# XXV. *On the Determination of the Constants of the Cup Anemometer by Experiments with a Whirling Machine*—Part II.

*By* T. R. Robinson, *D.D., F.R.S., &c.*

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(55.)* In the preceding Part (Philosophical Transactions for 1878, p. 777) I gave the results obtained by anemometers attached to a whirling machine, which substitutes motion through the air for real wind. If the air were quiescent this method would be nearly unexceptionable; but the whirling gives the air a vorticose motion for which it is impossible to make an exact allowance, and therefore some uncertainty affects these results. In the conclusion of that paper I expressed an opinion that greater certainty might be obtained by comparing two anemometers, similar and equal in every respect except friction; and stated that I would endeavour to carry this into effect. I propose now to give an account of my attempt to do so.

(56.) The instruments used, and their arrangement, are described in paragraph (51). The situation in which they are placed would be a good one but for the dome of the west equatorial, which in some points of the wind may interfere with its full action on one or the other of the instruments.

The diameter is 13′·6; the height of its summit above the platform is 15′·75; that of the arms of the instruments being 16′. The horizontal distance of its centre from the Kew instrument (K)=21′·5, bearing from it S.S.E., 2° S. The distance from the experimental one (E) is 23′, and its bearing S.W. b. S.

The distance between K and E=22′. Of course when the wind is S.S.E., K will be less acted on than E, and *vice versâ*, but probably the difference will be much less than that caused by fluctuations of the wind itself. When the wind is E. or W. the eddies caused by the windward anemometers may perhaps reach the leeward, but not, I think, to any great extent.

(57.) The chronograph record of each experiment was at first entered in groups during which $v$, the velocity of K, was nearly uniform; and A, the number of turns made by each instrument, was an integer. The length of the chronograph helix gives the time; it is measured in eighths of an inch (as the Observatory possesses a scale of eightieths) and when divided by the length of a second on the same scale, we have the number of seconds. As the chronograph in its present situation is exposed to considerable variations of temperature, its rate is not as regular as it was at Rathmines, but the second-space was determined each day of observations. The average in

* For facility of reference the numeration of the paragraphs and tables is in sequence to that of Part I.

winter is 1·665; and the times so deduced are certain to less than $0^{s}\cdot1$. Latterly the time was noted by a watch.

(58.) It was soon found that the method proposed in paragraph (52) is not available, for the wind is never uniform long enough to make two successive experiments fairly comparable. It was therefore necessary to use that of paragraph (53). Assuming such values of $\alpha$, $x$, and $y$, the constants of equation III. as will give V very nearly equal to V′ (the accented letters belong to E), we may correct them so that the mean $V'-V$ may vanish. This assumes first, that however the wind may vary in the course of an observation from one instrument to another, yet if the time be sufficient it comes to each of them with an equal amount; its deficiency at one part of the time being made up by its excess at another; and secondly, that the V computed for a mean value of $v$ will be its own mean value.

(59.) As to the first of these assumptions, I have come to the conclusion that if an observation lasts for nine or ten minutes, the average action of the wind on the two instruments will be nearly equal, though during portions of the time it may vary very much. This may be illustrated by the following table, which contains a set of $v$ and $v'$ taken with the normal frictions at K and E, which are 13·5 and 23·2; these were taken September 17, 1878, under unfavourable circumstances, for the wind was S.W. The $v$ and $v'$ ought to be nearly equal, for the difference of the friction will only diminish $v$ by 0·24.

TABLE XX.

| No. | Time. | $v$. | $v'$. | $v-v'$. | No. | Time. | $v$. | $v'$. | $v-v'$. | No. | Time. | $v$. | $v'$. | $v-v'$. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | s. | | | | | s. | | | | | s. | | | |
| 1 | 15·1 | 6·522 | 4·660 | 1·869 | 22 | 5·2 | 6·441 | 6·441 | 0·000 | 43 | 18·4 | 5·349 | 6·876 | −1·527 |
| 2 | 28·0 | 5·254 | 5·254 | 0·000 | 23 | 10·3 | 5·475 | 5·475 | 0·000 | 44 | 29·4 | 7·175 | 8·400 | −1·225 |
| 3 | 25·5 | 7·156 | 6·620 | 0·536 | 24 | 9·2 | 6·132 | 4·599 | 1·533 | 45 | 50·3 | 7·540 | 6·143 | 1·397 |
| 4 | 11·5 | 6·084 | 3·650 | 2·334 | 25 | 17·5 | 8·012 | 6·409 | 1·603 | 46 | 32·4 | 6·077 | 5·208 | 0·869 |
| 5 | 11·3 | 7·638 | 6·236 | 1·402 | 26 | 22·0 | 5·702 | 4·474 | 1·228 | 47 | 9·2 | 4·589 | 5·842 | −1·253 |
| 6 | 21·1 | 5·313 | 5·313 | 0·000 | 27 | 9·6 | 5·882 | 5·882 | 0·000 | 48 | 16·4 | 5·989 | 5·135 | 0·854 |
| 7 | 13·5 | 4·174 | 2·086 | 2·088 | 28 | 32·4 | 7·791 | 7·353 | 0·438 | 49 | 11·8 | 7·142 | 5·959 | 1·183 |
| 8 | 8·5 | 6·624 | 6·624 | 0·000 | 29 | 10·8 | 6·482 | 6·482 | 0·000 | 50 | 19·3 | 5·082 | 4·841 | 0·241 |
| 9 | 12·1 | 4·642 | 3·481 | 1·161 | 30 | 12·1 | 4·595 | 5·795 | −1·200 | 51 | 45·3 | 6·512 | 6·512 | 0·000 |
| 10 | 28·7 | 2·934 | 4·995 | 0·939 | 31 | 15·2 | 3·697 | 4·622 | −0·925 | 52 | 39·2 | 5·378 | 3·222 | 2·148 |
| 11 | 10·9 | 6·458 | 5·162 | 1·296 | 32 | 35·4 | 4·637 | 3·174 | 1·462 | 53 | 14·0 | 3·193 | 3·017 | 0·176 |
| 12 | 13·2 | 4·256 | 5·311 | −1·055 | 33 | 10·8 | 5·163 | 2·581 | 2·582 | 54 | 13·9 | 5·040 | 4·032 | 1·008 |
| 13 | 5·8 | 7·314 | 7·314 | 0·000 | 34 | 22·5 | 3·739 | 1·246 | 2·493 | 55 | 20·4 | 4·116 | 3·430 | 0·686 |
| 14 | 10·6 | 9·247 | 9·247 | 0·000 | 35 | 14·6 | 3·852 | 0·764 | 3·088 | 56 | 19·7 | 4·281 | 2·140 | 2·141 |
| 15 | 25·1 | 8·959 | 6·298 | 2·661 | 36 | 29·4 | 4·298 | 1·910 | 2·388 | 57 | 15·5 | 4·525 | 1·802 | 2·723 |
| 16 | 7·3 | 3·817 | 1·915 | 1·902 | 37 | 8·4 | 5·032 | 3·355 | 1·677 | 58 | 14·8 | 3·794 | 2·846 | 0·948 |
| 17 | 11·2 | 5·006 | 3·754 | 1·252 | 38 | 15·0 | 4·693 | 2·816 | 1·877 | 59 | 31·9 | 5·599 | 4·900 | 0·699 |
| 18 | 13·7 | 6·127 | 6·127 | 0·000 | 39 | 13·3 | 4·229 | 4·229 | 0·000 | 60 | 24·5 | 4·560 | 3·434 | 1·126 |
| 19 | 12·4 | 6·815 | 4·544 | 2·271 | 40 | 18·2 | 3·885 | 3·885 | 0·000 | 61 | 18·0 | 4·691 | 5·473 | −0·782 |
| 20 | 6·5 | 8·589 | 8·589 | 0·000 | 41 | 45·5 | 7·247 | 8·153 | −0·806 | 62 | 21·2 | 6·626 | 8·540 | −1·914 |
| 21 | 17·1 | 6·554 | 5·735 | 0·819 | 42 | 11·5 | 7·358 | 6·512 | 0·846 | | | | | |

Total time $=646^{s}\cdot7$; mean $v=5\cdot816$; mean $v'=5\cdot218$; mean $v-v'=0\cdot598$.

