' was much larger than its amount (22 grains) in ordinary instruments with axes vertical. For the different anemometers the following results were obtained:

Anemometer..	f'
	Grains.
I..............	113.2
II.............	101.3
III.............	201.3
IV	185.0

The lateral friction f_2'', produced in the ordinary use of anemometers by the wind pressing the anemometer axis against its bearing, and in Robinson's case by similar action of the resisting air, must be proportional to the areas of the cups d^2 and the square of the relative velocity of the wind and the anemometer cups, omitting the consideration of the arms and the axis so far as exposed; it may be represented by the expression

$$(\varepsilon V'^2-2kV'v+\varepsilon v^2)d^2$$

Its effect, therefore, will be merely to diminish α and β and to increase γ; but $-2kV'v+\varepsilon v^2$ is probably small in comparison with the first term, so that the pressure is approximately $\varepsilon V'^2d^2$. The value of ε was subjected to experimental measurement and found to be, for 9-inch cups, .004400; for 4-inch cups, .001387; and the resulting friction is

$$f_1''=\varepsilon v'^2\times 4.71$$

The gyroscopic friction f_2'', according to the principle of the gyroscope,* is proportional to $V'v$, and therefore its effect is merely to numerically increase the value of β; its absolute measurement was not attempted in Robinson's experiments.

* See article Gyroscope, by General Barnard, in Johnson's New Cyclopædia.

In the rotation of the anemometer on a whirling-machine the centrifugal force gives rise to an outward pressure, which must be resisted by some stop, and produces there the friction f'''.

This pressure varies as V^2, but, without sensible error, may be taken proportional to V'^2. The resulting friction could therefore be included in the value of α, but in order to have the experiment applicable to stationary anemometers and real wind, where f''' does not exist, Dr. Robinson measured it and added it to the other friction terms.

For each instrument the following results were obtained:

Anemometer.	P'''
I..............	$V'^2 \times \log [8.56260]$
II..............	$\times \log [8.46338]$
III..............	$\times \log [8.53793]$
IV..............	$\times \log [8.40158]$

The friction was then obtained by Dr. Robinson from the formula

$$f''' = aP''' + bP'''^2$$

in which for two different sets of friction rollers the values of a and b were, respectively,

$$a = 5.823 \qquad b = 0.127$$

and

$$a = 2.501 \qquad b = 0.051$$

(e_4) Direct experiments to obtain the value of α resulted as follows, noting that α is proportional to the areas of the cups.

Anemometer.	α	Barometer.	Thermometer.	No. of observations.
			°	
I..............	11.896	29.77	73.5	17
II..............	2.316	29.63	62.0	13
III..............	12.107	29.83	67.0	11
IV..............	2.502	29.66	75.0	6
[Reduced to the density of air prevailing for III and IV.]				
I..............	12.383	29.83	67.0	17
II..............	2.288	29.66	75.0	13

For various reasons the value of α for Anemometer IV is not considered by Dr. Robinson to be of as much weight as the determinations for the other three instruments. In the computation of each one of the observations the corresponding observed value of W was introduced. Much better results would doubtless have been obtained if the computations had been so conducted that for any given velocity V a normal computed value of W could have been used as before suggested.

(e_5) After this preliminary work the whirling experiments with each anemometer gave for different amounts of friction the quantities

$$\xi=\frac{v}{V} \qquad \eta=\frac{F}{V'^2}$$

in equation (II), where for F is taken the sum $F_B+f'+f''$, and where f_1'' and f_2'' are left to modify the resulting values of the coefficients α, β, and γ.

The friction f_1'' exists equally in the normal use of the fixed anemometer, and therefore the resulting values of the coefficients α, β, and γ, so far as affected by f_1'', will hold good as anemometer constants. The friction f_2'', which affects the value of β, is peculiar to the whirling-machine, and does not occur in a fixed anemometer. Its effect on β is, however, probably small, and within the limits of the errors of observations and of the uncertainties of the other observed frictions.

As α, β, and γ are functions of the density of the air equation (II) can be placed in the form

$$\eta=\alpha_0-2\beta_0\xi-\gamma_0\xi^2$$

in which α_0, β_0, and γ_0 are the values for a standard density, and where η has now the value $\frac{F}{\rho V^2}$; ρ is the coefficient for reduction to standard density.

This is an equation between two variables, η and ξ, and from their values, as given in Robinson's experiments, the constants α_0, β_0, γ_0 may be obtained. This was accomplished graphically by plotting the observed values of η and ξ. The plotted observations for Anemometer No. 1 were found to be best represented by a straight line. This result corresponds to the equation

$$\eta=\alpha_0'-2\beta_0'\xi$$

in which the term $\gamma_0\xi^2$ does not occur as being insensible. Restoring the values of η and ξ we now have

$$\rho\alpha_0' V'^2-2\rho\beta_0' V'v=F$$

This equation implies that the coefficient of that part of the resistance which depends on v^2 is relatively inappreciable, a result of Robinson's observations that was *a priori* not improbable.

The observations with Anemometer III gave the following values for α_0' and β_0', under the assumption that here also $\gamma=0$. The standard density to which they are reduced is not stated; the results are given for several groupings, according to the velocity v.

	$v<5$.	$v>5<9$.	$v>9$.	All.
α_0'	10.00	11.31	11.55	9.99
β_0'	11.64	12.77	13.04	10.93

(e_6) Upon comparing these computed values of α_0' with the measured values given in the preceding table Dr. Robinson decided to adopt 0.9

of the latter as the more reliable values, and with these the observations of Anemometers I and III were recomputed by the complete three-term formula, and gave the following results for the air density corresponding to barometric pressure 29.83 and temperature 67° F.:

Anemometer III:

$\alpha=10.90$ $\beta=11.88$ $\gamma=+1.60$

Anemometer I:

$\alpha=10.71$ $\beta=12.49$ $\gamma=-0.15$

With these values the limiting value of $\frac{V'}{v}$, when $F=0$, becomes for Anemometer III, 2.24; and for I, the Kew Anemometer, 2.32.

The plotting of the observations with Anemometers II and IV showed discordances too great to admit of determining valuable results. This failure of these observations arises from the fact that the impelling force $\alpha(V'^2+v^2)$ was only one-fifth of that for the anemometers with the larger cups, while the frictions were nearly the same.

The resulting limiting value of $\frac{V}{v}$ for an ideal frictionless Kew anemometer, namely, 2.32, agrees very well with the experiments by Scott and Jeffery, at the Crystal Palace, which gave the value of $\frac{V}{v}$ with the friction included, as 2.50 at high velocities.

(*f*) Robinson subsequently compared the above Anemometer I now called E_1, and other experimental anemometers set up in the open air with his permanent Kew standard; but first he entirely modified his above computation of the induced wind, the friction, and the quantity α, recognizing the mistakes of his hypotheses, and adopted the formula

$$\frac{V}{v}=2.831+\frac{0.355}{v^2}$$

for his stationary Kew. With this new assumption he deduced from the open-air work the following "limiting value" for the ratio $\frac{V}{v}$ for the six experimental anemometers when v is infinite or when friction is zero.

Anemometer.	Radii.		Limit.
	Cups.	Arms.	
	Inches.	*Inches.*	
E_1	4.5	24	2.826
E_2	4.5	12	3.163
E_3	4.5	8	4.047
E_4	2	10.67	4.961
E_5	2	26.75	3.430
E_6	6	23.17	2.820

(*g*) *Conclusion*.—From all this there results absolutely no determination, not even for the Kew pattern, of the true value of the "anemometer coefficient," but abundant evidence that it varies with the absolute and the relative dimensions of the instrument and with the velocity of the wind and density of the air.

117. INVESTIGATIONS OF ROBINSON'S ANEMOMETER AT ST. PETERSBURG.

In 1873 M. Dohrandt, of the Central Physical Observatory, undertook a series of whirling-machine experiments for the purpose of obtaining an empirical determination of the relation between the velocity of the wind and that of the cups in different patterns of Robinson's anemometer.

Previous to Dohrandt's work, Professor Wild had made one experiment, in 1871, in which an anemometer was exposed upon a railroad train in free air, but the labor attendant on similar railroad experiments was so great that Dohrandt decided to confine his work to the whirling apparatus and supplementary open-air comparisons of anemometers on the roof. Dohrandt's work extends over the years 1873 and 1874, and was taken up again in 1878; in 1882 the experiments were somewhat extended on the same plan by Stelling. The principal difference in the work done during these ten years was a steady increase in the velocities obtained by the whirling-machine, which, at first, was driven by hand, then by water, and finally by a gas motor, by which the highest velocities attained were equivalent to a wind of 70 kilometers per hour (19.44 meters per second).

(*a*) *The apparatus and method.*—The whirling-machine proper was established in the center of a hall whose diameters were 8.36 and 8.19 meters, and whose height was 8.03 meters at the center and 6.88 meters at the side.

The anemometers were fixed in their normal vertical position at the extremity of the horizontal arm of the whirling-machine, whose whole length was 6.88 meters, and whose center coincided with the center of the room. The vertical axis of an anemometer thus adjusted described a cylinder whose radius was 3.323 meters. The plane of the whirling arm was 6.19 meters above the floor, and when an anemometer was fastened to it the cups usually came within about 0.45 meter of the roof at the sides of the room, and the axis of the anemometer came to within 0.8 of a meter from the walls. In such close proximity to the walls the anemometers were not under conditions similar to an exposure in free air. Upon this point Dohrandt writes as follows:

> On the one hand the induced current that is produced by the rotation of the apparatus is diminished by friction on the walls and ceiling; on the other hand the air thrown out centrifugally by the cups of the anemometer can not flow away with perfect freedom.

In the course of his long series of experiments anemometers of different sizes and various patterns were placed successively upon the whirling-machine. In each experiment the whirling apparatus was rotated in opposite directions, NESW., and NWSE.

(*b*) *Preliminary work.*—The most important investigation, preliminary to the final reduction of the observations, consisted in the determination of the general motion of the air in the path of the cups induced by the whirling of the anemometers. To this end a Woltmann's anemometer was permanently established in the plane of rotation of Robinson's cups, with its axis so turned as to measure the horizontal component of the general motion of the air. Near this was placed a large and very light tablet anemometer, which also indicated the force of the same tangential component. The comparison of these instruments showed the presence of strong impulses, which could not be considered as existing in the air at any point at the moment preceding the passage of the advancing anemometer. In order, therefore, to obtain a more correct measure of the average condition of the air immediately in front of the anemometer a tablet anemometer was placed upon the opposite extremity of the whirling arm of the machine. The apparatus was now revolved, (1) without the Robinson anemometer, and (2), with the Robinson anemometer. The difference between the indications of the tablet anemometer in the two cases represents the velocity of the induced current caused by the whirling of the cup anemometer. The result of experiments with velocities of 11 to 20 kilometers per hour gave the motion of the induced current as probably not more than 5 per cent. of the velocity of whirl of the axis of the cup anemometer. This result was checked by allowing small balloons, so weighted as to stay nearly at the level of the whirling-machine, to move freely around the room. They generally described a half and in favorable circumstances sometimes a whole revolution without departing much from the path of the anemometers, showing a general movement of the air of 5 or 6 per cent. of the velocity of rotation. As the experiments with different anemometers placed on the machine gave very nearly the same result Dohrandt adopted a general correction for "mit-wind" of 5 per cent. in all his work.

Since by the introduction of centrifugal forces the circular motion of the anemometer might possibly affect the relation between the velocity of the cups and that of the machine special experiments were made with the anemometers whirled in circles of different radii. The following table presents the results thus obtained:

Anemometer.	Rotation NESW.		Rotation NWSE.		Mean.	
	a_1	b_1	a_2	b_2	a	b
	Radius of whirl=3.323 meters.					
Electrical register No. 4..	2.411	0.242806	2.855	0.278540	2.618	0.259448
Nowikof..................	1.570	0.367455	2.357	0.487783	1.908	0.419154
	Radius of whirl=2.253 meters.					
Electrical register No. 4..	1.995	0.235459	2.491	0.312941	2.208	0.268726
Nowikof..................	2.011	0.336971	2.837	0.538566	2.329	0.414560

In this table the last two columns give for the two radii 3.323 and 2.253 meters the resulting coefficients a and b in an empirical anemometer equation, $v=a+bn$, where n is the number of cup rotations in a unit time and v is the relative velocity of the wind.

Although the constant term a is quite different, yet the factor b does not vary for long and short radii more than 1 per cent. With the shorter radius the anemometers were farther from the walls of the room; the effect of this, however, is not specifically mentioned by Dohrandt, who concluded that reliable results might be obtained with the radius 3.323 used in his observations, and that this method of experiment, by taking the arithmetical mean of opposite rotations, gives results not very different from what would be obtained if the anemometers could be moved in straight lines instead of circles.

The large differences given by Dohrandt in the above table and in all his work for the results of rotations in opposite directions arise at least in part from the fact that he uses the number of apparent rotations of the cups instead of the number of true rotations; the latter are obtained by adding to or subtracting from the recorded rotations of the cups the number of rotations of the whirling-machine according as the machine revolves in the same or opposite direction to that of the cups.

In the reduction of his observations Dohrandt assumed that the velocity of the wind could be obtained from the motion of the cups by the empirical formula

$$v=a+bn+cn^2+\text{ etc.}$$

where v is the velocity of the wind, n the number of true rotations of the cups, and a, b, and c constants to be determined by the experiments. For four anemometers Dohrandt computed the coefficient, using both two and three terms of this formula (but using the apparent n); the resulting differences showed that with the use of three terms the observed values were represented with considerably greater accuracy than by the use of the first two terms only, and that the first or friction constant was evidently then much nearer the truth; but on account of the material increase in the labor of computation and the only slightly increased accuracy, and because the unreliability of the third term would be more largely felt in applying the formula to velocities beyond the limit of his experiments, Dohrandt adopted the formula with two terms, $v=a+bn$ or $=a+bk$, where k is the velocity of the center of the cups, and v that of the wind (*i. e.*, that of the whirl less 5 per cent. for "mit wind") in kilometers per hour.

(*c*) *First results.*—The results of experiments in 1873 and 1874 are presented in the following two groups:

I.—*Constants obtained by experiments with the whirling-machine.*

Anemometer.	r	R	a	B
	Meters.	*Meters.*		
Browning	0.3039	.0742	3.66	2.2271
Nowikof	0.2187	.0478	1.81	2.8979
Casella No. 318	0.1717	.0382	1.90	2.8742
Casella No. 317	0.1715	.0383	2.56	2.7548
Electrical register No. 4	0.1551	.0522	2.49	2.5293

Where r=radius of circle described by center of anemometer cup; R=radius of hemispherical cup.

II.—*Constants obtained by comparisons in free air on the roof.*

Anemometer.	r	R	a	B	Compared with—
Adie anemograph	0.6100	0.1046	5.25	2.144	Browning.
Breguet anemograph	0.2799	0.0542	3.69	2.501	Do.
Marine anemometer	0.1535	0.0467	3.08	2.548	Casella No. 318.

(*d*) *Second series.*—In 1877 and 1878 Dohrandt made a much more extensive series of experiments with the whirling-machine and also of comparisons on the roof. The computations were in all cases performed with both the two-term and the three-term formula above given. The details of the work were substantially as follows: Three sets of cups, whose sectional areas were to each other as 1, 3, and 5, were successively attached to three sets of arms, giving values of r, the length of arm to center of cup, in the ratio of 1, 1½, and 2. In this way the equivalent of nine anemometers was secured, for all of which the same registering apparatus was used. The radius of the whirling-machine was, as in the former work, 3.323 meters. The apparatus was rotated in opposite directions and an allowance of 5 per cent. for the velocity of the induced current was uniformly made, as in the previous work.

The mean results are presented in the following table, in which a_2, B_2; a_3, B_3, and C_3 are the constants computed by the two-term and three-term formula, respectively, *i. e.*, $v_2=a_2+B_2k$ and $v_3=a_3+B_3k+C_3k^2$.

III.—*Observed coefficients in the anemometer formula.*

Anemometer.	r	R	a_2	B_2	a_3	B_3	C_3
I..............	0.3000	.0364	2.21	2.8872	0.81	3.2698	−0.01953
II..............	0.3012	.0619	2.88	2.3933	0.89	2.9485	−0.02025
III..............	0.3005	.0804	2.12	2.2816	0.62	2.6600	−0.01857
IV..............	0.2256	.0364	2.36	2.7237	0.92	3.0967	−0.01804
V..............	0.2253	.0619	2.54	2.3796	1.42	2.6362	−0.01100
VI..............	0.2251	.0804	2.33	2.3256	1.30	2.5561	−0.00955
VII..............	0.1499	.0364	1.91	2.5871	0.70	2.8799	−0.01329
VIII..............	0.1507	.0619	1.65	2.4687	0.95	2.6306	−0.00699
IX..............	0.1506	.0804	0.71	2.5826	0.97	2.5248	+0.00256

Dohrandt concludes that the three-term formula has a decided advantage in accuracy over the two-term formula, and from the fact that the coefficient C_3 changes its sign from negative in No. VIII to positive in No. IX he concludes that between these two sizes of instruments there must be a size (one having a short arm and large cup) for which $C_3=0$, and for which, consequently, there subsists a linear relation between the velocity of the cups and the velocity of the wind, or where the two-term formula would be rigorous. Not being able to deduce any simple necessary relation between the factors B_3 and C_3 and the dimensions of the anemometer Dohrandt adopted the following empirical expression for B_2:

$$B_2=\alpha_1+\beta_1 \cdot \frac{R^2}{r}+\gamma_1 \cdot \frac{R^4}{r^3}$$

The following values of α_1, β_1, and γ_1 were determined from the observations of the above anemometers:

$$\alpha_1=+3.0133 \qquad \beta_1=-53.7367 \qquad \gamma_1=+1033.81$$

With these coefficients the observed and computed values of B_2 are as follows:

IV.—*Observed and computed anemometer coefficients.*

Anemometer.	$\frac{R}{r}$	B_2 (observed).	B_2 (computed).	Δ =obs−comp.	Per cent.
I..............	0.1213	2.8872	2.7962	+.0910	3.2
II..............	.2055	2.3933	2.4970	−.1037	4.3
III..............	.2676	2.2816	2.3357	− 541	2.3
IV..............	.1613	2.7237	2.7334	− 97	0.4
V..............	.2747	2.3796	2.3984	− 188	0.8
VI..............	.3572	2.3250	3.3227	+ 20	0.1
VII..............	.2428	2.5871	2.6191	− 320	1.2
VIII..............	.4107	2.4687	2.3154	+ 1533	6.2
IX..............	.5339	2.5826	2.6114	− 288	1.1

From this table Dohrandt concluded that the constant B_2 is represented by the above formula to within ±2 per cent. of its own value, and can therefore be computed with that degree of accuracy for any other anemometer for which the ratio $\frac{R^2}{r}$ lies within the limits of his experiments.

Besides the nine experimental anemometers Dohrandt repeated for several ordinary anemometers the determination of their constants both by the whirling-machine and by comparison on the roof, with the following results.

V.—*Whirling-machine experiments of* 1878.

Anemometer.	r	R	a_2	B_2 (observed).	B_2 (computed).
	Meters.	*Meters.*			
Browning	0.3030	0.0742	3.83	2.200	2.379
Casella No. 318	.1717	.0382	2.62	2.730	2.631
Electrical register No. 4	.1551	.0522	3.24	2.431	2.388
Salleron	.1562	.0521	3.06	2.441	2.392
Schultze No. 2	.2177	.0476	2.84	2.506	2.566
Schultze No. 5	0.2194	0.0474	2.54	2.519	2.571

VI.—*Roof comparisons of* 1878.

Anemometer.	r	R	a_2 (observed).	B_2 (observed).	B_2 (computed).	Compared with—
Adie anemograph	0.6100	0.1046	2.08	2.30	2.38	Browning.
Adie anemograph	....	....	5.48	2.06	2.38	Schultze No. 5.
Breguet anemograph	0.2799	0.0542	2.60	2.50	2.56	Browning.
Breguet anemograph	....	....	3.56	2.43	2.56	Schultze No. 5.
Munro anemograph	0.6082	0.1159	1.58	2.33	2.33	Schultze No. 5.
Schultze-Oettingen	0.4399	0.1492	0.62	2.47	2.94	Schultze No. 5.

The last column shows which of the two substandards was used in deriving the data for the observed a_2 and B_2.

The anemometers, whose constants were observed in 1873 only, show the following agreement with the constants computed by Dohrandt's formula:

VII.—*Comparison of computed and observed coefficients.*

	B_2 (observed).	B_2 (computed).
Casella No. 317	2.755	2.629
Nowikof	2.898	2.505
Marino No. 4	2.548	2.450

From all these Dohrandt concludes that, except for Browning and Nowikof, the computed B_2 agrees as closely with the observed value as in the case of the experimental anemometers, and that in general the constant B_2 can be computed with a certainty of ± 3 per cent.

(c) *Third series.*—In 1882 Dr. Stelling renewed this investigation, and by using a gas motor was enabled to obtain velocities of 70 kilometers per hour, or much higher than those attained by Dohrandt. He renewed the investigation as to the amount of the induced current so far as to confirm the previous result that it is proportional to the velocity of the whirling-machine; he therefore adopted the same 5 per cent. correction. The results of his experiments, computed by both the two and three-term formula, are contained in the following table:

VIII.—*Results of comparisons by Stelling.*

Anemometer.	r	R	a_2	B_2	a_3	B_3	C_3
Casella No. 318	0.172	0.038	3.64	2.65	1.06	3.23	0.02379
Schadewell No. 2	0.201	0.043	4.24	2.71	0.41	3.48	0.02803
Richter	0.265	0.050	3.63	2.55	2.36	2.80	0.00855

118. COMPARISON OF RESULTS BY STOW AND DOHRANDT.

The following table contains (1) the constants obtained from the observations of Rev. F. Stow, given in detail in section 116, by assuming as the equation of the Kew anemometer $V=1.50+2.40v$, and (2) the values of B_2 computed for the same anemometers by Dohrandt's formula:

Anemometer.	Stow's comparisons.		Dohrandt's formula.	Difference.
	a	B_2	B_2	
A, Kew standard	1.50	2.40	2.34	+0.06
B, Casella (Greenwich Observatory)	1.65	3.07	2.63	+0.44
C, Negretti and Zambra	0.14	3.33	2.50	+0.83
D, Adie	0.30	3.25	2.38	—0.13
E	0.00	3.53	2.32	+1.11
F	0.58	3.10	2.54	+0.56
G	0.18	3.08	2.52	+0.56

For the Kew anemometer the value of B_2 is nearly the same in both cases, but for all the other anemometers Dohrandt's formula gives values very different from those resulting from Stow's comparisons, discrepancies too large to result from any error in the assumed value for the friction term a of the Kew standard. The irregularities in the last column seem to me very likely to arise in part from the fact that anemometers in the open air (as were those of Stow) are subject to very local and large irregularities in the wind, so that this is not a refined method of comparing them, unless the observations extend over a period long enough to include equal amounts of wind from all directions.

Again, the Casella anemometers tested by Dohrandt were identical in size with the anemometer B of Stow's comparisons, whence the anemometer B, reduced by Dohrandt's observed constants for that instrument, may be used as a standard to obtain the constants of the other instruments in the following manner:

The values of B_2 obtained for the Casella anemometers in the St. Petersburg experiments were as follows:

1873, Casella No. 317, 2.755.
Casella No. 318, 2.874.
1878, Casella No. 318, 2.736.
1882, Casella No. 318, 2.650.

Using the mean of the values obtained in 1873 there results for the Casella velocities $V=1.37+2.814v_c$, with which as a standard Dohrandt has computed the constants for the remaining anemometers compared by Stow (see Wild, Bericht über Anemometrie, 1879). The resulting values for the Kew anemometer were $B_2=1.99$ and $a_2=2.48$; this value of B_2 differs largely from that obtained by Dohrandt for the similar instruments, "Munro" and "Adie," in his comparisons of 1878 (see Table VI), and from the value 2.34 derived from his formula.

This failure of the English observations to harmonize with Dohrandt's work is sensibly greater than the discrepancies appearing in the different parts of the St. Petersburg work itself. For example, in the latter work the constant B_2 for Casella No. 318 ranges in the different determinations from 2.65 to 2.87 and that for the Adie anemograph ranges from 2.06 to 2.30. These differences in the results at St. Petersburg may arise partly from errors in the experiments and comparisons and partly from the fact that the empirical formula

$$V=a_2+B_2v$$

does not represent with sufficient accuracy the equation of the anemometer. In fact, the agreements as to the factor for the Kew pattern must be considered as partly a matter of chance, and neither English nor Russian experimenters can be held to have as yet established a constant coefficient, B_2, for anemometers in the open air reliable to within 10 per cent. of its own value, though possibly the relation between the coefficients for different anemometers may have been fixed to within 5 per cent. by the Russian investigators.

It is evident that further experimental work is needed, and the experiences here detailed show that arrangements must be made to use a larger series of sizes and ratios in the dimensions of the cups and arms, and that the effects of such gusts as occur in open-air exposures, of lateral friction in anemometer spindles, of direction of rotation and of whirl, of the length of the whirling arm, of proximity to walls and obstacles, and of induced currents must be more carefully studied.

Finally, it is evident that as "eddy" friction can be greatly reduced by selecting a fair-shaped body for high velocities, while viscous friction and instrumental friction can not possibly be avoided, therefore our object should be to diminish the former, and with it the term depending on v^2. To accomplish this I must repeat my suggestion that >-shaped troughs or hemi-cylinders, open at each end and with their axes placed radially, will, as shown by Edgeworth (Phil. Trans, 1783), make an equally effective anemometer, whose theory and formula must undoubtedly be much simpler than that of the Robinson hemispherical cup form.

119. THIESEN'S THEORY OF THE ACTION OF THE ROBINSON CUP ANEMOMETER.

In connection with Dohrandt's experimental investigation Thiesen conducted a mathematical study in order to acquire at least a rational formula whose constants can be deduced from proper experiments and by the use of which the deduction of the wind velocities from the observed data might be made more reliable. In this work he was only partially successful and continued failures to accomplish this result must be regarded as indicating that the working of the instrument is too complex to allow of attaining high accuracy in the results and that it is very desirable to adopt some simpler apparatus, as suggested at the close of the preceding section, which will offer an equally efficient instrument and a simpler hydrodynamic problem. However, the wide use of the cup anemometer makes it important that all possible light should be thrown on its mode of action, therefore the following condensation of the elaborate analysis of Thiesen is here given:

Let v be the linear velocity of the center of the cups.
w the linear velocity of the wind.
z the ratio $\frac{v}{w}$.
r the distance of the center of the cups from the axis of rotation, the so-called arm of the anemometer.
R the radius of curvature of the hemispherical cups.
y the resultant velocity of the wind and the cups.
q the ratio $\frac{y}{w}$.
θ the angle at which the wind meets the arm of a cup.
ψ the angle which the resultant of moving wind and cup makes with the arm of a cup.
ε the density of the air.

(*a*) For the anemometer cup at rest the normal pressure is proportional to $\varepsilon w^2 R^2 F(\theta)$, if we consider only fluid friction and neglect viscosity and the elasticity of the air.

When the anemometer cup is in motion the pressure is proportional to $\varepsilon w^2 q^2 R^2 F(\psi)$, where $F(\theta)$ and $F(\psi)$ are undetermined functions of the angle between the wind and the surface of the cups, and we omit any resistance, such as viscosity, depending on the first power of the velocity.

If the point of application of the pressure be assumed to be at the center of the cups the moment of the gyratory force will be $\varepsilon w^2 q^2 R^2 F(\psi) r$. This, when summed for the four cups and neglecting friction, is equal to the product of the moment of inertia M of the instrument around its rotation axis into the acceleration $\frac{d^2\theta}{dt^2}$ of its angular velocity.

Recalling that

$$\frac{d\theta}{dt}=\frac{v}{r}=\frac{wz}{r}$$

we have, after multiplying by $d\theta$, the equation

$$M\frac{w^2}{r^2}zdz=\varepsilon w^2q^2rR^2F(\psi)d\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (I)$$

For a condition of equilibrium or uniform steady motion the acceleration must be zero, consequently the left-hand member must vanish and we shall have

$$0=\int_0^{2\pi}q^2F(\psi)d\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (II)$$

(*b*) Assume that z may be considered approximately constant during a rotation, which will be more nearly the case the greater the moment of inertia and the greater the number of the arms.

From the geometrical relation subsisting between v, w, y, θ, and ψ when z is constant we have

$$d\theta=\frac{qd\psi}{\sqrt{(1-z^2\sin^2\psi)}}$$

and the vanishing integral takes the form

$$0=\int_0^{2\pi}\frac{q^3F(\psi)d\psi}{\sqrt{1-z^2\sin^2\psi}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (III)$$

This integral is to be developed.

$F(\psi)$ is a function which may be developed in sines and cosines of multiple arcs.

Since $F(\psi)=F(-\psi)$ the sines disappear from the development, and we may place

$$F(\psi)=A_0+A_1\cos\psi+A_2\cos 2\psi+\text{ etc.}$$

in which the constants A are known quantities to be determined from experiments on the pressure of the wind on inclined surfaces. We have, therefore, to evaluate the sum of a series of integrals of the form

$$J_n=\int_0^{2\pi}d\psi\frac{q^3\cos n\psi}{\sqrt{1-z^2\sin^2\psi}}$$

in which q is known in terms of z and ψ from the following equation derived from the simple composition of motions:

$$q=\sqrt{1-z^2\sin^2\psi}-z\cos\psi$$

Substituting this value of q

$$J_n=\int_0^{2\pi}d\psi\cos n\psi\left(1+z^2+2z^2\cos 2\psi-z\cos\psi\times\frac{3-z^2+2z^3\cos 2\psi}{\sqrt{1-z^2\sin^2\psi}}\right)$$

If we place

$$J_n'=\int_0^{2\pi}d\psi\frac{\cos n\psi}{\sqrt{1-z^2\sin^2\psi}}$$

and recall that J_n' will vanish when n is odd, then J_n, except J_0 and J_2, vanishes when n is even. For odd values of n

$$J_n=-\frac{z}{2}\left[z^2(J'_{n+3}+J'_{n-3})+3(J'_{n+1}+J'_{n-1})\right]$$

The value of J_n' is obtained by a development of the radical in powers of z^2. If we place $z^2=4a^2$, then is

$$\begin{aligned}
\frac{1}{2\pi}J_0'&=1+a^2+\frac{9}{4}a^4+\frac{50}{8}a^6+\frac{245}{16}a^8+\frac{1134}{32}a^{10}+\ \ldots\\
-\frac{1}{2\pi}J_2'&=\frac{1}{2}a^2+\frac{12}{8}a^4+\frac{75}{16}a^6+\frac{392}{32}a^8+\frac{1890}{64}a^{10}+\ \ldots\\
\frac{1}{2\pi}J_4'&=\frac{3}{8}a^4+\frac{30}{16}a^6+\frac{196}{32}a^8+\frac{1080}{64}a^{10}+\ \ldots\\
-\frac{1}{2\pi}J_6'&=\frac{5}{16}a^6+\frac{56}{32}a^8+\frac{405}{64}a^{10}+\ \ldots\\
\frac{1}{2\pi}J_8'&=\frac{7}{32}a^8+\frac{90}{64}a^{10}+\ \ldots\\
-\frac{1}{2\pi}J_{10}'&=\frac{9}{64}a^{10}+\ \ldots
\end{aligned}$$

and therefore

$$\begin{aligned}
\frac{1}{2\pi}J_0&=1+4a^2\\
\frac{1}{2\pi}J_2&=4a^2\\
\frac{1}{2\pi}J_1&=-3a-\frac{3}{2}a^3-\frac{2}{8}a^5-\frac{3}{16}a^7+\frac{66}{32}a^9+\frac{434}{64}a^{11}+\ \ldots\\
-\frac{1}{2\pi}J_3&=\frac{5}{2}a^3+\frac{5}{8}a^5+\frac{9}{16}a^7+\frac{172}{32}a^9+\frac{1042}{64}a^{11}+\ \ldots\\
\frac{1}{2\pi}J_5&=\frac{7}{8}a^5+\frac{21}{16}a^7+\frac{180}{32}a^9+\frac{1055}{64}a^{11}+\ \ldots\\
-\frac{1}{2\pi}J_7&=\frac{9}{16}a^7+\frac{93}{32}a^9+\frac{62}{64}a^{11}+\ \ldots\\
\frac{1}{2\pi}J_9&=\frac{19}{32}a^9+\frac{205}{64}a^{11}+\ \ldots\\
-\frac{1}{2\pi}J_{11}&=\frac{29}{64}a^{11}+\ \ldots
\end{aligned}$$

Since, in general, a is equal to about one-sixth, we may neglect the terms involving values beyond its third power.

The equation of condition becomes then

$$0=A_0-3aA_1+4a^2(A_0+A_2)-\tfrac{1}{2}a^3(3A_1+5A_3)+ \quad . \quad . \quad (IV)$$

from which, when A_0, A_1, etc., are known, a is easily computed, whence results the desired value of the ratio z, if it be really constant, *i. e.*:

$$z=2a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (V)$$

(*c*) To this value a number of corrections are applicable:

First. A correction due to the fact that z is not constant, as previously assumed, but is a periodic function of θ. For a four-armed anemometer Thiesen makes this correction to the average value of z to be a factor of the form $1-\alpha^2c$, where

$$\alpha=\frac{\varepsilon r^3R^2}{M}$$

and c is a factor whose value may be determined.

Second. A correction depending on the inaccuracy of the assumption that the point of application of the wind pressure is at the centers of the cups, whereas it is at a variable distance from the center depending on the angle ψ, and may be expressed by the form $J(\psi)R$; for r in formula (I) we have, therefore,

$$r\left[1+\frac{R}{r}J(\psi)\right]$$

Developing in sines of multiple arcs the term $\frac{R}{r}$ drops out and leaves for the first approximation a term in $\frac{R^2}{r^2}$, neglecting higher powers.

The correction therefore to the average value of z may be expressed in the form of a factor

$$1+b\frac{R^2}{r^2}$$

where b is to be determined from the observations.

Third. A correction due to the resistance of the arms of the anemometer. This introduces into the right side of the differential equation (I) a term containing their dimensions, and the correction to the expression for z is shown to be a factor of the form

$$\left[1-c\frac{\rho^2}{R^2}\left(1-\frac{R}{r}\right)\right]$$

in which ρ is the thickness of a section of the arms.

(*d*) The assumption that the wind pressure is strictly as the square of the velocity continues as a first approximation, and therefore these three correction factors give the following expression for the ratio of the mean rotation velocity of the cup centers to the velocity of the wind:

$$z=2a\left[1-\alpha^2e+b\frac{R^2}{r^2}-c\frac{\rho^2}{R^2}\left(1-\frac{R}{r}\right)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(VI)}$$

in which a, b, c, and e are constants, positive with the exception of b, whose values remain undetermined, $\alpha=\frac{\varepsilon r^3R^2}{M}$, ε=the density of the air, R the radius of hemispherical cups, r distance from axis to cup centers, M the moment of inertia of the instrument around its axis, and ρ the mean thickness of the arms.

(e) Up to this point the frictional forces have not been considered, and, in so far as by them z is made a function of the velocity itself, our problem (to determine the mean wind velocity for any period without the knowledge of the velocity prevailing at each moment) is, from an analytical point of view, illusory. These frictional forces have a double origin and consist of the viscosity of the air and the frictional resistances within the anemometer; the former is neglected by Thiesen, the latter is approximately assumed by him to be proportional to the velocity of rotation wz. The coefficient of friction is made up of two parts: (1) That due to the weight of the instrument (neglecting the slight effect of inclination of the wind to the horizon) and which is constant; (2) that produced by that component of the wind pressure on the cups which passes through the axis, and thereby develops lateral friction on the side bearings of the spindle. Since this part is produced by the pressure of the wind it is proportional to w^2.* The resulting total moment of the two frictions is therefore of the form $-wz(\beta+\gamma w^2)$, where β and, approximately, γ may be considered as constant. The differential equation (I) for determining z by adding this term becomes.

$$zdz=\alpha\left[q^2F(\psi)-z\frac{\beta+\gamma w^2}{\varepsilon wrR^2}\right]d\theta \quad . \quad . \quad . \quad . \quad . \quad \text{(VII)}$$

The effect of this friction term is to add to that term in the development of $q^2F(\psi)$ in z and θ that contains the first power of z independent of θ, the quantity

$$-\frac{\beta'}{w}+\gamma' w$$

where, for simplicity, we have inserted

$$\beta'=\frac{\beta}{\varepsilon rR^2} \qquad\qquad \gamma'=\frac{\gamma}{\varepsilon rR^2}$$

* If Thiesen had also considered the compressibility and the viscous friction this would have added to the term given in paragraph (a), others depending on the temperature of the air, on the surface or perimeter of the cups, and on the first power of the ratio q; but omitting this it results as above.

Now, the coefficient of z in the development of $q^2F(\psi)$ equals

$$-\sin\theta F'(\theta)-2\cos\theta F(\theta)$$

and the constant term is $-A_1$, or for an n-armed anemometer $-nA_1$. To A_1, therefore, must be added the quantity

$$\frac{1}{n}\left(\frac{\beta'}{w}+\gamma' w\right)$$

This is consequently also required in the equation (IV) which determines a. Thus a becomes

$$a\left[1-d\left(\frac{\beta'}{w}+\gamma' w\right)\right]$$

where d is a factor to be determined by the observations; neglecting the higher powers of the quantity multiplied by d there results

$$d=\frac{1}{n}\cdot\frac{3a+\frac{3}{2}a^3+\quad\cdot\quad\cdot\quad\cdot\quad\cdot}{3aA_1-Sa^2(A_0-A_2)+\frac{3}{2}a^3(3A_1+5A^3)+\text{ etc.}}$$

For very small wind velocities near or below that for which the anemometer begins to move the assumption made above as to the friction does not hold good. For if the friction forces are strictly proportional to the velocity of rotation the anemometer could not come to rest. A small constant term must therefore be added to express the general principle that the work done by the extraneous forces must not be less than a small positive quantity, if the anemometer is to keep in motion. Taking this into consideration there results for the dependence of z on the wind velocity w the following:

Up to a certain small value of w, z has the constant value zero; then it rapidly increases until for $w=\sqrt{\frac{\beta'}{\gamma'}}$ it has reached the value

$$z=2a(1-2d\sqrt{\beta'\gamma'}\quad.\quad.\quad.\quad.\quad.\quad.\quad.\qquad\text{(VIII)}$$

all which follows from the formula for z that holds good at these values, viz:

$$z=2a\left[1-d\left(\frac{\beta'}{w}+\gamma' w\right)\right]\quad.\quad.\quad.\quad.\quad.\quad.\quad.\qquad\text{(IX)}$$

From this maximum value z again diminishes, and for $w=\infty$ vanishes.

The consideration of instrumental friction in the theory of this anemometer shows that the instrument can never give precise results, for as soon as z has to be considered as dependent on w, the object of the instrument becomes (analytically) unattainable, or, in other words, without the knowledge of the wind velocity prevailing at each moment, it is impossible to obtain the mean velocity for a period of time. It is only possible to give certain limits, between which the mean wind velocity must lie.

(*f*) If the wind velocity w is dependent on the time t we have $\int_0^T w dt$, for the total movement in any period T, and the mean wind velocity

$$W=\frac{1}{T}\int_0^T w dt$$

This is the quantity that is to be determined by the cup anemometer. Observation gives the number of anemometer turns during this period, and thence, by multiplying by $2\pi r$, is obtained the path traveled by the centers of the cups at the average velocity V,

$$TV=\int_0^T wz dt$$

In order that in general W shall be determined by V then there must exist between w and z only a relation of the form

$$z=\frac{1}{i}\left(1-\frac{h}{w}\right)$$

for only in this case can one integral be expressed by the other without involving indeterminate quantities.