These show plainly both the variation of wind at one anemometer and the difference at the two. In No. 14, $v=9\cdot247$; in No. 10, it is 2·934. These represent $V=26\cdot264$, and 8·551. If we look to the column $v-v'$, at No. 35 we find $+3\cdot088$, at No. 62

$-1{\cdot}914$; fourteen are $=0$, and nine are negative. But if we divide them into four consecutive and nearly isochronal groups the discordance is much less.

| | | | |
|---|---|---|---|
| Time $=$ 161·4 s. | $v=6{\cdot}219$ | $v'=5{\cdot}203$ | $v-v'=1{\cdot}016$ |
| 159·8 | 5·508 | 4·375 | 1·133 |
| 160·6 | 6·241 | 5·926 | 0·315 |
| 164·9 | 5·369 | 5·067 | 0·302 |

The extreme range here is 0·831 instead of 5·002, grouping them in pairs

| | | | |
|---|---|---|---|
| T$=$321·2 | $v=5{\cdot}864$ | $v'=4{\cdot}791$ | $v-v'=1{\cdot}073$ |
| 325·5 | 5·799 | 5·489 | 0·310 |

There can be little doubt that the total means are nearly correct, and these values of $v-v'$ differ from the mean one by $+0{\cdot}475$ and $-0{\cdot}289$. In general, $v-v'$ will be less than this; and if it be observed by inspecting the chronograph while an observation is proceeding that the ratio of A to A′ varies notably, a longer time should be taken.

(60.) As to the second point it is easily shown that no great errors can arise from assuming that V is truly given by the mean $v$. The mean V of a series is, taking the time into account,

$$=\frac{\text{SVT}}{\text{ST}}=\frac{x\text{S}v\text{T}}{\text{ST}}+\frac{\text{S}v\text{T}\times\sqrt{z+\frac{\phi}{v^2}}}{\text{ST}}$$

Now the first of these $=x\times$ mean $v$. In instruments like K where $\phi$ is small, if we develop the radical in powers of $\phi$, the second term becomes

$$\text{S}v\text{T}\times\left(\sqrt{z}+\frac{\vartheta\phi}{2v^2\sqrt{z}}\right)$$

and as the $\phi$ term may be neglected the mean of radical becomes $\sqrt{z}\times$ mean velocity. When $\phi$ is large the simplest course is (calling the radical R) to compute $\frac{\text{S}v\text{T}\times\text{R}}{\text{ST}}$, or what comes to the same thing $\frac{\text{S}v\text{CR}}{\text{SC}}$, C being the time-space, and compare this with the R computed with the mean $v$. Taking at random No. X. of Table XXI. whose $\phi=343{\cdot}28$, we have for the separate groups whose A and A′ are nearly uniform

| No. | C. | $v'$. |
|---|---|---|
| I. | 31·8 | 6·081 |
| II. | 95·2 | 7·411 |
| III. | 88·3 | 7·350 |
| IV. | 49·35 | 4·886 |
| V. | 32·5 | 7·364 |
| VI. | 18·68 | 6·042 |
| | 315·83 | 6·835 |

$\frac{\text{sum } v\text{CR}}{\text{sum C}}=20{\cdot}633.$

R for mean $v'=20{\cdot}689$.

Requiring the correction $=0{\cdot}056$.

Here the $v'$'s do not range very widely, and I take a more aberrant set observed September 16, 1878, $\phi'=173{\cdot}80$.

| No. | C. | $v'$. |
|---|---|---|
| I. | 15·38 | 3·768 |
| II. | 107·86 | 9·537 |
| III. | 99·83 | 4·644 |
| IV. | 55·11 | 3·155 |
| V. | 48·47 | 1·196 |
| VI. | 60·66 | 4·060 |
| VII. | 63·16 | 1·606 |
| VIII. | 36·39 | 1·5925 |
| IX. | 54·87 | 7·393 |
| X. | 64·98 | 5·351 |
| XI. | 15·05 | 0·9625 |
| XII. | 16·02 | 6·331 |
| XIII. | 23·09 | 6·275 |
| XIV. | 43·58 | 4·987 |
| | 704·45 | 4·854 |

$$\frac{\text{sum } v'\text{RC}}{\text{sum C}}=14·984.$$

R for mean $v$=14·728.

Correction =+0·156.

Even here the error is not of a nature to interfere with the determination of the constants, though in such work terms like V. and XI. had better be omitted. If it were thought necessary the exact computation is not difficult.

(61.) At first the additional friction was applied by the brake-levers, and was measured by the process described in the note to paragraph (19); but it was soon found to be irregular on account of the rusting of the cast-iron disc on which the rubbers pressed. This could not be prevented in the present location of the instrument.

The rust wore off in the course of an experiment and filled the pores of the cloth on the rubbers. Yet more, it became evident that the constants which in the whirling experiments had given V−W pretty fairly, fail totally here: for instance, with the set last given they give V=14·605, V′=20·066, the difference being far too great to be caused by any error of the friction.

(62.) I intended to remove the uncertainty caused by the rust by substituting for the iron disc one of bell metal of the same diameter; it is, however, some 20 oz. heavier, and the normal friction of E is now 30·4 grains, and its moment =27542. But while it was being prepared it occurred to me that instead of measuring the brake friction first and assuming its permanence during a series of observations it would be better to record and measure it during the entire time of each observation. PRONY's brake afforded a ready means of effecting this, and was thus applied: a ring of iron an inch deep and $\frac{1}{16}$ inch thick is divided into two semicircles held together by screws tapped in the lugs; when these are removed it can be got on the axle, lowered to the disc, and is made to clamp it with any required pressure by replacing and tightening the screws. The ring has an arm which carries an arc of the same depth concentric with the disc and of 8″ radius. It is obvious that when the anemometer is turning, a cord attached to this arc will be pulled by a force =the moment of the ring friction at 8″. This pull is measured by a spring balance which I made with one of the clock springs described in paragraph (26).

(63.) It consists of an iron spindle 6″ long, turning on a small toe below, and above in a brass collar carried by a transverse piece of wood supported on two uprights. It has a projecting piece to which the inner end of the spring is attached by a screw secured by a check nut; the outer end is fixed to one of the uprights. On the top of the spindle is screwed a disc of mahogany 13″ diameter and 0″·5 thick, on the edge of which is turned a groove in which the thread that connects it with the brake arc is wound. On this disc is fitted one of the papers used with my original anemometer, which has its circumference graduated to half degrees, and is covered with circles 0″·05 apart, every tenth one stronger than the rest. By pulling the cord the spindle is turned and the spring tended, the number of its revolutions is shown by a tell-tale fixed to one of the uprights, and the degrees by a pointer.

(64.) It is thus used: tighten the clamp screws so that when the arm is held fast the anemometer shall turn without coming to a stop; pass the cord of the balance through a hole in the remote end of the arc, and tighten it till the increased tension keeps the arm nearly in the same position, then secure it to a pin provided for the purpose. In this state of things it is evident that the tension corresponding to $\theta$, the angle through which the balance has been turned, is the moment of the friction at 8″, from which the moment at the cups is known, to which must be added the normal friction. The brake-ring weighs 14 oz., which would increase the friction a few grains, but this was obviated by hanging an equivalent weight on the relieving apparatus mentioned in paragraph (51). The ring was at first lined with cloth, but as it slightly abraded the bell-metal, I removed it and used the iron surface lubricated with lard.

(65.) The relation between the tension of the spring and $\theta$ was thus obtained: the balance being clamped to a table its cord was passed over a pulley; nineteen weights in regular succession from 2 oz. to 36 oz. were hung to it; and to eliminate the effect of friction the disc was turned a few degrees in advance and in rear of the positions of rest, when they were attained, the mean of the $\theta$'s was taken as that due to the tension. From ten to thirteen sets were taken for each weight. I had expected that the tension would be very nearly as $\theta$, but with this spring such is not the case; at the beginning $1^\circ = 13$ grains, at 4 rev. it $= 20$, and the change is not uniform; so I formed from the series an interpolation table with $\theta$ argument, from which T is easily computed by a formula analogous to that given by STIRLING for equal intervals.

(66.) In carrying out the work I was met by an unexpected difficulty: friction applied in this way is not constant; and I found that in strong breezes (when the wind is always fitful) the arm oscillated more than 90°, the utmost range which the opening of the iron box (paragraph 2) permitted unless the friction was reduced. These oscillations made it necessary to have a record of the tension, which was provided by clockwork moving a pencil from the centre to the circumference of the graduated paper at the rate of 0″·5 per minute. This traces an irregular sector from which the mean $\theta$ is easily obtained. But, besides this, it is necessary to reduce the oscillations below 90°, so that they all may be recorded. This was effected by connecting with the arc an auxiliar balance, so that its action would begin only at the minor limit of the arm's motion, and increase, so as to prevent it from reaching the major limit. It consists of an iron tube 0″·75 diameter, containing 12″·5 deep of mercury. In this is immersed a rod of iron 0″·3 diameter, reaching to the bottom, and with a cross piece at top resting on the tube; from this cross piece descend two wires carrying weights just sufficient to balance the flotation of the mercury. To the top of the rod is attached a cord, passing over a pulley to the arc. It is easily shown that if the rod be raised an inch the cord will be pulled with a force-weight of a cylinder of mercury 0″·3 diameter and 1″·191 high. (For this also I formed an interpolation table, but in it $\delta^2$ is nearly insensible.)

To use it the spring balance is tended till it keeps the arm near the middle of the opening of the box; the arm is then pressed back to touch the box, the cord is looped on the pin already mentioned and shortened till the cross just rests on the tube, and the $\theta$ read which gives the zero of the auxiliar. Deducting this from the mean $\theta$, we have $\theta'$, the measure of the auxiliar tension.*

The largest oscillation which I have observed under this arrangement was 54°; the wind was moderate, V being only $16^{\mathrm{m}}$. This is equivalent to a change of tension =764 grains at the cups, nearly 0·4 of the entire tension there. I cannot account for the great irregularities of this friction, but I believe similar facts have been observed on a large scale in applying PRONY's brake to machinery. The extent and frequency of the oscillations do not seem to follow any regular law of V or $v$, though they evidently are related to them.

(67.) The process described in paragraph (58) gives for each observation an equation containing three unknown quantities, $a$, $x$, $y$, and two unknown variables, V and V′, or $\mathrm{V}\pm w$, $w$ being the difference of wind at the two instruments. It is shown by Table XX. that $w$ may be considerable for a few seconds, but when the time is a few minutes it is probably confined within the limits $\pm 3$. Even when (as in the whirling experiments) we know V approximately, and have not V′ to consider, the coefficients are so related that it is impossible to get accurate values of the constants by the usual methods of elimination, and here the difficulty is still greater. I have therefore thought it best to assume probable values for $a$ and $y$, and determine $x$ so that the mean V−V′ may vanish. In the first approximation to this, supposing U the true value of the wind at K, we have $\mathrm{U}=\mathrm{V}+e dx=\mathrm{V}\pm w+e' dx$; $e$ being $=\frac{\mathrm{V}}{\sqrt{z+\frac{\phi}{v^2}}}$.

Hence $\mathrm{S}(\mathrm{V}'-\mathrm{V})\pm \mathrm{S}w=\Delta x\times \mathrm{S}(e-e')$.

* A far better mode of retarding the motion of E would be to have on its shaft a sheave connected with an apparatus like that described in paragraph (6), so that the instrument in revolving might draw up the driving weight. The moment of this at the cups would be constant and accurately known, and the observer would only have to continually unwind the cord. Unluckily, this did not occur to me till the series of Table XXI. was nearly completed; and I was unwilling to repeat the measures, for, owing to deficiency of wind, that series had occupied several months.

If the number of observations be sufficient, $S(\pm w)=0$, and we have $\Delta x=\frac{S(V'-V)}{S(e-e')}$. This will give an $x$ nearer the truth (not exact unless $\frac{\Delta x^2}{n}\times\frac{d^2V}{dx^2}$ be insensible), and a second computation will in general be sufficient. When the constants give the mean $V-V'=0$, the V so obtained must be very nearly $=U$, as shall be shown presently.

(68.) First as to $\alpha$: in the case of $v=0$, we have the measure of it given in paragraph (27). These must be reduced by some hypothesis as to the action of friction. In the first part of my paper I assumed that the momentum of the cups carried them past the point of equilibrium, induced to this by the small value of $\alpha$ given by min. squares (paragraph 39), $=99$. This gives $\alpha=\frac{T-f}{(V-W)^2}$. My preliminary work with the two instruments showed that this was too small, and I recurred to the more natural supposition that the cups stop when the wind's force $=T+f$. This gives $\alpha=15{\cdot}315$ at Bar. 30° and Therm. 32°.* For 4″ cups it is 3·357, very nearly in the proportion of the areas. I know no means of determining whether this constant varies with $v$; the individual measures seem to show that it does not change with V. The lateral pressure on the upper bearing of the shaft causes a resistance as $V^2$, and will diminish $\alpha$; but the probable value of its coefficient is $\alpha\times 0{\cdot}00051$, which may safely be neglected. The change of $\alpha$, if it exist, cannot have much influence on V; for $\frac{dV}{d\alpha}=\frac{\phi}{2\alpha\sqrt{z+\frac{\phi}{v^2}}}$. Taking I. and X. of Table XXI., where $\phi$ is a minimum and maximum, we have $dV=d\alpha\times 0{\cdot}0573$ and $d\alpha\times 0{\cdot}1603$.