In this case $W=h+iV$; but Dohrandt's observations give

$$z=\frac{1}{i}[1-\frac{h}{w}+f(w)]$$

therefore

$$W=h+iV-\frac{1}{T}\int_0^t wf(w)dt$$

or

$$W=\frac{h+iV}{1+f_0(w)}$$

where all that can be said of the value of $f_0(w)$ is that it must be between the largest and smallest values which the function $f(w)$ can assume, and that it is conditioned by the way in which w depends on t. Upon these limits, between which $f(w)$ can vary, depends the accuracy with which W can be determined; but if the variations of the velocity of the wind are rapid and exceed a certain limit the accurate determination of the mean wind velocity becomes impossible.

This result of Thiesen's thoughtful study confirms the conclusion stated at the close of section 118 as to the necessity of seeking to so modify the cups as to make the rotations depend less than now on the square of the relative velocity.

120. ROTATION ANEMOMETERS AT SEA.

The force of the wind at sea is estimated almost universally by mariners in terms of the Beaufort scale (see Chapter XIV). In this scale as originally used the object of observation is the varying effect of the wind on the sails of sailing vessels, and experienced seamen are able to observe such relative effects of the wind with great precision and uniformity of results. With the introduction of steam-ships, however, the amount of sail and its appearance no longer continues to be an index of the wind velocity, and the observer is unable to designate the proper Beaufort number with the accuracy previously attained. It is therefore desirable, if possible, to measure the wind velocity instrumentally. To this end experimental observations on shipboard have been made with the Robinson anemometer,* the general result being to indicate that, under proper establishment and supervision, good results may be obtained.

(c) *Smyth's method.*—The first systematic application of the Edgeworth-Robinson anemometer to observations at sea was devised by Prof. C. P. Smyth (see the Transactions of the Royal Society of Edinburgh, XVI, p. 455) in 1847. Having tested a number of cup anemometers and assured himself that the best radii for the cups and the arms were about 2 and 6 inches, respectively, he proposed to observe the apparent direction of wind on shipboard, the anemometric velocity, the ship's direction, and its velocity per hour. Then, to avoid computations, he proposed the following mechanical device for resolving the parallelogram of motions.

Let AB, in the accompanying Fig. 67, represent the direction (towards which) and the velocity of the ship's motion, as set off on a graduated bar, to which the movable bars AC and BC are hinged at A and B, respectively. Set AC so that the angle CAB is the apparent bearing, viz, the relative direction from which the wind appears to come. Adjust the angle CBA so that BC shall cross AC at that point on the scale AC that corresponds to the apparent wind velocity. Then will the angle CBA be the true bearing from which the wind is coming, and the length CB as given on the scale will be the true velocity. The bearing CBA, added to the direction from which the ship is moving, will give the true direction from which the wind comes.

(b) *Welsh's method.*—An early proposition for the use of the Robinson anemometer at sea is due to Mr. Welsh (see the Report of the British Association, 1856, II, page 38). The following is an extract from a let-

* The more important references are to the following memoirs:

MOHN, H.: Den Norske Nord-Atlantiske-Expedition, 1876–'78. Meteorologie. Christiania, 1883.

RYKATSCHEW, M.: Beobachtungen der Richtung und Stärke des Windes auf Schiffen. Repertorium für Meteorologie, Bd. 8, 1881.

WALDO, F.: Anemometer Observations at Sea, Signal Service Monthly Weather Review, p. 31, January, 1887.

ter addressed by him to the chairman of the Kew committee, describing his method:

By means of a portable Robinson's anemometer, provided with a means of observing the total number of turns made by the rotating part in any given time, observe the apparent velocity of the wind and record it in knots per hour. By an anemoscope of any kind register the apparent direction of the wind. From the log-book take the rate and direction of the ship's motion. On a slate or other similar surface scratch a permanent compass circle. Set off from the center of the circle on the radius of the direction of the ship's head by any convenient scale the number of knots per hour the ship is going. From this point draw a pencil line parallel to the direction of the wind as observed by the anemoscope (*i. e.*, the apparent direction to which the wind is going). Set off on this line the number of knots per hour as shown by the anemometer. Draw a line from the center of the circle to the last point. The length of this line by the scale adopted gives the true velocity of the wind, and its direction (carried backwards) shows the point from which the wind is coming. A parallel ruler divided on the edge is all that is required besides the slate. It would be easy enough to contrive some mechanism to save the trouble of drawing the lines, but it would not, I believe, be any real simplification, and would increase the expense. The train of indicating wheels might be so arranged that they at once indicate knots per hour without reference to the table, and can readily be set to zero for a fresh observation.

[NOTE.—The author's expression, "knots per hour," has been retained here, although it is tautological, as a knot is a velocity of 1 nautical mile per hour, and therefore nautical records are kept in "knots" simply.]

(*c*) The following is a fuller development of this subject, based partly on the works of Rykatschew, Mohn, and Waldo:

The direction and velocity of the wind as felt on board a ship in motion are, in general, both different from the true direction and velocity, owing to the motion of the vessel itself. From the observed direction and velocity the true result is to be computed.

Let the rate and direction of a vessel (see Fig. 68) be indicated by the line *ab*, then this motion acts to produce a head-wind, which will be represented by *ad*. If the true wind is blowing with velocity and direction *ac* the resultant apparent motion of the air felt on board will be *ac*.

Observation having given the apparent wind velocity w the rate of the ship s, and the angle γ, between the direction of the apparent wind and the course of the ship, the true wind velocity W may be obtained from the triangle *adc*, either by numerical computation or geometrical construction.

Convenient formulæ for the solution of the triangle are given by ordinary trigonometry, viz, compute $\frac{1}{2}(\alpha-\beta)$ from

$$\tan\frac{1}{2}(\alpha-\beta)=\frac{s-w}{s+w}\cot\frac{1}{2}\gamma=\frac{\frac{s}{w}-1}{\frac{s}{w}+1}\cdot\cot\frac{1}{2}\gamma$$

Thence α is found from $\alpha=\frac{1}{2}(\alpha-\beta)+90^\circ-\frac{1}{2}\gamma$; thence the full velocity of the wind is found from

$$W=s\frac{\sin\gamma}{\sin\alpha}=w\frac{\sin\gamma}{\sin\beta}$$

In cases where the apparent wind direction γ is not observed, but, as is usual in ships' logs, the actual direction β, either true or magnetic, is estimated by observations of waves and froth, we have the formula

$$W = \sqrt{w^2 - s^2 + s^2 \cos^2 \beta} - s \cos \beta$$

where β is the angle between the actual direction of wind and the course of the vessel.

The apparent wind direction may be observed with the compass by turning the face to the wind and projecting this direction on the compass card. The apparent direction of the smoke, or of a vane or pennant attached to the mast or to a pole, may be also similarly observed and transferred to the compass.

The true rate of the ship is determined by nautical methods. Small errors in the observed quantities, as well as in the observed wind velocities, introduce corresponding errors in the computed true wind velocity.

The preceding analysis considers only the effect of the forward motion of the vessel on the apparent velocity as observed on board.

If this velocity is measured by a Robinson anemometer its indications will require special corrections for the rolling, pitching, and slewing motions of the vessel and the average careen.

(1) *Angle of roll and pitch.*—If the anemometer spindle is fixed to the vessel, and rolls and pitches with it, then it is continually oscillating about an average position through an angle depending on the type of construction and the loading of the vessel and the roughness of the sea. The average careen of the spindle, depending on the direction of the wind and on its force, introduces an error, since the anemometer is standardized only with its axis vertical. If the anemometer is hung on gimbals, so that the wind does not tip it, then its axis oscillates only slightly and around a vertical position. By thus mounting it the error incident to the careening of the spindles is approximately eliminated.

(2) *Linear roll and pitch.*—In proportion as the anemometer is distant from the center of oscillation of the vessel it is carried bodily to and fro through the air, and the linear distance thus traveled, by reason of the pitching and rolling, is added to the effect of the wind. The amount of this oscillation may be measured by apparatus devised by Antoine (1878), and by methods of reduction due to Bertin (1874), also by an apparatus due to Madamet (1876), and by a still better one due indpendently to W. Froude (Brit. Assoc. Adv. Sci., Report, 1872, II, page 243), and to Admiral Paris (see Revue Maritime et Coloniale, 1876 and 1887). By methods there given the amount of the motion may also be expressed approximately by an empirical rule, which gives the average angular movement of vessels of different types and for different conditions of the sea. Knowing the average angular movement the linear movement at the anemometer may be approximately computed.

(3) *Slewing and swinging.*—The movements of the vessel called "slewing," as it glides down the side of a wave, and the movement called "swinging," when the vessel rides at anchor, introduce slight additions to the path of an anemometer that may be generally neglected, but if in any special case it is desired to measure their effect this may be approximately done by establishing anemometers at the same height above the sea, but at the stern, centre, and prow of the vessel, namely, at three different distances from the axis of angular motion.

(4) *Altitude and location.*—The determination of the wind velocity at several altitudes above the land shows a steady increase in velocity up to considerable heights, which increase is usually attributed to the frictional resistance or drag of the air on the ground. It is desirable to determine the relative velocity of the wind at different altitudes above the ocean, but anemometers placed at different heights on the vessel, even if the exposures be perfectly comparable, are differently affected by the rolling and pitching, and these must be fully allowed for before any inference can be drawn as to the relative velocities of the wind. If the effects of rolling and pitching are proportional to the total angular movement α and to the heights d, d', and d'' above the center of buoyancy then the records A_0, A_1, and A_2 of three anemometers will be

$$A_0 = W_0 + \alpha d \qquad A_1 = W' + \alpha d' \qquad A_2 = W'' + \alpha d'' \quad . \quad . \quad (1)$$

where W_0, W', and W'' are the true wind or apparent winds at the three heights.

With these conditions is to be combined the law according to which the wind velocity diminishes. This law may be either an increase proportional to the height or it may be an increase such as the ordinate of a parabola bears to the abscissa.

For the first case we have

$$W' = [1 + \beta(d' - d)]\,W \qquad W'' = [1 + \beta(d'' - d)]\,W$$

If the parabolic law be preferred, we have

$$W' = W[1 + \beta'(\sqrt{d'} - \sqrt{d})]$$
$$W'' = W[1 + \beta'\sqrt{d''} - \sqrt{d})]$$

By substitution of either of these in the preceding equations (1) we obtain three equations with three unknown quantities W, β, and α. If α has been determined, as in paragraph (2), we need only two equations.

Owing to the interference of sails, pipes, deck-houses, and the hull of the vessel the lower anemometer should be at a considerable elevation; for example, at the cross-trees, and in any case the sails should be furled when a determination of β is attempted.

(5) If it is desired to determine at sea not only the horizontal movement, but also the vertical component due to any possible slight inclination of the wind, then an arrangement that could possibly give an

approximate determination would seem to be attained by the establishment at two altitudes of the triple anemometer described at the end of section 122 on analyzing and integrating apparatus.

121. ANALYZING AND INTEGRATING ANEMOMETERS.

Besides the temporary direction and velocity of any wind meteorology also needs the resultant direction and movement of all the winds that have blown during any given period.

This can be obtained by the numerical analysis and composition of the different winds or by a graphical process or by mechanical analysis.

(*a*) *Graphic method.*—In the graphic method, which is the simplest, the given directions and movements of the wind for any interval of time are plotted, as illustrated in Fig. 69, which shows that a particle moving with the wind would, during a given time, have described a path, *abcde* . . . . ., and finally stopped at *o*. The straight line *ao* is therefore the resultant, both as to length and direction. If during this time the wind has several times blown from the same direction, as shown by the plotted lines, these may be numerically added together before drawing the diagram. Therefore the most expeditious method of graphic construction consists in adding up all the movements for each direction, thus forming a rose of wind movement; then subtract the movements that are directly opposed, such as the north from the south, east from the west, etc., and combine the remainders together. When the actual movement has not been measured it is customary to assume that the velocities in all directions have been the same, and thus make the total movement of wind for each direction proportional to the length of time during which the wind has blown from that direction.

(*b*) *Lambert's method.*—The numerical method proposed by Lambert in 1777 is exactly equivalent to the preceding graphic process. Let α be the inclination of any wind to the meridian, counting westward from the south to the point from which the wind blows, and m the corresponding total movement of that wind. Then will the resultant of all winds be for the meridian movement $\Sigma m \cos \alpha$, positive southward; for the prime vertical movement $\Sigma m \sin \alpha$, positive westward. Hence the bearing and length of the resultant total movement are given by

$$\tan \beta = \frac{\Sigma m \sin \alpha}{\Sigma m \cos \alpha}$$

$$R = \sqrt{(\Sigma m \sin \alpha)^2 + (\Sigma m \cos \alpha)^2}$$

(*c*) *The mechanical method.*—Analyzing anemographs have been devised to facilitate the above computations, and are described by Radau, Wheatstone, Du Moncel, Kuhn, and Laughton.

In Beaudoux's and various modifications of this form the relative duration of each wind is recorded by surrounding the shaft of a wind vane with a horizontal circular trough divided into compartments corresponding to the wind directions and which are successively presented to a fixed spout, from which sand flows at a uniform rate. The quantity of sand in any compartment indicates the duration of the corresponding wind direction without regard to its velocity.

In Lomonosoff's register, which, however, never worked successfully, it was designed that the flow of mercury, instead of sand, should vary as the velocity of the wind; therefore the quantity of mercury in any compartment would be a measure of the total movement for the corresponding direction. A similar idea is embodied in Craveni's and Goddard's registers, in which the pressure of the wind regulates the flow of grain and water, respectively, into the compartments.

Von Oettingen.—The only analyzer at present in active operation that takes account of wind velocity and fully represents Lambert's formula is that devised by von Oettingen. In this construction the vertical spindle of a Robinson anemometer gives a continuous register of winds from all directions. Connected with the lower end of this spindle is a horizontal plate that revolves with a velocity proportional to that of the wind. Upon this plate there roll and slide four rollers, whose position upon the plate is determined by means of a wind vane. These rollers, by their friction on the plate, are set in rotation by it with velocities depending upon their positions, and as these latter depend upon the wind vane their resulting motions are directly proportional to the north, south, east, and west components. The rotations of the four rollers are recorded separately by electric contacts upon a sheet of paper.

For a full description of the rather complex apparatus see von Oettingen, Wind-Component-Integrator, Reportorium für Meteorologie, vol. 5, 1887, § No. 10.

The records of this anemograph give directly the total east, west, north, south movements, which are equivalent to the elements required in Lambert's formula for computing the resultant wind.

Stanley.—A simpler form of integrating anemometer is described (Quarterly Journal Royal Meteorological Society, 1883, Vol. IX, page 208) by A. Stanley, but it seems that this has not yet been constructed. In this form (see Fig. 70) the vertical axis *a* of a Robinson anemometer is so attached to a wind vane that it describes a cylinder about the shaft of the vane which carries it, and which is the principal vertical axis *jj* of the analyzer. The pressure on the Robinson's cups and axis *a* is sufficient to keep the latter on the leeward side of *jj*, even without using additional vane wings. At the lower end of *a* is a disk-wheel, *b*, and below that a Hooke's universal joint, *d*. The disk *b* communicates the weight of *a* and the cups to the friction wheels *cc*, so that *a* moves with great freedom. The joint *d* turns the rod *d*, whose lower end rests upon an agate bearing, *i*, attached to the axis *jj*, and moving with it, so that *i*, *d*, and *a* are always in the same vertical plane with *jj*. The rod *d* carries a conoid or beveled wheel, *e*, which moves upon the upper surface of the crown-wheel *f*, and thus moves the shaft *g*. A series of these shafts and crown-wheels is arranged in a circle completely around the vane-shaft *jj*, so that in any position of the vane and the beveled wheel *e* there will be a shaft, *g*, to be set in motion. Thus each shaft gives the total movement of the wind for the direction corresponding to the location of the shaft with respect to the vane-shaft *jj* at the center.

Electric analyzer.—Instruments embodying separately the following ideas were constructed by Wild-Hassler, Du Moncel, Salleron, and other European workers, and by A. Eccard for use at the Signal Office, Washington.

The Robinson anemometer with electric contacts ordinarily records each mile of wind upon the chronograph by a single pen. Let now eight or sixteen pens be placed side by side to record the movements for the respective winds, and let the wires for the respective magnets go thence to the contact disks of the electric wind-vane register (see section 87); from the shaft of the vane the wire goes to the Robinson anemometer and thence to the battery and ground; the contact disks of the vane determine which of the magnets and pens shall make a record, while the anemometer determines when and how often the dashes for the miles of wind shall be recorded. Thus the chronograph sheet gives, by its daily countings on eight or sixteen parallel lines the number of miles of wind from each point of the compass.

The contact disks of the wind vane are made substantially in the following manner: A horizontal disk is attached to the vane-shaft, and is divided into four or eight insulated sections; below this a similar horizontal disk, divided into the same number of insulated sections, but fixed in its position with reference to the meridian. A single contact spring joins the edges of the two disks; from each of the sections of the lower disk a wire proceeds to the corresponding electro-magnet at the register.

122. MEASUREMENT OF MOVEMENT IN VERTICAL PLANE.

The problem of constructing an anemometer which shall analyze all winds and give the three rectangular components, wherefrom to compute the resultant, would be still more simply solved if three anemometers could be so co-ordinated that each would give only the total movement of the air in or normal to its own plane of rotation; but such anemometers do not exist, and all anemometers are affected by winds that are inclined to the planes of action. Thus the Robinson integrates not only the movement of the air perpendicular to the spindle, but also some component of any wind inclined at an angle, α; similarly the Casella air meter gives a certain component of the wind whose direction is inclined to the axis at an angle, β.

Again, if the effective component were simply $v \cos \alpha$ then the indication of each of the three co-ordinate anemometers for a long period, including many winds, would be, respectively, the horizontal component, the meridional component, and the prime-vertical component, and these are what are required by Lambert's formula in order to deduce the resultant wind direction and movement. Unfortunately the anemometer cups do not give the simple cosine component, so that this method gives results whose accuracy depends upon the closeness with which this law obtains. No experimental determinations have been made of the relation between the relative indications of an anemometer for winds at different inclinations to the plane of rotation, but if for small inclinations the record varies nearly as the cosine, then the vertical component of motions in the atmosphere may be approximately determined with two rotation anemometers, one of which is attached to the wind vane.

(*a*) *Cacciatore's anemometer.*—An apparatus on this principle was set up by Cacciatore at the Palermo Observatory about 1830, and used for many years (see Annuario della società Meteorologica Italiana, I, 1878, 201).

Two anemometers were used, having four cylindrical paddles similar in pattern to those of Gärthner, Leutmann, and d'Ons-en-Bray, but without the exterior screen. One of these was established with the axis vertical, and so registered, like the ordinary Robinson anemometer, the horizontal motion of the wind. A second was so attached to the frame of a wind vane that its horizontal axis was parallel to the vane, and therefore nearly parallel to the wind. If the wind has an inclination, α, to the horizon the first anemometer is assumed to give, approximately, $H=R\cos\alpha$, and the second gives $V=R\sin\alpha$, whence $\tan\alpha=\frac{V}{H}$. Two assumptions underlie these formulæ: First, that the vane and its paddles follow the wind without following the ordinary oscillations of the vane. If, by reason of these oscillations, there is, as usual, a small average angle, β, between the horizontal wind and the axis of rotation, then the vertical paddles record an additional component, and the second anemometer gives $V=R\sin\alpha+H\sin\beta$. It follows, then, that this arrangement by Cacciatore does not determine α, but an angle that is a function of α and β.

The second assumption that underlies these formulæ is, as before stated, that the rotating anemometer accurately gives the components $R\sin\alpha$ and $R\cos\alpha$. This is probably not strictly true, but neither experiment nor analysis has yet indicated the precise value of the components thus measured.

Finally, a descending current gives the same result as an ascending current; these two, therefore, can not be discriminated.

The Cacciatore instrument is now replaced at Palermo by one in which the Robinson cups replace the cylindrical vanes. It is evident that the air meter or Woltmann's anemometer could be used in a similar manner, but to these, as to all other forms, the same two objections as above given will apply.

If the plane of rotation of the movable anemometer be vertical and in the direction of the wind instead of perpendicular to it, as in Cacciatore's arrangement, then will it give $V'=R\cos\beta$.

Considerable changes, therefore, in the angle β affect the results but little, and the approximate value of β can be taken from the register of the wind direction. The value of

$$R=\frac{V'}{\cos \beta}$$

being thus known, the value of α is deduced from

$$\cos \alpha=\frac{H}{V'} \cos \beta$$

(*b*) *Triplex co-ordinate anemometer.*—Whenever for a given form of rotating anemometer the effect of inclined winds has been determined satisfactorily, then it would seem that three such anemometers, set so as to revolve in three fixed planes at right angles to each other, should give three components from which by some method to be devised, there can be deduced approximately for each the values of

$$\Sigma V \cos \alpha \qquad \Sigma V \cos \beta \qquad \Sigma V \cos \gamma$$

The simplest method of doing this would consist in taking hourly readings from the sheets. During short periods of time α and V change slowly, so that the hourly resultants will be given with sufficient approximation; but in order to determine whether the wind is ascending or descending some form of rotating anemometer must be used (such as Woltmann's meter) that distinguishes between winds in direct and reverse directions, which is not the case with Robinson's anemometer.

CHAPTER XVIII.

THE MOVEMENTS OF THE UPPER CURRENTS.

123. ELEMENTARY METHODS FOR SMALL HEIGHTS.

According to the distinction made in the beginning of Section C we have hitherto treated of apparatus on or attached to the ground and entirely within the control of the observer, by means of which we determine the motion of the air at a small height above the observer's station. Whenever we exceed these heights, or, in general, when we launch our apparatus freely into the air, so that its motion is beyond our control, and can take place at any altitude, we have a new and important branch of meteorological study, viz, the movement of clouds and upper currents. The movement of the center of a free object like a balloon, smoke, or cloud is undoubtedly the same as the average motion of the mass of air immediately around it, and may be determined by some one of the methods enumerated in the following section.

One-fourth of the atmosphere is above the highest clouds or balloon ascensions, and the vertical height to which the atmosphere extends is as yet undetermined. Within this higher stratum important movements of the thin air doubtless do occur, but as yet no method has been devised by which to observe and measure them. The only direct indications that we have of the existence and movements of the air at very great elevations are given by the phenomena of the shooting stars and the trails left behind them, and to a limited extent by the transfer of smoke, dust, and vapor, from great volcanic outbursts, or of fine dust from arid regions. Further researches at these elevations are greatly to be desired, although, doubtless the most important phenomena are those that occur in the lower portion of our atmosphere.

(*a*) *Methods of observing free-floating objects at small heights.*—The discharge by day of smoke clouds from chimneys, and from bombs or rockets sent up into the air, and the discharge of small signal balloons by day or night are methods that have been employed for ascertaining both direction and velocity of the air currents; the bombshell sent up vertically to heights not exceeding 1,500 feet has of late been especially employed by the London Meteorological Office. Small signal balloons have long been used by aeronauts previous to the ascension of a large balloon, and this method, which is not expensive, is worthy of being made a special feature of all first-class meteorological stations and national services. The flight of such a balloon during five or ten minutes would be an item more important for the daily weather map than the usual local record of wind, and, as recommended by me in 1872, should be added to the ordinary weather telegram. To this end the small Mongolfier may be used in dry weather or at night time, and the small gas balloon in rainy or foggy weather and in the day time. Such balloons should be numbered consecutively and carry a corresponding numbered postal card, requesting the finder to inscribe the exact time and place of finding, and return it to the central office. The balloon should carry a suspended light thread from 50 to 500 feet long, at the bottom of which hangs suspended a light object. The observer can at any time ascertain the linear distance and altitude of the balloon by observing the apparent angular altitude of the upper and lower end of the vertical line thus carried by the balloon, the formula for the computation being as follows:

Let l be the length of the vertical thread.

α and α' the angular altitude of the upper and lower ends.

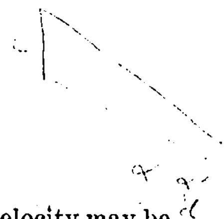
h the height of the upper end above ground.

d the horizontal distance from the observer.

Then will

$$d=\frac{l}{\tan\alpha-\tan\alpha'}=\frac{l\cos\alpha\cos\alpha'}{\sin(\alpha-\alpha')}$$

$$h=\frac{l\tan\alpha}{\tan\alpha-\tan\alpha'}=\frac{l\sin\alpha\cos\alpha'}{\sin(\alpha-\alpha')}$$



From two observations of the azimuth and altitude the velocity may be obtained by methods subsequently given (see section 127).

The height of a bank of fog or of the lower surface of a cloud may also be determined approximately by means of these balloons by the device described in 1877 in my suggestions for the guidance of observers on the schooner *Florence*, namely: Observe the rate of the vertical ascent and the gradient of the first 100 (or 500) feet as the balloon leaves the surface of the ground, and the gradient at the time when the balloon disappears in a cloud or fog, we easily compute the height at the moment of disappearance, as the rate of ascent may be assumed uniform.

(*b*) *Captive balloons and kites.*—A captive balloon by the strain upon its cable can give some idea of the horizontal direction and velocity of the wind at the level of the balloon, and observations of this kind have been made; but, as in the case of a kite, this method involves too many hypotheses, and it is best to utilize the captive balloon or the kite as the carrier of anemometric apparatus, and not as giving directly the force of the wind. This is the method that has been carried out by Archibald in his important measurements of the velocities at different altitudes. The heights of the clouds can, however, be determined by the disappearance of kites, as was first done by Alexander Wilson in 1749 in the course of his observations on temperature at high altitudes, when thermometers were carried up by a series of kites flying tandem, one above the other.

(c) *Aeronautic voyages.*—The observer voyaging in a balloon car determines his altitude by the barometer, and his horizontal movements by observation of the ground beneath him. When the ground is hidden by clouds his motion can be determined only by means of astronomical observations of latitude and longitude as when at sea. In this case he may use the cloud horizon as being approximately the same as the sea horizon, excepting only that his correction for dip of the horizon is very uncertain. He may with advantage employ an artificial horizon, consisting of a mirror floating on a small surface of water or attached to a pendulum, the balloon being in general too steady to require the gyrostatic apparatus designed by Piazzi-Smythe and recently revived by the French observers, and, on the other hand, too unsteady to allow of the simple mercurial artificial horizon. This elegant method of determining upper-air currents is, however, too expensive to be available, except on very special occasions and in limited regions.

124. METHODS OF OBSERVING CLOUD MOTIONS.

The following methods of determining the motions of balloons, clouds, or other objects, by means of observations at stations on the ground, are very generally available to ordinary observers:

(a) *Riccioli's or the direct trigonometric method.*—In this method, which was first used about 1650 by Riccioli, at least two distant observers must determine simultaneously the apparent angular altitude and azimuth of the same object. The angles are measured either with the surveyor's altitude and azimuth instruments or with the sextant, or, more conveniently, by an application of the photographic camera.

(b) *The photographic method.*—The application of photography for this purpose was contemplated by me in 1858, the plan being to photograph on one plate simultaneously two images of the same cloud reflected from two distant convex mirrors. In 1871 apparatus was purchased for photographic work by the Signal Office, but important work has been done with it only very recently by German and English observers. The photographic method was also suggested in 1860 by Dr. Zenker, who, however, first applied it in 1882, while Captain Abney, for the British Meteorological Office, has conducted his work with more complete appliances since 1883.

In 1881 J. Harmer proposed to place two photographic cameras at the extremities of a long horizontal beam that could be set perpendicular to the direction of the cloud, so that the measurement of the differences in the positions of the two photographs would give directly the parallactic angle precisely as in the use of the telemeter or in my method of 1858.

For the study of clouds by photography the camera is mounted on an altitude and azimuth stand, so that the direction of its optical axis can be determined in any position. The sensitive plate is crossed through the center by two lines, representing the horizontal and vertical planes, whose intersection determines the axis of collimation. By preliminary experiments in photographing a standard surface, crossed by a known system of lines, the amount of angular distortion due to the photographic lens is determined for each portion of the picture on the sensitive plate, and the corrections thus given enable one to convert the measured apparent angles into true altitudes and azimuths for any point in the sky. The application of photography has the great advantage that subsequent measurements may be taken at leisure and may cover every portion of a cloud, instead of being confined to one or two points. It also renders easier the identification of any spot observed in the two simultaneous pictures. The following methods of utilizing photography have been suggested:

1 If one camera is used the use of modern "very rapid" sensitive plates, as prepared by the methods of Vogel, Moureaux, and others, allows of taking upon the same plate, in the same position of the camera, two or more instantaneous impressions of a cloud at an interval of a few seconds. The measurement of the line joining the centers of two such pictures gives the angular velocity and position-angle of the apparent motion, which, with the angular azimuth and zenith distance, give all the data required for computing the azimuth and zenithal velocity of the cloud by the method given in section 140 on Vettin's cloud camera without photography.

2 If two cameras are used at a known distance apart the exposure must be simultaneous; this is best insured by employing a telegraph signal to move the shutters that expose the plates. In the absence of a connecting telegraph the observers must use accurate watches very carefully compared. If preferred the cameras may be habitually directed in some one definite position, such as the zenith, the sun, the horizon, etc., each of which has its special advantages, as shown by Dr. Zenker: (1) If pointed to the zenith the instrument may be permanently established with great accuracy, and the manipulation of the plate and the computation of the results become the easiest possible. The exact location on the plate of the observed zenithal or antipodal point is obtained by observing a star, the sun, or other object near the zenith in two positions of the camera, differing 180° in azimuth. The value in degrees of arc of a unit length on the plate is found by taking two or more successive pictures of the sun at known intervals of time. Two such cameras at a few hundred feet distance give the angular azimuths and the zenith distance of a cloud simultaneously photographed at the two places, whence the altitude is computed by the method of section 125. (2) If the cameras be pointed to the sun, or so that the sun always appears somewhere on the sensitive plate, we have always the means of

determining the azimuth at the time. By measuring from the sun's center as a base point the parallactic angle at the cloud is determined; this is particularly adapted to portable apparatus and traveling observers. (3) The camera may be mounted with its axis permanently horizontal. A vertical circle is thus dispensed with, and the reading on the plate, corrected for the optical distortion, gives directly the angular altitude of a cloud. A distant terrestrial object, whose azimuth is known, may always be included in the field of view, from which the azimuth of the cloud may be determined. By this arrangement the altitudes of the lower surface of the cirri and the tops of the distant cumuli may be ascertained quite accurately.

The apparatus for photographic study of the clouds, devised by Captain Abney, and established at Kew since 1883, is designated by him as the "photo-nepho-graph," but only a general account of it has as yet been received. It consists of twin cameras, 800 yards apart. Two sets of photographs are taken simultaneously, at an interval of about one minute of time. The numerical computations have been replaced by a mechanical and graphic process embodied in an apparatus devised by Professor Stokes, and called a "projector," by which the heights and motions of the clouds are obtained in a much simpler manner than by ordinary measurements and computations. The original sketch of this method is given by Professor Stokes at pages 22, 23, Report of the Meteorological Council to the Royal Society, March 11, 1886, but the further details will hardly be of value here until the method has been actually put in practice.

(c) *Kämtz' method.*—When there is but one observer with optical apparatus and one camera, as in the method proposed by Kämtz, the observer, after finishing the work at the station *A*, passes quickly to station *B*, observes, and then returns to station *A*. By interpolation between the two observations at *A* he deduces data that may be considered simultaneous with that obtained at station *B*, and thence computes the cloud height by section 125.

In order to secure perfect agreement between two observers Pouillet (1853) recommended that the observers have between them a railroad or other method of quick transit (the modern bicycle answers the demands perfectly), and that, having met at the central point to agree upon the point of observation, they should separate quickly, observe simultaneously twice, and then return to compare notes and assure themselves of the success of the work. The observations to be made in this case consist in simultaneous altitudes and azimuths or bearings. In 1872 Prestel executed a series of measures in which the two observers kept up constant communication by electric telegraphy.

Observations at two stations were made during the Swedish international polar expedition of 1882 to Spitzbergen, and also for several years subsequent at Upsala, and were greatly facilitated by the use of a telephone between the two stations that were about 500 meters apart.

This distance being, however, too short for accurate determinations of the higher clouds, a third station was subsequently added at a distance of 1,300 meters, by means of which the observations made in 1885 determined the altitude, horizontal and vertical velocity of clouds with a precision that left nothing to be desired. The Swedish observers Ekholm and Hägstrom conclude from the experiments that stations should be so arranged that a base of 500 to 2,000 meters shall be at their disposal, and that each observer while at work with his altitude and azimuth instrument shall have a telephone in his hand, in order to secure by conversation a perfect agreement as to the point under observation. They estimate that with a base of 1,300 meters and two theodolites of the special construction devised by Mohn for observing auroras the probable error of the resulting altitude of a cirrus is only 3 per cent., and that of the lower clouds about 2 per cent.; with a base of 500 meters the probable error of the upper clouds is 9 per cent., and of the lower clouds 4 per cent.

125. FORMULÆ FOR COMPUTATION.

Whenever by the preceding or other method we obtain the angular altitudes α and α', corrected for terrestrial refraction, if necessary, and azimuths A and A' of a point, K (see Fig. 71), as seen from two ends, S and S', of a known line, whose horizontal projection is l and whose azimuth is A_0, the computation of the linear distances d and d' and altitude h above S is given by the following formulæ:

Let $b = S_0S'$ = altitude of S' above the level of S.

$$B = A_0 - A \qquad B_1 = 180^\circ - (A_0 - A')$$

$$d = \frac{l \sin B'}{\sin (B + B')} \qquad d' = \frac{l \sin B}{\sin (B + B')}$$

$$h = d \tan \alpha \qquad h - c = d' \tan \alpha'$$

Owing to errors of pointing and observing the two values of h deduced from these formulæ will in general differ somewhat, and the combination of the four observations, α and B, α' and B', so as to deduce the most probable result, is to be made according to the following method used by Ekholm and Hägstrom:

The known quantities being as before l, α, B, α', and B', and assuming that the visual lines SK and $S'K$ do not intersect at the point K but pass each other at the points K_1 and K_2, so that their least distance apart is K_1K_2, whose center is C, we now first compute the following seven quantities:

$$l_1 = \cos \alpha \cos B \qquad l_2 = \cos \alpha' \cos (180^\circ - B')$$

$$m_1 = \cos \alpha \sin B \qquad m_2 = \cos \alpha' \sin (180^\circ - B')$$

$$n_1 = \sin \alpha \qquad n_2 = \sin \alpha'$$

$$\cos \theta = \sin \alpha \sin \alpha' + \cos \alpha \cos \alpha' \cos (180^\circ - B' - B)$$

Find r_1 and r_2 from the equations

$$\tfrac{1}{2}(r_1+r_2)=[\tfrac{1}{4}l(l_1-l_2)+\tfrac{1}{4}b(n_1-n)]\operatorname{cosec}^2\tfrac{1}{2}\theta$$

$$\tfrac{1}{2}(r_1-r_2)=[\tfrac{1}{4}l(l_1+l_2)+\tfrac{1}{4}b(n_1+n_2)]\sec^2\tfrac{1}{2}\theta$$

Knowing r_1 and r_2 we compute

$$x_1=l_1r_1 \qquad y_1=m_1r_1 \qquad z_1=n_1r_1$$
$$x_2=l_2r_2+l \qquad y_2=m_2r_2 \qquad z_2=n_2r_2+b$$

Whence we find

$$x=\tfrac{1}{2}(x_1+x_2) \qquad y=\tfrac{1}{2}(y_1+y_2) \qquad z=\tfrac{1}{2}(z_1+z_2)$$

Let verticals fall from K_1CK_2 to $K'_1K'_cK'_2$ and thence horizontals to $P_1P_cP_2$, then will these quantities represent respectively the following:

x=horizontal distance from S to P_c, which is the foot of the perpendicular K'_cP_c.

y=horizontal distance from K'_c to P_c.

z=vertical distance CK' up to a point that is midway on the shortest line that can be drawn, joining the two lines of sight SK_1 and $S'K_2$.

In order to be sure that the lines SK_1 and $S'K_2$ do really come very near actually crossing each other at the point that was intended to be observed the following formula should be computed:

$$\Delta=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

which gives the vertical length of the shortest line, and of which C is assumed to be the center.

An elaborate illustration of the great value of this method will be found in Mesures des Hauteurs et des Mouvemens des Nuages, par N. Ekholm and K. L. Hägstrom, Upsala, 1885, Nova Acta R. S. S.

(*a*) A computed altitude generally relates to a horizontal plane through the observer's station. If the cloud is very distant horizontally there must be added a correction for the elevation of the extended observer's horizon above the ground immediately beneath the cloud. This is given (see Fig. 72) by the following formula:

Let O be the center of the earth; we have, approximately,

$$\theta=SOK=\frac{SK'}{SO}\times 57.296^\circ$$

The triangle OSK'' gives $OK''=R+K_0K''=R\sec\theta$, whence $K_0K''=R(\sec\theta-1)=2R\sec\theta\sin^2\frac{\theta}{2}$. From the triangle $KK'K''$ we have $KK''=KK'\sec\theta$. Hence the total height of the cloud above the earth beneath it is

$$KK_0=h\left(2R\sin^2\frac{\theta}{2}\right)\sec\theta$$

(*b*) The accuracy of the determination of the altitude or distance by means of two observing stations depends upon the location of the cloud with reference to the line joining them. If instead of two stations, *A* and *B*, four, *A*, *A'*, *B*, and *B'*, could be occupied, so that *AB* is perpendicular to *A'B'*, or nearly so, we have then the most advantageous arrangement for determining both altitude and distance.

Instead of the four observers we may substitute one who shall successively occupy the four stations, at each of which he has only to observe one angle, namely, *KAC*, *KA'C*, *KBC*, *KB'C*. If the stations are at the corners of a square inscribed within a circle (see Fig. 73) the formula for the computation of *KC* and *KCB*, or still better, of *KK'*, and the azimuth of *CK'* become quite symmetrical and admit of very convenient arrangement.

(*c*) The labor of any method involving two distinct stations may be overcome if it be practicable to use the ordinary telemeter, in which two mirrors are so placed at a short horizontal distance apart that one observer, looking into both simultaneously, sees in each the image of the same cloud. The observation consists in measuring the angle (see Fig. 74) between the lines *KM* and *KM'*, the mirrors being so placed that *M'* is perpendicular to *KM*. The angle is measured by turning the mirror *M'* through a small angle until the image reflected from it coincides exactly with that reflected from *M*. This method is in principle the same as that adopted by Krecke (Utrecht, 1849), whose mirrors were about 2 meters apart. The difficulty in the use of the telemeter arises partly from the small angle to be measured micrometrically, and largely from the high magnifying power and small field of view, which render it difficult to identify the points observed in the clouds, but make the method easy of application when observing very definite terrestrial objects. When the parallactic angle *MKM'* has been satisfactorily measured the distance of the cloud is obtained by the simple formula

$$MK = mm' \cot mKm'$$

For clouds near the zenith this distance is approximately the same as the vertical altitude.

126. WARTMANN-BRAVAIS METHOD.

An elegant method of determining the successive positions of the cloud, whence the movement and altitude may be found, was devised independently by Wartmann (Geneva, 1842), and Bravais (Lyons, 1842), and Whewell (Cambridge, 1846), but it seems to have been extensively used by Bravais only. In this method the observer looks from an elevated position downward perhaps 50 meters upon a horizontal reflecting surface, such as a pond of still water or a level mirror (see Fig. 75), and with some altitude instrument observes the vertical angles *ZOK* and *ZOR* in the plane of the diagram, as well as the azimuth of the plane

ZOK. From such observations he computes the distance and altitude as follows: Let

$$BR = X = OB \tan BOR = a \tan (180^\circ - \zeta')$$
$$OR = OB \sec BOR = a \sec (180^\circ - \zeta')$$

The triangle *KOR* gives

$$KR = OR \cdot \frac{\sin (\zeta' - \zeta)}{\sin (\zeta' + \zeta - 180^\circ)}$$

The right angled triangle *KRK'* gives

$$KK' = h = KR \sin KRK' = \frac{a \sin (\zeta' - \zeta)(-\cos \zeta')}{-\sin (\zeta' + \zeta)(-\cos \zeta')} = \frac{a \sin (\zeta - \zeta')}{\sin (\zeta' + \zeta)}$$

The same triangle also gives *RK'*, which, added to *BR*, gives

$$BK' = \frac{2a \sin \zeta \sin \zeta'}{\sin (\zeta + \zeta')}$$

Two successive observations of time, altitude, and azimuth will thus give the exact location of the cloud, by which its velocity and direction of motion is computed by the following general method.

127. CONVERSION OF BEARINGS INTO MOTION.

Whenever by any method we know the azimuths A and A' measured from south to west, and the linear distances d and d' of the horizontal projections K and K' of a cloud or other object at two moments t and t', we determine by the following formula the actual horizontal movement *KK'* and the azimuth *AK* of that movement, also measured from south to west: Let

$$x = d \cos A \qquad y = d \sin A$$

$$x' = d' \cos A' \qquad y' = d' \sin A'$$

then we shall have

$$\tan A_k = \frac{y' - y}{x' - x} \qquad KK' \begin{cases} = \sqrt{(y' - y)^2 + (x' - x)^2} \\ = (x' - x) \sec A_k \\ = (y' - y) \operatorname{cosec} A_k \end{cases}$$

This solution enables the computer to secure the proper quadrant for the resulting direction by giving careful attention to the signs of the circular functions.

128. BRAVAIS METHOD.

The computations involved in the preceding paragraphs become simplified when we desire only the height of the cloud. To this end

Bravais always selected for observation a cloud spot reflected from the same point of the mirror R (Fig. 75), thereby keeping the angles ZOR and ORK constant. By this method the observer, after identifying in the sky the cloud seen in the mirror, has only to observe one vertical angle ZOK.

If the ordinary vertical circle be used the observed angle will be ζ, from which the height is computed by the formula for h in section 126.

If a sextant is used the angle of observation will be $\theta=KOR=\zeta'-\zeta$, and if we make the constant angle $ORB=k=\zeta'-90°$ the formula of section 126 becomes

$$h=\frac{a \sin \theta}{\sin (\theta+2k)}$$

where the only variable is $\theta=KOR$, so that the right-hand member of the equation can be tabulated for successive values of this variable, and the desired altitude can easily be obtained by interpolation.

For his own use Bravais devised the following special method of observation and computation:

To a vertical circle he attached a plane mirror, a portion of which is unsilvered, as in the sextant. The observer looking upon the silvered portion sees the cloud reflected therefrom, and looking through the unsilvered portion sees the reflection of the same cloud from the distant mirror or reflecting surface, R (Fig. 75). The two images are brought into coincidence by turning the vertical circle, so that its mirror makes a small angle, ω, with the horizon, whence

$$2\omega=\zeta+\zeta'-180°$$

With $k=\zeta-90°$, as in the preceding case, the formula of section 126 gives

$$h=a\frac{\sin (2k-2\omega)}{\sin 2\omega}$$

The use of the small horizontal mirror in this method makes it easy to identify the cloud.

If in the preceding methods we add to the determination of the vertical angles also the measurement of the apparent linear velocity, with which the reflected image crosses the distant mirror R, we have a simple method of determining the absolute velocity and direction of the motion of the cloud. This idea was carried out by Bravais in 1842; his mirror, consisting of a mixture of water and ink, was at a distance of 21.8 meters below him, and had a radius of about 1 meter. He observed n, the time in seconds required by the image to describe a path, l, of about 1 meter on the distant mirror, and recorded the azimuth of that motion, which is the same as the azimuth of the actual direction of motion of the cloud. Knowing the cloud height h, as above computed, and the vertical distance a of the mirror below him he deduced the linear velocity v of the cloud per second in its horizontal movement by the formula

$$v=\frac{l}{n}\times\frac{h+a}{a}$$

It may be noticed that the normal (in this case over 20 meters) from the observer to the mirror is the base of the angle OKR. If this distance is diminished, so that the angle becomes inappreciable, then we have an arrangement that can give us the correct azimuth, but only the apparent altitudes and velocities, and this idea was embodied by Aimé, in 1846, in his "reflecting anemometer" (Ann. de Chimie T., XVII, p. 498), and is the underlying principle of the modern nephoscope.

129. LAMBERT'S METHOD.

The simplest method of determining the absolute linear velocity of upper air currents is that first proposed by Lambert in 1773, and consists in observing from an elevated point the velocity of the shadow of a cloud moving over the landscape; for this purpose the observer is provided with a detailed topographic map of the surrounding region, on which he can locate the position of the cloud at any moment. The same observation thus gives at once the direction and linear velocity of the air current. As it is important to know also the altitude for which these data hold good the observer can determine it very easily from an observation of the apparent angular velocity of the same or associated clouds. For low clouds the apparent angular velocity can often be obtained with sufficient accuracy by timing their transit over an estimated angle of 15° or 30°, but for clouds far from the zenith, and for high clouds, the use of some measuring apparatus is necessary, and such will be found especially described in section 135.

Knowing the apparent angular velocity γ in a unit of time, and the linear velocity v, measured on the map for the same unit of time, we have the altitude $h=\frac{v}{\tan\gamma}$

130. BRANDES'S METHOD.

In Brandes's modification of Lambert's method, as published in 1820, a cloud (or hole in a cloud) casting a definite shadow is observed when it lies in the same vertical plane with the observer and the sun. The point P (see Fig. 76) upon which its shadow is cast is identified and located upon the map. The observer being elevated somewhat above the plane of the shadowed spot measures the zenith distance of the sun $ZOS=\zeta$, of the cloud $ZOK=\zeta'$, and of its shadow $ZOP=\zeta''$, while the map gives the linear distance BP, whence the horizontal distance BK'

and altitude KK' of the cloud above a horizontal plane through P are computed by the following formulæ:

$$PK=\frac{OP \sin KOP}{\sin PKO}$$

$$PK=\frac{BP \sec(\zeta''-90^\circ)\sin(\zeta''-\zeta')}{\sin(\zeta'-\zeta)}$$

$$BK'=BP+PK'=BP+PK\sin\zeta$$

$$KK'=PK\cos\zeta$$

The measurements of the sun may be simplified by using an apparatus equivalent to a sun dial, by which the observer reads directly the azimuth and length of the shadow of a gnomon.

If the sun is obscured the observer must compute ζ by knowing the time of his observation. In this, as in the following section, the computations may be replaced by a graphic construction.

131. FEUSSNER'S METHOD.

Feussner (1872) has developed an important method, suggested in part by the earlier methods of Brandes (1800), Wrede (1826), and Prestel (1863), which is entitled to a high rank on account of its simplicity. The execution of Feussner's method requires only one observer with a good pocket watch, a simple plumb-line, and an accurate map of the region, and gives all the accuracy that is attainable in the observation of anything so indefinite as a cloud. Brandes's method is restricted to clouds in the vertical of the sun; Feussner's method allows the clouds to be in any vertical plane.

In the accompanying diagram (Fig. 77) let the vertical through the cloud and observer be the plane $OKBK'$, where O is the location of the observer and K that of the cloud; let KP be the solar ray, casting a shadow on the ground at P; let OP be the visual ray to P, and BPK' a horizontal plane through the point P, which latter may be at any level below or above the observer. The field-work of the observer consists in the following: Standing behind his plumb-line, which may hang in the open air from the limb of a tree, and sighting beyond it upon the horizon he identifies some point by which he may locate a definite azimuth, $N'S'$, upon his topographic chart. Assuring himself that he identifies a given cloud and its shadow he proceeds as follows: First, records the time; second, locates the shadow spot P by a mark on his map; third, places his eye so that the plumb-line bisects the cloud K, and identifies some point in the landscape beyond the plumb-line, by means of which he draws upon the map the line BK', which gives him the azimuth of the cloud; fourth, bisects the sun by his plumb-line and similarly draws the line BS_0, representing the azimuth of the sun; finally, observes the time.

These observations and records are then to be immediately repeated in inverse order, namely, time, azimuth of sun, azimuth of cloud, location of shadow, time. The mean of each pair of observations corresponds to one complete simultaneous set. He now at his leisure reads off the azimuths S_0BK' and $PBK'=\alpha$, measured from the south point of the meridian line $N'S'$, and draws the line PK', so that the angle $PK'B$ shall be equal to $K'BS_0$. If the bearing of the sun is not observed directly, then the angle $S'BS_0$ can be computed, and the line PK' be drawn parallel to B through the fixed point P, whence the distances BK' and $K'P$ can be measured from the map.

The altitude of the cloud is computed by the formula

$$KK'=\frac{BP\sin\alpha\tan\gamma}{\sin\beta}=PK'\tan\gamma$$

The measurement of PK' should be compared with the computation as a check upon the work. We have thus a complete determination of the altitude, the distance, and azimuth of the cloud at the time t, and two such determinations, or still simpler, a second location of the shadow P' gives the means of computing the direction and velocity of motion of the cloud.

As the clouds change their shape rapidly it is often difficult to obtain two determinations from which to compute the intervening motion. I therefore suggest that it is sufficient to combine with one observation of absolute height, distance, or velocity a determination of the reduced linear velocity, as given by the nephoscope to be described in section 135. One observation with this instrument gives the true direction of horizontal motion of the cloud and the linear velocity $\frac{a}{n}$ of the motion of the cloud image, as seen in the mirror at the distance b below the observer's eye. The Feussner method gives KK' the elevation of the cloud above the plane of its shadow; a topographic map gives h the elevation of the shadow spot below or above the observer at O, whence the cloud is $H=KK'-h$ above the observer; we have, therefore, as in Bravais's method, the linear velocity V of the cloud and $\frac{a}{n}$ that of its image proportional to $H+b$ and b, respectively; whence

$$V=\frac{a}{n}\times\frac{H+b}{b}=\frac{a}{n}\left(1+\frac{H}{b}\right)=\frac{a}{n}+\frac{a}{n}\frac{H}{b}$$

132. CAMERA LUCIDA METHOD.

For zenithal clouds much of the trouble attending the use of a photographic camera may be avoided by adopting a portable camera lucida, in which the lens is placed vertically above a horizontal table within a small observatory, tent, or sentry-box (see Fig. 78). Below it is placed horizontally a properly ruled sheet of paper, so that the observer at O

can easily see and record the motions of the clouds that are depicted thereon. The central spot of the paper corresponds to the vertical axis of collimation of the lens. Two observers, with two similar cameras and measuring apparatus, in tents a few hundred yards apart and in telephonic communication, can rapidly accumulate a large number of observations of the motions and positions of the clouds within 30° of the zenith.

If above the lens just described there is mounted a mirror inclined at an angle of 45° to the vertical the observers can then simultaneously observe and determine the position and motion of a cloud near the horizon.

133. METHODS FOR OBSERVATIONS AT NIGHT.

The preceding methods generally fail at night, owing to the absence of the necessary shadows. In such cases if signal balloons can not be utilized any method of determining the altitude that may be found practicable may be combined with the nephoscopic observation. Among such altitude methods are the following:

(*a*) M. Paul de la Cour, in Denmark, 1860, determined the altitude of the under surfaces of layers of cloud (see Fig. 79) by observations of the apparent altitude of the patch of light reflected thereon from the lamps illuminating a distant city.

In this method he assumed that the illuminated cloud spot K was midway between himself and the source of light L, whence $h=\frac{1}{2}OL \tan \alpha$. This method can be relied upon only when there is no doubt as to the source of light and the angle of reflection.

(*b*) The present writer in 1872, among methods suggested for use in the Signal Service, proposed that the illumination be effected by a beam of light specially controlled by the observer and preferably vertical; the nearly parallel beams reflected from the mirrors of the calcium or electric-light apparatus are best, but feebler lights will do. In this method if I be the location of the light it is necessary to know only the distance OI (Fig. 72) and the apparent angular altitude of the spot. The method serves peculiarly well when the sky is covered with a uniform sheet of cloud or high haze, in which no definite points occur for the application of other methods.

134. BERNOULLI'S METHOD.

The method proposed by J. Bernoulli (1744), being specially applicable to the time of sunset and sunrise, can be used to fill in the gap between the observations by day and by night, although it is of inferior accuracy, owing to the assumptions that underlie its formulæ of reduction. This method consists in observing the time at which the sun's rays first or last touch a given cloud, whose angular altitude and azimuth is also observed at that moment. It is, however, extremely difficult to decide whether the cloud is illuminated by the first rays of the sun or by reflection from other portions of the sky, and even after the illumination by the direct rays of the sun seems to be assured it is still impossible to say whether the rays come from the whole sun, or the upper two-thirds, or the central half, etc. As a general rule, however, when in the morning the cloud has been rapidly growing bright, but not yet attained its full brightness, or when in the evening the cloud has begun to fade, but has only lost a little of its brightness, we may in both cases assume that at any moment the center of the sun is in the apparent horizon of that cloud. By running through the following computations for the moments when the cloud is brightest and faintest the observer may obtain an idea of the possible errors of his results. In a general way there must always be some uncertainty as to whether the rays of the sun that illuminate the cloud are grazing the sea horizon or the hills of *terra firma* or the top of a stratum of clouds. The observer, therefore, at best, will feel uncertain as to whether his computation gives him the altitude of the cloud above the top of some other clouds or above the sea. And this uncertainty is only slightly relieved by the consideration that if the tint of the observed cloud is decidedly pinkish the illuminating beam of light probably grazed the earth's surface, while if decidedly white the beam undoubtedly passed over the tops of clouds and above the dust and haze of the lower air.

If the sky is covered with a horizontal cloud stratum whose under side lighted up by the sun furnishes the points that are being observed, and the observer can obtain a number of successive determinations of their height, the agreement among themselves may serve to give a satisfactory determination of the general level of the lower surface. Fig. 80 illustrates the general method of computation.

Let A be the observer, AL his horizon, AB the ocean surface, w the cloud, h a point below the cloud on the earth's surface, and SBW the sun's rays touching the earth at B. The observed quantities are $LAW = \theta$, or the angular altitude of the cloud, and t the hour-angle of the sun corresponding to the time of observation, whence the sun's zenith distance is computed. If the sun and cloud are in the same vertical circle we have the geocentric angle ACB equal to $2LAB$, or to the zenith distance of the sun, as deduced from the observed hour-angle t less twice the horizontal refraction at B, and less 90°. But $ALW = ACB = \alpha$, because the including lines are respectively perpendicular. Therefore in the triangle AWB we know AB and the angle $BAW = \frac{1}{2}\alpha + \theta$ and $ABW = \frac{1}{2}\alpha$, whence we compute AW. The triangle WAC, in which two sides and the included angle are known, gives WC, whence $WH = WC - R$.

The complicated computation thus indicated is reducible to the following formulæ, which are given by Zenker in his Meteorological Calendar for 1887 as an improvement upon those given by Vettin in the Zeit. Oest. Gesell. Met., 1883, in which the effect of atmospheric refraction is approximately allowed for.

Let t_0 be the sun's hour-angle at the moment of disappearance on the sea horizon, computed from the data of the Astronomical Ephemeris, by taking account of parallax, refraction, solar radius, and dip. If the sea horizon is visible t_0 may be obtained from the observed time of sunset; t represents the same datum for the time when the sun is in the horizon of the cloud; $T=t-t_0$ represents the angle by which the earth has turned on its axis since sunset. Let H=the linear height of the cloud above the sea level beneath it, φ the latitude, δ the sun's declination, and R=the earth's radius. For clouds within 60° of the zenith the altitude is given by the following formula:

$$H=\frac{7}{4}R\cos^2\delta\cos^2\varphi\sin^2\frac{T}{2}\sin^2\left(t_0+\frac{T}{2}\right)\times\left[\frac{\sin^2\left(\frac{\alpha}{2}+\theta\right)}{\sin^2(\alpha+\theta)}\right]$$

in which the last bracket is negligible when the cloud is within 30° of the zenith. The effect of terrestrial refraction has been approximately allowed for by introducing the factor $\frac{7}{4}$ instead of that otherwise obtained, *i. e.*, $\frac{8}{4}$. This altitude belongs not merely to a cloud in the vertical plane between the observer and the sun, but to all those clouds on either side to which the sun sets at the same moment.

The altitude as above computed assumes the sun's rays to be tangent to the ocean at B. If this is not so the observer must determine the elevation h' of the mountain, plateau, or cloud to which they are tangent, and add to his computed H a correction which is approximately equal to $h'\sec\alpha$, but for which we may generally write h' itself, since α is rarely more than 5°. The uncertainty as to this last correction is undoubtedly the weakest feature of this method. A tabulation of the first part of the formula for every five days of the year and for every four minutes of time in the value of T, followed by a small correction representing the factor in [——], as is done by Zenker, renders the method very easy of application.

135. NEPHOSCOPIC METHODS.

The preceding methods contemplate the determination of the absolute position and motion of a moving cloud, but when this is not practicable the apparent and relative motions may almost always be determined. For this purpose, in 1845, Aimé devised an instrument, now known as the nephoscope, whose use is urgently recommended to all observers. If, as Aimé suggested, we combine with this some of the preceding methods for the absolute determination either of linear velocity by Lambert's method, or of height by Feussner's method, or of distance by trigonometrical methods, we have at once a complete solution of our problem. Moreover, the nephoscope is available for determining the apparent position of any celestial phenomenon, such as a halo, aurora, or zodiacal light, and is therefore an instrument of wide range and usefulness.

This instrument, whose name is due originally to Braun (1865), has been modified and used in several distinct ways by Marié-Davy (1870), Linss and Cecchi (1872), Fornioni (1880), and Finemann (1882). The latter comes back to the original form and use, as described by Aimé [see Ann. de Chimie (3), XXVII, 1846, p. 498.] The principles of the method of observation with the nephoscope are those of the method already mentioned (see section 126) as having been invented in 1842 by Wartmann and Bravais. This instrument has been widely used since 1872 in Italy. It is equally applicable to observations on land and sea, and is strongly recommended to all observers by the permanent committee of the International Congress of Meteorology.

The simple fundamental principle here utilized is as follows (see Fig. 81): When a horizontal cloud path, K_1K_2, is seen by the observer at O, reflected from the points $M_1 M_2$ on a horizontal mirror, the distance $M_1 M_2$ is parallel to $K_1 K_2$, and is shorter than it in the ratio of the vertical altitude OB to K_1K_1', or in the ratio of the distances OM_1 to M_1K_1, or in the ratio of the horizontal projection B_1M_1 to M_1K_1. The various methods of using this apparatus are essentially as follows:

(*a*) In Braun's method of 1865 a sight-point, P (see Fig. 82), is adjusted at a proper elevation above the mirror, the elevation being increased when the clouds move slowly, and the observer at O brings a horizontal circle, LL, with cross wires AB and LL, so that the center of the cross covers the reflection in the mirror of the sight-point, which latter covers the still more distant reflection K_1' of the cloud K. Holding the eye in this line $OFM_1P'K_1'$, and as nearly stationary as possible, the observer turns the circle LL, so that a graduated diameter, AB, coincides with the motion of the reflected clouds; he times the rate of progress, and thus determines an angle whose apex is at O by reading off the distance traversed on the scale AB. The accuracy of the resulting azimuth and velocity depends upon the steadiness with which the eye is always held in the same position. In this respect Braun's method of using the apparatus was inferior to that of Aimé, which latter was reinvented and used by Cecchi in 1872 and Linss in 1878. In Braun's method the observed horizontal linear distance AB described in t seconds gave an apparent velocity, c; from this, if H be the height of the cloud above the mirror, the actual cloud velocity v will be found by the formula $v=c\frac{H}{h}$. In order to express all observed apparent velocities in one standard Braun arbitrarily assumed as such standard the apparent velocity of a cloud 2,000 feet in altitude, whose absolute velocity is 1 foot per second, or the value of v, given by him, expresses the linear velocity in feet per second of a cloud 2,000 feet high, whose apparent angular velocity is the same as that of the cloud actually observed.

(*b*) Cecchi in 1872 adopted a form almost identical with that of Aimé in 1846, a full description of which, as given by Cecchi, is here quoted:

A horizontal disk of wood or metal, on the edge of which are inscribed the points of the compass (see Fig. 83), is fastened to a support on a foot furnished with three screws, which serve to level the instrument. On this disk, which has a diameter of 20 centimeters, is placed a plane circular mirror 15 centimeters in diameter, which, by means of a convenient pivot, fitted centrally into this disk, can be made to rotate horizontally. The mirror has on it a line through the center *ab*, and at the center a short traverse line, forming with the first a cross, as can be seen at *c*; these lines are drawn on the amalgam of the mirror. The line *ab* has also a certain number of divisions, which may, for example, be centimeters. Under the disk there extends a brass arm that can be made to rotate freely around by means of a movable ring at the extremity of the support which sustains it. On the extremity *A* of this arm the upright *AB* rotates freely, causing the attached horizontal brass rod *BP* to describe a horizontal circle. This rod of brass has at *P* a diminutive ball, similar to the head of a large pin, and is fastened to the upright *AB* by means of a clamp-screw with a ring, *B*, which permits it to be clamped at any altitude. *AB* has a certain number of divisions that serve to measure the height of the spherule *P* above the plane of the mirror.

To observe the motion of the clouds with this instrument one ought to begin by orienting it by means of a compass when the magnetic declination of the place is known, or by means of a meridian line, which can be provisionally traced on the ground. The instrument may be placed on a window-sill or the balustrade of a piazza, or it may be fastened on a wooden tripod, at the height of at least a meter, with three leveling screws at the base. When once oriented three little holes should be made on the window-sill in the first case, or on the floor in the other case, precisely at the points touched by the three leveling screws. Thus it will suffice in the future always to place the three points of the same screws in the same three holes and the instrument will remain oriented; otherwise it will have to be verified every time an observation is made.

To make an observation the image of a point of a cloud is observed reflected from the mirror with the eye placed at a point, *O* (Fig. 83), so as to perceive the image with sufficient distinctness back of the center *c* of the mirror. Then the little metal disk, which is soldered to the foot of the pointer *ABP*, is taken in the hand, and the upright *AB* is made to rotate about its axis, while the lower horizontal brass arm is turned around the center *c*. In this manner the spherule *P* is made to describe the arc of an epicycloid in a plane parallel to the mirror, and is so placed that one may perceive its image at the point *P′* in the same direction *OcP′* as that in which the image of the cloud is found.

The pointer is left in this position until the cloud has changed its position. Then replacing the eye in the same direction *OcP′*, and observing anew the center of the mirror and the image *P′* of the spherule, the displacement of the cloud is perceived; the mirror is quickly turned until the image of the point of the cloud is seen on the diameter *ab* and travels along it. The position which is thus given to *ab* indicates the true direction of movement of the cloud. Thus, if the image is seen to move from the center *c* toward the NE., it follows that the upper wind is blowing from SW., as is shown in Fig. 83.

The precision attainable in this way, depending on the rotation of the mirror and on making the point of the cloud traverse the diameter *ab*, is very great, and this, therefore, becomes an instrument of precision without great cost when the edge of the disk is divided to degrees and when the mirror carries a little metal plate projecting from one extremity, *b*, of the diameter, and provided with an index mark for reading or with a vernier if desired.

With the nephoscope above described the apparent velocity of the motion of the cloud can also be determined. For this purpose the pointer *ABP* is moved as above indicated so as to see in the same direction *OcP′* the center of the mirror, the image *P′* of the spherule, and that of the point of the cloud; then the mirror is turned as described in such a manner that the point of the cloud selected runs along the diameter *AB*.

The motion of the cloud is then followed, not keeping the eye fixed in the direction *OcP′*, as in the above first method, but moving the head so as to see the image of the point of the cloud always behind the image *P′* of the spherule. The time required by the image of the point of the cloud to travel from the center of the circumference of the mirror, or to traverse a certain number of divisions of the radius, *cA*, is noted in seconds. Since the height of the spherule *P* above the plane of the mirror remains constant during this operation, therefore this second method gives sufficient data for determining the apparent velocity of the cloud.

In the absence of a clock beating seconds there is added a little pendulum, made of a ball of lead about 3 centimeters in diameter, attached to a little chain of such a length that the pendulum beats half seconds. This pendulum can be put on the instrument itself or on any object in the vicinity, or may be kept in the hand, holding the end of the chain between the thumb and the index finger.

As above stated, when it is desired to determine the apparent velocity of the cloud the motion of the cloud must be followed, not by keeping the eye fixed in the direction *OcP′*, but by moving the head so as to keep the image of the point of the cloud always behind the image *P′* of the spherule. In this second method the apparent velocity of a cloud is independent of the height of the eye above the plane of the mirror, which is not the case in the first method. This second method is far more reliable than the first.

To understand this better, consider Fig. 84, in which the length of the arrow *AB* represents the displacement of the cloud observed by reflection in *A′B′* behind the mirror of the nephoscope; *P* represents the spherule of the pointer and *P′* its image. Keeping the eye fixed in the direction *OcA'* during the motion of the cloud from *A* to *B* we estimate the displacement by means of the length *ca*, measured on the diameter of the mirror. But if the eye is at *O''*, nearer to the mirror, then we estimate the same displacement *A'B'* by means of the length *ci*, which is less than *ca*. Since, then, we can not be certain in different observations of always placing the eye at the same distance from the mirror, it is clear that this method may give erroneous results for the apparent velocity of the clouds, while at the same time it is a very easy method for determining the direction only. The second method, on the contrary, while it serves to indicate the direction of the cloud, also suffices to determine its apparent velocity. Let the eye be put either at *O* or at *O''* to observe the cloud at *A'*; then, moving the head, the eye is carried to *O'* to observe the same cloud at *B'*. The displacement *A'B'* will be estimated by means of the length *cb*, which is the base of the triangle *P'cb* and which remains the same whatever may be the distance of the point *O'* from the mirror.

(*c*) In 1878 Linss published a method of using the nephoscope substantially the same as that of Aimé and Cecchi, and with the additional proviso that the sight-point *P* be kept at a constant height above the mirror, or at most that only two or three definite heights be allowed. He insists especially upon the importance of moving the eye so as to keep it always in a line with the reflected images of the cloud and the sight-point; thus the sight-point always covers the cloud, and therefore the angle upon which the distances *MM* or *FF'* depend has its vertex at the sight point *P*, and not at the observer's eye, as in Braun's method; in this method of observation, as Linss very clearly shows, the large error that may be caused by bringing the eye unintentionally nearer or further from the mirror is avoided, and the similarity of the triangles *P'bc* and *P'A'B'* (Fig. 84) is fully assured.

(*d*) Linss further adopts as a standard of velocities the angle that would be described in 1 second by the cloud starting from the zenith. In the accompanying diagram (Fig. 85) let Z, W_1, and W_2 be, respectively, the zenith and the first and second positions of the cloud, mm' the path observed in the mirror. We have then the following relations: $mm'=a$, the observed length of the path described in n seconds; $BB'=\frac{a}{n}$, the path described in 1 second if the cloud had started from the zenith;

$$\frac{BB'}{BP}=\frac{a}{n}\cdot\frac{1}{b}=\tan\,\omega$$

where ω is the angular velocity at the zenith in a unit of time. The velocity expressed in radii, *i. e.*,

$$c=\tan\,\omega=\frac{a}{b}\cdot\frac{1}{n}$$

can easily be tabulated, and is used by Linss in the place of ω in arc.

The angular velocity thus computed is available for any comparisons of the apparent cloud velocities, and should be entered in the observing journal as the apparent angular zenithal velocity for the adopted unit of time. It has the advantage of depending on no arbitrary assumptions except that of the unit of time, which is taken to be 1 second.

(*e*) From two similar determinations of ω_1 and ω_2 at stations of which one is elevated D meters above the other, for which, therefore, the height of the cloud is H and $H+D$, we get for the lower station the actual velocity of the cloud $V=(H+D)\tan\omega_1$, and for the upper station $V=H\tan\omega_2$, whence we deduce the true altitude

$$H=\frac{D\tan\omega_1}{\tan\omega_2-\tan\omega_1}$$

and thence the velocity

$$V=\frac{D\ \mathrm{tg}\ \omega_2\ \mathrm{tg}\ \omega_1}{\mathrm{tg}\ \omega_2-\mathrm{tg}\ \omega_1}$$

This method may be used at sea when a vessel is becalmed if observations of ω are conducted long enough at the top and bottom of a mast to eliminate the effect of ship's rolling.

(*f*) If at one station we observe the azimuth and the above quantity $c=\tan\omega$, and also determine the linear velocity V of the motion of the shadow on the ground, we get the true altitude of the cloud by the formula $H=b\frac{V}{c}$. This method also is available at sea when the vessel is not advancing.

(*g*) In 1880 Fornioni described his portable pocket nephoscope or "nephodoscope," and introduced it among Italian observers.

This instrument is shown in Fig. 86; it is adapted for observing only the direction of motion of the clouds, and is so portable, simple, and inexpensive that it should be in the hands of every observer who has not the more complete apparatus, but who desires to observe at least one element correctly. The following is Fornioni's description of his nephodoscope:

Inside of a box of wood or metal, *S*, 15 centimeters in diameter, there moves freely on opposing pivots a magnetic needle. Above this needle, and at a convenient distance, there is fixed horizontally a plane mirror, *BB*, which occupies the whole of the inside of the box.

On the polished surface of the glass are traced lines corresponding to the points of the compass, and the space between the north and the northwest, from which the amalgam has been removed, is divided to whole degrees, and permits of seeing the extremity of the magnetic needle for the purpose of orienting the apparatus more easily.

A species of pointer *T*, free to move around the edge of the box, completes the instrument.

When it is desired to observe the direction of a given cloud the nephodoscope is held approximately horizontal, or it is put on a level surface and oriented with reference to the true, not the magnetic meridian.

The pointer *T* is then moved so that the eye forms the third point in a visual line passing through the aperture of the pointer, the center of the mirror, and the reflected image of the selected point of a cloud.

The direction of the displacement that the reflected image will have experienced after a time, proportional to the velocity of the cloud, or inversely as its distance, will constitute the exact direction of the movement of the cloud.

This nephodoscope is in use with satisfactory results at the Royal Observatory at Brera, the Central Meteorological Office at Rome, the observatories of Velletri and Vicenza, as well as among various amateur meteorologists.

(*h*) In 1882, as the result of much experience, Finemann recommended to the Meteorological Committee in session at Paris, a form of nephoscope not very different from that used by Aimé and by Garnier, and which Finemann thinks most practical. Owing to the fact that no uniformity in apparatus or methods of using has as yet been widely enforced upon observers his recommendation is here given in full. The adoption by Finemann of the time required to describe an angle of 15° as the standard of apparent velocity is excellent, but is not so essential as that the nephoscope itself be used as an instrument of measurement to remedy the present general neglect of this very important subject.

Finemann's nephoscope is of the following construction.* A vertical scale (Fig. 87) divided into millimeters, standing perpendicular to the plane of the mirror and entirely to one side of it, can be raised and lowered by means of a rack or button and slide at *B*. This scale is attached to a horizontal circle movable about the mirror, as in the apparatus of Fornioni, so that the scale can be placed at any azimuth with respect to the center of the mirror. As in the instrument of Fornioni, the mirror is graduated like a magnetic compass card, and below it is placed a magnetic needle, so that the mirror can be oriented. Around the center *n* of the mirror is traced a circle having a radius of *b* millimeters.

By raising or lowering the scale *AB* (see Fig. 88) it can be so set that upon the line *AD* a point, *K*, of a cloud can be seen reflected at the center *D* of the mirror. If the point *K* moves in the direction *KL* this direction is found immediately by observing the radius *DF*, along which the image moves. In order, then, to determine the relative velocity the time is noted that elapses between the moments when the image has been observed upon the line *AD* and when it is seen in the direction *AF*.

It is easily seen that an image traversing a certain distance describes across the mirror a line whose length is independent of the azimuthal direction of the motion.

From the observed time *t* necessary for the image to move along the

*Instruments of this construction are now made by J. L. Rose, of Upsala. American observers can undoubtedly get good ones made by any good mechanician.

radius a of the circle it is easy to calculate the time T necessary for the cloud to move from the zenith to a point 15° distant.

Let $MR=BC=AB=b$, then in the vertical plane KLR we have $mn=DF$ and $\omega=KRL$, whence

$$\tan \omega=\frac{MN}{b}=\frac{a}{b}$$

If ω is described in the time t then 15° will be described in the time

$$T=\frac{\text{tg } 15^{\circ}}{\frac{1}{t}\text{tg } \omega}=\frac{0.268}{a}\times bt$$

The radius a is a known constant, and the quantities b and t are to be observed. By making $a=26.8$ we have simply

$$T=\frac{bt}{100}=0.01bt$$

136. OBSERVATION OF CLOUDS AT SEA.

Observations of cloud motion may be taken at sea if the nephoscope be held in the hand or mounted on gimbals.

The apparent velocity and direction thus observed is, of course, a resultant obtained by combining the movements of the cloud and the ship. Let H be the height of the cloud and V_1 the ship's velocity, moving from the direction of the azimuth α_1, then $\frac{V_1}{H}=\tan \theta_1$, where θ_1 is the apparent angular velocity of the ship, as seen from the height of the cloud.

Let V_2 be the actual linear velocity of the cloud, then $\frac{V_2}{H}=\tan \theta_2$ will give θ_2, the apparent angular zenithal velocity of the cloud, as seen from a stationary ship.

The observer on a moving ship (see Fig. 89) records the apparent angular zenithal velocity θ of the cloud and the apparent bearing α of the azimuth whence the cloud comes, which are connected with $\alpha_1\alpha_2\theta_1\theta_2$ by the equations

$$\tan \theta \cos \alpha=\text{tg } \theta_2 \cos \alpha_2-\text{tg } \theta_1 \cos \alpha_1$$

$$\tan \theta \sin \alpha=\text{tg } \theta_2 \sin \alpha_2-\text{tg } \theta_1 \sin \alpha_1$$

whence

$$\text{tg } \alpha=\frac{\frac{V_2}{H}\sin \alpha_2-\frac{V_1}{H}\sin \alpha_1}{\frac{V_2}{H}\cos \alpha_2-\frac{V_2}{H}\cos \alpha_1}=\frac{\frac{V_2}{V_1}\sin \alpha_2-\sin \alpha_1}{\frac{V_2}{V_1}\cos \alpha_2-\cos \alpha_1}$$

Therefore, unless we can obtain the altitude H, our cloud observation at sea can only give a result depending on the velocity of the vessel. Fortunately the ratio $\frac{V_1}{V_2}$ may be made small, and the effect of the ship's motion is therefore negligible or can be estimated for, so that the observations are nearly as valuable as though the vessel were stationary; consequently the use of the nephoscope at sea is to be strongly recommended.

The altitude of clouds may frequently be determined at sea by the method of section 130, the observer being high up the mast and observing apparent altitudes above the sea horizon. This method was used on the voyage of the French corvette *Venus* in 1837–'39.

137. THE NEPHOSCOPE AS AN AZIMUTH INSTRUMENT.

The nephoscope can easily be used so as to give the apparent angular altitude and azimuth of the cloud, as well as the apparent velocity and direction of its motion, by which means it leads to absolute measures. The following remarks by Finemann are therefore as applicable to observers furnished with nephoscopes as those furnished with more elaborate altitude and azimuth instruments:

The observations hitherto made on the clouds consider only the horizontal component of their motion and the relative horizontal velocity. This is all that a nephoscope can give, and it is the only result that can be expected from stations of the second order or from the deck of a vessel. These results are without doubt of the greatest importance, but it is evident that the determination of the true height, velocity, and path in space of the clouds, especially of the upper clouds, is a problem still more important. Thanks to the invention of the telephone, two observers placed at different stations can easily communicate with each other, and thus the principal difficulty of making direct measures is eliminated. Some experiments made in this respect in 1882 and 1883 at the Swedish polar station at Spitzbergen seem to show that the difficulties in these measurements are not insurmountable. I have had established at Upsala two stations, distant from each other about half a kilometer, furnished with instruments necessary for these observations, and connected by a telephonic line. The first trials showed at once that it was not difficult for the observers to agree upon the same point of a cloud and to measure the angular co-ordinates with sufficient exactness. The execution of these experiments was entrusted to Messrs. Ekholm and Hägstrom, who, in a preliminary treatise, have already given an account of the method of observation employed by them, of the manner in which the observations have been calculated, and of the first results. Our base being too short, I established a third station at a distance of 1,302 meters from the further of the two stations just spoken of, in order to be able to determine more exactly the position of the highest cirrus. The observations have been made during the summer three times daily, and they have already given several valuable results. It is not to be doubted that the execution of such absolute measurements in other countries will be of the greatest usefulness.

138. OBSERVATIONS OF THE VANISHING POINT.

A comprehensive view of the directions of the motions of the clouds over the whole sky is easily obtained by the following method, which was tried by me in a crude way in 1857 or 1858, but with better means in 1871, and frequently used between 1871 and 1880 by me at the Signal Office. It is based upon the principle that all planes containing the observer's eye and the horizontal paths of clouds moving parallel to each other will intersect the horizon in a common vanishing point, whose bearing is the azimuthal direction of the motion of the clouds.

Upon this principle both a portable and a fixed apparatus were designed by me, of which the latter was constructed in the following manner (see Fig. 90): A fixed horizontal graduated circle, CC, of several feet radius was supported at the height of the observer's eye. Upon this a similar circle, $C'C'$, rested, turning around easily in a groove. The semicircle SZ attached to $C'C'$ represented the vertical circle through the observer's zenith, and could be set at any azimuth by moving $C'C'$. A second graduated semicircle, PSP', was pivoted at P and P', and could be set at any angle of elevation SOC', where it was held by clamping to the semicircle $C'ZC'$. By adjusting the movable circle CC' and semicircle PSP' the observer at O is able to bring the plane PSP' to coincide with the path $W_1 W_2$ of any cloud in any part of the sky, and then determines its apparent angular velocity $v = K_1 O K_2$ in that plane. The graduated semicircle SZ gives the angle SOC', and the graduated circle CC gives the direction of the horizontal cloud motion. From the observed angular velocity v the apparent velocity V of a zenithal cloud may be deduced by the following approximate formula:

$$V = v \operatorname{cosec} \alpha \operatorname{cosec}^2 \theta$$

where α is the angular altitude SOC', and θ the angle SOK measured on the semicircle PSP' from S to the point at which the cloud velocity is observed.

This apparatus is especially convenient for observations of auroras and their streamers, parallel bands of cirri, location of the zodiacal light, and the paths of meteors. Having set the plane PKP' so as to show the direction of motion of the lower clouds over any special portion of the sky, the author has frequently found that clouds belonging apparently to the same stratum have a perceptibly different direction of motion in different parts of the sky.

This diversity of directions has sometimes been explicable by recalling the fact that the apparatus can give only the "trace" of the planes, including the cloud motion, the observer, and the horizon, so that any ascending motions in different parts of the surrounding region will affect the traces for the respective clouds. At other times this variation has been explicable by the assumption that clouds in different portions of the sky are not moving in precisely parallel rectilinear directions, but presumably in circles about some neighboring storm center, precisely as was subsequently demonstrated by the Swedish observers Hildebrandsson, Ekholm, and Hägstrom.

139. OBSERVATION OF POSITION ANGLE—MOHN'S ALT-AZIMUTH.

The following method is recommended by Ekholm and Hägstrom as giving by very simple observation with ordinary measuring apparatus the means of obtaining apparent zenithal velocities, and consequently linear velocities if we know the altitude of the clouds (see Fig. 91). A telescope with micrometer position circle, or the equivalent Mohn "theodolite" for observing the aurora borealis, or the equivalent Neumayer apparatus for observing shooting stars, is arranged so as to give directly the altitude and azimuth of a cloud and the position angle of the cloud's apparent motion across the field of view, the latter being determined by setting the micrometer thread of the position circle so that the cloud follows it across the field of view. The apparent velocity is obtained by timing the passage of the cloud along a portion of the thread.