(69.) As to $y$, if there be no resistance as $v^2$, except what appears in the resultant, the equation of motion is $V^2+v^2-2Vvx-\frac{f}{\alpha}=0$, from which we see that $y$, the coefficient of $v^2$, $=1$. This is its major limit; if we diminish $y$ by $\Delta y$, the equality of V and V′ may still be preserved by diminishing $x$. But the value of V is a little decreased: so the $\Delta V=\sqrt{V^2-\Delta y\times vv'}-V$, or, in the case of K, $-\frac{\Delta y\times vv'}{2V}$. Such diminution can only be affected by an expenditure of power in driving air before the cups, or throwing it outwards; and I tried to find a limit by making them revolve in quiescent air. For this purpose I mounted four forms of E on a vertical spindle driven by HUYGHEN'S maintaining apparatus, and noted the time and moment at the cups. The resistance was always more than twice the action of direct wind on the convex sides, and I think its excess may be taken as the extreme possible value of $y$ in the

* I tried to measure it by the spring balance, but the oscillations were too extensive to permit any continuous observations. By noting the time, and counting the turns of K while the oscillations of $E_2$ were clear of the sides of the box, I got two values of $\alpha$, 15·165 and 19·562; but the possible difference of wind must be remembered.

negative direction. It would give for $y \frac{a-r}{a}$; but I think this action must be small in a current of wind moving with nearly three times the velocity of the cups. It is found to increase as the diameter of the cups and the length of the arms diminish; for E it gives $y=+0{\cdot}0546$, but for $E_4$ (to be soon described) $-3{\cdot}3406$. This supposition would give smaller values of V; for No. II. of Table XXI., where V with $y=1$ is 35·255, the difference is 1·871, and the true value is certainly between these. I will use $y=1$ as certainly known.

(70.) For $x$, as K and $E_1$ are similar and equal, it must be the same in both, and the means of obtaining it are explained in paragraph (67). Here I need only show how its first approximation is got: Supposing $w=0$, we have $2\text{V}vx=\frac{\phi'-\phi}{v-v'}+(v+v')$; but as in K $\phi$ is small, and may here be neglected, we have $\text{V}=v(x\pm\sqrt{x^2-1})$, and the sum of the equation becomes

$$2x(\sqrt{x^2-1}+x)\times\text{S}v=\text{S}\frac{\phi'-\phi}{v-v'}+\text{S}(v+v') \text{ (VII.)}$$

from which $x$ is easily found. When E is not similar to K the process is simpler: the reading of K gives V, and we have $2x\text{V}v'=\text{V}^2+yv'^2-\phi'$, whence

$$2x\times\text{S}v=\text{SV}+y\text{S}\frac{v'^2}{\text{V}}-\text{S}\frac{\phi'}{\text{V}} \text{ (VIII.)}$$

Both these formulæ are defective from omitting $w$, but are near enough to begin with.

(71.) The following table gives the results of the comparison of K and $E_1$, which is equal and similar to K. The second column gives the wind's direction; the third log. air's density; the fourth the time in seconds; the two next A and A′, the number of turns made by K and $E_1$; the seventh log. of $\frac{f'}{a\times\text{D}}$ or $\phi'$; the two next the velocities of the centres of K and E; the two next the *computed* velocities of the wind; and the twelfth V′−V. V and V′ were computed by the formulæ $\text{V}=v(x+\sqrt{z})+\frac{\phi}{2v\sqrt{z}}$; $\text{V}'=v'\left(x+\sqrt{z+\frac{\phi'}{v'^2}}\right)$ when $z=x^2-1$.

TABLE XXI.

| No. | Dir. | L. Dens. | Time. | A. | A′. | Log. φ′. | v. | v′. | V. | V′. | V′ − V. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | s. | | | | | | | | |
| I. | S.W. | 9·97776 | 176·1 | 74 | 51 | 1·42156 | 3·601 | 2·482 | 10·291 | 10·030 | −0·261 |
| II. | S.W. b. S. | 9·97760 | 391·8 | 569 | 511 | 1·56164 | 12·444 | 11·176 | 35·255 | 32·471 | −2·784 |
| III. | S.W. | 9·99124 | 353·9 | 226 | 130 | 1·88336 | 5·393 | 3·102 | 15·327 | 14·380 | −0·947 |
| IV. | E. b. N. | 9·99320 | 359·9 | 268·3 | 170 | 2·07307 | 6·389 | 4·047 | 18·137 | 18·294 | +0·157 |
| V. | S.W. b. S. | 9·97684 | 334·4 | 322·8 | 194 | 2·25147 | 8·271 | 4·971 | 23·453 | 22·606 | −0·847 |
| VI. | S.E. b. E. | 9·98104 | 278·0 | 215·9 | 134 | 1·97602 | 6·659 | 4·133 | 18·874 | 17·512 | −1·302 |
| VII. | „ | 0·98132 | 330·9 | 260·7 | 161 | 2·21161 | 6·749 | 4·168 | 19·334 | 20·397 | +1·063 |
| VIII. | „ | „ | 210·1 | 189·5 | 114 | 2·27685 | 7·733 | 4·652 | 21·932 | 22·315 | +0·383 |
| IX. | S.E. | 9·97757 | 237·0 | 306·8 | 192 | 2·50054 | 11·092 | 6·941 | 31·428 | 30·812 | −0·616 |
| X. | „ | „ | 176·1 | 234·3 | 153 | 2·53450 | 10·446 | 6·835 | 29·664 | 31·249 | +1·585 |
| XI. | „ | „ | 197·3 | 213·6 | 152 | 2·34007 | 9·252 | 6·599 | 26·228 | 27·405 | +1·177 |
| XII. | „ | „ | 154·4 | 180 | 122 | 2·39493 | 10·102 | 6·772 | 28·631 | 28·762 | +0·131 |
| XIII. | „ | „ | 131·1 | 157·3 | 104 | 2·41366 | 10·285 | 6·800 | 29·149 | 28·994 | −0·155 |
| XIV. | N. | 9·98883 | 455·3 | 369·7 | 211 | 2·16403 | 6·957 | 3·970 | 19·741 | 19·363 | −0·378 |
| XV. | S.E. b. S. | 9·97683 | 278·6 | 231·1 | 160 | 2·01434 | 7·108 | 6·920 | 20·169 | 19·441 | −0·728 |
| XVI. | „ | „ | 259·9 | 204·4 | 134 | 2·10555 | 6·738 | 4·417 | 19·116 | 19·570 | +0·454 |
| XVII. | S. b. E. | 9·96418 | 646·9 | 639 | 512 | 2·13691 | 8·462 | 6·780 | 23·994 | 25·202 | +1·208 |
| XVIII. | S. | 9·96469 | 414·3 | 209 | 126 | 2·01367 | 4·321 | 2·605 | 12·313 | 14·807 | +2·494 |
| XIX. | S.W. | 9·96609 | 684·9 | 708·6 | 489 | 2·26543 | 8·867 | 6·119 | 25·141 | 25·293 | +0·152 |
| XX. | „ | 9·96872 | 404·6 | 310·3 | 187 | 2·10790 | 6·571 | 3·960 | 18·656 | 18·643 | −0·013 |
| XXI. | „ | 9·95783 | 713·5 | 477·5 | 251·2 | 1·90599 | 5·734 | 3·003 | 16·290 | 14·523 | −1·767 |

These were computed with $x=1{\cdot}5920$ and $z=1{\cdot}534$; S (V′−V)$=-0{\cdot}994$, which being divided by S $(e-e)=164{\cdot}56$, we have $dx=-0{\cdot}0060$; $x=1{\cdot}5860$; $z=1{\cdot}515$; $x+\sqrt{z}=2{\cdot}826$; log. 0·45111; limit of $\frac{V}{v}=2{\cdot}826$.

For K, $V=v\times 2{\cdot}831+\frac{0{\cdot}355}{v}$.

It is evident from the values of V′−V that the constants do not change with $v$ or $v'$; but that their differences are casual owing to the difference of wind at the two instruments. They differ when the $v$'s are nearly equal: For instance, I. and VIII. differ by 2·995; VII. and XIV. by 1·441, and IX. and X. by 2·181; and that such differences of wind may exist for some time is shown by Table XX. where during the first 321 seconds V′−V$=-2{\cdot}888$, and during the following 325$^s$ it is $-0{\cdot}784$.*

The minus values predominate during S.W. winds as might be expected from paragraph (56).

This $x$ and $z$ are larger than those given in paragraph (40), namely, $x=1{\cdot}2282$, and $z=1{\cdot}340$, which give for the limit 2·286.

This difference is due partly to my having then used an $\alpha$ only two-thirds of what I believe to be its real value, partly to the uncertainty of the frictions employed and of W, and partly to the defect of the method of minimum squares in such a case.

* As these instruments are generally constructed to register $V=3v'$, their readings should be corrected by subtracting 0·056 of the recorded V.

Reducing the first 21 of Table X. by formula XIII., and with my present values of $\alpha$ and $y$ I get $x=1\cdot3744$, $z=0\cdot889$, and the limit $=2\cdot317$.

The W's used in computing these constants were certainly inaccurate. I measured them in the plane of the centre of the anemometer, but as the disturbance of the air will be as $V\mp v\times\sin\theta$, W must be less in the upper semicircle than in the lower, while it acts with less mechanical advantage in lessening $v$. It must also be kept in mind that any measure of W is an average one, and that it may have very different values in parts of the air vortex.

(72.) In $E_2$, the cross remaining the same the 9″ cups were set at 12″ from the axis; it is my No. III. In it the constant for $v'$ is half that for K's $v$, and the normal friction is double $=60\cdot8$. With the approximate $x=1\cdot7481$ and $z=2\cdot056$, the results are given in the following Table, in which the densities are omitted as involved in $\phi'$.

TABLE XXII.

| No. | Dir. | Time. | A. | A′. | Log. $\phi'$. | $v$. | $v'$. | V. | V′. | V − V′. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | s. | | | | | | | | |
| I. | N.E. | 474 | 108 | 165 | 0·67241 | 0·976 | 1·491 | 3·151 | 5·590 | −2·439 |
| II. | N.E. | 605·6 | 313 | 540·5 | 0·62808 | 4·434 | 3·828 | 12·637 | 12·539 | +0·098 |
| III. | S.E. | 541·7 | 281·5 | 505·5 | 0·64167 | 4·453 | 3·998 | 12·693 | 13·093 | −0·400 |
| IV. | S.E. b. S. | 550·9 | 570 | 1011·0 | 0·63633 | 8·864 | 7·861 | 25·135 | 25·206 | −0·071 |
| V. | S.W. b. S. | 503·6 | 203·0 | 342·0 | 0·63394 | 3·453 | 2·909 | 9·795 | 9·853 | −0·058 |
| VI. | S.E. | 446·2 | 261·6 | 453 | 0·63185 | 5·0235 | 4·350 | 14·298 | 14·179 | +0·119 |
| VII. | S. | 601·3 | 611 | 1134 | 0·68300 | 8·705 | 8·078 | 24·686 | 25·891 | −1·205 |
| VIII. | S. | 547·1 | 440·3 | 752 | 0·68300 | 6·895 | 5·888 | 19·562 | 18·971 | +0·591 |
| IX. | S.W. | 607·8 | 453·5 | 782 | 0·63397 | 6·348 | 5·551 | 18·020 | 17·779 | +0·241 |
| X. | W. | 552·1 | 380 | 647 | 0·63127 | 5·896 | 5·019 | 16·754 | 16·083 | +0·671 |
| XI. | S.W. | 599·4 | 133 | 240 | 0·63644 | 1·914 | 1·727 | 5·617 | 6·254 | −0·637 |
| XII. | S.W. | 475·8 | 340 | 576·5 | 0·63295 | 6·121 | 5·190 | 17·389 | 16·797 | +0·592 |
| XIII. | S.W. | 480·8 | 330 | 582 | 0·63966 | 5·345 | 5·185 | 15·201 | 16·624 | −1·423 |
| XIV. | W. b. N. | 572·1 | 192 | 331·5 | 0·63283 | 3·212 | 2·773 | 9·212 | 8·938 | +0·274 |
| XV. | N.W. | 663·5 | 314 | 535·5 | 0·62821 | 4·045 | 3·4495 | 11·584 | 11·381 | +0·203 |
| XVI. | N.W. | 515·2 | 290 | 497 | 0·62623 | 4·3765 | 3·674 | 12·474 | 12·068 | +0·406 |
| XVII. | N.W. | 600 | 248·2 | 412·5 | 0·62710 | 3·544 | 2·945 | 10·139 | 9·845 | +0·294 |
| XVIII. | N.W. | 660 | 178 | 313·8 | 0·62130 | 2·311 | 2·037 | 6·704 | 7·127 | −0·423 |
| XIX. | S.W. | 946·6 | 488·7 | 752 | 0·63008 | 4·4205 | 3·401 | 12·598 | 11·243 | +1·355 |
| XX. | S. | 720 | 354 | 599 | 0·63364 | 4·212 | 3·564 | 12·014 | 11·453 | +0·561 |

$S(V-V')=-1\cdot251$ which divided by $Se'=145\cdot185$, we have $dx=-0\cdot0086$ $x=1\cdot7395$, $z=2\cdot026$, and the limit $=3\cdot163$. These are larger than those of $E_1$. The results obtained in paragraphs (38) and (39) would make it less, but in the present work it is the rule that diminishing the length of the arms increases $x$.

(73.) In $E_3$ the 9″ inch cups were fixed 8″ distance from the axis: too near for good work, but I wished to see the effect on $x$. With its approximate values $x=2\cdot1359$ and $z=3\cdot562$, I computed Table XXIII.