Let $\varphi = SOP$ = the desired angle that the azimuthal direction, whence the cloud comes, makes with the meridian; this angle is measured on the horizontal circle from the south point through the west.

$\gamma = HCM$ = the observed position angle of that end of the micrometer thread that shows the apparent direction whence the cloud comes as it crosses the field of view; this angle is to be measured from the lowest point of the position circle towards the right as the observer stands facing the cloud, *i. e.*, in the same direction as azimuths are counted. If the telescope be directed to the zenith the position angle and the azimuth angle will coincide.

α = the observed altitude of the cloud $= KOH$.

θ = the observed azimuth of the cloud $= SOH$.

$\beta = HOP = \varphi - \theta$ = the difference in azimuth between the direction of the cloud OH and of its starting point P.

Direction.—We have, then, first to compute β from the equation

$$\tan \beta = \tan \gamma \sin \alpha$$

where β is always in the same quadrant as γ; then compute $\varphi = \theta + \beta$.

Velocity.—Let the observed angular movement along the micrometer thread be χ degrees in one second, counting from the center c of the field of view. The corresponding linear motion L of the cloud is assumed to be horizontal.

Let H = the linear altitude of the cloud above the observer, V = the angular zenithal velocity corresponding to the linear velocity L at an altitude, H, expressed in degrees per second.

Let the auxiliary angle δ, which is always between $+90^\circ$ and -90°, be found from the formula

$$\tan \delta = \cos \gamma \cot \alpha$$

we then find V and L from the formulæ

$$\tan V = \frac{\sin \chi}{\sin \alpha \cos (\delta - \chi)} \qquad L = H \tan V$$

140. VETTIN'S CLOUD CAMERA.

The preceding method of observation and computation, as recommended in 1884 by the Swedish observers, is substantially that which Vettin had already many years before embodied in his "cloud camera," with which simple and convenient apparatus he has collected many thousand observations. His apparatus is shown in the accompanying sketch (see Fig. 92), which gives a side view of the *camera obscura* supported on its tripod or stand. *C* is the object glass, *a* a mirror at the other end of a rectangular box; the lens forms an image of the cloud upon the ground-glass plate *e*, which is circular, and whose circumference is divided into angles or compass points, so that its 0° or south point is at the upper edge of *e*. The center of this circle is shown by a cross-mark on the glass. A plummet hangs from *i* and the divided quadrant *f* gives the angle of elevation of the cloud. The radius of *e* subtends an angle of $\frac{1}{2}\alpha$ whose vertex is at *c*, and whose value in arc is determined by pointing the apparatus upon some known terrestrial angle or, still better, upon the sun; in the latter case, observing *t* in minutes, the time required for the solar image to pass centrally across the circle *e*, we have in minutes of arc $\frac{1}{2}\alpha=15t\cos\delta$, where δ is the declination of the sun. A value of the radius such that this angle is between 150′ and 300′ of arc will be found convenient in practice.

By means of a light movable index lying on *e*, or by making *e* rotate in its own plane, the angle of position of the apparent motion of the cloud is easily read off. After reading off the altitude on the quadrant *f* the camera is brought to the horizontal position, and then a compass needle and circle on top of the camera, as at *h*, gives the azimuth of the cloud.

Instead of using the formulæ of section 139, based on the altitude, azimuth, and position angle, Vettin has employed other devices, which are, however, not to be recommended as desirable for general use.

For the purpose of comparing the relative velocities of clouds, instead of taking the angular zenithal velocity in minutes of arc per second of time, as is generally recommended by the Permanent International Committee, Vettin takes the corresponding linear velocity at an assumed unit altitude of 24,000 German feet or 1 German mile. This he calls the "projected velocity." (See Zeit. Oest. Gesell. für Met., 1883.) It is obtained by the formula

$$V'=24{,}000\times\frac{1}{t}\times\tan\frac{1}{2}\alpha$$

This "projected velocity" can be most easily obtained from observation if the cloud appears to be moving horizontally, *i. e.*, if the position angle $\gamma=90°$ or $270°$; therefore Vettin's usage is restricted to clouds lying in the plane perpendicular to the direction of the vanishing points, for which case, however, the angular zenithal velocity *V* can also just as easily be obtained from the formula of section 139, which in this case becomes

$$\gamma=90° \text{ or } 270° \qquad \text{tg } \delta=\pm\cot\alpha \qquad \delta=90°-\alpha$$

$$\text{tg } V=\frac{\sin\chi}{\sin\alpha\cos(90-\alpha-\chi)}=\frac{\sin\chi}{\sin\alpha\sin(\alpha+\chi)}$$

This formula for *V* can easily be tabulated for the arguments *a* and χ. The use of the angular velocity (expressed in radii per second of time) of a zenithal cloud having the same altitude and linear velocity as the observed cloud is to be recommended as the most appropriate method of expressing the relative velocities of apparent cloud motion.

SECTION D.

THE MEASUREMENT OF AQUEOUS VAPOR.

CHAPTER XIX.

THE MEASUREMENT OF AQUEOUS VAPOR.

141. HYGROMETRY IN GENERAL.

Water is present in the air in two conditions, both as an invisible vapor and as visible particles of water and ice, and exerts important modifications in the properties of the otherwise dry atmosphere. Some of the phenomena that are due to true vapor offer convenient means of determining the quantity of water present. Other phenomena, especially the optical, depending on the particles of condensed vapor, give indications as to the distribution of the particles, their shape, and size, which latter again depend largely on whether they are in a solid or liquid condition, namely, whether they are water or ice. The various methods of making all such determinations constitutes hygrometry in its broadest sense, namely, the art of determining everything relating to the quantity and condition of atmospheric moisture. These methods will eventually be treated of in detail, and may be summarized in general terms under the following paragraphs:

(1) The weight of the total vapor and fog present in the atmosphere is added to the weight of the dry air to give the total weight of the atmosphere. Therefore the weight of the water in a given volume of air is determined by exposing the measured volume of air to the action of chemicals that absorb all the moisture, and whose increase of weight, as shown by a delicate balance, gives the desired quantity of water actually present. This is known as the chemical method of determining the absolute moisture. It was first used by de Morveau, in 1808, who used calcium chloride as the dryer.

(2) When no fog is present the elastic pressure of all the vapor is added to that of the dry gaseous atmosphere. This pressure may therefore be determined, if no fog is present, by exposing a measured volume of air contained within a rigid inclosure to the action of chemical absorbents, and measuring by the manometer the consequent diminution of pressure within the inclosure. This constitutes Edelmann's method.

(3) In practicing the preceding method when any fog is present the air, before admittance to the inclosure, must be warmed artificially to

a measured temperature, such that the water exists only in a gaseous condition. The measurement of the change in pressure produced by the chemical absorbents give then a means of determining the total amount of vapor present whence the amount present as fog in the original condition of the air can be calculated.

(4) The volume of the vapor when no fog is present is simply added to the volume of the dry air. If, therefore, the absorbents act upon a measured volume contained in an adjustable inclosure, and we measure the resulting change in volume, we determine the volume occupied by the vapor. This constitutes the principle of the method elaborated by Schwackhofer.

(5) If fog be present in the initial condition of the air it must be warmed up until all fog has disappeared; the total volume occupied by the vapor can then be determined by Schwackhofer's process, and the quantity originally present as fog be calculated as before.

(6) The last three methods for determining the amount of fog present in the air are based on the assumption that the space between the particles of visible fog must be also saturated with invisible vapor, and on the further assumption that we know the relations between the temperature and the density, the elastic pressure, and the volume of the vapor in a saturated space. These latter relations are known only by means of special determinations made in physical laboratories; usually the results published by Regnault, as revised by Broch, for the International Bureau of Weights and Measures are adopted. Henry, in 1855, explained why the space between fog particles is not wholly saturated, and subsequent researches by Wullner,* W. Thomson, W. Gibbs, R. Helmholtz, Duhem, Sprung, etc.,† show that certain questions still need investigation; but in general Regnault's table gives approximately the tension or elastic pressure of vapor in a space saturated with aqueous vapor at a known temperature. This latter temperature is known as the temperature of saturation or the dew-point; the density of vapor is approximately 0.623 that of pure dry air of the same temperature and elastic pressure. The possession of Regnault's table suggests a secondary method of determining the amount of vapor present exclusive of any rain or fog, namely, by causing the air to become saturated, and then determining the changes in temperature or pressure or volume needed to produce such saturated air at any moment and place.

(7) The temperature of saturation is determined by cooling a mass of moist air containing a thermometer until it reaches the temperature at which dew is deposited or at which fog begins to become visible. This temperature is called the dew-point, and is at least very approximately the temperature of saturation, as given in Regnault's tables. With the known dew-point these tables give the tension, weight, etc., of the vapor in a unit volume of air saturated at that temperature. Dew-point ap-

* Lehrbuch, Vol. III, 4th ed., 1885, §§ 82, 86, 87.

† See Galitzine Wied., Ann. 1888, B. XXXV, p. 200.

paratus of various forms have been devised by Le Roy (1751), Berzelius (1807),* and Daniel (1818). Among the improved forms are those of Regnault, Bache, Alluard, Crova, etc. Espy, in his nephelometer of 1835, and lately Helmholtz and Sprung, have determined the temperature needed to produce fog by expansion.

(8) The change in volume or pressure consequent on saturating the air is the reverse of Edelmann's and Schwackhofer's methods, and is used by Maytern.

(9) *Hygroscopes.*—The depression of the dew-point below the temperature of the air shows the fact that in general the air is not completely saturated; rigorously speaking, the existing deficiency of vapor is the difference between that due to the temperature of the air and that due to the temperature of the dew-point. Thus, for air whose temperature is T and dew-point t, Regnault's tables give, respectively, W as the weight of vapor that could exist if the air were saturated at the temperature T, and w as that which does exist in the actual atmosphere, which becomes saturated when cooled to the temperature t; therefore the existing deficiency is $W-w$. This latter is the absolute dryness of the air just as w is the absolute moistness. The ratio $\frac{w}{W}$ is known as the relative moisture or humidity; this ratio is 1 or 100 per cent. when the air is completely saturated at the temperature t, as is frequently the case in rainy or foggy weather. The relative humidity $\frac{w}{W}$ is the complement of the relative dryness $\frac{W-w}{W}$.

The absorption of vapor from the air by vegetable and animal substances, porous earthen diaphrams, etc., is so largely dependent upon the relative humidity that these are used as a means of directly observing the approximate value of this element. In most cases a correction to their indications is needed, depending on the temperature or the absolute humidity.

These instruments are known as hygroscopes, and were first made by Deluc and Saussure about 1781. Their scales are graduated empirically from 0 to 100 per cent., and any further corrections to this graduation are determined by means of comparative observations with standard apparatus.

(10) *Sulphuric acid hygrometer.*—If a thin film of strong sulphuric acid be spread over the bulb of a thermometer by simply raising it up out of the acid bath into the free air, a rise of temperature due to the absorption of moisture from the air takes place, and its maximum value is easily observed. This rise is apparently approximately proportional to the absolute humidity, and can be converted into vapor tension by a few comparative observations with dew-point apparatus. This method constitutes Regnault's sulphuric acid hygrometer.

(11) *Psychrometer.*—The cooling of the surface of evaporating water increases with the rate of evaporation, and therefore with the dryness

* Berzelius, Fortschritte d. Physik, Vol. III, p. 63.

of the air and the strength of the wind. When a thin film of water at the surface of a thermometer cools by evaporation, the temperature of the surface of the water is given by this thermometer, while that of the air is given by a second thermometer. This combination, known as the August psychrometer, was used by Baumé (1767), Saussure (1786), James Hutton (1792), and Leslie (1799), but re-invented by E. F. August (1825), who first gave the tables for determining directly by means of it the absolute humidity or dew-point.

As the rate of evaporation and consequent cooling depends largely on the velocity of the wind, a form was devised by Belli (1831) in which the psychrometer was placed in a well-regulated current of air, but Saussure's whirling wet-bulb was preferred by the French observers, who have adopted the sling psychrometer of Bravais (1836) or the equivalent whirled psychrometer, which latter is now also adopted by the Signal Service.

(12) *Spectroscope.*—The presence of moisture in the air affects the dark absorption bands and lines both in the visible and invisible portions of the solar spectrum, in proportion to the absolute quantity of vapor through which the light passes. Any method of detecting this influence by optical, thermal, or photographic means constitutes a spectroscopic method of determining the amount of vapor through which the light has traveled. The apparatus for measuring the intensity of the aqueous bands usually gives only relative intensities with respect to other portions of the spectrum.

(13) *Refraction methods.*—The vapor proper affects the refractive index, and therefore the refraction of the atmosphere, but these delicate influences, although allowed for in astronomical observations, have not yet been made useful to practical hygrometry.

(14) *Color and polarization.*—The minute particles present in the atmosphere, whether as globules of hail, fog, or mist, drops of rain, or crystals of ice or snow, produce special optical effects that give indications as to the condition of the water and its amount. The condensation of vapor begins with the segregation of its molecules into minute particles that may, so far as the optical phenomena are concerned, be either crystals or spheres, provided that they have diameters smaller than the wave lengths of light. Such small particles are invisible under the microscope, but when illumined by white light they send out a delicate light that is blue in the case of aqueous vapor, and is polarized in planes normal to the direction of the illuminating beam. To this effect of these particles is due the blue light of the ordinary clear sky and the blue haze of afternoon and evening in moist climates and the ordinary polarization of sky-light. Ultra-microscopic particles of any substance will apparently produce the same phenomena, but aqueous particles are the only ones likely to be present in such regularity and quantity as to take any important part in the formation of the polarized blue-skylight. The blue light that issues from these minute

molecular spheres is not to be regarded as the blue of the upper end of the solar spectrum diffusely reflected by the surfaces of the small particles, since all wave lengths would equally share in such reflection and thus produce merely a delicate white light. Neither is this blue the result of dispersion by refraction of rays through aqueous spherules; neither is it due to the diffraction of light passing between minute fixed spheres, but to a modified form of diffraction in which the obstructing particles are movable and each becomes the center, whence emanates such light as it can emit; it selectively absorbs vibrations of a certain rapidity in the blue end of the spectrum, and emits these in all directions, while the remaining rays pass on. The vapor particles are no longer gaseous and able to support many rates of vibrations, but have become those of a liquid or a solid, and with difficulty vibrate at other rates than the few that are peculiarly adapted to their molecular structure.

The preceding is in outline the result of the studies of Stokes (1849), Tyndall (1869), Rayleigh (1871), and Bosanquet (1875), but the definitive utilization of their work for hygrometry still remains to be developed. [See also Soret in Ann. Chim. et de Phys., August, 1888 (6), XIV, pp. 503–540; Heidinger (1852), Brücke (1852), and Rubenson (1864).]

(15) *Cyanometry.*—The blue light comes from the smallest spheres only, and these extend to great heights in the atmosphere, while the layers that reflect the white light are much nearer the earth's surface. The addition of larger particles of water introduces unpolarized white light reflected from them and that dilutes the blue light from the finer particles. The relative amount of the latter is shown by the measured intensity of the blueness and the peculiarities of the polarization.

(16) *Photometry.*—When the aggregation of vapor has produced spheres whose diameter is greater than that of a given wave length the ordinary reflection and refraction becomes possible for all lesser wave lengths, and the blue gives place to the ordinary white light; therefore the haze gives a diffused white color, whose intensity depends upon the size and distribution of the particles and the direction of the lines of illumination and vision. Photometric observations, such as those of Pickering, will develop this method in hygrometry.

(17) *Diffraction.*—When the layer of haze is not too thick, and the particles are small and uniformly distributed, diffraction rings, known as coronæ and halos, are observable. The angular dimensions of these rings depend on the relative size and distance of particles; the distinctness of the rings depends on the uniformity in size and distribution.

When the particles are fine and rather more nearly crowded similar rings are observed, known as fog bows and diffraction rings, or aureolas, due to regularly reflected light and whose size depends on the size and arrangement of the particles.

(18) *Rainbow.*—Still larger particles give the ordinary rainbow and its supernumerary bow, but large particles in falling lose their spherical shape and go through a system of vibrations that have been investigated by physicists, and that introduce appreciable modifications into the elementary theory of the formation of the rainbow.

(19) *Parhelia.*—Reflections and refractions within crystaline forms of ice give parhelia and anthelia, the accurate observation of which gives much information in regard to the condition of the aqueous vapor.

(20) *Ice spherules.*—Reflections and refractions from fine particles, which, although frozen, yet have, because of the strong surface tension, retained the spherical form of the original water particle, may introduce a small percentage of polarized light into the light from clouds and haze, owing to the crystaline structure of ice.

(21) *Cloud colors.*—The thickness of a cloud gives it a dark under surface, which is, however, materially lightened by the reflection at the under surface of light from distant terrestrial objects and from the clouds themselves; thus, we see, under favorable circumstances, blue and green spots in clouds during the brighter afternoons of summer days, and the various tints reflected by low clouds at the time of sunset.

(22) *Absorption colors.*—As the sun descends below the horizon its transmitted light is modified by the moisture and dust in the air. The more there is of these so much the more is the blue and yellow light absorbed, and the transmitted light becomes successively ashey, lemon, orange, amber, rosy-red, bright red, blood-red, copper color, and dark; these are the colors of the sun's disk as we see it by transmitted light. The mountain tops, snowy peaks, haze, and clouds receive and reflect this beautiful transmitted light; the higher haze diffracts it into successive rings, as seen in the sunset glows of 1884. Colorimetric observations, such as with Maxwell's color box, promise to render these phenomena useful in ascertaining the general hygrometric condition. Indeed, in a crude way these colors have been for ages used as local weather signs.

(23) *The heights of clouds.*—The cooling of ascending moist air produces by condensation the cumulus and other forms of clouds; the rate of cooling can be approximately calculated by thermo-dynamic methods, thus determining the altitude at which the dew-point is reached in a mass of rising air. Conversely the known height and thickness of clouds gives data for approximately determining the humidity of the surface air at the point that the cloud started from, and the average condition of the air up to the lower limit of the clouds.

(24) *Actinometry.*—The radiation received hourly from the sun decreases as the sun's altitude decreases, owing to the slight absorption of heat by the dry air, and the far more effective absorption by the aqueous vapor and haze. The actinometer has therefore been used to measure the solar radiation, and after allowing for the absorption of

the pure air, a maximum value is deduced of the average effect due to the vapor, whence some idea of its amount and character may be obtained.

(25) *Fog scale.*—An arbitrary scale of the density of the fog, as observed at sea, based on the distance at which two standard objects can be distinguished from each other, can, by experimental comparisons, be converted into a rough measure of the quantity of moisture present as fog.

(26) *Scintillation.*—The initial process of condensation apparently produces a minute disturbance of refraction and dispersion that combines with the larger temperature changes to affect somewhat the twinkling of the starlight; observations of scintillation, therefore, may reveal something as to hygrometric conditions above us, although this phenomenon is mostly due to refractions that are independent of atmospheric vapor. The views of students on this subject have presented many contradictions, but probably those advanced by Exner (1886) are substantially correct.

(27) *Aurora.*—This silent discharge of electricity apparently depends essentially on the quantity and conditions of the moisture in the air; there are many indications that the auroral phenomena are due to such discharges, and already some general relations have been indicated between the observed phenomena and hygrometric conditions.

The preceding twenty-seven paragraphs show the great variety and complexity of the phenomena introduced into meteorology by the presence of water in the atmosphere. The general development of the broad field of optical hygrometry for the use of meteorologists will be postponed to a future occasion; for the present we will analyze only the ordinary physical apparatus and methods used in obtaining strictly local conditions.

142. GENERAL RELATIONS BETWEEN VAPOR AND HEAT.

The weight, tension, and temperature of the vapor in the atmosphere are related to each other in such a way as to give rise to the following generalizations:

(*a*) *Elastic pressure.*—The tension or elastic pressure of aqueous vapor in a saturated atmosphere depends upon its temperature only, and is known accurately from laboratory measurements; those made by Regnault are adopted by meteorologists. An illustration is given in the following table, which is an abstract of the larger table given by Broch in the first volume of the Travaux et Mémoires of the International Committee of Weights and Measures; the pressures are expressed by the corresponding heights of the mercurial barometric column at standard temperature, density, and gravity, and are computed by a formula that represents the whole range of Regnault's observed pressures:

Metric system.			English system.		
Temperature.	Elastic pressure.	Weight of vapor in a saturated cubic meter.	Temperature.	Elastic pressure.	Weight of vapor in a saturated cubic foot.
°C.	*mm.*	*Grams.*	°F.	*Inches.*	*Grains.*
−30	0.381	0.43	−30	0.069	0.12
−20	0.944	1.02	−20	.016	0.21
−10	2.151	2.26	−10	.027	0.35
0	4.569	4.83	0	.045	0.54
+10	9.140	9.33	+10	.071	0.84
20	17.363	17.12	20	.109	1.30
30	31.510	30.05	30	.167	1.97
40	54.865	50.63	40	.246	2.86
			50	.360	4.09
			60	0.517	5.76
			70	0.732	7.99
			80	1.021	10.95
			90	1.407	14.81
			100	1.913	19.79

The agreement of the results of the measures of elastic pressure, as made almost simultaneously by Regnault and Magnus, is such as to show that the numbers given by either do not differ 0.05^{mm} at temperatures below 30° C., and that Regnault's are not likely to be in error by more than that amount, which is equivalent to an uncertainty in the observed temperatures of 0.02° C. at 30° C., but of 0.15° C. at 0° C.

(*b*) *Density*.—The density of aqueous vapor, relative to pure dry air at the same pressure and temperature, has been directly determined by Regnault, Wullner, and others with sufficient precision to justify the adoption of 0.623 as correct and constant within the ordinary range of temperature and pressure. The value 0.622, deduced from Gay Lussac's law of volumes, is, however, preferred by some as equally reliable.

The volume of dry air at a pressure of 760^{mm} of mercury under standard gravity and temperature 0° C. being taken as unity its relative volume for any other temperature and at the same pressure is in English measure $1+0.0020361(t-32)$, or in metric measure $1+0.003667\times t$. The relative density is the reciprocal of the volume, whence the following table results:

For dry air under pressure of 760^{mm}.			For dry air under pressure of 30 inches.		
Temperature.	Volume.	Density.	Temperature.	Volume.	Density.
°*C.*			°*F.*		
−30	0.88999	1.12361	−30	0.87377	1.14447
−20	0.92666	1.07914	−20	0.89413	1.11840
−10	0.96333	1.03807	−10	0.91449	1.09360
0	1.00000	1.00000	0	0.93485	1.06969
+10	1.03667	0.96462	+10	0.95521	1.04689
20	1.07334	0.93167	20	0.97557	1.02504
+30	1.11001	0.90089	30	0.99593	1.00410
			40	1.01629	0.98394
			50	1.03664	0.96465
			60	1.05700	0.94607
			70	1.07736	0.92819
			80	1.09772	0.91098
			90	1.11808	0.89439
			+100	1.13845	0.87839

(*c*) *Weight.*—From the preceding it follows that the weight of vapor is known as soon as the weight of a unit volume of air at standard temperature and pressure is known. The latter datum has been given by Regnault, who found that 1 liter of pure dry air, at 0° C. and 760^{mm}, at his laboratory, weighed 1.29321 grams, which figures, as recomputed by Broch for the International Bureau, for a pressure of 760^{mm} under standard gravity (see Trav. et Mém., Tome I), becomes 1.293052 for ordinary dry air containing 0.0004 parts of carbonic acid gas. Hence a cubic foot at 32° F. and 30 inches pressure under standard gravity weighs 536.565 grains Troy. The complete expression for the weight of a unit volume of vapor therefore becomes as follows:

French measures:

$$0.623 \times 1.29305 \times \frac{\text{vapor pressure}}{760} \text{ or } [7.025340] \times \text{vapor pressure}$$

English measures:

$$0.623 \times 536.565 \times \frac{\text{vapor pressure}}{30} \text{ or } [1.070617] \times \text{vapor pressure}$$

With these formulæ are computed the weight of vapor in a cubic meter or in a cubic foot, as given in the preceding tables.

Omitting the factors 0.623 and substituting air pressures for vapor pressures we have the following formulæ for the weight of the dry air alone:

French measures: Weight in kilograms of a cubic meter or in grams of a liter of air

$$= 1.29305 \times \frac{\text{air pressure}}{760} \text{ or } [7.230852] \times \text{air pressure}$$

English measures:

Weight in grams of cubic foot of air=[1.276129]× air pressure

(*d*) *Relative humidity.*—The relative humidity has been described as the ratio or percentage of the vapor actually present at any time in a given volume of air to that required for the full saturation of the same volume. But, strictly speaking, it is a relation between the vapor and the space within which it is contained, and should be defined independently of any reference to air or other gases that may co-exist in that space.

The preceding formulæ show that for a given temperature of the vapor contained in any space the relative humidity may be computed either as a ratio of weights or a ratio of elastic pressures; therefore the formula for the relative humidity is

$$RH=\frac{\text{Prevailing elastic pressure of the vapor at temperature } t \text{ or elastic pressure corresponding to temperature } t_0 \text{ of the dew-point}}{\text{Elastic pressure corresponding to the temperature } t \text{ of the vapor.}}$$

thus, if the observed dew-point or temperature of saturation is +10° C., but the observed temperature of the vapor and air is +20° C., we have

$$\text{Relative humidity}=\frac{9.140}{17.363}=0.53, \text{ or 53 per cent. of saturation.}$$

(*e*) *Volume of air and vapor.*—If water evaporates into dry air the elastic force of the mixture is increased by the amount of the vapor tension, and if free to expand, as is ordinarily the case in the atmosphere, the mixed vapor and gas will occupy a larger volume. If not free to expand the mixture will press outward with additional force. Let the mixture be free to expand to the volume n', so as to maintain the same external pressure p and temperature; if the mixture is saturated with vapor a unit volume will consist of n volumes of dry air, whose tension is $p\frac{n}{n'}$, and $(n'-n)$ volumes of vapor whose tension is e; we shall then have the equation

$$p=e+p\frac{n}{n'}$$

from which

$$n'=n\frac{1}{1-\frac{e}{p}}$$

By this formula is calculated the following table, which shows the increased volume n' that is occupied by the saturated mixture at different temperatures when the original volume of dry air at the same temperature is unity:

English measures. $p=30$ inches.		Metric measures. $p=760^{mm}$.	
Temperature.	$\frac{n'}{n}$	Temperature.	$\frac{n'}{n}$
°F.		°C.	
0	1.0015	−30	1.0005
10	1.0023	−20	1.0012
20	1.0036	−10	1.0028
30	1.0056	0	1.0060
40	1.0083	+10	1.0122
50	1.0120	20	1.0234
60	1.0172	30	1.0432
70	1.0254	40	1.0778
80	1.0352		
90	1.0492		
100	1.0682		

(*f*) *Weight of air and vapor.*—The combined weights of a volume of air and of the vapor that would saturate that volume at a given temperature, as given in preceding tables, divided by the combined volumes given in the last table (by which the volume of the mixture is seen to be greater than either one alone), gives the weight of a unit volume of the mixture, namely, a unit volume of saturated air, as in the following table (see Glaishier Hygrometric Tables, 5th ed., 1869):

Weight of a cubic foot of saturated air under a pressure of 30 inches.

Temperature.	Weight.
°F.	*Grains.*
0	606.0
10	592.9
20	580.3
30	568.0
40	556.0
50	544.3
60	532.7
70	521.1
80	509.4
90	497.3
100	486.7

143. DIRECT DETERMINATION OF THE WEIGHT OF WATER IN THE AIR.

If a definite volume of air at a known temperature and pressure be drawn through a drying tube, namely, one containing some chemical that has a powerful affinity for water, nearly all the water is absorbed and retained by the chemical; therefore the increase in weight of this substance shows the total weight of the vapor and fog formerly present in the air. The weight thus determined, when divided by the original volume of the mixture, gives the absolute quantity in a unit volume.

The limit of accuracy with which the weight of the atmospheric water may thus be determined, although barely such as to satisfy certain physical inquiries, is in excess of the ordinary needs of meteorology. Our knowledge of this subject has been carefully reviewed by Morley [Am. Journ. Sci. (3), XXX, p. 140, and XXXIV, p. 199], from which it would appear that air passing in a slow current of 1 or 2 liters per hour through a fine tube, and then through larger drying tubes over a large surface of glass-wool, covered with strong pure sulphuric acid of a density of 1.84, has left in it not more than the 0.0025 of a milligram of water per liter of air.

A similar experiment, in which the air passes at the rate of 2 or 3 liters per hour through a tube 2 centimeters in diameter and 8 centimeters long, filled with phosphorous pentoxide and glass-wool, shows that the air does not then retain more than the 0.0025 of a milligram of water per liter of air.

The determination of the quantity of water by chemical absorption and weighing, requiring as it does delicate and expensive apparatus, is used only in determinations of fundamental physical data and by physicists more than by meteorologists. To a certain extent it can be considered as an integrating hygrometer when the air is drawn in slowly from the outside free air during a stated interval of time.

144. THE DIRECT OBSERVATION OF THE TENSION AND VOLUME OF VAPOR.

The methods proper for this object are embodied in the apparatus of Edelmann (modified by Voller and Hasselt), of Renoux and Matern, and of Schwackhofer, respectively.

(*a*) *Edelmann's method* (Zeit. Oest. Ges. Met., 1879, XIV, p. 54).—In this method a definite volume of air is drawn into the receptacle *R* (see Fig. 93) through the two-way cock *c* by suction at the smaller cock *a*, and its temperature and the pressure are accurately determined after closing both stop-cocks by means of the thermometer *T*, an exterior barometer, and the attached manometer *M*. The air within *R* is then subjected to the action of a drier (sulphuric acid) that flows from *S* through *c* into *R*, while a little air flows from *R* through *b* and *k* into *S*, the upper part of which latter is closed by the glass stopper. The absorption of the moisture in the air causes a partial deficiency of pressure in *R*, which is measured by the manometer *M*, and gives the means of ascertaining the volume of vapor absorbed.

The accuracy of this apparatus is limited by the completeness with which the chemical dries the air and the accuracy of the observations of the manometer and the temperature of the air within the receiver; to secure the latter the glass receiver *R* contains a thermometer and is inclosed in a non-conducting case, *HH*. For convenience of cleaning and drying preparatory to a second measurement the receiver has at each end large openings A and B, which are closed by large rubber stoppers, into which the smaller parts are permanently set, as shown in the figure. The stoppers are easily removed and replaced. The formula for the computation of the vapor tension is derived as follows:

Let V be the volume of the receiver, b the pressure, T the temperature, and e the unknown vapor tension of the confined air. Let $\frac{s}{\sigma}$ be the density of the liquid (oil) in the manometric tube, whose section is q; let h be the change in the height of the column observed by the manometer and expressed in millimeters and T the temperature, and assume that the external pressure, as shown by the barometer, has changed to b' during the observation. The pressure within the vessel becomes $(b-e)$ after the drying operation, but the air is now distributed through a new volume depending on the expansion of the vessel by temperature and the slight contraction due to inflow of manometric liquid; therefore the change in pressure, expressed in millimeters of mercury, becomes

$$(b'-b)+h\frac{s}{\sigma}=(b-e)\frac{1+\alpha T}{1+\alpha t}\cdot\frac{V}{V-\frac{1}{2}qh}$$

whence

$$e=\frac{b\alpha\frac{T-t}{1+\alpha t}+h\left(\frac{s}{\sigma}+\frac{b'-b}{h}+\frac{bq}{2V}\right)}{1+\alpha(T-t)}$$

The accuracy of this method is seriously impaired by the fact that the air in the receiver is by no means completely dried within a few minutes, but may require several hours, unless means are taken to produce a thorough exposure of every portion to the sulphuric acid, and this remark is true to a less extent of the modified form of Edelmann's apparatus, suggested by Hasselt, in which phosphoric acid is used.

The above method measures only the vapor pressure due to the vapor proper, and takes no account of the fog particles. If the latter are to be measured the foggy atmosphere must be warmed up to a higher temperature and made perfectly gaseous before entering the receiver.

(*b*) *Matern's method.*—Instead of absorbing the moisture and measuring the diminution in volume or tension one may bring the air to saturation either by cooling it directly to the dew-point or by cooling it by expansion, as in Espy's nepheloscope, or by adding as much water vapor as the air requires to become perfectly saturated. This latter is Matern's method (see Wiedemann's Annalen, 1880, IX, p. 159, and X, p. 149), which consists essentially of the following processes:

In an open cylindrical vessel a piece of paper saturated with water is placed; then the cylinder is closed by laying its cover upon it, to which a thermometer and manometer are attached as in Edelmann's apparatus. The evaporated liquid adds its tension to that of the original air, and when saturated the increase in pressure, together with that due to any change in temperature or of external pressure, is shown by the manometer. These last-mentioned changes in external pressure and temperature can, however, be rendered zero so far as affects the manometer, by setting the whole of the preceding apparatus within a larger vessel, which is hermetically sealed, or by simply attaching the open end of the manometer to such a closed vessel; the manometer then gives the changes due only to what occurs within the inner vessel. If the paper be saturated with sulphuric acid instead of water, then the inner vessel shows lower pressure and the method becomes one of absorption, but with this advantage over Edelmann's that the annulment of the change in outer temperatures and pressures produces a simplification of the formula for computing the vapor tension from the observed barometric change.

Let V and V' be the volumes of air in the inner or saturating and the outer or pressure vessels, respectively; let b, t, and e be the pressure temperature and vapor tension within V at the time of closing it up; $\frac{s}{\sigma}$ be the density of the oil or other liquid in the manometer tube in terms of the density of mercury for which $\sigma=13.6$; let q be the area of the section of the manometer tube, h the observed change in the height of the manometer liquid when the saturation is complete, T the new temperature of the saturated air, and E the corresponding Regnault's vapor tension that must prevail in the interior of the vessel. The coefficient of expansion of the gas for 1° C. is $\alpha=0.003665$.

The pressure in the saturation vessel rises from e to

$$E+(b-e)\frac{V}{v+\frac{1}{2}qh}\cdot\frac{1+\alpha T}{1+\alpha t}$$

The pressure in the outside or manometer vessel rises from b to

$$b\cdot\frac{V'}{V'-\frac{qh}{2}}\cdot\frac{1+\alpha T}{1+\alpha t}$$

In both of these the factors in V and V' take account of the changes in volume due to the slight movement of the liquid in the manometer tube.

The change in pressure observed by the manometer is $h\frac{s}{\sigma}$, and is the difference of the two preceding formulæ; the equation obtained by equating these quantities after some simplification by omitting small fractions gives

$$e=\frac{E-hc}{1+\alpha(T-t)}$$

where

$$c=\frac{s}{\sigma}+\frac{bq}{2}\cdot\frac{V+V'}{VV'}$$

c is so nearly constant that its mean value may be used without incurring an error of 1 per cent. The neglect of the denominator in the value of e involves an error of 1 per cent. when the change in temperature $(T-t)$ is $2\frac{3}{4}$° C. The multiplication of h by c can be avoided if the divisions on the manometer tube be $\frac{1^{mm}}{c}$ instead of whole millimeters.

In the apparatus as used by Matern $V=279.5$ and $V'=319.7$ cubic centimeters, $q=0.0560$ square centimeters; whence for $b=760$ millimeters there results $\frac{1}{c}=12.15$.

If the outer or concentric vessel is not used, and there be a small change in the outer pressure, then the above formula for e becomes more complex, while by the very construction of the apparatus the uncertainty in the accuracy of the temperature also becomes greater; therefore this form is not to be recommended.

[NOTE.—August, 1888. Since writing the above I discover that in 1858 Renoux developed a method equivalent to Matern's (see Paris C. R., XLVII, p. 354). Consequently Matern's is only to be considered as an improvement by reason of the external vessel added by him.]

E and e are now known in millimeters, but if the weight of vapor required for the process of saturation and corresponding to $E-e$ is desired then the density of saturated vapor is given by the expression

$$D=\frac{0.000106E}{1+\alpha T}$$

Let g=the weight of a unit volume of air; consequently the total weight corresponding to the tension $E-e$ becomes

$$W=DVg$$

As the operation of evaporation cools the wet paper, and therefore the air around it, this might be supposed to possibly affect slightly the temperature of the whole; but the preceding value of W multiplied by the latent heat in calories ($606.5-0.695\ T$, according to Regnault) shows that in extreme cases the resulting effect on the temperature of the internal air is not greater than 0.5° C. for $V=300$ centimeters, and this is so rapidly communicated to the walls of the vessel as to produce no appreciable cooling.

If sulphuric acid be used to dry the inclosed air then the pressure in the vessel v becomes diminished and the manometer shows a fall h'. The equations for this case are the same as for the preceding if only the sign of h be changed and the quantity E be considered as zero, and we have thus the final equation

$$e=\frac{hc}{1+\alpha(T-t)}$$

where c has precisely the same significance as before.

If, however, in this latter case we omit the outer or manometer vessel and allow free communication with the air we have returned to the form recommended by Edelmann.

The accuracy of this method of saturation is limited by the difficulty with which the air becomes completely saturated by merely standing in contact with a wet surface. Evaporation is a very slow process, depending on the surface tension of the liquid, the temperature, moisture, and motion of the air; in fact the air may be full of moisture in the shape of globules of water for hours without being saturated with the true aqueous vapor. There is, moreover, some doubt as to whether the Regnault vapor tensions for an atmosphere of pure aqueous vapor hold good exactly for mixtures of air and vapor at the same temperature. Regnault himself found that vapor in the presence of air gave tensions appreciably smaller up to 0.8^{mm} than for pure vapor without air [Annales de Chimie et Physique (3) XV, p. 130], but Wüllner (Lehrbuch der Exp. Physik, 4th ed., Vol. III, 1885), has explained this otherwise, and the effect of the presence of air is considered inappreciable.

(c) *Schwackhofer's method.*—This is illustrated by Fig. 94. In this method a definite volume of air is drawn through the openings *a* and *b* and the three-way stop-cock χ into the receiver *B*, which is inclosed in a chamber full of glycerine for security in determining the temperature by means of the thermometer *t*. A definite volume of air extends from χ through *B* down through the graduated tube *c* to near the scale 0 at its bottom, where the air is limited by a surface of mercury that fills the small space *y* and the bottom *R* of the glass cylinder *P*. By means of the loose wooden piston *F* and the screw at its upper end the mercury can be pressed down and out at *R* and up into *B*, while it also rises to the same height in the narrow ring of space between *F* and the walls of the cylinder. In fact it was by raising the piston at first and allowing the mercury to flow down from *B* that the air was drawn from α into *B*; when the air had filled *c* the stop-cock α was closed, and the temperature in *B* and scale reading on *c* were noted when they had become stationary. The stop-cocks α and β are now so set that communication is opened between *B* and the drying chamber *A*, which is a cylinder full of glass rods and strong sulphuric acid. The latter reaches up to a certain limiting mark, *m*. *A* has two small openings at *o* by which it communicates with a larger vessel half full of sulphuric acid. By grasping the elastic bag *z* more air can be forced into this vessel, thus driving the sulphuric acid through *o* into *A* up to *m*, where it stays when the stop-cock *q* is closed. The air is now pushed from *B* into *A* by the action of the piston *F*, which pushes the mercury up *c* until *B* is filled. The sulphuric acid in *A* is pushed down into the outer vessel, the stop-cock *q* is opened, and the air in the vessel escapes into *z*. When the air in *A* is dried sufficiently the observer grasps the elastic bag *z*, forces the sulphuric acid into *A*, and that in *A* into *m*, and at the same time turns the handle that raises *F*, so that the air returns to

B almost perfectly dried and under the same pressure as before. But when the sulphuric acid has ascended to *m* the air in *B* having lost its volume of vapor will not reach down to the original reading on the scale *c* unless such agreement be forced by a change of temperature or pressure. A change of temperature, if any occurs, is shown by the thermometer *t*. A change of pressure is counteracted by the use of the pressure tube *d*. This latter is parallel and similar to *c*, but needs no special graduation. It opens at the bottom like *c* into the mercurial cistern *y*, but at the top it opens into the free air, excepting that some cotton-wool at *W* intercepts the dust and insects. When *F* is turned so as to lower the mercury and force it up the tubes *c* and *d* the mercury in *c* will stand at a greater height than that in *d* so long as the dry air in *d* is at less pressure than the free air at *W*. This difference will become, however, rapidly diminished as the mercury rises in *c*. The observer therefore raises the two columns of mercury until they coincide as to height and then reads off the divisions of the scale *c*. This gives in hundredths and thousandths the value of the volume of the original bulk of air that has been abstracted from *c* and *B* by the absorption of vapor. The final act consists in connecting *B* with *a* or *b* by the stop-cock *α*, and expelling the air from *B*, bringing the mercury up to the level *nd*. The apparatus is then ready for a new observation.