TABLE XXIII.

| No. | Dir. | Time. | A. | A′. | Log. $\phi'$. | $v$. | $v'$. | V. | V′. | V − V′. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | s. | | | | | | | | |
| I. | S. | 660 | 348·5 | 677·5 | 0·81086 | 4·524 | 2·931 | 12·891 | 12·349 | +0·542 |
| II. | W. b. N. | 475·4 | 131 | 237 | 0·80236 | 2·359 | 1·423 | 6·836 | 6·721 | +0·115 |
| III. | W. b. N. | 600 | 238 | 433 | 0·80390 | 3·399 | 2·061 | 9·731 | 9·037 | +0·694 |
| IV. | W. | 600 | 192 | 360 | 0·80174 | 2·742 | 1·714 | 7·900 | 7·761 | +0·139 |
| V. | S.E. | 540 | 458 | 940 | 0·81188 | 7·267 | 4·972 | 20·623 | 20·424 | +0·199 |
| VI. | S. b. E. | 646·9 | 494·5 | 1011·3 | 0·81938 | 6·631 | 4·972 | 19·088 | 20·343 | −1·255 |
| VII. | S.W. | 334·8 | 227·3 | 471 | 0·80834 | 5·883 | 4·063 | 16·718 | 16·757 | −0·039 |
| VIII. | S. b. W. | 600 | 300·5 | 599 | 0·80480 | 4·291 | 2·851 | 12·233 | 12·032 | +0·201 |
| IX. | S.S.W. | 600 | 260 | 465·5 | 0·82025 | 3·713 | 2·216 | 10·616 | 9·629 | +0·987 |
| X. | S.S.W. | 720 | 540 | 1047 | 0·81604 | 6·426 | 4·153 | 18·251 | 17·115 | +1·136 |
| XI. | S.W. | 600 | 286 | 593·5 | 0·81637 | 4·084 | 2·825 | 11·682 | 11·927 | −0·245 |
| XII. | S.W. | 600 | 285 | 609 | 0·81150 | 4·070 | 2·898 | 11·596 | 12·223 | −0·627 |
| XIII. | S.W. | 420 | 316 | 638 | 0·81497 | 6·446 | 4·336 | 18·307 | 17·623 | +0·684 |
| XIV. | S. | 600 | 466·5 | 910 | 0·81140 | 6·662 | 4·331 | 18·915 | 17·817 | +1·098 |
| XV. | W. | 720 | 291 | 574 | 0·80694 | 3·462 | 2·277 | 9·910 | 9·839 | +0·071 |
| XVI. | N.W. | 600 | 235·5 | 445 | 0·80694 | 3·362 | 2·118 | 9·631 | 9·255 | +0·376 |
| XVII. | S.W. b. W. | 511 | 463·3 | 959·5 | 0·79931 | 7·767 | 5·363 | 22·033 | 21·883 | +0·150 |
| XVIII. | S.W. b. W. | 509·1 | 395 | 883·7 | 0·79712 | 6·960 | 4·957 | 20·029 | 20·271 | −0·242 |
| XIX. | S.W. | 600 | 350·3 | 727 | 0·80133 | 5·002 | 3·460 | 14·235 | 14·389 | −0·154 |
| XX. | S. b. W. | 491·9 | 312·5 | 751 | 0·80780 | 5·443 | 4·361 | 15·477 | 17·937 | −2·460 |

The sum of V−V′=+1·370, S$e'$=119·53. Hence $\Delta x$=+0·0114, $x$=2·1473, $z$=3·611 and the limit =4·047. The residue is a little too large; but I did not think it necessary to pursue the approximation farther. The great increase of $x$ is remarkable, and I think shows that when the cups are so near the axis of rotation they disturb the regular action of the wind. Even with the 12″ arms this effect is sensible.

(74.) $E_4$. I now fixed the 4″ cups on the cross at 10″$\frac{2}{3}$ from the axis. This arrangement is *similar* to the 9″ cups at 24″, and I thought that the same $x$ might serve for both, but it was far otherwise.

The measures in paragraph (27) give for 4″ cups $\alpha$=3·357 at the normal pressure and temperature. For the first ten observations $f'$=35·19; but as these cups are 35·5 oz. lighter than the 9″ ones, I thought there was too little pressure on the toe, and changed the relieving weight from the 11·5 lbs. to 9 lbs. This made the friction =68·89. I computed with $x$=2·57 and $z$=5·405 Table XXIV.

TABLE XXIV.

| No. | Dir. | Time. | A. | A′. | Log. φ′. | v. | v′. | V. | V′. | V−V′. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | s. | | | | | | | | |
| I. | S.W. | 600 | 291·5 | 343 | 1·04878 | 4·156 | 2·177 | 11·853 | 11·719 | +0·134 |
| II. | S.W. | 420 | 202·5 | 211 | 1·04675 | 4·131 | 1·946 | 11·785 | 10·516 | +1·219 |
| III. | S.W. | 225·03* | 138·6 | 147 | 1·05066 | 5·526 | 2·488 | 15·710 | 13·169 | +2·541 |
| IV. | S.W. | 588·7 | 433 | 554·5 | 1·05384 | 6·302 | 3·587 | 17·919 | 19·696 | −1·777 |
| V. | W. b. N. | 540 | 156 | 139·5 | 1·04918 | 2·475 | 0·990 | 7·159 | 6·676 | +0·483 |
| VI. | S.W. | 600 | 313·5 | 414·5 | 1·04366 | 4·477 | 2·631 | 12·750 | 13·362 | −0·612 |
| VII. | S.W. | 600 | 263 | 306·3 | 1·04293 | 3·756 | 1·941 | 10·730 | 10·674 | +0·056 |
| VIII. | S. b. W. | 780 | 391·7 | 449 | 1·04205 | 4·303 | 2·192 | 12·266 | 11·796 | +0·470 |
| IX. | S. b. W. | 600 | 371·5 | 449 | 1·04491 | 5·305 | 2·850 | 15·096 | 15·314 | −0·218 |
| X. | S. S. W. | 660 | 402 | 499 | 1·04389 | 5·219 | 2·879 | 14·844 | 14·969 | −0·125 |
| XI. | S. b. E. | 840 | 430·3 | 471 | 1·33626 | 4·389 | 2·185 | 12·509 | 12·575 | −0·066 |
| XII. | W.S.W. | 532·3 | 261·1 | 290·4 | 1·33929 | 4·145 | 2·078 | 11·824 | 12·112 | −0·288 |
| XIII. | S.W. b. S. | 600 | 420·3 | 535·5 | 1·33648 | 6·002 | 2·085 | 17·052 | 18·201 | −1·149 |
| XIV. | S.W. b. W. | 600 | 419·5 | 503 | 1·33768 | 5·949 | 3·193 | 17·019 | 17·117 | −0·098 |
| XV. | S.W. | 608·1 | 362·3 | 408·5 | 1·33538 | 5·105 | 2·558 | 14·524 | 14·211 | +0·313 |
| XVI. | N. | 540 | 320·5 | 407 | 1·32198 | 5·085 | 2·870 | 14·470 | 15·480 | −1·011 |
| XVII. | N. | 600 | 363·5 | 430 | 1·32039 | 5·191 | 2·729 | 14·763 | 14·933 | −0·170 |
| XVIII. | N. | 600 | 382·5 | 493 | 1·32025 | 5·462 | 3·129 | 15·527 | 16·741 | −1·214 |
| XIX. | W. | 666 | 493·5 | 512·5 | 1·33003 | 6·407 | 2·957 | 18·193 | 15·991 | +2·202 |
| XX. | W. b. N. | 600 | 461·6 | 550·5 | 1·33073 | 6·592 | 3·494 | 18·714 | 18·456 | +0·258 |

The sum of V−V′=+0·948; Sc′=83·121, so $\Delta x$=+0·0114, $x$=2·5814, $z$=5·664, and the limit =4·961. This great excess of $x$ above that of K is very remarkable, and shows not only that anemometers must be equal as well as similar to have the same constants, but also that $x$ depends on the diameter of the cups as well as the length of the arms; for here it is greater than in $E_3$, though the arms are longer.

(75.) $E_5$. The 4″ cups were fixed as far out on the cross as possible, the distance from the axis being 26″·75; 2·75 greater than that of K, and I expected its $x$ would be less. At first it was mounted on the axle of E, but it moved so much slower than K that I thought its friction =27·42 was too much for the small cups. I therefore mounted it on the spindle already mentioned with friction =4·72, and with *its* results (VI., XXIII.) computed Table XXV.

TABLE XXV.

| No. | Dir. | Time. | A. | A′. | Log. φ′. | v. | v′. | V. | V′. | V−V′. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | s. | | | | | | | | |
| I. | S. | 600 | 547 | 378 | 0·94639 | 7·812 | 6·016 | 22·163 | 21·194 | +0·969 |
| II. | S.W. b. S. | 600 | 444·5 | 292·3 | 0·94689 | 6·347 | 4·652 | 18·026 | 16·617 | +1·409 |
| III. | W. b. N. | 600 | 302 | 172·5 | 0·92562 | 4·313 | 2·825 | 12·293 | 10·599 | +1·694 |
| IV. | N.W. | 300 | 261·8 | 183 | 0·92552 | 7·477 | 5·825 | 21·207 | 20·527 | +0·680 |
| V. | S.W. | 600 | 547 | 387 | 0·94124 | 7·811 | 6·160 | 22·159 | 21·674 | +0·485 |
| VI. | S.W. | 660 | 646 | 484 | 9·97483 | 8·386 | 7·003 | 23·783 | 24·291 | −0·508 |
| VII. | S.W. | 660 | 723·5 | 561 | 0·16863 | 9·3925 | 8·117 | 26·627 | 28·134 | −1·507 |
| VIII. | S.W. | 600 | 679 | 513 | 0·16820 | 9·696 | 8·300 | 27·485 | 28·300 | −0·815 |
| IX. | S.W. | 600 | 309·8 | 209·5 | 0·17272 | 4·424 | 3·334 | 12·609 | 11·616 | +0·993 |
| X. | W. | 1200 | 922 | 660·5 | 0·16968 | 6·583 | 5·256 | 18·689 | 18·269 | +0·420 |
| XI. | W. | 480 | 250 | 162·25 | 0·16849 | 4·452 | 3·288 | 12·686 | 11·512 | +1·174 |
| XII. | S.W. | 600 | 521 | 384 | 0·16846 | 7·440 | 6·112 | 21·110 | 21·219 | −0·109 |
| XIII. | S.W. | 600 | 458 | 320·5 | 0·16785 | 6·540 | 5·101 | 18·570 | 17·736 | +0·834 |
| XIV. | S.W. b. S. | 600 | 473 | 320 | 0·16788 | 6·754 | 5·093 | 19·174 | 17·710 | +1·464 |
| XV. | S. | 600 | 432 | 314·5 | 0·16727 | 6·056 | 5·007 | 17·205 | 17·412 | −0·207 |
| XVI. | S. | 600 | 555 | 437·5 | 0·16864 | 7·925 | 6·963 | 22·481 | 24·450 | −1·969 |
| XVII. | S.E. | 600 | 388 | 302 | 0·15962 | 5·542 | 4·817 | 15·749 | 16·720 | −0·971 |
| XVIII. | S.E. | 600 | 357·3 | 268 | 0·16041 | 5·103 | 4·266 | 14·514 | 14·870 | −0·356 |
| XIX. | .. | 1200 | 908 | 725 | 0·16274 | 6·483 | 5·770 | 18·408 | 20·037 | −1·629 |
| XX. | S.W. | 660 | 659 | 495·5 | 0·17775 | 8·555 | 7·169 | 21·261 | 22·902 | −1·641 |
| XXI. | W. b. S. | 600 | 417 | 291·5 | 0·17795 | 5·956 | 4·639 | 16·920 | 16·152 | +0·768 |
| XXII. | S.W. | 600 | 459·5 | 322 | 0·17390 | 6·561 | 5·125 | 18·632 | 17·833 | +0·799 |
| XXIII. | S.W. | 600 | 362·5 | 264 | 0·17404 | 5·176 | 4·202 | 14·726 | 14·645 | +0·081 |

* Time short because it was at the end of the chronograph sheet.

Omitting the first five S(V−V′)=−3·179, Se′=269·80. Hence $\Delta x=-0\cdot0117$ and $x=1\cdot8624$ and $z=2\cdot468$ and the limit =3·436. This result surprised me, for the friction was so small that no irregularity of it could have any sensible influence, nor does it seem probable that the pressures on the surfaces of the two sets of cups are in any other ratio than that of the surfaces. The $x$ is actually larger than that of $E_2$. The five first V″s were computed with the final $x$. They give V′ rather too small, but in three of them the wind was S.W.

(76.) $E_6$. I now placed my old anemometer, cups 12″, arms 23″·17, on the axle of E. The $\alpha$ of these cups (if as their area)=27·227* and their $f$=29·0. With the second approximation, $x=1\cdot5897$, $z=1\cdot527$, I recomputed the V and V′ of Table XXVI.

TABLE XXVI.

| No. | Dir. | Time. | A. | A′. | Log. φ′. | v. | v′. | V. | V′. | V − V′. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | s. | | | | | | | | |
| I. | N.W. | 600 | 313 | 339 | 0·05547 | 4·470 | 4·674 | 12·737 | 13·415 | −0·678 |
| II. | N. | 600 | 332·5 | 386·5 | 0·04264 | 4·747 | 5·328 | 13·519 | 15·133 | −1·614 |
| III. | N. | 1200 | 695 | 784 | 0·05223 | 4·962 | 5·404 | 14·122 | 15·351 | −1·229 |
| IV. | N. b. E. | 600 | 240 | 286 | 0·04524 | 3·427 | 3·943 | 9·809 | 11·247 | −1·438 |
| V. | N.E. | 600 | 382 | 378 | 0·04701 | 5·455 | 5·211 | 15·509 | 14·806 | +0·703 |
| VI. | W. b. N. | 600 | 292 | 300 | 0·05411 | 4·170 | 4·136 | 11·896 | 11·795 | +0·101 |
| VII. | S.W. | 600 | 398 | 368 | 0·05435 | 5·683 | 5·074 | 16·379 | 14·404 | +1·975 |
| VIII. | S.E. | 600 | 751 | 783 | 0·05273 | 10·725 | 10·796 | 30·392 | 30·464 | −0·072 |
| IX. | S. | 600 | 726 | 751·7 | 0·05918 | 8·640 | 8·636 | 24·499 | 24·484 | +0·015 |
| X. | S. | 660 | 620 | 615·8 | 0·05918 | 8·048 | 7·778 | 22·830 | 22·014 | +0·816 |
| XI. | S. | 660 | 659 | 679·6 | 0·05918 | 8·555 | 8·517 | 24·261 | 24·120 | +0·141 |
| XII. | S. | 660 | 502 | 516·7 | 0·05918 | 6·516 | 6·476 | 18·506 | 18·365 | +0·141 |
| XIII. | N.E. | 600 | 286 | 282 | 0·04772 | 4·277 | 3·887 | 11·652 | 11·591 | +0·061 |
| XIV. | E. | 600 | 340 | 329 | 0·04556 | 4·855 | 4·535 | 15·497 | 12·900 | +2·597 |
| XV. | E. | 600 | 358 | 348·5 | 0·04833 | 5·112 | 4·935 | 14·454 | 14·037 | +0·417 |
| XVI. | E. | 1200 | 793 | 860 | 0·04435 | 5·662 | 5·928 | 16·093 | 16·890 | −0·797 |
| XVII. | N.E. b. E. | 600 | 279·7 | 294·5 | 0·04666 | 3·994 | 4·060 | 11·399 | 11·578 | −0·179 |
| XVIII. | N.E. b. E. | 600 | 278·5 | 277·8 | 0·04797 | 3·977 | 3·830 | 11·351 | 10·934 | +0·417 |

It will be observed here, as in Table XX., that $v'$ is sometimes greater, sometimes less than $v$; the near equality of the constants of the 12″ cups to those of K makes the irregularities of the wind manifest.† The S (V−V′) giving III. and XVI. (double weight) =−0·649, Se′=282·65, therefore $dx=-0\cdot0023$, $x=1\cdot5874$, $z=1\cdot520$, and the limit 2·8202. This $x$ is a little less than that of K; which shows that the influence of the diameter of the cups is felt even here, overpowering the effect of the shorter ones.