In using this apparatus care must be taken not to draw in air from a distant place through tubes that will alter its hygrometric condition. Before drawing in the air, and especially before setting the stop-cock to drive the air from *B* into *A*, the sulphuric acid must be carefully adjusted to the mark *m* and the mercury must be at the line *n* at first, and then after air is drawn in the mercury must be lowered to the 0 division on *c*. The indraft and subsequent expulsion of fresh air should be repeated until any moisture left in *B* by previous observations is expelled. The mercury in *c* and *d* and the temperature *t* should be stationary at the moment when the stop-cock *α* is closed. The expulsion of *B* into *A* and its return should be repeated in order that the air may be perfectly dried, and all air bubbles in *A* should be allowed to rise above the mark *m*. When the heights in the columns *d* and *c* have been made the same by raising the mercury in both, the stop-cock *β* being closed, a slight correction for the defective pressure in the tube above the mark *m* is needed. This is avoided by opening *β* and again adjusting the sulphuric-acid column to the mark *m* by squeezing *z*, while also by the piston *F* adjusting *d* and *c* to equality. Any doubt as to whether the air is thoroughly dried, and, in fact, an estimate as to how completely the air can be dried by one exposure in the chamber *A*, is easily got by making a series of transfers from *B* to *A* and return, and making a complete computation of the vapor tension after each of these processes. Sworykin finds that air containing 2 per cent. in volume of vapor requires six successive exposures in *A* to be sufficiently dried. One charge of 600 grams of sulphuric acid suffices for many thousand determinations.

The external barometric pressure is to be read off as usual after the beginning and end of the process, if the vapor tension and other data are to be determined, but this is not needed if the object is merely to determine the percentage of volume of vapor and dry air.

The Schwackhofer apparatus has been used with much care for comparative observations by Sworykin, whose method of computation of the humidity of the air is as follows: Let V, H, and t be the volume, temperature, and pressure of the air inclosed in the receiver B; let V_1, H_1, and t_1 be the same data after the air has been dried and returned to the receiver. At the beginning H is the same as that of the outer air, but the outer H and therefore the inner pressure H_1 may have changed in the interval. The effect of such a change amounts frequently to 0.1^{mm} in the resulting vapor tension. The new volume V_1 differs from V by the change due to temperature $\frac{1+\alpha_1 t_1}{1+\alpha t}$ and pressure $\frac{H}{H_1}$ and to drying $\frac{H-x}{H}$, where x is the tension of the vapor that is withdrawn by the absorbent; therefore V becomes V_1, or

$$V_1 = V \times \frac{1+\alpha t_1}{1+\alpha t} \cdot \frac{H}{H_1} \cdot \frac{H-x}{H}$$

whence

$$x = H - H_1 \cdot \frac{V_1}{V} \cdot \frac{1+\alpha t}{1+\alpha t_1}$$

Although the changes in pressure and temperature are small, yet they must be carefully attended to. The effect of such changes is shown by writing the preceding expression in the following form:

Let

$$H_1 = H + \varDelta H$$
$$V_1 = V - \varDelta V = V - V\frac{\delta v}{100}$$

where δv is the percentage, as read directly from the graduated tube of the Schwackhofer apparatus, by which the original volume V has been diminished by absorption or other causes; the expression for x now becomes

$$x = \frac{H_1}{100}\left(\frac{100\alpha(t_1-t)}{1+\alpha t_1} + \frac{1+\alpha t}{1+\alpha t_1}\delta v\right) - \varDelta H$$

Therefore at ordinary pressures an error of 0.013 in δv entails an error of 0.1^{mm} in the tension x. An error of 1^{mm} in the same direction in both the pressures H and H_1 entails an error in the resulting x, which, for a maximum velocity of $\delta v = 2$, is about 0.2^{mm}. An error in $H_1 - H$ or in $\varDelta H$ of 0.1^{mm} affects x by its full amount. An error of 0.036° C. temperature produces an error of 0.1^{mm} in the vapor tension.

From a series of six successive dryings and measurings of the same volume of air Sworykin concludes that the last three successive dryings removed in all 0.08^{mm} of vapor tension from what had been left in the air after previous dryings, and that the final remainder must be less than this. This remainder is the constant error of the apparatus, and to diminish it as much as possible he always passed the air six times in and out of the drying chamber before completing his measurement of x. He states that the probable accidental error of each measurement is $\pm 0.024^{mm}$ over and above this outstanding small constant error.

The above measurement takes account only of the gaseous vapor. Any fog that is present in the original volume of air in B affects the result only to the extent of the inappreciable volume occupied by the minute particles, but the quantity of vapor then present can be determined if the air be warmed until the fog evaporates completely. To this end the apparatus is kept in a warm place at the temperature of the warmed air that is to enter B. Let the original barometric pressure be H, the temperature of the fog be t, and the corresponding vapor tension e, as given by Regnault's tables for saturated pure vapor; let the temperature of the warmed air be T and the new vapor tension be $e+E$, where E is that due only to the evaporated fog at the temperature T. Let the volume of the vapor in the warmed air, as measured by the Schwackhofer apparatus, be v, we have

$$v=100\frac{V_1-V}{V}=100\frac{x}{H}=100\frac{E+e}{H}$$

whence

$$E=\frac{Hv}{100}-e$$

145. CONDENSATION METHODS.

By cooling a space containing vapor until the temperature of saturation is reached phenomena are produced that enable the observer to determine this temperature.

Tyndall has shown that when a tube full of an invisible and nearly saturated mixture of vapor and gas, perfectly free from dust, is exposed to the action of a beam of light the vapor immediately begins to condense and a beam of polarized blue light, that gradually changes to ordinary white light, is emitted. This not only affords a delicate test of the condition of saturation, but also shows that air may be nearly saturated in the dark and yet appear foggy or supersaturated in a strong light. It would appear, therefore, that strong light produces a deposition of fine particles, such as themselves produce the blue color and other optical effects, even when the air is far from being saturated in the sense of the word as used in the physical laboratory.

Aitken and others have shown that the presence of hygroscopic dust particles in the air accelerates the formation of visible fog and rain particles. Thomson, Duhem, Gibbs, Terquem, and others have shown that visible fog may exist in the air, even though it may not be otherwise saturated, by reason of the relations between surface tension and diffusion.

Regnault has shown [Annales de Chimie et de Physique (3), XV, p. 130] that vapor tension is apparently less, or the temperature of saturation is lower when vapor is mixed with air, nitrogen, or other gases than when it is pure. Wüllner and his students, however, have made it probable that this diminished tension may be a phenomenon of surface condensation, etc., in the apparatus.

Joseph Henry (Scientific Writings, II, p. 5, or Smithsonian Report, 1855, p. 214) showed that fog particles, if sufficiently small, may, by reason of surface tension, exist indefinitely in an atmosphere that is not saturated. [See also Galitzine, Wied., Ann. 1888, B. XXXV.]

It would seem, therefore, from the preceding physical peculiarities that the temperature of saturation with which the meteorologist has to do in the presence of the sunlight, the dust, and the air may, for all these reasons, be lower and may occur with a lower vapor tension than was the case in the fundamental experiments of Regnault on the tension of steam, where diffuse light occurred with little or no dust or air.

When, therefore, the meteorologist observes that the air is misty, whether the natural air or that which he has inclosed and cooled in order to determine its temperature as being that of apparent saturation, he has need to recall that there may be a slight correction to the Regnault vapor tensions corresponding to the temperature observed by himself before concluding as to the correct elastic pressure of the atmospheric vapor.

As regards the amount of the preceding corrections no very definite data has yet been furnished by physicists. Regnault's observations indicated that the effect of the admixture of gas may at ordinary temperatures amount to as much as 1^{mm}, but Wüllner's studies reduce this to an inappreciable amount.

The preceding considerations suggest that there must be a limit to the accuracy of those condensation methods that rely upon the formation of fog. On the other hand the dew-point, as determined by the deposition of moisture in the shape of dew or frost upon a metallic surface, is affected by the hygroscopic quality of the surface, and only neutral surfaces, if such there be, or, failing this, the average between a repulsive and an attractive surface, should be used. The quantitative effect on observed dew-points of the nature and condition of the surfaces ordinarily employed has not yet to my knowledge been accurately investigated, but much data bearing on this point has been collected by Tomlinson [see Phil. Mag., 1868 (4), XXXV, p. 241]. All these points therefore require further study in order to increase the accuracy of meteorological data.

The principal forms of condensation apparatus are the two following:

(1) *The nepheloscope.*—This instrument was invented by Espy, about 1835, to show the formation of cloud by dynamic cooling, due to expan-

sion. With it he made a determination of the volumetric expansions and corresponding dew-points. His method depended for its accuracy upon the detection of the first formation of mist in expanding air and the immediate measure of temperature. It was not sufficiently accurate to require more than this passing notice, but as a convenient method of obtaining by one direct experiment a datum most interesting to meteorology, and as the first experimental determination of the law of cooling of ascending moist air, it must always have great historic interest. [Since writing this Sprung and R. Helmholtz have announced their success in attaining accurate results with a convenient instrument of this character, of whose construction I have as yet no further details.]

(2) *The dew-point apparatus.*—In Dobereiner's and Regnault's form of this apparatus a polished metallic vessel called a "thimble," containing sulphuric ether, rhigolene, or other volatile liquid, is cooled by evaporation of the liquid until a thin film of dew is seen to be deposited on the polished surface. The evaporation is then immediately checked slightly and the surface of the thimble allowed to warm slowly until the film of dew disappears; a thermometer gives the temperature at the moments of appearance and disappearance of the dew. The mean of these two readings is adopted as the temperature of the dew-point.

In Daniel's form of this apparatus, as first used by him in England, the cooled bulb was of glass. This was subsequently gilded, as constructed by Greiner, but the rate of cooling was not sufficiently great to give the dew-point in the drier climates of continental Europe; therefore Daniel's apparatus was modified as above by using a thimble of polished silver or gold, filled with ether, in which the thermometer is immersed and through which a current of air is forced so as to control the rate of evaporation and cooling. In Crova's form (Journal de Physique, 1884) the cooling air has access to the inside of a polished silver tube, which is cooled by the ether by which it is surrounded; the deposit upon this tube is not affected by the wind or the radiation of heat from external objects, and is very quickly perceived.

In Belli's and Bache's form (see Belli, Fisica, Vol. II, p. 522, and Bache, Obs. Girard College, Vol. I, Intro., p. 9) a horizontal metallic trough is kept full of mercury. One end of this liquid bath is kept at a very low temperature; the other is at the temperature of the air. At some intermediate point the temperature of the dew-point prevails, as is shown by the deposit of dew on the mercury and the trough; a thermometer bulb dipped in the mercury at this point gives the temperature of the dew-point. Bache found it necessary to shield the apparatus from strong winds that warmed the polished trough above the temperature of the dew-point and evaporated the dew.

As the Regnault dew-point apparatus, especially with various modifications introduced by the makers, is very widely used a general analysis of its action and errors is desirable. The air enters either by pressure through *ab* or by suction through *C* (see Fig. 95), bringing heat and moisture to the ether *E*; it loses a part of these as it bubbles up from *b* and before it leaves the ether, and finally passes away through *CC*, carrying away whatever portion of the ether has been evaporated.

The outward surface of the thimble *SS*, on which dew is deposited, radiates and absorbs heat; the very deposit of dew itself communicates some of the latent heat of vaporization of water, and heat is also communicated to the ether by any condensation of vapor from the air that flows in through *ab*. A slight amount of heat is conducted downwards through the supports of the apparatus. The great consumption of heat in the evaporation of the ether ordinarily overpowers the inflow of heat, and the thermometer *T* is cooled rapidly down to a very low temperature. Only in the very dryest climates and during fresh hot winds (and although fresh supplies of ether are poured in through the tube *F*) does this apparatus occasionally fail to give a deposit of dew, owing to the fact that the ether is used up before the necessary reduction of temperature has been accomplished; such failure, however, can perhaps be avoided by modifying the apparatus, as hereafter explained. The temperature of the ether is usually brought rapidly down to the dew-point in order to save time, but the apparatus must be capable of then maintaining this temperature constant for a minute while the formation and disappearance of the dew is being observed; therefore the condition under which the apparatus will work successfully is that the polished surface must maintain the temperature of the dew-point by means of the balance between the heat consumed by evaporation per unit of time and the heat gained in that time from all other sources. The commotion within the ether, produced by the bubbles of air, keeps the thermometer bulb at nearly the same temperature as that of the polished surface, but the bulb usually lags a little behind the surface temperature; therefore the mean of the temperatures observed for appearance of dew with falling thermometer and disappearance of dew with rising thermometer should be taken as the true dew-point.

The balance between the heat lost and gained involves a consideration of the following details:

(*a*) $W(t_a - t_0)\gamma$ or the heat brought in a unit of time by *W* grams of the air that flows in through *ab*, having a specific heat γ, and assuming that it is cooled from its temperature t_a to the temperature t_0 of the ether. If the air thus forced in is, as is often the case, the breath from the observer's lungs its high temperature and dew-point will greatly retard the time required to obtain the deposit of dew, beside which the absorbed organic material from the lungs converts the ether into a viscous mass, such that evaporation takes place more slowly. The air thus employed to reduce the temperature of the ether should be as nearly as possible at the temperature of the required dew-point, and should have its own dew-point somewhat lower than that of the outside air, so that none of its own moisture need be condensed. This is nearly realized if, instead of forcing in the breath through *A*, one applies suction to the tube *c*, and thus draws in the ordinary free air through the tube *ab*, which may be coiled about in the space above the ether so as to cool the indrawn air before it bubbles up at the bottom of the thimble.

(*b*) $C\ (t_a-t_0)\lambda$ or the latent heat evolved within the ether by the condensation of moisture brought in through the tube *ab* by air whose dew-point temperature is higher than that of the ether. If very dry air is used this term becomes zero, and if the ordinary air is drawn in it is so small as to be negligible, but if the warm moist breath is forced in it becomes appreciable. C is the average excess of moisture in a unit weight of air saturated at the temperature t_a over that at t_0 and λ is the latent heat of condensation.

(*c*). The latent heat evolved by condensation of the dew on the exterior polished surface of the thimble; this is conducted inward, and the passing wind or the descending convection currents immediately communicate to the dew sufficient heat to partially evaporate it, so that the observer will see only alternate momentary deposits or dulling and brightening of the surface of the thimble, if the experiment is properly conducted. This small portion of heat diffused through the whole tumultuous mass can hardly affect the thermometer. If the internal current is increased and the ether brought still lower in temperature, and the dew allowed to accumulate on the surface, the thimble becomes equivalent to a wet-bulb thermometer and tends to rise to the temperature of evaporation. If, now, the velocity of the internal current is such as to continue lowering the temperature of the ether, and therefore of the exterior dew, the latter absorbs the heat that comes to it by external radiation and by conduction from the air, but conducts it very slowly to the ether; the gradient of temperature between the outer and inner surface of the dew deposit becomes large, the dew accumulates rapidly in quantity and may easily become frozen. It is possible to so adjust the external wind and the internal current through *AB* that the whole shall be kept at any required temperature, and it is frequently easy to greatly lower the temperature below the dew-point, but this should be avoided. The successful use of this instrument depends on the maintenance of the balance between little dew and no dew. As a strong warm dry wind may furnish enough heat to greatly retard if not wholly prevent the deposit of dew, and will in any case contribute to cause the polished exterior surface to be warmer than the interior thermometer, it is therefore best to have the external current as well as the internal under the control of the observer. This is easily done by means of a thin glass tube (as at *GG HH*, Fig. 95), bent at right angles and so mounted that its vertical part incloses the polished thimble, while its mouth *HH* can be turned toward or away from the wind at pleasure; the wind entering the mouth flows up between the tube and the thimble, and has its velocity checked, while its temperature and moisture are not appreciably altered. It may be best that the current from HH should be just strong enough to overcome the downflow of the heavier cooling air and push a slow upward current out at GG, thus preventing any fumes of ether or moisture from the breath from affecting the thimble.

Equally as important as the wind is the nature of the polished surface; if it has a strong affinity for water it will tarnish before its temperature is lowered to the dew-point, and *vice versa*. The properties of various bodies in this respect are apparently not yet known very precisely, but by common consent gold and silver surfaces have been adopted, so that if these are not neutral they at least introduce only an error common to all observers. The surface should be kept chemically clean, according to Tomlinson's methods, *i. e.*, washed with sulphuric acid, a strong caustic solution, and then rinsed with water and alcohol. On the other hand the tarnished and roughened spots sometimes show deposits of dew sooner than the rest of the surface, which may be due both to the altered hygroscopic and absorbing properties and to the relative slowness of the currents produced by the greater friction.

(*d*) $SV(t_a-t_0)\varkappa\frac{R}{K}$.—The warm air, cooling as it flows past the thimble, gives up its heat by conduction in proportion to the reduction of its temperature $\frac{1}{K}(t_a-t_0)$ to the quantity of air VS (due to the velocity V and the area of the surface S), the coefficient $\varkappa$ of conductivity from the air to the metal, and to the relative capacity R of the air for heat.

This term, as stated in previous sections, becomes very large for large surfaces or high velocities or large depressions of dew-point, and it should be kept as small as possible by diminishing the ratio of the surface to the volume of the thimble, and especially by diminishing the velocity V, as is done by the screen $GGHH$.

(*e*) $S(t_a-t_0)\alpha$.—The cold thimble, when in the open air, is entirely surrounded by objects whose surface temperatures may be either higher or lower, but to prevent the large correction due to radiation of sky, sun, hot ground, etc., the thimble should be so sheltered that the radiation comes from objects whose surface temperatures agree nearly with t_a or preferably t_0. If the coefficient of absorption of heat for the metal surface is α, and the total surface S, the quantity of heat absorbed will be $\alpha S(t_a-t_0)$. As t_a is greater than t_0 this quantity is brought to a minimum by using a highly polished surface whose coefficient α is small [the coefficient for polished silver is 0.03 for obscure heat and 0.08 for the solar rays]. It is not allowable to use a non-conducting substance for the thimble, otherwise the temperature of the surface and the thermometer will differ too much. But it has been considered proper (as in Alluard's and Dines' hygrometers) to reduce the metallic surface, by a non-conducting covering, in so far as this does not affect the deposit too much; apparently the thin glass tube proposed above as a means of regulating the effect of the wind may be made so poor a conductor that the interchange of heat by radiation between its inner surface and the thimble will be considerably reduced and quite under control.

(*f*) $(t_m - t_0)\mu k d^{-1}$.—The upper portion of the thimble and the thermometer stem contain heat that is conducted to the liquid. In well-made apparatus this is reduced to a minimum by making the thimble above the ether out of some non-conducting substance. If the upper mass be M, its temperature t_m, its distance from the thermometer bulb d, and the coefficient of conductivity of the silver thimble k, and its section μ, then the expression for the amount of heat conducted per unit of time will be approximately as above.

(*g*) $E\varepsilon$.—The abstraction of latent heat by vaporization of the ether first reduces its temperature to t_0, and then the steady current of air keeps it at that temperature. Therefore in this condition the weight E of ether evaporated in a unit of time and multiplied by the latent heat of vaporization ε represents the quantity of heat abstracted in order that the observer may maintain the condition just favorable to the deposition of the dew; but this quantity is the sum of the preceding six items, which leads us to the following equation for the case when ordinary air enters through *ab*, and in which we have omitted the third and the sixth terms as ordinarily negligible:

$$E\varepsilon = W\gamma(t_a - t_0) + C\lambda(t_a - t_0) + SV\frac{R}{K}\varkappa(t_a - t_0) + S\alpha(t_a - t_0)$$

(*h*) The quantity of air W forced through the ether in a unit of time produces evaporation by a diffusion whose rapidity is in proportion to the surface exposed, and therefore in proportion to the number and minuteness of the small bubbles into which the air is broken up as it rises through the ether. The quantity that a unit weight of air will thus evaporate depends therefore on the number and size of the bubbles, the temperature, purity, and depth of the ether through which they rise. But the purity, depth, and temperature of the ether diminishes as the process approaches the dew-point, so that E diminishes with the initial temperature t_a of the air and with the relative temperature or $(t_a - t_0)$. Approximately we have

$$E = Wx[1 + y(t_a - t_0)]$$

where W is the weight of the air per unit of time and x and y must be determined experimentally for the liquid and the apparatus.

As the evaporation is most rapid at the point where the bubbles enter the liquid, namely, near the bottom, the ether is therefore coolest there, and the thermometer elevated some distance above should show a somewhat higher temperature. The amount of this difference can easily be found by experiment in any special case. An ordinary unprotected Regnault dew-point thimble was examined with this point in view, at the Signal Office, in 1886, by Mendenhall and Marvin, who explored all parts of the surface with a thermo-electric junction, determining thereby the lines of equal temperature, and finding a difference of several hundredths of a degree centigrade between the top and bottom of the liquid.

It is therefore important to decide as to what portion of the surface shall be chosen to watch for the appearance and disappearance of the dew. Evidently this should be the spot whose temperature is nearest to that of the thermometer, and the current of outside air should therefore be directed, not as now, downward against the central portion, but upward, against the lowest and coldest portion, thus tending to make that the warmer and equalizing the temperature of the whole thimble. The difference of temperature between the thermometer bulb and the dew-spot, if appreciable, must be determined empirically for the given instrument and conditions.

The observer will not fail, of course, to apply to the dew-point thermometer reading the various corrections already indicated in the chapter on thermometers, and among which those for sluggishness and temperature of protruding stem, and the Poggendorff corrections are likely to be appreciable.

146. EVAPORATION METHODS—THE TEMPERATURE OF EVAPORATION.

The evaporation of the free surface of water in the open air must in some way be an indication of the dryness of the air or the readiness with which it absorbs moisture. From this point of view both Baumé, Saussure, Hutton, and Leslie had employed a wet-bulb thermometer, whose temperature, lowered by the evaporation of the water, gave them a rough indication of the dryness of the air.

About 1825 E. F. August thought of utilizing Daniel's dew-point apparatus to measure the temperature of evaporation instead of that of the dew-point. He therefore covered one of the two thermometers with wet muslin, hung it beside the dry bulb, and observed that there was a simple relation between the depression of the wet bulb and that of the dew-point below the temperature of the air, the latter depression being twice the former. This constituted his psychrometer and his first method. The factor 2 he subsequently found to be a variable quantity. In 1822 Ivory and in 1825 August published a rational theory as to the mode of action of the psychrometer, very much as subsequently adopted and slightly improved upon by Regnault. Belli, in 1830, adopted an improved arrangement of apparatus for regulating the radiation and the convection of heat. Apjohn, in 1834, adopted an approximate formula similar to August's, and gave tables for the reduction of the observation. On account of the difficulty of obtaining uniformity in observations of the dew-point apparatus, and especially because of their labor and expense, much thought and care has been given during the past fifty years to the study of the proper methods of using the psychrometer and of computing the moisture in the air. The results now obtainable by the proper use of this instrument are to be considered as reliable as our knowledge of vapor tensions and other data will permit. The principal studies upon this subject are enumerated in the appended note, and the following is brief synopsis of past progress and present views on this important apparatus.

(1) Ivory (Phil. Mag, 1822, LX, p. 81) gave a theoretical formula for computing the vapor tension essentially the same as that of August, but in the absence of the necessary tables his results did not come to be accepted so widely as the more practical work of the latter.

(2) August (Pogg. Ann., 1825 and 1828) considered especially the convection of heat from the wet bulb by the flow of air past it; he assumed that a steady flow of partly saturated air, whose temperature is t and whose dew-point is t_0, touched the bulb and immediately left it

NOTE.

(1) J. Ivory. On the Hygrometer by evaporation. Phil, Mag., 1822, LX., p. 81.

(2) E. F. August. Ueber die Verdunstungs-Kälte. Pogg. Ann., 1825, Band V, p. 69.

(3) E. F. August. Ueber die Verdunstungs-Kälte. Pogg. Ann., 1825, Band V, p. 335.

(4) E. F. August. Ueber die Anwendung des Psychrometers. Pogg. Ann., 1828, B. XIV.

(5) E. F. August. Ueber die Fortschritte der Hygrometrie, Berlin, 1830.

(6) E. F. August. Psychrometer-Tafeln, Berlin, 1848. 2d. ed., 1862.

(7) Belli. Corso di fisica sperimentale, Tomo II, Milano, 1831.

(8) Belli. Di un nuovo psicrometro. Nuove Saggi Accad. Padua, VI, 1847.

(9) Apjohn. The Wet-Bulb Thermometer. Trans. Roy. Irish Acad., 1834.

(10) Regnault. Étude sur l'hygrométrie (first memoir). Ann. de Chim. et de Physique (3), XV, 1845. Taylor's Memoirs, IV, 1846.

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*Regnault made this error, which was recopied into Guyot

fully saturated due to evaporation at the temperature t_1 of the wet bulb; thence he deduced the formula

$$x=f'-\frac{0.568(t-t_1)b}{640-t_1}$$

or approximately

$$x=f_1-0.000685(t-t_1)b$$

where x is the vapor tension of the free air corresponding to the dew-point t_0, f_1 is the vapor tension, according to August's tables, for the temperature t_1; b is the observed barometric pressure in millimeters.

The constant coefficient was computed by August from the above theory and is adapted to the Reaumur scale; for the centigrade scale it would be 0.000708. In his subsequent tables of 1848 August adopted a constant that agreed more nearly with his comparative observations, namely, for a bulb covered with water 0.000952 for the Reaumur and 0.000762 for the centigrade scale; for a bulb covered with ice his figures were seven-eighths of these, namely, 0.000833 and 0.000668, respectively.

(3) Apjohn, in 1834, seems to have adopted the idea that the heat required for evaporation is given to the water by the surrounding air, which, having itself cooled, then cools the bulb. He was led to a formula of the same style as Ivory's and August's, namely,

$$x=f_1-A(t-t_1)\,b$$

where the pressure is in English inches and the temperature by Fahrenheit scale. The constant coefficient A is $\frac{1}{30}\times\frac{1}{88}$ for water and $\frac{1}{30}\times\frac{1}{96}$ for ice.

(4) Belli, about 1830, sought to cut off all noxious radiation and to control the convection by surrounding the wet bulb by a wetted cylinder, through which the air is drawn in regular current; his formula for the computation of x had, however, the same essential form as August's and the same underlying theoretical basis. He simply provided for the control of the two important sources of error by constructing an apparatus to which August's theory, as well as his own, was strictly applicable. In fact the errors of modern hygrometry consist essentially in applying Regnault's tables to cases for which they are confessedly not strictly adapted. Arago, 1830, and Bravais, 1836, Espy, about 1840, probably at Henry's suggestion, and Dr. B. F. Craig, surgeon, U. S. Army, about 1867, adopted a uniform rapid ventilation by whirling the wet bulb as Saussure had done in 1786.

(5) Regnault also, about 1845, adopted August's view that the equilibrium established by convection alone is of prime importance; he introduced the best numerical results as to fundamental data, such as the

* August (Ueber die Fortschritte 1830) gives the formula:

$x = f' - \frac{0.558(t-t_1)b}{512-t_1}$ but he does not deduce (derive) it. (Reverse 572 for ice)

He says "Ich werde an einem andern Orte die Art der Entwickelung

= Assuming August (1830) is referred to, the above equation (A) as originally printed should have 512 instead of 640 if Reamur calories are to be used. This would have made the

heat of vaporization, density of vapor, etc., and recomputed August's formula, putting it finally in the following shape:

For bulbs covered with water:

constant in degrees Reamur .001090 and in deg. cent .000872.

$$x=f'-\frac{0.429(t-t_1)b}{610-t_1}$$

For bulbs covered with ice:

$$x=f'-\frac{0.429(t-t_1)b}{689-t_1}$$

where the denominators express the fact that the latent heat of vaporization from ice is greater than that from water. Regnault determined in a general way the effect of calms, light and strong winds, and of the error of the crude hypothesis that the air is saturated at the temperature t' before leaving the bulb. He finally adopts 0.480 instead of 0.429 as representing his observations in the still air of a room, whence his formula can with sufficient approximation be written

For bulb covered with water:

$$x=f'-0.000800(t-t_1)h$$

For bulb covered with ice:

$$x=f'-0.000706(t-t_1)h$$

These latter formulæ have been the basis of most meteorological reductions since 1848, but as Regnault recognized that they do not take full account of radiation, relative humidity, or velocity of the wind, and as he despaired of ever being able to do so, he, in his last memoir (1871), recommended his sulphuric acid thermometer as a better method.

(6) Glaisher by collating a vast number of observations was led to think that the formulæ and constants for the water-covered bulb apply also to the ice-covered, and that the formula for high pressure is applicable to the low pressures experienced in balloons and on mountains. But these conclusions can only be adopted as a rough approximation to the physical facts.

In a similar manner by collating simultaneous observations of dew-point and wet bulb he determined a series of factors by which the depression of the dew-point may be directly computed from the depression of the wet bulb (although with no great degree of accuracy) by the formula

$$(t-t_0)=G(t-t_1)$$

very much as Leslie and August had also done in the early history of

den neuen Formel mittheilen, und führe sie daher nur als Resultat an.
Perhaps he never got around to publishing the derivation of his 1830 formula. At least it cannot be found in Washington D.C.

the psychrometer. Glaisher's factor G varies with the temperature, as is shown in the following table:

Temperature of air.	Glaisher's factor.
t	G
10	8.78
20	8.14
30	4.15
40	2.29
50	2.06
60	1.88
70	1.77
80	1.68
90	1.63
100	1.57

(7) *Kämtz.*—The difficulty of obtaining consistent results with the wet bulb increases remarkably at temperatures below freezing; this is undoubtedly aggravated by the fact that the wet covering is not pure water, but a wetted muslin. This covering, or any device like a roughened or dimpled surface, must alter at once the conducting power of the layer between the mercury and the air, and must affect the sluggishness of the thermometer as well as the radiation and absorption of its surface. The effect of muslin as such relatively to that of the water increases slowly as the temperature lowers, since the lower the temperature so much less the evaporation and heat that is involved. Kämtz proposed to diminish the discrepancies between the results of observations with psychrometers and dew-point apparatus by subtracting 0.5° C. from the wet-bulb thermometer for all readings below freezing.

(8) Wüllner, as quoted by Sworykin, shows that *a priori* it is not evident that the change from water to ice should change the coefficient so greatly as is done in Regnault's formulæ for these two conditions, inasmuch as the coefficient A is essentially equal to $\frac{E}{c\lambda}$, which represents the quantity of heat (E) communicated in a unit of time to a unit mass of wind divided by the product of the quantity of water c evaporated in a unit of time and the latent heat λ of the vapor, where both E and c probably change with the change from water to ice, and, as they both vary with the velocity of the wind, this may explain the observed dependence of A upon the wind.

(9) Stefan, theoretically and experimentally, developed the laws of diffusion and convection of vapor in the process of evaporation, and subsequently applied them to the theory of the pyschrometer with spherical bulbs.

(10) Maxwell, in 1880, gave independently a similar mathematical analysis of the action of radiation, convection, and conduction, with a brief reference to numerical data.

(11) Chistoni, about 1878, in view of the uncertainties attending Regnault's determination of the constant A even for moderate velocities, attempted its determination for a definite low velocity. This gave him 0.00800 for a velocity of about 2 meters per second, and its great variability for small changes in the low velocities that he used became apparent, as had indeed been before shown by Regnault from his observations.

(12) Doyére, Angot, and Sworykin experimentally showed the value and amount of the variation in the constant A for various pressures and for the higher velocities given by the sling psychrometer; Sworykin gave the fullest accurate measures of the relation between A and the velocity of ventilation, showing that the ventilation must be known and allowed for numerically unless it be controlled to a standard uniformity, as in Belli's apparatus.

(13) Pernter, 1882, adopted Kämtz's correction to the wet-bulb temperature, explaining it as a sort of sluggishness, but making it proportional to $(t-t_1)$ so that for $(t-t_1)$ he substituted

$$\left[(t-t_1)+\frac{0.5}{(t-t_1)+1}\right]$$

in Regnault's formula. He also experimentally showed the effect of pressure, and by combining the determinations of Chistoni, Stefan, and Angot he concluded $A=0.000843$ for the constant coefficient at normal pressure and average ventilation. His final formula for centigrade thermometer and millimeter barometer was

$$x=f_1-0.000630\left[(t-t_1)+\frac{0.5}{(t-t_1)+1}\right]\left[1+\frac{760}{3b}\right]b$$

(14) Ferrel has, in his Recent Advances, given a very complete analysis of the physical questions involved in the pyschrometer, and in his Report on Psychrometric Tables for the Whirled Psychrometer (Annual Report Chief Signal Officer, 1886, p. 233) he has embodied his more refined theory in tables based on extensive special observations and that apparently render the psychrometer as accurate an instrument as our knowledge of the tension of aqueous vapor and the accuracy of our thermometers renders possible. Ferrel's analysis of the action of the wet bulb, as elaborated in these two works, is here presented with the fullness that its importance demands. When the wet bulb is cooling to the temperature of equilibrium it is giving its own heat by conduction to the evaporating water. The last step in this process, when but a few tenths of a degree remain to be annulled, takes place quite slowly, because of the thickness of the film of muslin and water; but eventually a stationary condition is reached such that the heat consumed in vaporization no longer comes from the bulb, but is almost entirely that given by conduction from the adjacent air and by radiation from

the surrounding inclosure. The departing wind carries away a lower temperature than that of the arriving wind, but slightly more heat, the latter being now latent in the vapor that it takes away. The quantity of heat required is determined by the amount of evaporation that is possible, and the latter is determined by the amount of diffusion that is possible, and by the rate of convection due to the ventilation. Ferrel therefore adopts Stefan's formula for evaporation by diffusion in still air, and Sworykin's observations on the action of the wind.

Let L=the latent heat of evaporation of water at the temperature θ_1 of the wet bulb.

Then, according to Regnault,

For water:

$$L=606.5-0.695\theta_1$$

For ice:

$$L=685.75-0.695\theta_1$$

Let ρ=the density of dry air at the standard pressure and temperature. According to Regnault 1 cubic centimeter of dry air weighs 0.00129276 grams at 45° latitude and sea level.

σ=the density of vapor relative to the air=0.623.

Q=the volume of aqueous vapor diffused through the still air in a unit of time at the temperature θ_1 from the unit of surface; $[Q]$ will be the total volume for the whole surface.

H=the quantity of heat conducted to a unit of surface of the wet-bulb thermometer in a unit of time by the adjacent air; $[H]$ is the total quantity for the whole surface.

h=quantity of heat absorbed by a unit of surface at the wet bulb in a unit of time from the external radiation; $[h]$ is the same quantity for the whole surface.

The slight amount of heat conducted down the glass stem to the wet bulb and that conducted and conveyed to it by the wet wicking are neglected by Ferrel, although they apparently may become appreciable in very dry air for large values of $\theta-\theta_1$. This may, however, be considered as merged in H. The following formula for still air expresses the fact that the receipt of heat equals the loss, which is the physical condition of equilibrium in the temperature of the wet bulb where the values of $[H]$, $[h]$, and $[Q]$ are to be given by the laws of conduction, radiation, and diffusion:

$$L\rho\sigma[Q]=[H]+[h] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

These quantities can be expressed in terms of the temperature by the following considerations:

(*a*) *Conduction.*—Let a=the area of the outer surface of the wet covering of the bulb; θ=the temperature; r=a distance measured from the center of the bulb; r_0=the value of r at the limit reached by the particles of diffusing vapor; r_1=the distance of the surface of the wet bulb from the center of the bulb; θ_1=the temperature of the air around the wet bulb at the distance r_1; θ_0=the temperature of the air around the wet bulb at the distance r_0, where the temperature is that of the external dry air; k=the coefficient of conductivity for heat, for a unit area and a unit gradient=0.000055 calories per square centimeter.

The laws of conduction give the general law

$$H=ak\frac{d\theta}{dr}$$

but for spherical bulbs $a=4\pi r_1^2$. Substituting this and integrating for the whole surface we have

$$[H]=4\pi k\frac{r_1 r_0}{r_0-r_1}(\theta_0-\theta_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

for shperical bulbs. For cylindrical bulbs, neglecting the ends of the cylinders, let l=the length of the bulb and we have $a=2\pi r_1 l$. Substituting this and integrating we have

$$[H]=2\pi kl(\theta_0-\theta_1)\times \text{mod}\times\log\frac{r_1}{r_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2')$$

(b) *Radiation.*—Let e=radiating power of the wet bulb, so far as relates to surrounding objects; B=radiating power of a unit of surface of lamp-black=1.146 calories per square centimeter per minute, approximately; μ=1.0077, the constant determined by Dulong and Petit.

Assuming that the bulb is wholly inclosed by objects whose temperature is θ_0, as in Belli's psychrometer, we have

$$h=Be(\mu^{\theta_1}-\mu^{\theta_0})$$

If the bulb is not contained in such an inclosure, or an equivalent fog, or under a cloudy sky, the value of θ_0 is subject to an unknown amount of variation, and it is best that the radiation should be regulated by inclosing the psychrometer in some form of shelter.

For a spherical bulb and for a cylindrical one the values of a are as before given; therefore the integration for the whole bulb is simply the product ha, which gives

For spherical bulbs:

$$[h]=4\pi r_1^2 Be\times 0.0077(\theta_0-\theta_1)\mu^{\theta_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For cylindrical bulbs:

$$[h]=2\pi r_1 lBe\times 0.0077(\theta_0-\theta_1)\mu^{\theta_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3')$$

(c) *Diffusion.*—Let D=Stefan's coefficient of diffusion for aqueous vapor in still air=0.18 grams per minute per square centimeter for ordinary temperatures and varying slightly, but to an unknown extent, with temperature; P=barometric pressure of the air; p_1=elastic force of vapor for temperature θ_1.

According to Stefan's researches the law of diffusion is expressed by

$$Q=aD\frac{d\log(P-p_1)}{dr}$$

Substitute the value of a, as before given in this value of Q, and integrate for the whole bulb, and we have

For spherical:

$$[Q]=4\pi D\frac{r_1r_0}{r_0-r_1}\times\frac{p_1-p_0}{P} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For cylindrical:

$$[Q]=2\pi lD\frac{p_1-p_0}{P}\times \text{mod}\times\log\frac{r_1}{r_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4')$$

(*d*) *Summary.*—Substitute in the original equation (1) the values given by equations (2), (3), and (4) and we get

$$p_0=p_1-A(\theta_0-\theta_1)P \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where

$$A=\frac{S}{L\sigma}\left[\frac{K'}{D}+c\mu^{\theta_1}\right]=\frac{S}{L\sigma}\left[\frac{K'}{D}+c(1+0.0077\theta_1+\text{etc.})\right] \quad . \quad . \quad (6)$$

S=specific heat of air=0.2375 calories per liter

$$K'=\frac{k}{\rho S}=\frac{0.000055}{0.2375\times0.00129276}=0.18 \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

For spherical bulbs:

$$c=\frac{0.0077Be}{\rho SD}\times\frac{r_1(r_0-r_1)}{r_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

For cylindrical bulbs:

$$c=\frac{0.0077Be}{\rho SD}\times\frac{r_1}{M}\log\frac{r_0}{r_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where M=modulus=0.434294.

(*e*) The quantities D, S, and K_1 probably vary with the temperature, but, neglecting these variations as quantities at present unknown and apparently of small amount, we may substitute the approximate constant values that obtain for ordinary pressures and temperatures, and thus put the previous formula in numerical shape. Neglecting the term $0.695\theta_1$ in the value of L we have the following:

For spherical bulbs covered with water in the still air:

$$c=2.66e\,\frac{r_1(r_0-r_1)}{r_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$A=0.000630[1+c(1+0.0077\theta_1)] \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$p_0=p_1-A(\theta_0-\theta_1)P \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

For cylindrical bulbs covered with water in the still air:

$$c=0.612r_1 \log \frac{r_1}{r_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

A and p_0 as before.

For bulbs of either shape covered with ice in the still air:

$$A=0.000557[1+c(1+0.0077\theta_1)] \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

c and p_0 as before.

These formulæ give no support to the idea that the coefficient of A is the same for water and ice. On the contrary, they show that this should not be so in perfectly still air; but still air in this sense can rarely exist, and the slightest current suffices to render the preceding formulæ more or less inapplicable. It is therefore necessary to allow for the effect of convection.

(*f*) *Influence of ventilation.*—The outside surface of the water on the wet bulb is that designated by a in paragraph (a) as being that at which conduction, absorption, radiation, and diffusion all take place in the still air; but if a current prevails (even the light convection current of cooler air falling from the wet bulb when a calm prevails in the surrounding air as a whole) then this ceases to become the bounding surface. In the preceding analysis r_0-r_1 is the depth of the stratum of air and water within which conduction of heat and diffusion of vapor are simultaneously taking place; if we assume that this depth varies inversely as the velocity of the wind, which is likely to be a first approximation, we may then put

$$r_0-r_1=\frac{K_0}{2.66eV} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where K_0 is an unknown constant and V the velocity of ventilation.

The introduction of the constant $2.66e$ into the denominator merely alters the constant K_0, and is done here since it leads to subsequent simplification in the formula. If we substitute r_0 from this equation in the expression for c in equations (10) and (11) we get

For spherical bulbs:

$$c=\frac{r_1}{r_0} \cdot \frac{K_0}{V}=\frac{2.66er_1K_0}{2.66er_1V+K_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

For cylindrical bulbs:

$$c=0.612r_1 \log \frac{r_1}{r_0}=0.612r_1 \log \frac{2.66er_1V}{2.66er_1V+K_0} \quad . \quad . \quad . \quad (17)$$

The value of K_0 must be determined by comparative observations between standard absolute hygrometers and wet bulbs of various sizes and shapes. Such observations are furnished especially by the work of Sworykin. If the adopted expression (15) for r_0-r_1 is allowable, and the resulting expression for c is approximately correct, then a single determination of c by Sworykin for a given value of the ventilation velocity and any special thermometer should determine the value of K_0 for that thermometer whose correctness can then be tested for other values of the velocity. Thus, from Sworykin's observations, Ferrel finds that $K_0=0.25$ for a large spherical bulb whose $r_1=0.5$ centimeter, and again $K_0=0.11$ for a small cylindrical bulb whose $r_1=0.2$ and $l=0.8$ centimeter.

With these values of K_0 Ferrel computes the values of c and of A by formulæ 16, 17, 11, and 13 for these two bulbs and various velocities, and shows that they agree closely with all Sworykin's determinations, as is illustrated in the following table:

Velocity of ventilation.	Spherical bulb. $r_1=5^{mm}$		Cylindrical bulb. $r_1=2^{mm}$ $l=8^{mm}$	
	A observed.	A computed.	A observed.	A computed.
Meters per second.				
0.0	....	0.001658	....	Infinite.
0.2	0.001120	.001084	0.000854	0.000893
1.0	0.000774	.000780	0.000700	0.000699
2.0	0.000712	0.000711	0.000670	0.000677
6.0	0.000656	0.000658	0.000640	0.000643
Infinite.	....	0.000630	....	0.000630

The large differences of the K_0 and A for these two bulbs show that these constants must be determined for each instrument, or at least for typical samples of the styles used in any meteorological bureau.

In this computation of K_0, and therefore in the ordinary use of the above formulæ, Ferrel employs a constant value of the factor $(1+0.0077\theta_1)$, as computed for a mean value of θ_1, thus throwing a small but probably inappreciable residual term depending on θ_1 into the value of K_0.

Ferrel follows Pernter in showing that the reason why the convection theory of August and Regnault and the present more elaborate analysis have led to the same form of formula is the accidental fact that K' and D have the same value, *i. e.*, 0.18. If K' had been equal to $0.77D$, as supposed by Maxwell, the coefficient 0.000630 would have become 0.000482, which, according to Sworykin's observations, is smaller than is consistent with any, even an infinite, amount of ventilation.

For velocities of 3 meters per second or 10 feet per second and over, the variations in A become so small as to have an inappreciable effect on the resulting value of the vapor tension p. This velocity is easily secured by whirling the wet bulb, as in the sling psychrometer, and Ferrel's tables, as adopted by the Signal Service, January 1, 1886, are adapted to psychrometers whirled with this velocity within a light shelter. The coefficients are so chosen as to satisfy the dew-point observations made by Professor Marvin on Pike's Peak, and by Professors Hazen and Marvin at lower altitudes over a wide range of temperature.

(*g*) *Conclusion.*—The resulting formula is

For metric measures:

$$p=p_1-0.000660P(\theta-\theta_1)\times(1+0.00115\theta_1)$$

For English measures:

$$p=p_1-0.000367P(\tau-\tau_1)\times(1+0.00064\tau_1)$$

which is applicable to all velocities over 3 meters per second, and very approximately to all shapes and sizes of bulbs, and to both water and freshly formed ice, provided that in this latter case the velocity of 5 meters or more is attained.

For convenience of tabulation these formulæ may be transformed, as recommended by Kämtz, into the approximate expressions

For metric measures:

$$p=p_1-0.000660P(\theta-\theta_1)\times[1+0.00115(\theta-\theta_1)]$$

For English measures:

$$p=p_1-0.000367P(\tau-\tau_1)\times[1+0.00064(\tau-\tau_1)]$$

which may differ from the preceding by 0.05^{mm}, but in the present work on the average differed ±0.016. This approximate formula represents all the 791 comparative dew-point observations of Hazen and Marvin with a probable error of $\pm0.084^{mm}$, the range of the discordances being from -0.49 to $+0.44$. Any error in the dew-point apparatus has not been considered, as two different instruments were used by the observers. Special investigations showed that the whirling should be continuous for one minute at the rate of 8 meters per second, or for two minutes at the rate of 3 meters per second in order to attain the true temperature of evaporation, and should not be continued much longer because of the drying of the muslin and the consequent rise in temperature.

In the use of the psychrometer attention must finally be called to the importance of attending to the corrections elucidated in chapter III, for the case in which the temperature of the bulb differs greatly from that of the stem, and to the Poggendorff and other corrections. These corrections are usually neglected, but as the adopted table of vapor tensions implies the use of the air thermometer, therefore the mercurials used in psychrometers and dew-point apparatus should be reduced to that standard.

147. EVAPORATION METHODS—THE RATE OF EVAPORATION.

Instead of measuring the temperature of the surface of evaporating water, as in the psychrometer, it is appropriate to measure the rate of the resulting evaporation, as in the evaporimeter. This measurement is usually considered as a means of ascertaining for engineering purposes the quantity of water lost by the earth or by reservoirs, etc., and as giving the meteorologists a crude approximation to the quantity of water daily thrown into the atmosphere. But the rate and the temperature of evaporation are equally dependent on the dryness of the air and the velocity of the wind, and are therefore equally available as means of determining the hygrometric condition. Owing to the small mass involved in the temperature observations by the wet-bulb thermometer that instrument is adapted to give the momentary condition of the atmosphere. On the other hand, the large masses required in the measuring operations of the evaporimeter renders this instrument important to the meteorologists as a means of ascertaining the average hygrometric condition of the air during a long interval. From this point of view, therefore, this becomes an integrating hygrometer, and demands a more minute theoretical investigation than has as yet been given to it.

At present we can only indicate the basis of this investigation. The most accurate observations available are those made in 1876 to 1882 by Desmond Fitz-Gerald, civil engineer, engineer in charge of the Chestnut Hill Reservoir, near Boston, Mass. (see Transactions Amer. Soc. C. E., 1886, Vol. XIV). Fitz-Gerald's observations combined with Stefan's work on diffusion should give a first approximate formula for the utilization of the evaporimeter as an integrating hygrometer. Of its use as an evaporimeter only, the works of Dalton, Lamont, Weilenmann, Stelling, and others will be discussed in a future volume.

Fitz-Gerald's measurements of the evaporation of water in pans 14.85 inches in diameter, in which 1 ounce of water is represented by a depth of 0.01 inch, gave him the value of E or the depth of water in inches evaporated in one hour. These measures were made at the ordinary atmospheric pressures, and do not show any appreciable effect due to the ordinary range of the barometer at Boston. They are represented quite closely by an empirical formula similar to those deduced by other investigators, namely,

$$E=[0.014(V-v)+0.0012(V-v)^2]\,[1+0.67\,W^{\frac{2}{3}}]$$

for which Fitz-Gerald uses the approximate expression

$$E=0.0166(V-v)\,(1+\tfrac{1}{2}W)$$

in which

V=vapor tension in inches of mercury corresponding to the temperature of the water.

v=vapor tension corresponding to the dew-point in the free air.

W=velocity of the wind in miles per hour measured by the Robinson anemometer at the level of the water surface. Fitz-Gerald finds that the velocity recorded by an anemometer 30.5 feet above the water is three times that prevailing at the surface.

Comparative observations made in the sunshine and the shade are equally well represented by the above formula. The evaporation from snow and ice was also measured with minuteness and found to be well represented by this formula.

If, then, the temperature of the water is observed so that V, E, and W are known, then the Fitz-Gerald formula gives the average vapor tension in the free air during the time in which the evaporation was effected; for which purpose it may be written

$$v = V - \frac{60E}{1+\frac{1}{2}W}$$

This use of the evaporimeter, therefore, is additional to its ordinary use for engineering purposes, but implies that the temperature of the water and the velocity of the air at its surface be observed.

148. DIFFUSION METHODS.

Under this general term I include two apparently dissimilar pieces of apparatus.

(*a*) *Dufour's hygrometer.*—The rate at which vapor diffuses through a porous diaphragm depends on the difference of the elastic pressure of the vapor on either side of the diaphragm. The laws of this form of diffusion known as endosmose and exosmose have been studied by Dufour,* Reusch, Merget, and Kundt, and may be applied to the construction of a form of hygrometer.

Let there be a porous vessel provided with a manometer, so that the changes of the elastic force within may be easily measured; when the vapor tension of the outside air is less than that of the inside vapor the outer air passes through the porous vessel and the manometer shows a higher pressure within and *vice versa* when the internal air is moister. These variations in the manometer are temporary, and eventually an equilibrium is attained in accord with Graham's laws of diffusion. If the inner air is kept saturated by water in the vessel at a known temperature, shown by the thermometer, then the internal vapor tension is always known, and the external tension becomes known by converting into hygrometric values the manometer readings taken soon after exposing the porous cylinder to the air.

[NOTE.—August, 1888. Since writing the above I find that Schidlowski (Wied., Beiblätter, 1887) has used a modification of this form, consisting of a metallic vessel, whose open end is covered with a porous plate. If the vessel contains water the diffusion causes the pressure within to rise. If, on the other hand, the vessel contains sulphuric acid as an absorbent the diffusion causes the pressure within to fall. The graduation of the scale of relative humidity is empirical.]

Undoubtedly a similar diffusion into animal and vegetable cells through their thin walls is the basis of the hygrometric action that is utilized in the hair hygrometer of Saussure.

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* Bibl. Univ. Rev. Sci., 1874, XLIX; 1875, LIII.

(*b*) *Saussure's hair hygrometer.*—The earliest forms of hygroscopes depended on the effect of atmospheric moisture in expanding animal and vegetable substances. The hair hygrometer of Saussure is found to be the most uniform in its indications. The most approved form of construction is that known as the Goldschmid-Koppe hygrometer (see Fig. 96). In this a delicate single hair about 10 inches long extends vertically from the summit to the base of a light brass frame; at the base the free end is wound around an axis, which carries a light and well-balanced index, and then continuing below is held taut by the action of a light spiral spring, whose tension is about equal to the weight of one-half gram, and can be lessened at will. The upper end of the bar is fastened to an adjusting screw, by moving which the index can be set at 100 per cent. on the arbitrary scale whenever the air within the case is known to be thoroughly saturated. This adjustment to 100 is made at any moment by slipping a sheet of wet cloth within the box containing the apparatus, thus causing the air to be quickly saturated. The perfection to which this form of hygrometer has attained depends largely on the convenience of this test and on the introduction of the tension spring below, but especially in preparing the hair itself. Human hair is chosen, well cleaned of all fatty substances and having a breaking strength of 100 grams and an elastic stretch of 33 per cent. The lower axis, about which this is wound, and by which it turns the index, has a diameter of 4 millimeters. The arc which the index describes is about 1 quadrant, divided into a scale of 100 parts, of which the zero point is at the left hand and the 100 per cent. on the right hand, the right-hand divisions being much compressed, as compared with the left hand and as concerns the size of the degree mark. The average expansion in the length of the hair for a change of 1 per cent. is 0.03 millimeter, whence it may be inferred that a very large per cent. of change in the dryness of the air will not cause the hair to materially change its hygroscopic peculiarities, although the zero point of the scale may change. Observations show that for the same hair the same scale is correct whenever the point of saturation is adjusted anew after any derangement.

The fact that the hair has become unserviceable is known whenever its hygroscopic peculiarities become so changed (as, for instance, by undue stretching or by boiling in too strong alkaline solution) that it ceases to lengthen or may even shorten in very moist air. As a dry hair loses its strength the mistake is often made of applying heavier weights; instead of this it is best to apply no weight exceeding a half-gram. The scale is to be empirically graduated by comparing at a few points with standard instruments. From such comparisons made by Koppe in 1877 he concluded an error of 1 per cent., but from other comparisons a probable error of 2 per cent. has been found for the best hair hygrometers. For low temperatures this apparatus is apparently subject to much larger errors, but these may in part be due to the standard instruments themselves.

SECTION E.

THE MEASUREMENT OF PRECIPITATION.

CHAPTER XX.

THE MEASUREMENT OF RAIN AND SNOW.

149. THE PROBLEM.

The aqueous vapor in the atmosphere, after being condensed into cloud, falls to the earth in the forms of rain, hail, and snow. The measurement of such precipitation constitutes a class of observations in meteorology which has most extensive practical applications.

The precipitation of aqueous vapor is usually held to include all its visible forms, as the cloud, fog, dew, and haze, but rain, hail, and snow are the most important, and will alone be considered in this section.

The natural unit of measurement would be by volumes, but just as the barometric height has been adopted instead of pound's weight for expressing the pressure of the air, so the depth of the fallen water when spread uniformly over the horizontal projection of the surface on which it fell has been adopted as the conventional unit by which to measure precipitation; this horizontal projection of the surface corresponds to the legal method of measuring the area of land. The object of the rain-gauge is to collect the rain falling upon a given small horizontal area, and by its measurement to determine the height of rain-fall over that area.

150. ORDINARY APPARATUS.

The observation is of the simplest character when the instrument is a receiver with horizontal bottom and vertical sides.

Inasmuch, however, as for light rains the measurement of the height of the water is subject to a relatively large error, it has been customary to measure the "catch" in a gauge of smaller sectional area than the receiving surface. By this means the measured height is magnified and the real height is obtained with greater precision.

(*a*) *The gauge—Forms.*—Innumerable forms of receivers and measuring apparatus have been used. The receivers have been funnels, cylinders, or boxes, with or without beveled rims; their size has been a few square inches or square feet in area; their material has been sheet metal, porcelain, wood, or glass; their color has been light or dark, painted or unpainted.

Methods of measurement.—The height has been obtained by an immersed stick; the volume has been obtained by pouring the water into a graduated glass tube; the height has been computed from the volume; the water has been weighed by a balance and the height computed therefrom. The first method is the simplest and can be made sufficiently accurate.

The accompanying diagram (Fig. 97) shows the Signal Service standard rain-gauge. The diameter of the receiver is 8 inches, and the height of the instrument is 2 feet. The rim of the receiver is made of brass, beveled to a sharp edge and accurately circular. This brass rim is soldered to a galvanized iron funnel.

The collecting tube is made of seamless brass tubing No. 16 of 2.53 inches inside diameter and 20 inches deep. With these dimensions the area of the measuring tube is to the nearest hundredth of an inch, one-tenth of the area of the receiving surface.

The measuring stick is of well-seasoned cedar $\frac{1}{2}$ inch wide $\frac{3}{32}$ inch thick and carefully graduated.

Each instrument is tested to determine the outstanding errors in the accuracy of the measuring tube and its ratio to the receiving surface, and corrections are furnished for any errors greater than one-half of 1 per cent. of the measured rain-fall.

The size and quality of the measuring stick is such that the possible errors incident to its use need no special consideration.

Although the sectional area of the stick is small, yet the small displacement of the water in the tube is allowed for in the ratio of the receiving surface to the measuring tube. The graduations of the scale are made with all the accuracy needed for rain-fall measurement. The cedar wood out of which the sticks are made has a small capillarity, and gives a clear sharp line of wetting, whose position on the scale can be read off with all requisite accuracy.

The Signal Service rain-gauge needs therefore no purely instrumental corrections additional to those furnished by its correction card in order to measure the catch to an accuracy of 1 per cent., and this should be true of any other form of rain-gauge.

But more important than those of the instrument itself are the errors arising from its exposure, which are large and difficult of determination.

(*b*) *The exposure.*—Gauges have been exposed in all locations; on roofs, poles, towers, on the surface of the ground, and buried to the rim of the gauge. All these and numerous other modifications have given rise to a large literature on the whole subject, the details of which have mostly but little value to future observers, but are of vital importance to the proper interpretation of past records. Owing to peculiarities of exposure the amount of rain caught in the mouth of the gauge is not the same as would have fallen upon the same surface in the absence of the gauge itself, and, further, after correcting for this the rain falling at the place of exposure may not be with any accuracy the average amount of rain falling in the immediate vicinity. These sources of error, depending on exposure and location, respectively, have been shown to be ultimately due to the influence of the wind, which in blowing against obstacles suffers variations in velocity and deviations in direction, and thereby brings about local inequalities in the distribution of the rain-fall.

That the catch of a gauge varies according to its location was noted by Dr. Heberden in 1766 by observing that gauges placed on the ground collect, in general, a larger amount of rain than gauges on roofs and high towers. In lack of any satisfactory explanation of this variation Dr. Heberden says "It is probable that some hitherto unknown property of electricity is concerned in this phenomenon." Some years later Benjamin Franklin suggested that the increase of rain collected in gauges on the ground might be due to the augmentation of the rain-drops by the condensation of moisture on their surfaces during their fall. Though this hypothesis was considered by Dr. Franklin himself to be inadequate and open to objection it became the generally accepted explanation of the phenomenon.

The insufficiency of this explanation was afterward shown by others, notably Sir John Herschel, who demonstrated that the increase of a rain-drop by condensation or otherwise within the last few hundred feet of descent must be absolutely inappreciable.

The first step towards a true explanation supported by the results of observation was made by Prof. A. D. Bache, of Philadelphia, in a paper presented in 1837 to the British Association, entitled "Note on the effect of deflected currents of air on the quantity of rain collected by a rain-gauge." Professor Bache made the important discovery that of four gauges placed at the corners of a roof, in general, the gauges to leeward received more rain than those to windward. From this difference he was led to the belief that the effect of eddy-winds is a subject of primary consideration in the whole question of rain-gauge exposure. In 1855 and 1859 Henry fully explained the errors of the Franklin hypothesis and gave the correct explanation of Bache's results.* A further elucidation of the variations in collection observed by Bache was furnished in 1861 by W. Stanley Jevons (Phil. Magazine, 1861: Signal Service Notes, No. XVI), who showed that when by reason of an obstacle of any kind the wind is forced into currents of varying velocity, greater and less than the normal, an irregular distribution of rain-fall will result. At places where the wind velocity is increased the rain will be carried over and past, and a deficiency will be collected, in like manner where the wind velocity is diminished an excess is collected. The rain-gauge is itself to be considered as such an obstacle to the wind, and the currents around it divert the rain-drops from the mouth of the gauge, causing a deficiency that is proportional to the wind velocity. Buildings, towers, fences, and trees are still greater obstructions, causing irregularities in distribution that vary with the direction and force of the wind, so that gauges located on or near them may give either deficient or excessive amounts, but more generally the

* See also, Maille; Ann. Soc. Met. de France, 1855, III., p. 165.

former. The smaller quantity of rain collected on a roof than on the adjacent ground is undoubtedly due to the deflections and eddies about the building and the gauge, which cause such locations to be classed as very "poor exposures" compared with the center of a cleared field.

Signal Service observations show that this deficiency in the rain caught by a roof gauge frequently amounts to 25 per cent. of the true total rain-fall, even in monthly and yearly averages. European observers have found that for gauges of different sizes exposed to the wind the very small gauges have the greater deficiencies, evidently because the eddies about them have a larger ratio to the receiving surface. Extended comparisons made by the Signal Service between 3-inch and 8-inch gauges when exposed to high winds have shown, in conformity with European experiments, the superiority of the larger gauge.

(c) *The standard exposure.*—The preceding considerations lead to the definition of the standard exposure as one in which no obstacles, including the gauge itself, have any influence on the catch. This criterion is attainable by placing a gauge in a large level space, and buried in the ground so as not to offer any obstruction to the wind. In practice the top of the gauge must, of course, be sufficiently above the surface of the surrounding ground to prevent any spattering drops from falling into the gauge.

Such an exposure was recommended about 1850 by Joseph Henry in his instructions (Tenth Annual Report of the Smithsonian Institution, 1855) to observers of the Smithsonian Institution. There are, however, several practical objections to such an exposure; gauges whose mouths are near the surface of the ground frequently become filled with leaves, dirt, and rubbish of all kinds, and are liable to accidental injury, and to disturbance from animals and unauthorized persons. It is also often impracticable to find such locations, especially in large cities. Therefore in 1858, in his memoirs on meteorology, Henry recommended what is now known as the shielded gauge (see his Scientific Writings, pp. 260–262).

151. THE SHIELDED GAUGE.

(a) *Henry's gauge.*—In view of the difficulties introduced by gauge-eddies Henry recommended a simple cylindrical gauge 2 inches in diameter, near whose mouth is soldered a collar, consisting of a horizontal sheet of metal "like the rim of an inverted hat." Comparative observations are said to have been made with this form.

(b) *Nipher's shielded gauge.*—As a solution of nearly all difficulties Professor Nipher recommends a post or pole exposure, and has sought to eliminate the effect of the gauge itself as an obstacle to the wind when in an elevated position by surrounding it with a shield that deflects the wind downward instead of allowing it to sweep up over the mouth of the gauge. The following description (Am. Assoc. Ad. Sci., 1878, p. 106) of this is given by Professor Nipher (see Fig. 98): Six inches from the lower end of the cylindrical rain-gauge a false bottom is placed, and the cylinder is set over a turned post for support. Around this gauge a trumpet-shaped shield is placed with the mouth or flaring part opening upward. The shield is furnished with a clamp screw at the bottom, and is braced near the top by metal strips set with their edges up and reaching from the shield to a sliding collar, which encircles the gauge. By this means the shield can be set at any desired altitude on the gauge. The upper part of the shield terminates in a horizontal annulus of copper wire cloth. All splashing is avoided as all drops fall through the cloth, and in order that this end may be secured the wire must be small (No. 20, B. W. G.), the meshes running about 8 to the inch. Comparisons between a shielded and unshielded gauge set 6 feet above the ground showed in the latter a rain-fall of 97 per cent. of that collected in the former. Further experiments made by Nipher on the roof of a tower confirmed the advantage of the shields, but nowhere on the roof could uniform results be attained, owing to the irregular distribution of rain over its surface. The gauge was then raised to an elevation of 18 feet above the roof and 118 above the ground. An unshielded gauge thus placed showed a catch of 90 to 50 per cent. of the catch of the shielded ground gauge. The shield was then placed around this upper gauge and clamped, so that its rim should be at various elevations above the top of the gauge; when this elevation amounts to an inch it is necessary to take precautions against splashing into the gauge. This was done by placing inside the shield a concentric cone of copper wire cloth separated from the shield by an interspace of half an inch. From the results of the experiments it appeared that when the shield is placed at an elevation of 3½ inches the elevated gauge, 118 feet above the ground and 18 feet above the roof, gives about the same indications as the common unshielded gauge at the ground, although differing in level 112 feet.

Recent similar comparisons by Wild corroborate Nipher's conclusion that such a shield by annulling the eddies about the mouth of the gauge renders its records almost wholly independent of the wind that strikes it, so that wherever located it gives the true rain-fall for that spot. With Nipher's forms of gauge, therefore, we can examine the distribution of rain in the neighborhood of a building, tree, hill, or other obstacle, and for each direction of the wind, and decide upon the correction needed to reduce any exposure to the correct average of the neighborhood. In general, with regard to the roof exposure adopted in cities, there is little doubt that by raising the gauge above the roof and providing it with a shield the catch will approximate more nearly to the true rain-fall.

152. THE MEASUREMENT OF SNOW.

For collecting snow a simple cylinder 2 feet deep is used by the Signal Service in place of the funnel rain-gauge, which would be unsuited for the purpose. In cases of light dry snow-flakes and high wind, however, all gauges frequently fail to collect or to retain the proper amount of snow, and the catch should, when possible, be checked by measurement of the snow on an undrifted uniform flat surface. This is best done by plunging the cylinder vertically into the level snow until its lower edge reaches either the ground or the upper surface of the snow that fell since the last measurement. A sheet of tin is then slipped under the gauge; the snow thus collected is melted, and its amount represents the height of the snow-fall in equivalent inches of water.

As preparatory to such measures a small spot of ground should be kept covered with a sheet of tin or a few sheets of paper, so that one of these may be lifted up with the inverted gauge.

The measurement of the height of the snow and the reduction of this height to an equivalent height of water by assuming 10 inches of snow to 1 of water or some other factor, is subject to a large range of error because of the wide variability of this ratio for different kinds of snow. The rain and snow-fall should be melted and measured at every observation and not merely once a day.

The depth of the snow in inches as it lies on the ground is frequently wanted in weather predictions, and should be recorded at every observation in addition to the measure of snow-fall during the preceding few hours.

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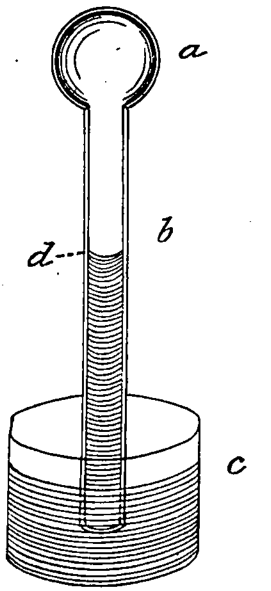
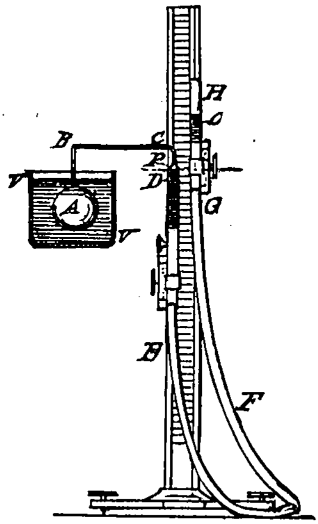
Fig. 1.



GALILEO'S AIR THERMOMETER.

Fig. 2.



JOLLY'S AIR THERMOMETER.

Fig. 3.

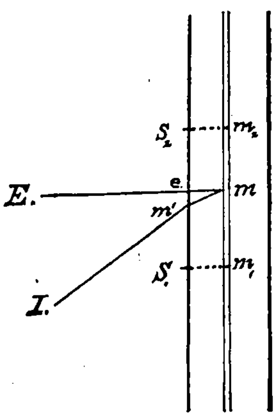


Fig. 4.

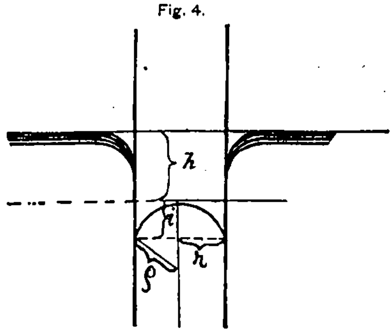
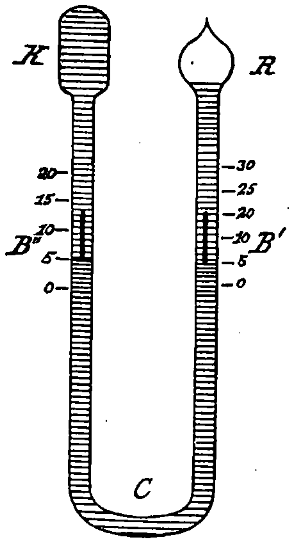


Fig. 5.



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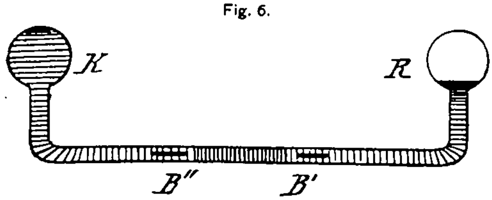
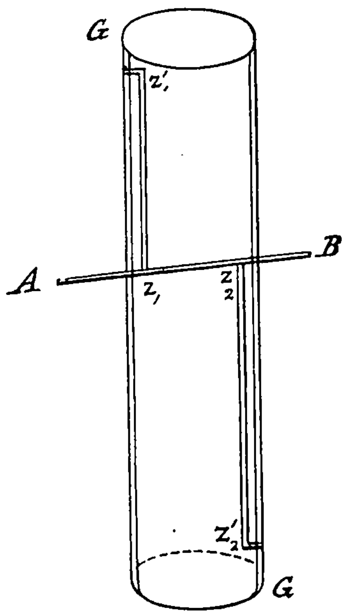
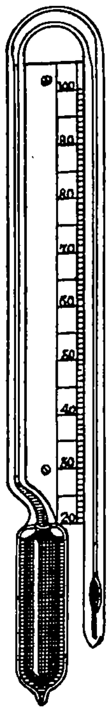


Fig. 7.



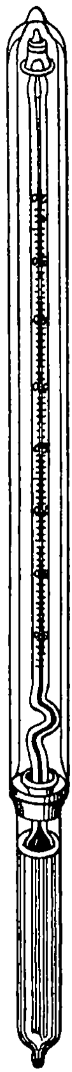
UPSET-THERMOMETERS.

Fig. 8, *a*.



NEGRETTI AND ZAMBRA'S EARLIEST FORM.

Fig. 8, *b*.



NEGRETTI AND ZAMBRA'S RECENT FORM.

Fig. 9.

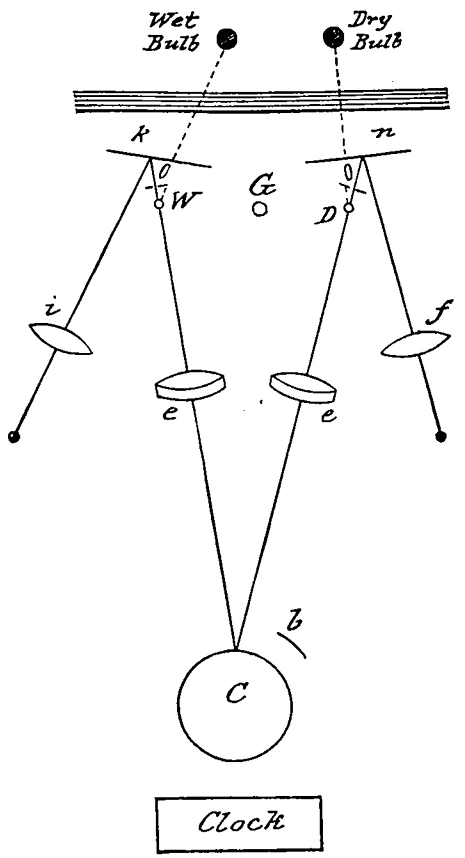


Fig. 10.

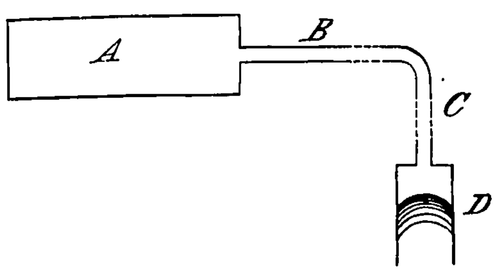


Fig. 11.

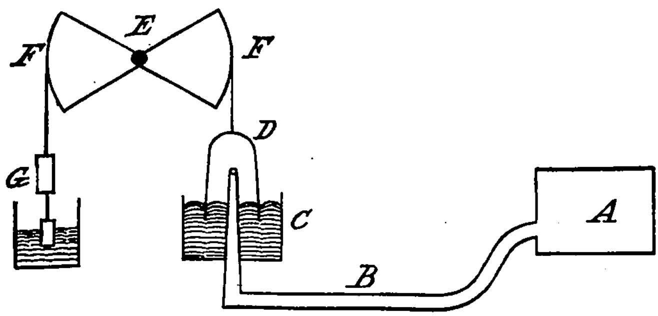


Fig. 12.

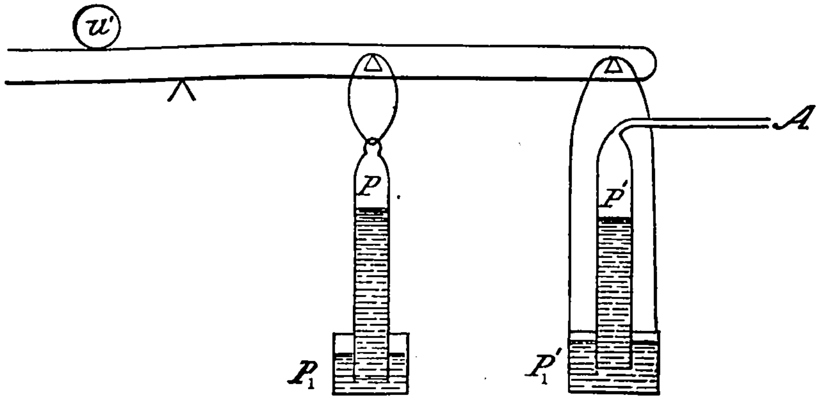


Fig. 13.

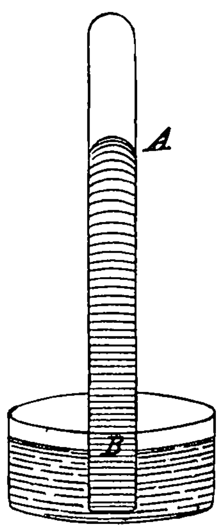
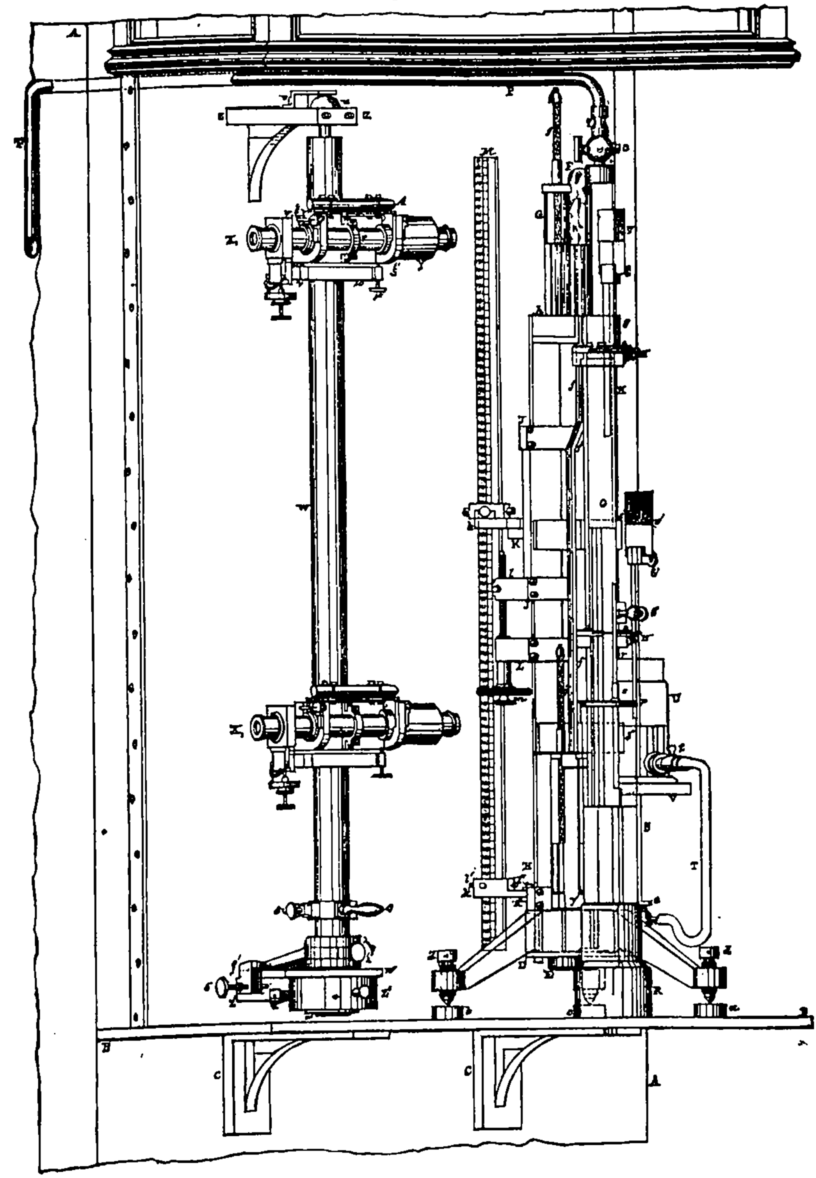
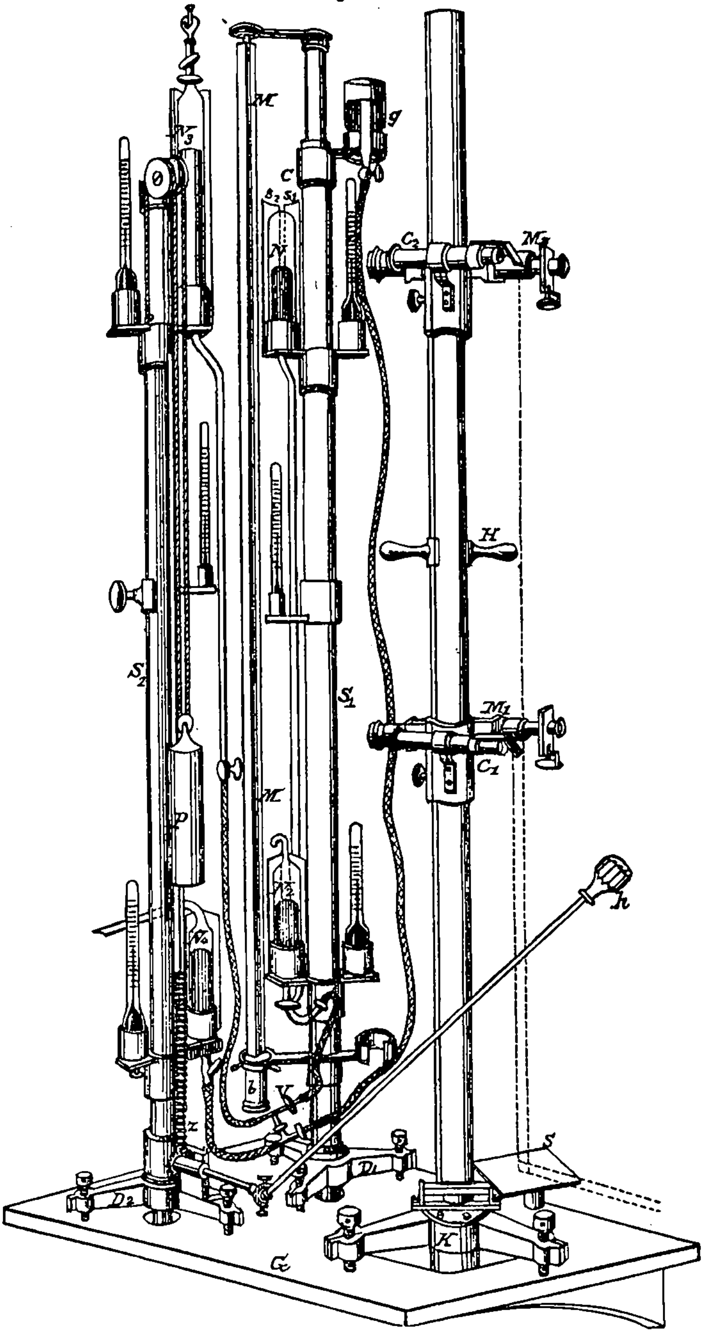


Fig. 14.