(77.) I shall conclude with a few remarks on the preceding results.

The process by which the $x$ of K is determined seems liable to but two objections :

* In my original paper "On the Cup Anemometer" (Trans. R.I.A., vol. xxii., p. 170) I have mentioned some trials to measure $\alpha$. As the V's given there were doubtful, I have recomputed them with these constants and the friction of that instrument =48·61. The six give for $\alpha$ (at normal pressure and temperature) 27·898, agreeing fairly with that given in the text.

† I may mention here, as further proof of the unsteadiness of the wind, that on one occasion I reversed two cups of this anemometer so that all the convexes were opposed to the wind; I expected they would remain at rest, but they were in continual oscillation through many degrees, so that in the limited area 5′ × 1′ there must have been differences of V able to overcome a friction of 53 grains.

the assumption that $y=1$, and the possibility that the wind errors are not eliminated. As to the first, I have shown in paragraph (69) that it cannot be far astray; even were the extreme diminution of it which is mentioned there to occur, the error for $V=100$ would only be 6·1; but such is not likely to be the case, and $y=1$ may be accepted as a major limit and one not far from the truth. As to the second it is certain that in a sufficient number of observations the + and − errors must balance each other; but it may be a question whether the XXI. of Table XXI. were enough. Still, it is evident, from inspection of the column $V-V'$ that there cannot be any large outstanding residue. I have pointed out the defects of the situation. Could I have had my wish I would have established the instruments on spars 20′ high, erected on a level ground away from any influence of houses or trees, and used a better mode of applying friction to $E_1$. I also regret that no strong gale occurred during these experiments to verify the formula for a very large $v$. Under favourable circumstances, I think this mode of determining $x$ the best that is known. I have stated reasons for distrusting the results obtained by the whirling machine, and as yet no unexceptional mode of carrying an anemometer through the air in a right line has been devised. Even could we get a locomotive which could travel without disturbing the air through which it passes (and perhaps the new electric motors might effect this), and a line of rails certainly screened from wind, there would still remain the doubt whether the pressure is the same when a body is moved through a quiescent fluid or a current impinges on the body.

(78.) The results obtained with other anemometers show that $x$ is a function both of R, the length of the arm, and $r$, the radius of the cups. I subjoin its values.

| | | |
|---|---|---|
| $R=23·17$ | $r=6$ | $x=1·5880$ |
| 24 | 4·5 | 1·5919 |
| 12 | 4·5 | 1·7463 |
| 8 | 4·5 | 2·1488 |
| 26·75 | 2 | 1·8587 |
| 10·67 | 2 | 2·5798 |

If we take Nos. 2, 3, and 4 in which the cups are equal, the dependence of $x$ on R is manifest. In No. 3 it is $\frac{1}{10}$ larger than No. 2, and in No. 4 $\frac{1}{3}$. This is partly due to the air's escape before the convexes being less easy as the circle described by them is less. Such a fact is strikingly shown by the whirling experiments (paragraph 69) which I made in search of a minor limit for $y$, in which I found the resistance =30·61, and 79 for the three respectively. This was in quiescent air; but a similar though much smaller effect must occur in the actual working of an anemometer. Its influence can be obtained only by experiments, such as the present.*

* I thought to test this by removing two opposite cups in $E_3$. As in this case there is only one cup in each semicircle at a time, the probability of their mutual disturbance was small. A set of ten gave the

(79.) It is more difficult to account for the similar dependence of $x$ on the size of the cups; *à priori,* there seems no reason why small cups should be more resisted than large ones, but such is evidently the case. Unluckily I did not place the 12″ and 4″ cups at the same distances as the 9″, so that the effect of the cups on $x$ is mixed with that of R. I tried to eliminate the latter by interpolating for the values that the 9″$x$ would have at the R's of Nos. 1, 5, and 6, but this could not be done very exactly from the three values. However, this gives $x$ for the 12″ 0·005 less than for the 9″; for the 4″, in No. 5, 0·2894 greater, and in No. 6, 0·7517. The only way in which I can conceive the possibility of such an occurrence is the existence of powers of $r$ and R in the factors, which express the mean effects of wind on the concave and convex surfaces of the cups. In equation III. I suppose the mean $v$ to be that of the centre of the cups, and that the mean impulse and resistance act at these points. But this is not necessarily the case. The effect of the resultant on an element of the cup is (1) as the square of that resultant; (2) as the perpendicular pressure on the element; (3) as the resultant of that pressure perpendicular to the plane of the cup's mouth; (4) as the distance from the axis at which the projection of that resultant meets the arm; and (5) as the magnitude of the element. Of these five factors the first contains $v$ and $v^2$. Now $v$ as the element $=v\times\frac{(4)}{\mathrm{R}}$ which contains R and $r$; $r^2$ also enters the fifth, so that the differential may contain $\mathrm{R}^3$ and $r^5$. As to the second we are ignorant of its formula, and it is pretty certain that it will depend on powers of the sine and cosine of incidence and (at least for the concave) on the curvature. If we knew its exact form we could integrate the differential which they form and get the impulse and resistance for a given $\theta$, and multiplying this by $d\theta$, and again integrating from 0 to $\pi$ we should find their mean values. Of the terms in this last integration those which have $\sin^2\theta$ as a factor disappear; $\pi r^2$ (the surface) will be a factor of the others, among which *may* be the three first powers of $\frac{r}{\mathrm{R}}$; and these may produce the change of $x$.

(80.) In paragraph (41) I inferred from the work with the whirling machine that with 9″ cups the $x$ is the same for 24″ and 12″ arms; but what precedes shows that this is not the fact, and that each type of anemometer has a special $x$. I would therefore suggest to meteorologists and opticians the propriety of confining themselves to two types: one for fixed instruments, the other for portable ones. For the first the Kew type should, I think, be adopted; if the determination of its constants, given in paragraph (70), be not thought sufficiently exact, there would be little difficulty in making more observations like those described there, and under more

$x$=2·0709 less than in No. 4, but so large as to make it evident that there must be some other cause of the increased value of $x$.

favourable circumstances; but I think they would make very little change in my number. For the portable instrument, the only one of which I have experience is CASELLA's 3″ cups and 6″ arms, and I found it very convenient: its $x$ should be determined as above. Some such arrangement seems necessary to ensure a uniformity of velocity measures.