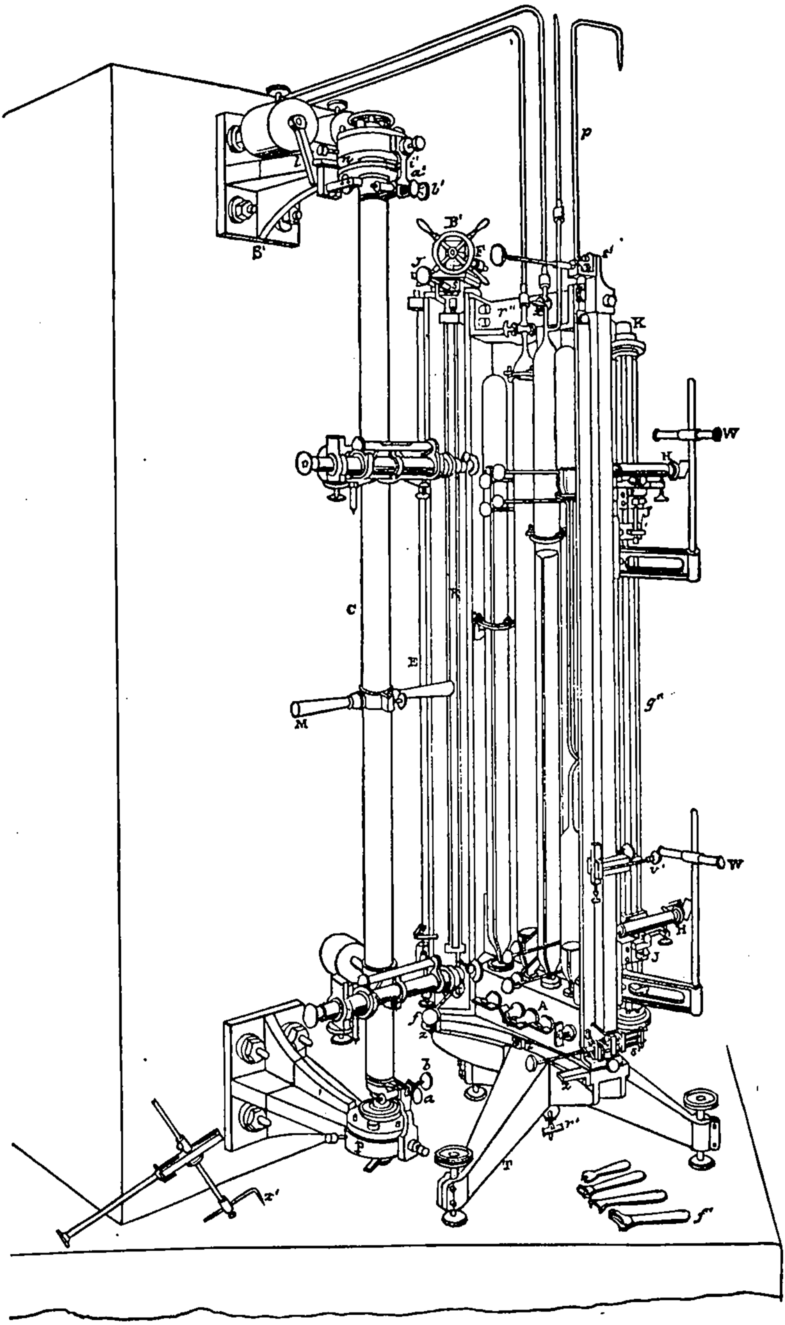
WILD'S NORMAL BAROMETER.

Fig. 15.



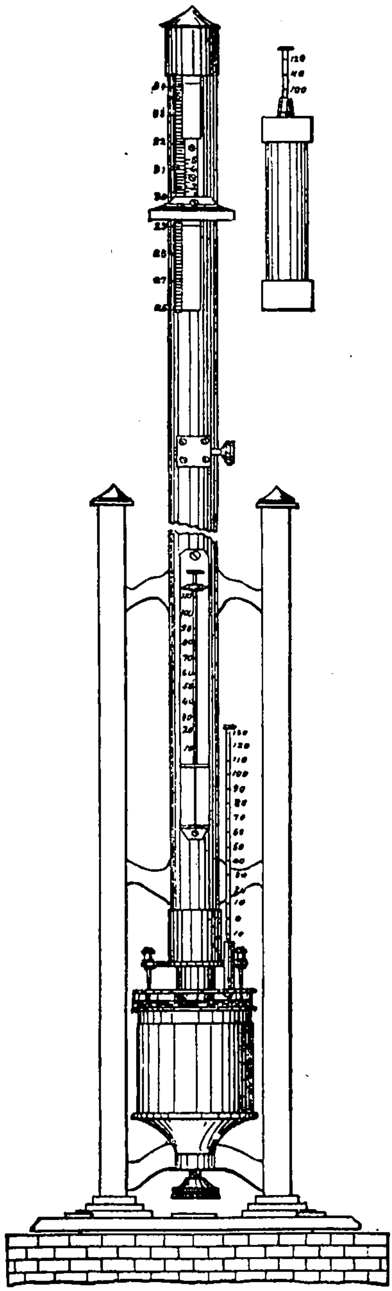
FUESS NORMAL BAROMETER.

Fig. 16.



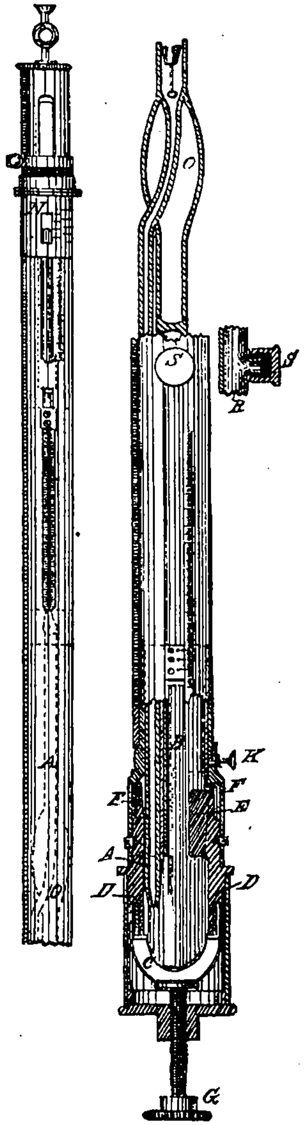
MAREK'S NORMAL BAROMETER.

Fig. 17.



GREEN'S STANDARD BAROMETER OF 1879.

Fig. 18.



WILD-FUESS CONTROL BAROMETER.

Fig. 19.

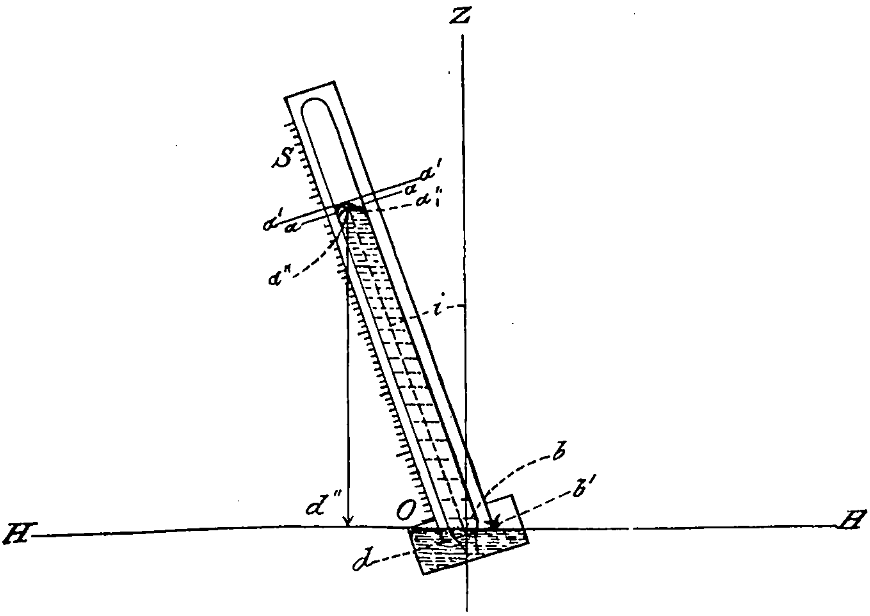


Fig. 20.

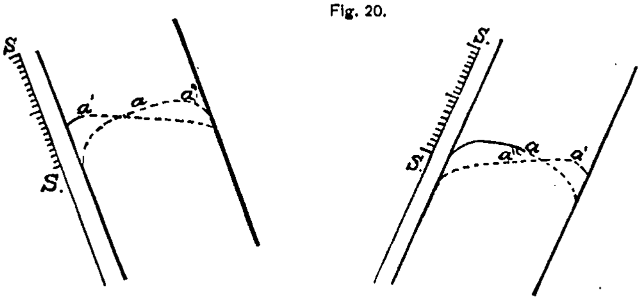


Fig. 21, *a*.

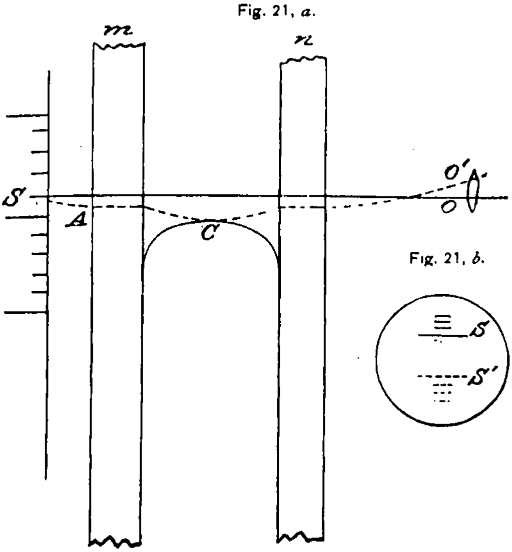


Fig. 21, *b*.

Fig. 22.

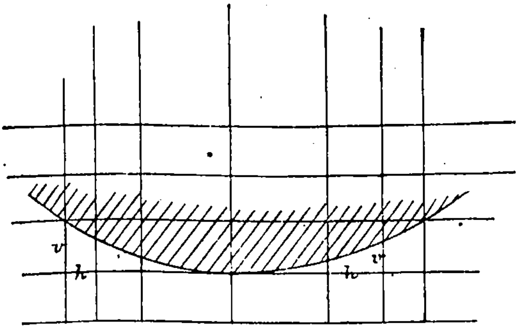


Fig. 23.

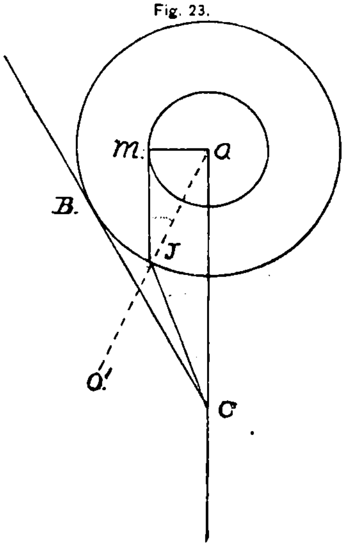


Fig. 24

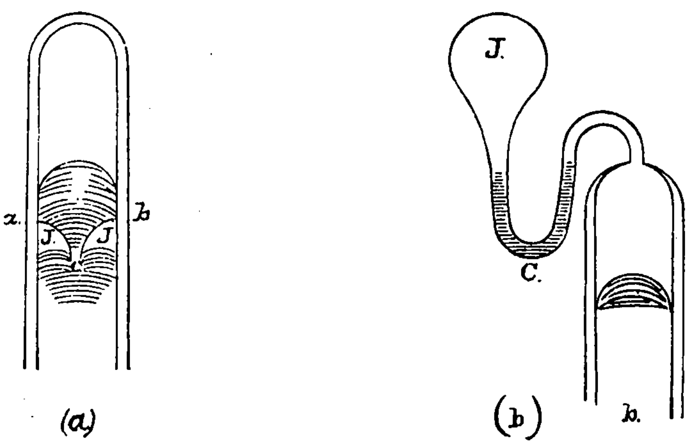


Fig 25.

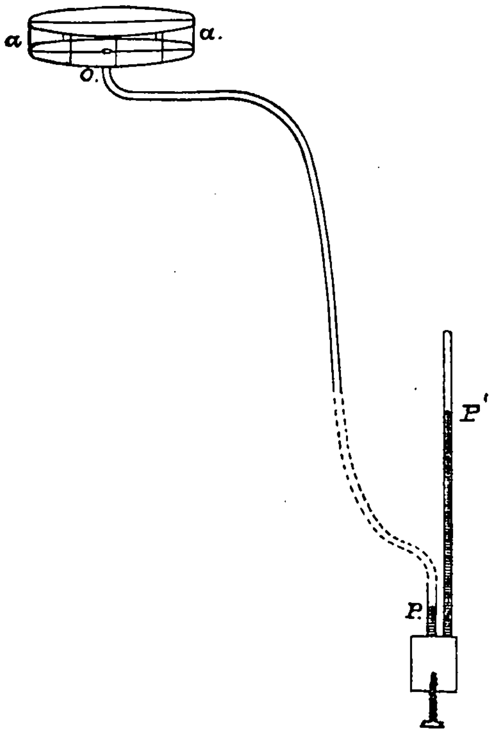


Fig. 26.

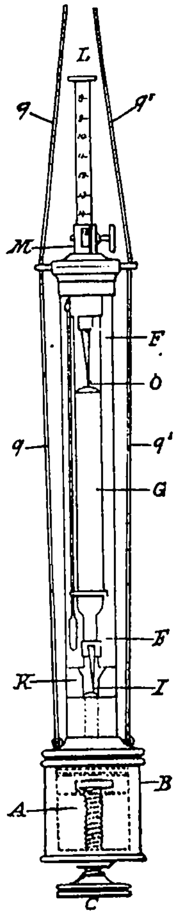


Fig. 27.

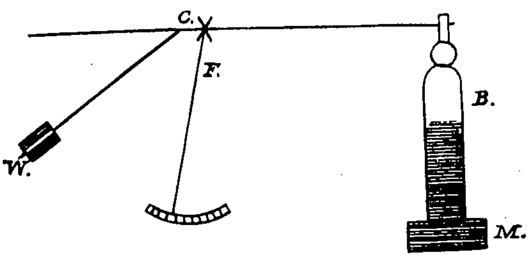


Fig. 28.

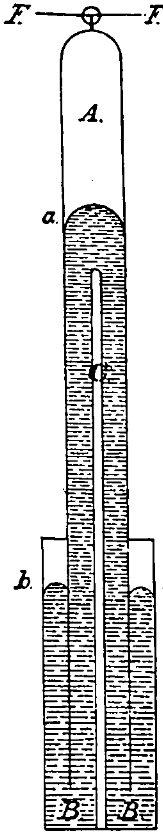
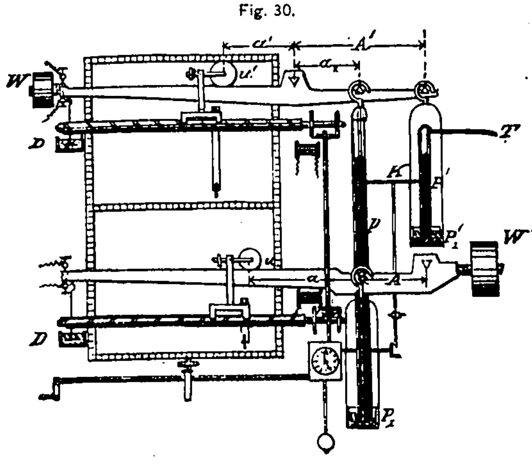
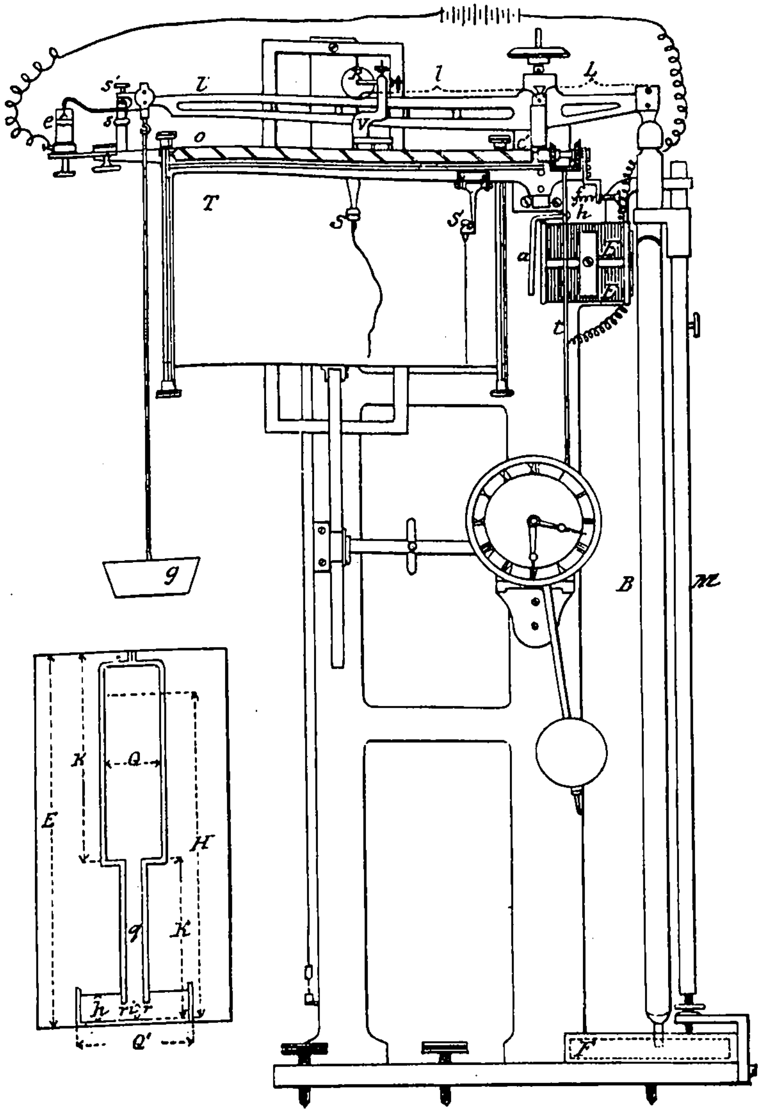


Fig. 30.



SPRUNG'S NEW THERMO-BAROGRAPH.

Fig. 29.



SPRUNG'S FIRST BAROGRAPH.

Fig. 31.

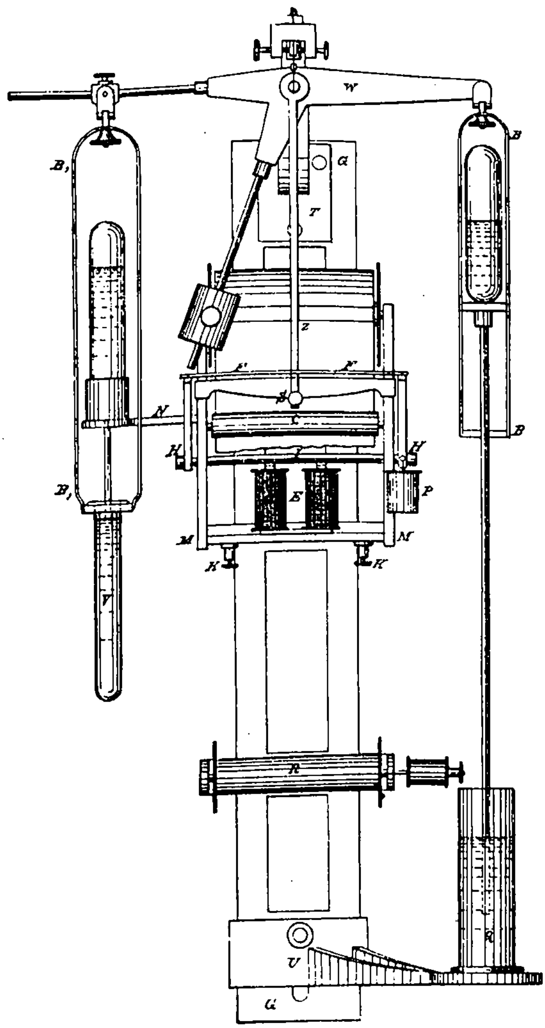
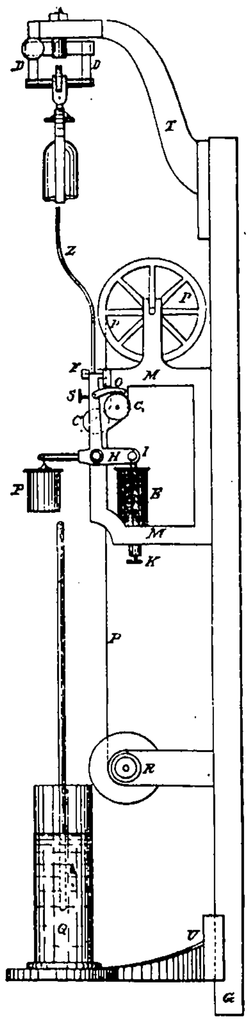


Fig. 32.



WILD-HASLER BAROGRAPH.

Fig. 33.

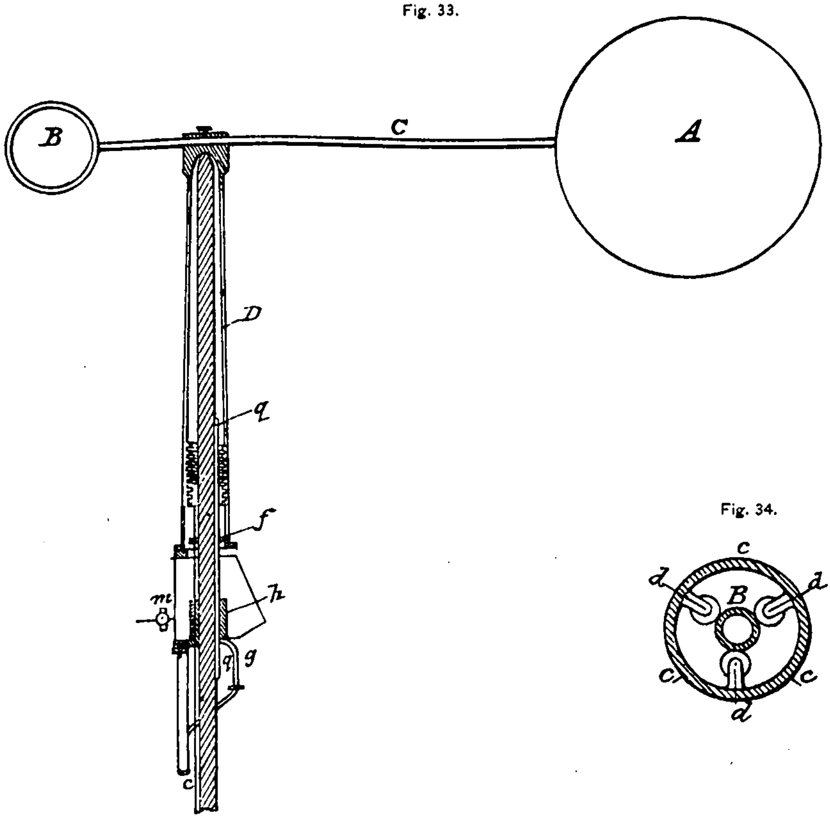


Fig. 34.

Fig. 35

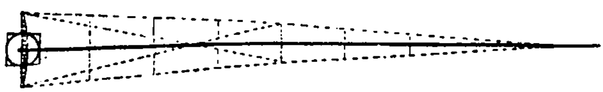


Fig. 36.

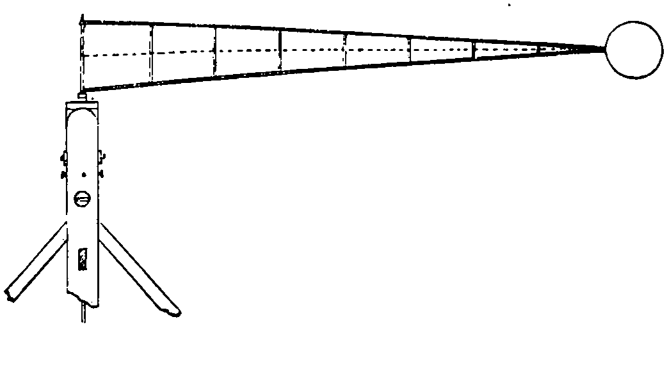


Fig. 37.

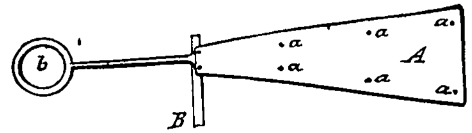


Fig. 41.

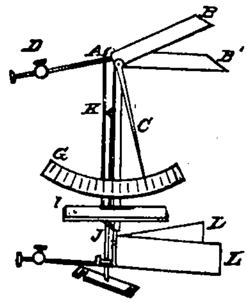


Fig. 38.

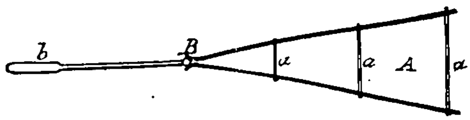


Fig. 40.

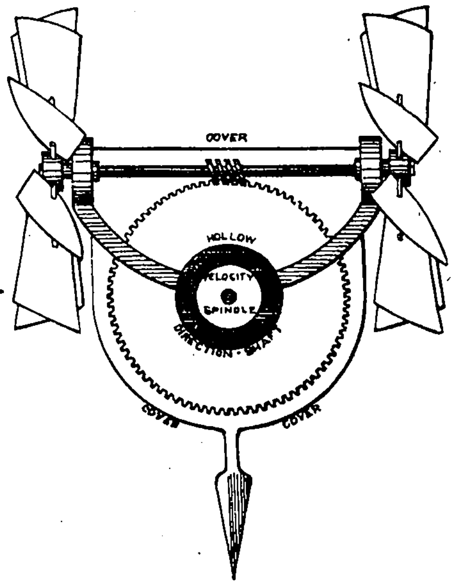
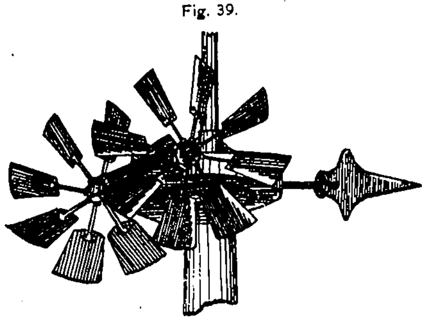
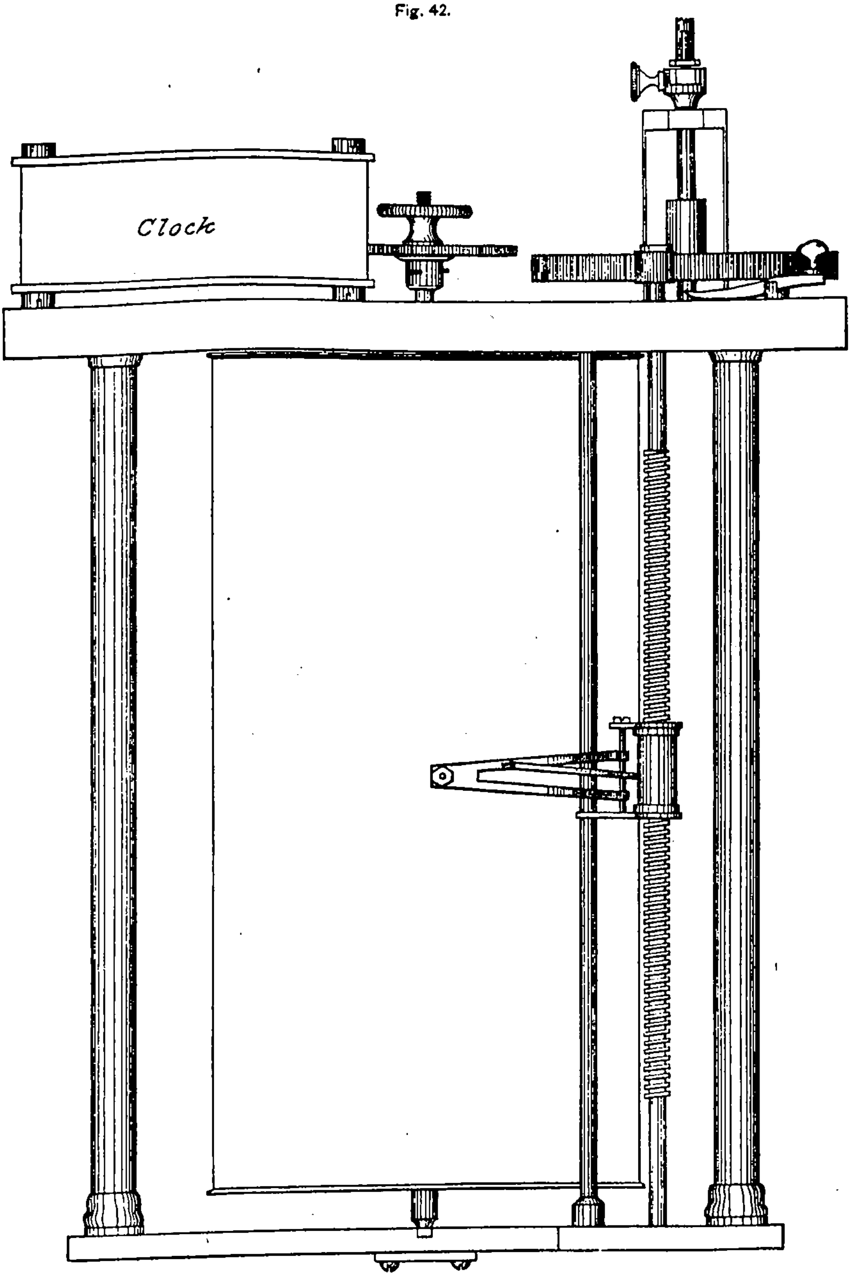


Fig. 39.



WINDMILL VANE.

Fig. 42.



SIGNAL SERVICE ANEMOGRAPH.

Fig. 43.

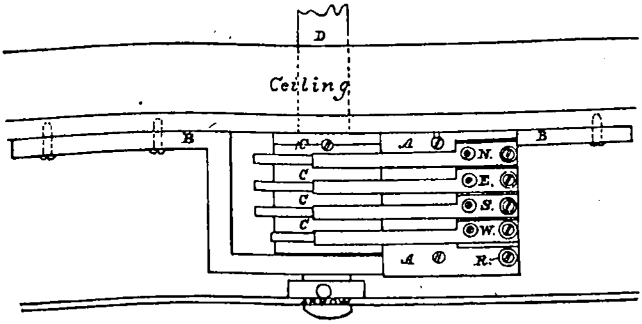


Fig. 44.

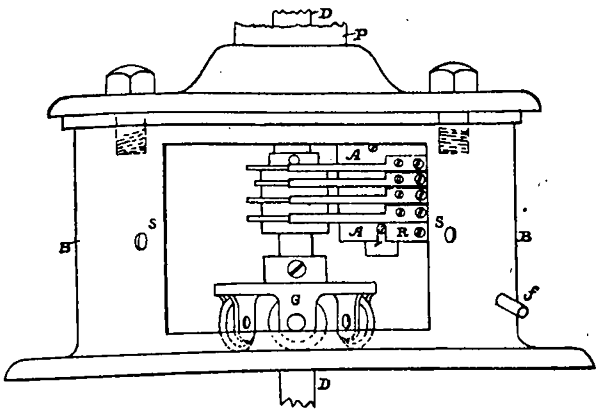


Fig. 45.

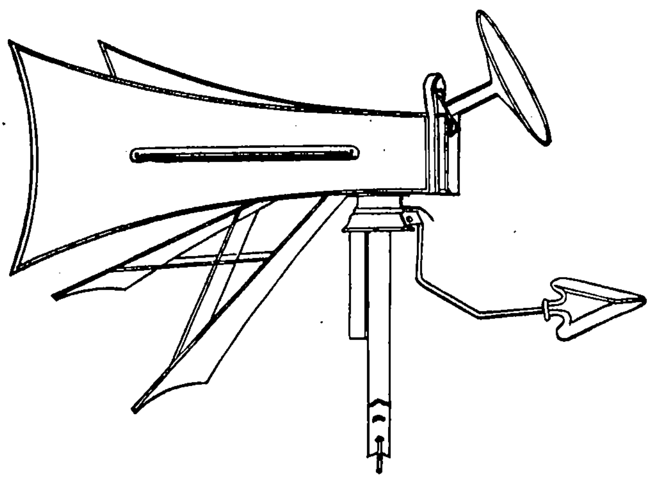


Fig. 46.

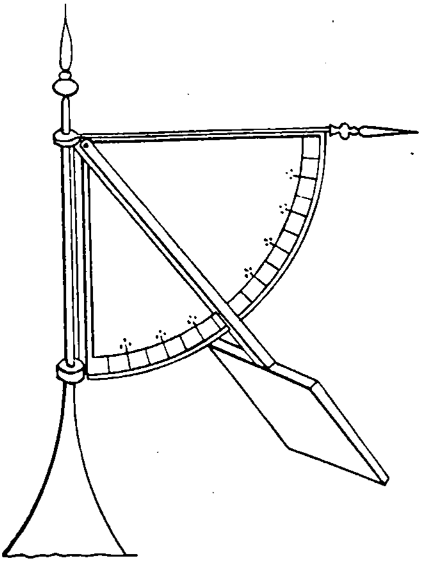


Fig. 47.

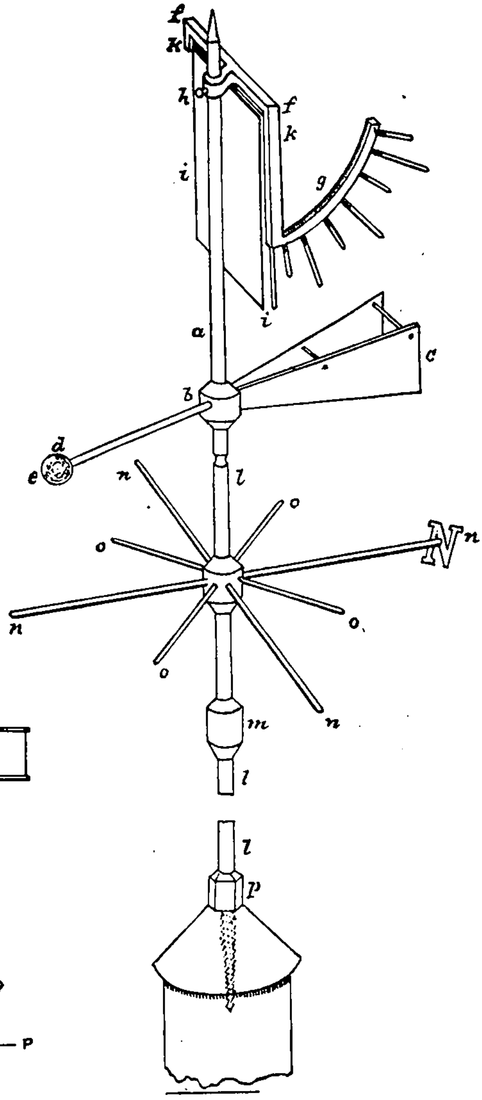


Fig. 48.

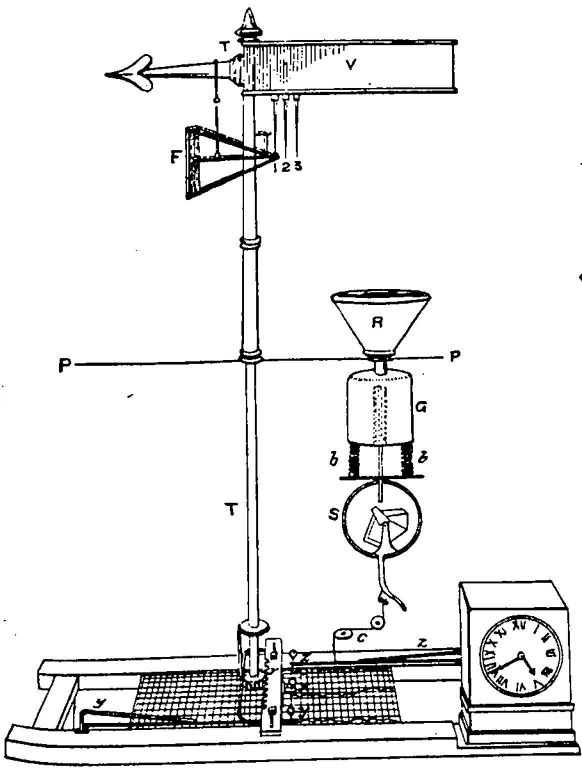


Fig. 49.

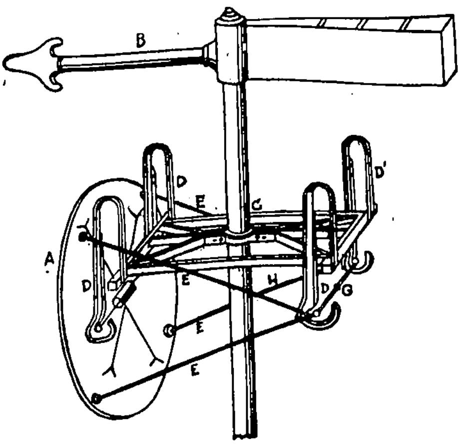


Fig. 50.

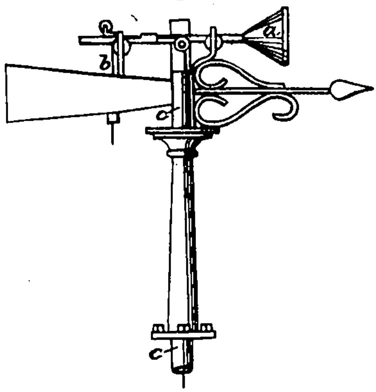


Fig. 51.

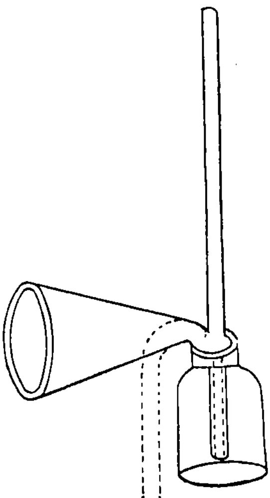


Fig. 52, *a*.

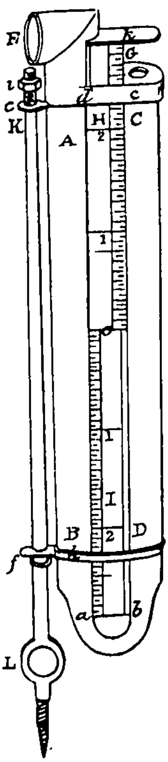


Fig. 52, *b*.

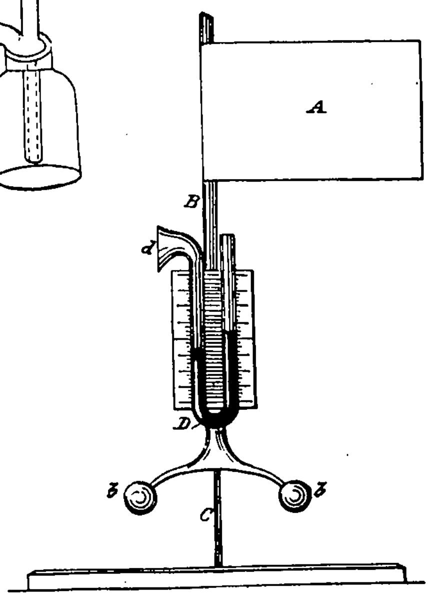


Fig. 53.

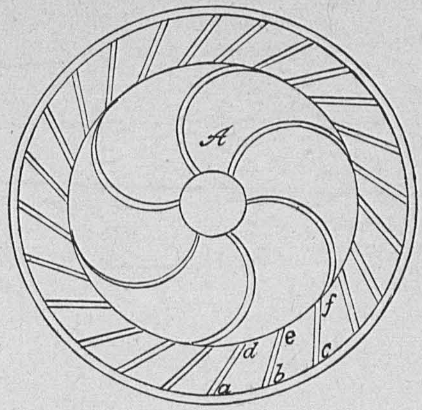


Fig. 54.

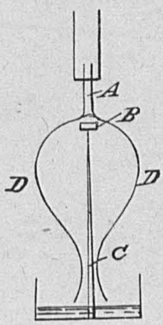


Fig. 55.

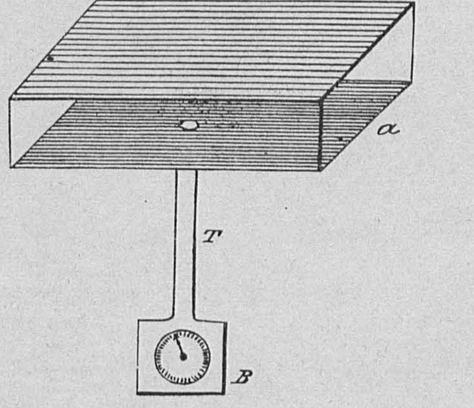


Fig. 56.

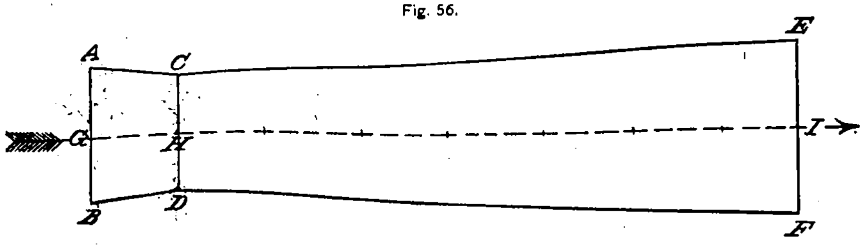


Fig. 57.

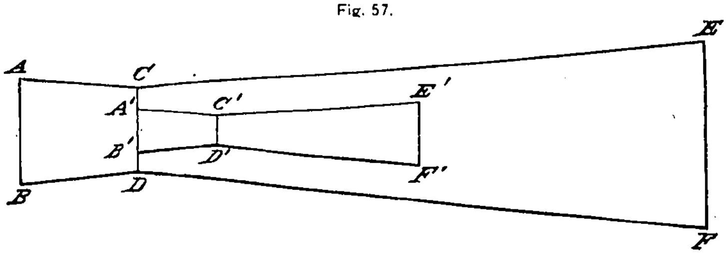


Fig. 58.

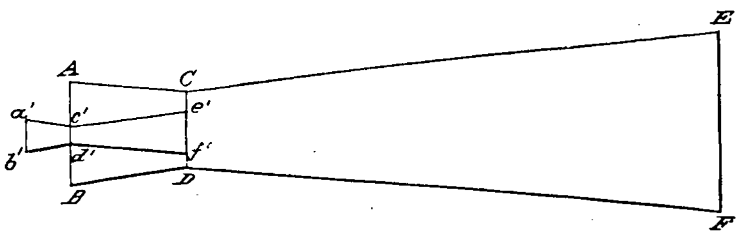


Fig. 59.

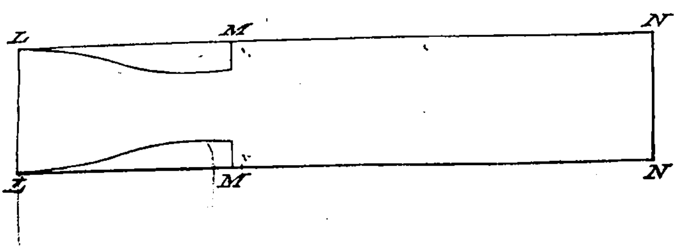


Fig. 60.

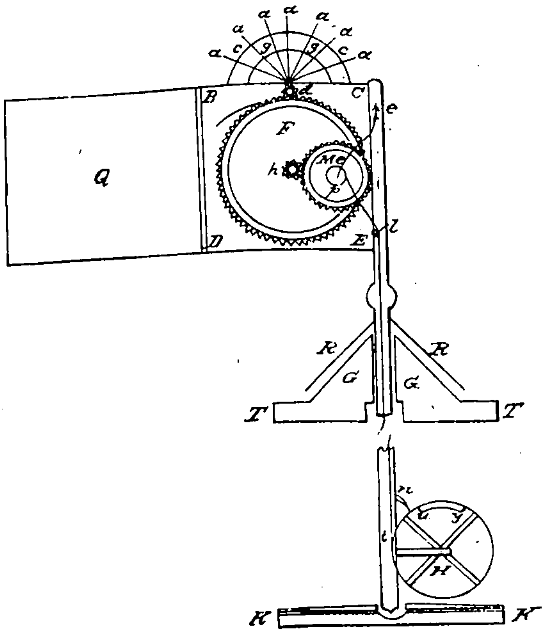


Fig. 61.

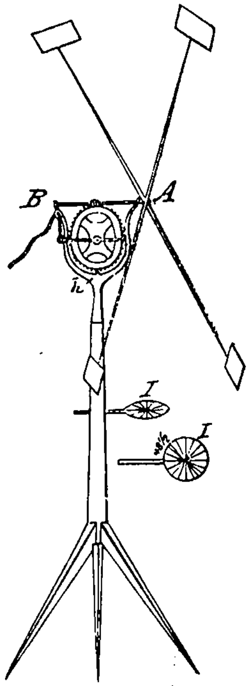


Fig. 63.

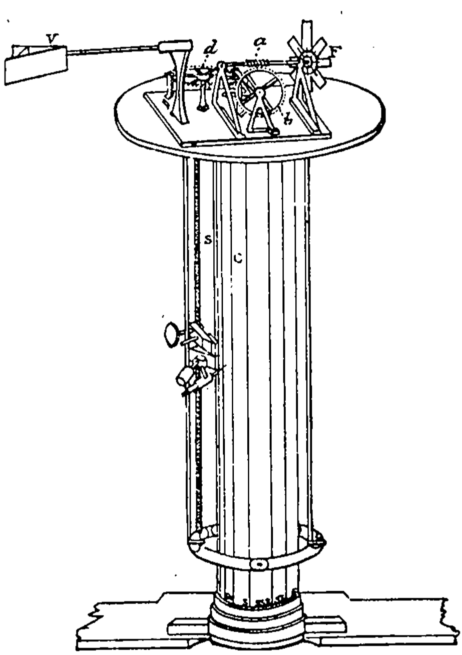


Fig. 62.

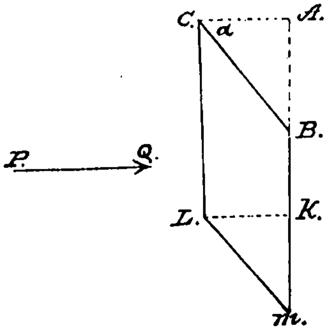


Fig. 65.

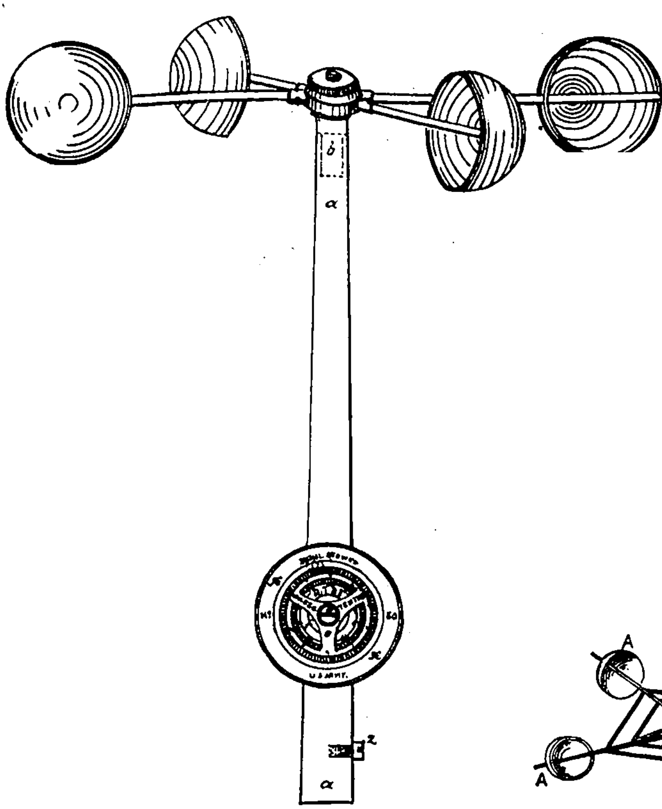


Fig. 66.

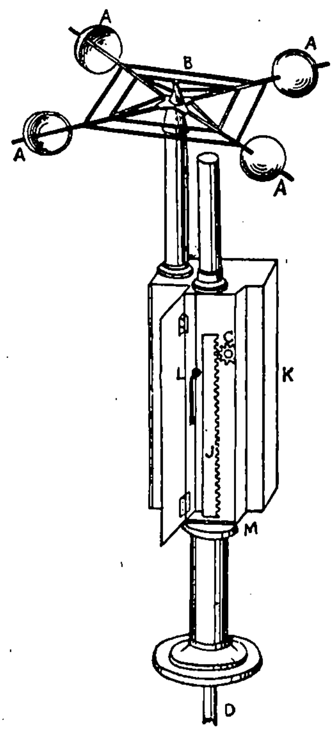


Fig. 64.

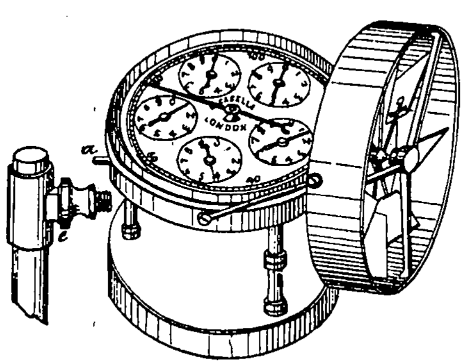


PLATE XXVII.

Fig. 67.

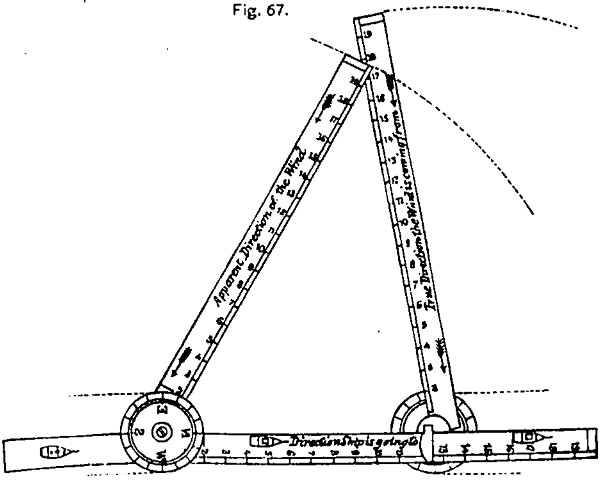


Fig. 68.

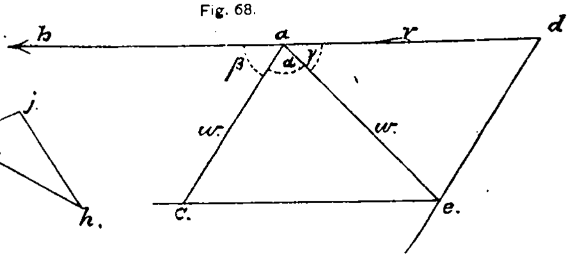


Fig. 69.

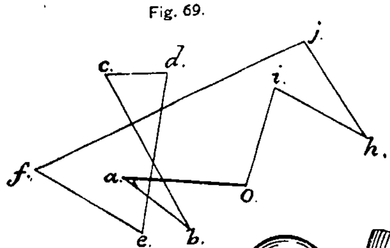


Fig. 70.

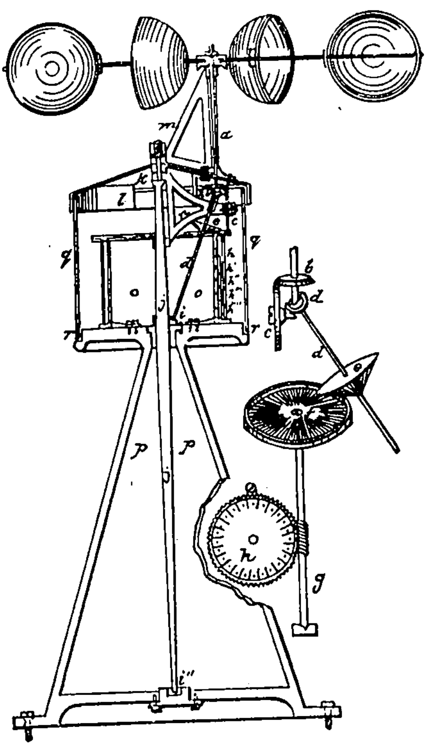


Fig. 71.

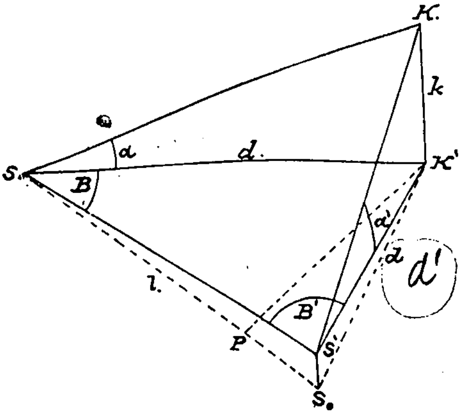


Fig. 72.

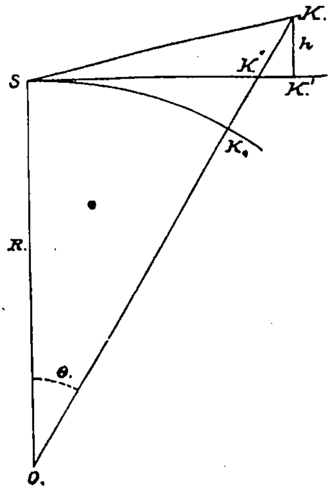


Fig. 73.

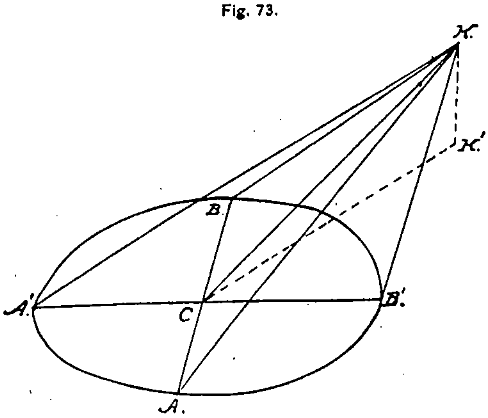


Fig. 74.

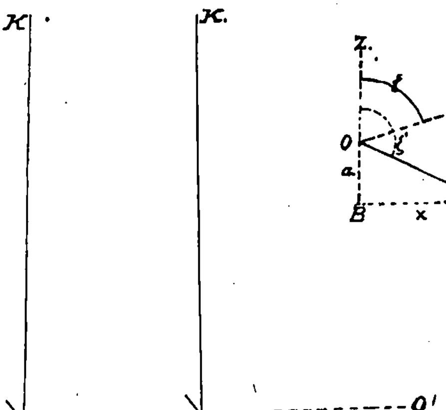


Fig. 75.

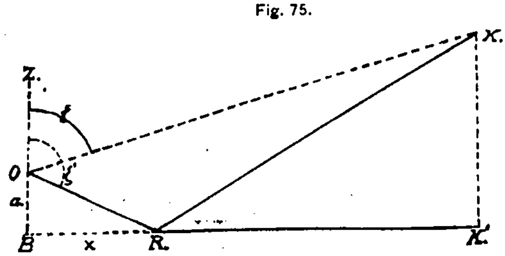


Fig. 76.

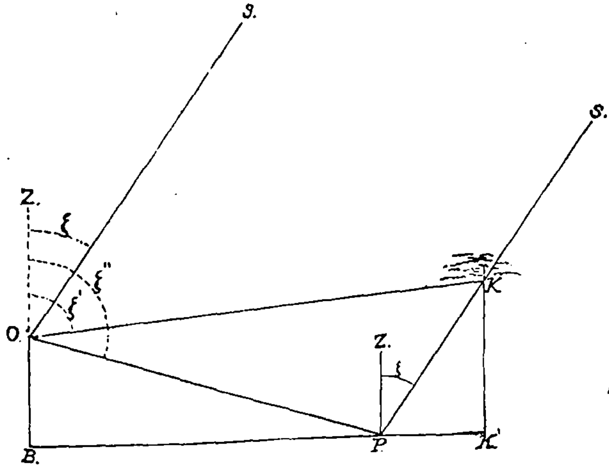


Fig. 78.

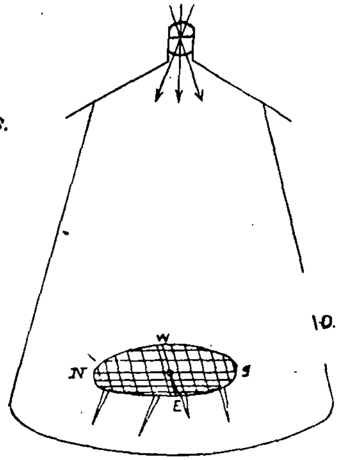


Fig. 77.

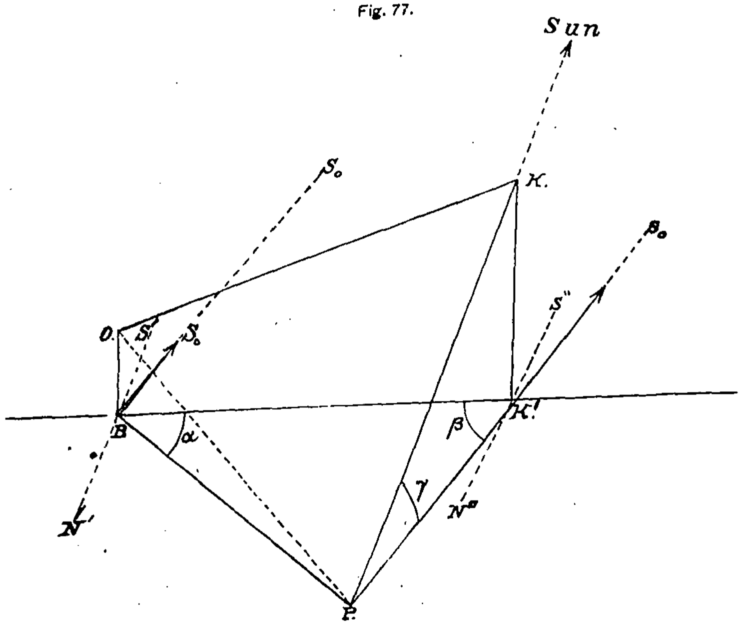


Fig. 79.

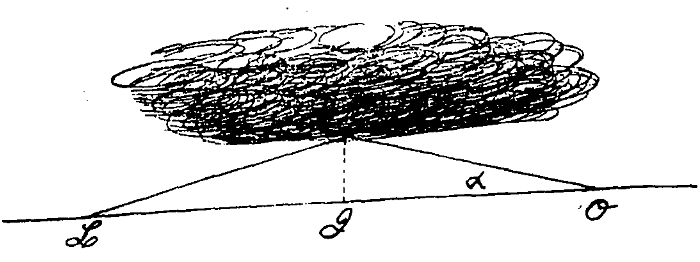


Fig. 80.

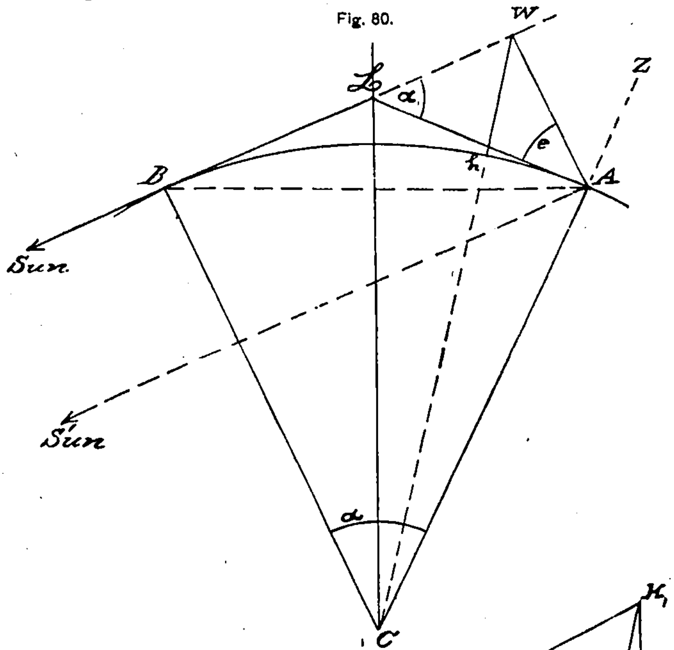


Fig. 81.

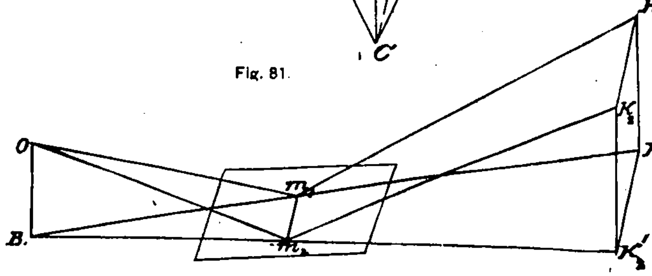
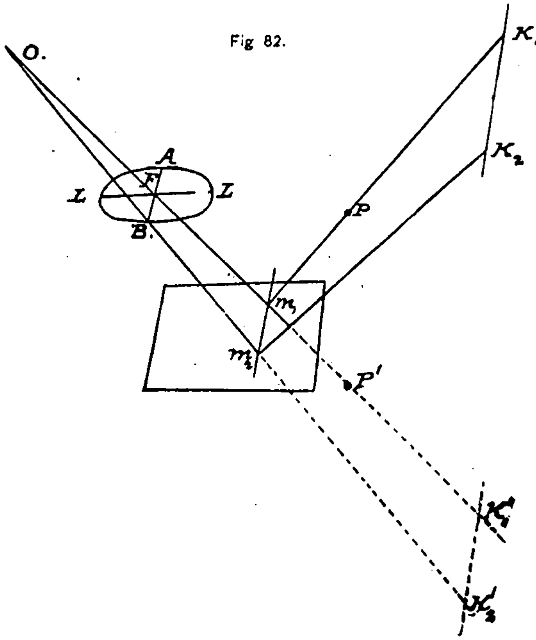


Fig 82.



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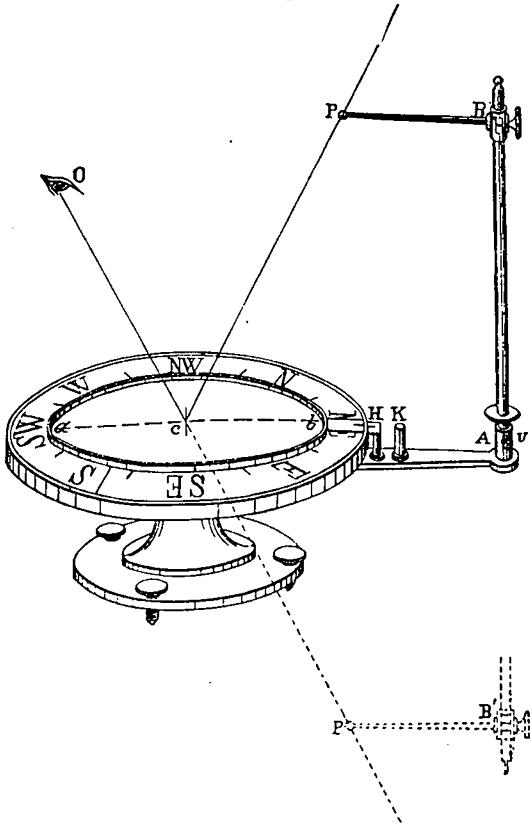

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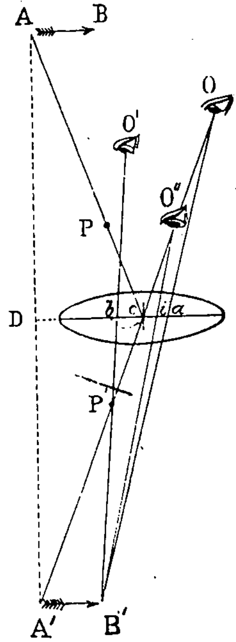
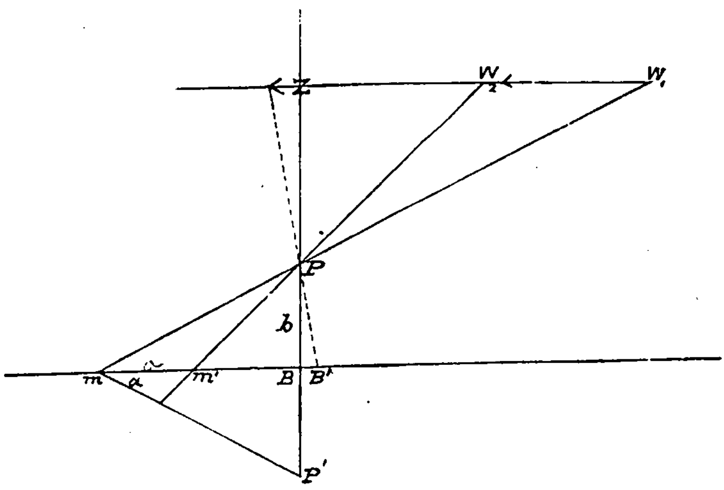


THE CECCHI NEPHOSCOPE.

Fig. 85.



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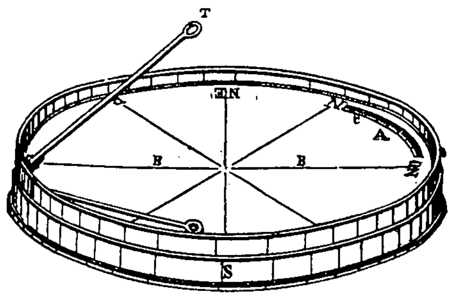
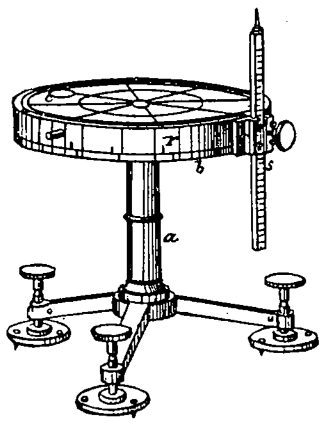


THE FORNIONI NEPHOSCOPE.

Fig. 87.



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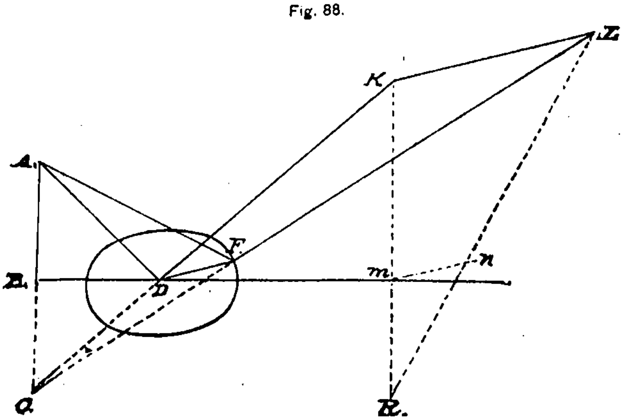

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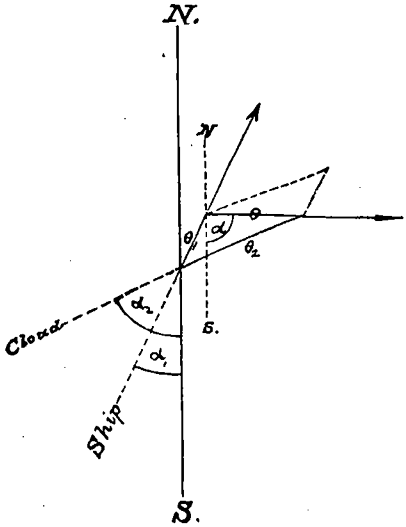

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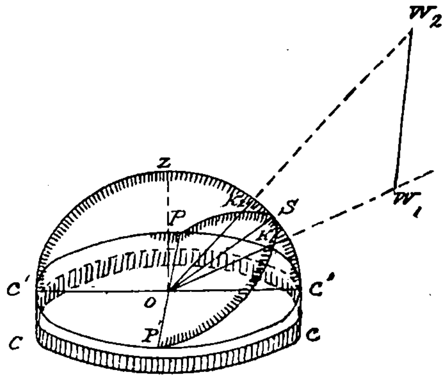

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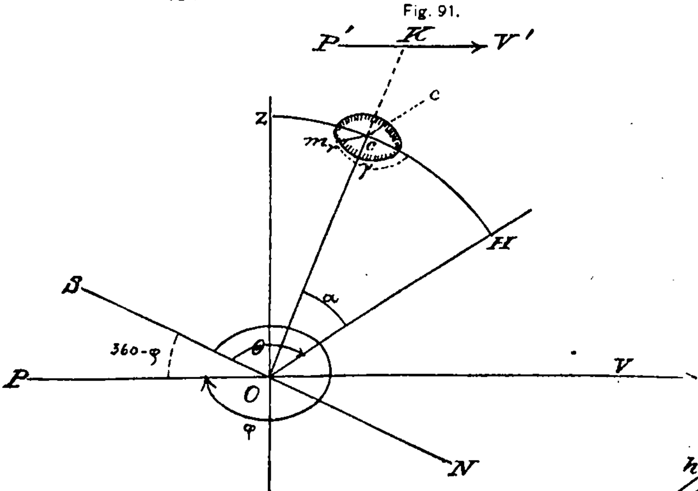

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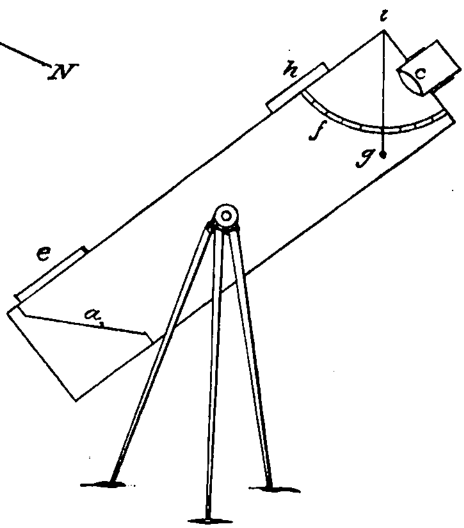

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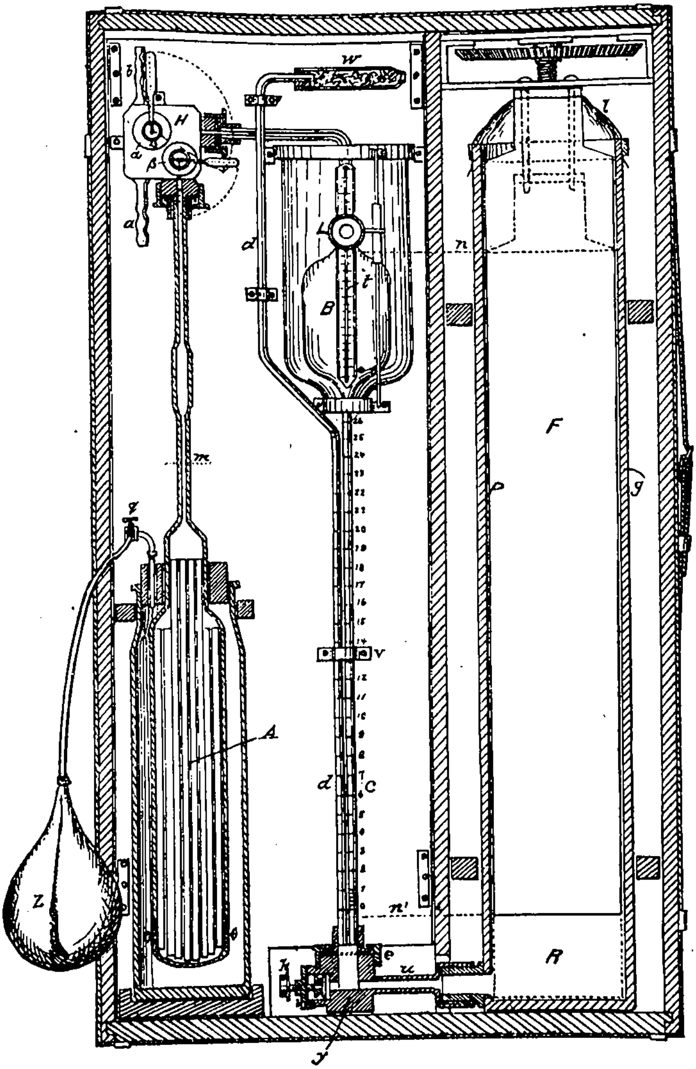

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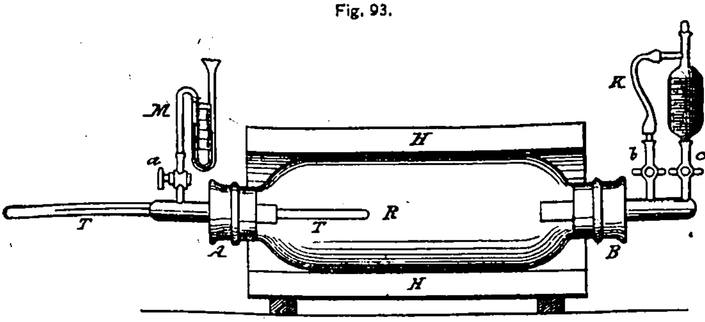

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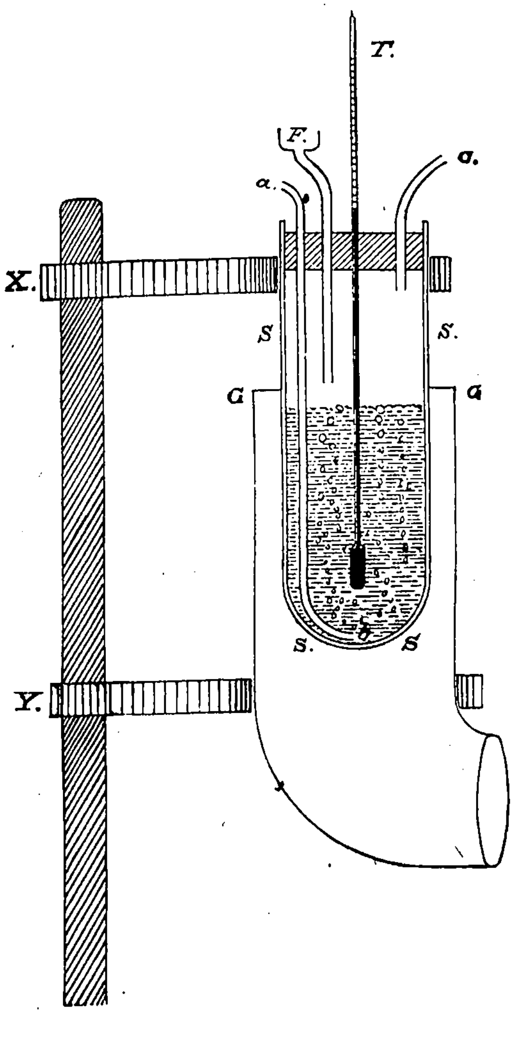

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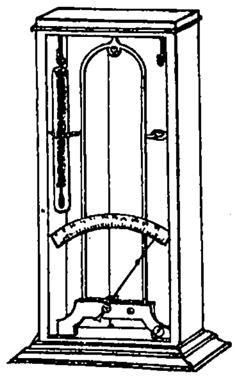

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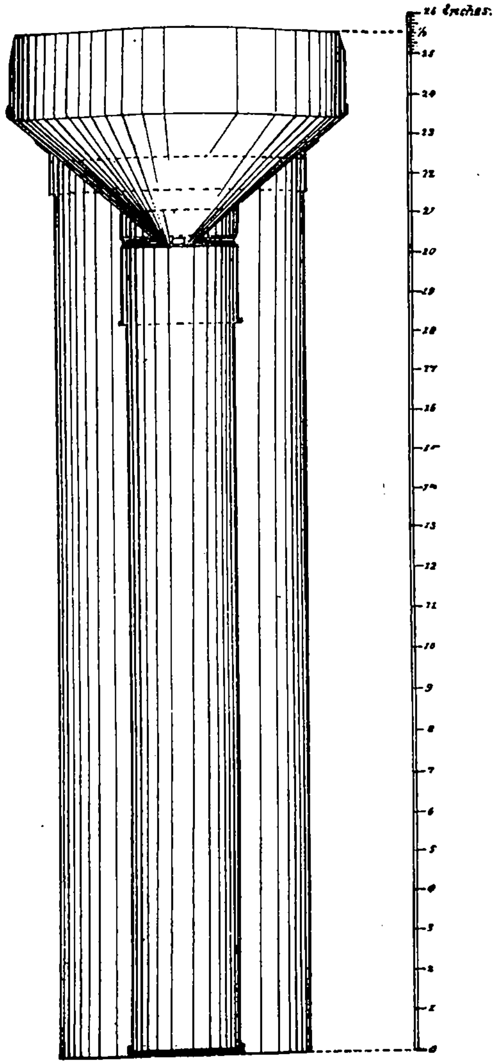

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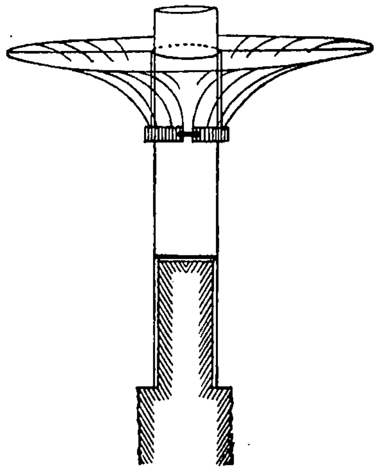

NIPHER'S SHIELDED GAGE.

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