SOME CHARACTERISTICS OF THE MARVIN PYRHELIOMETER

By Paul D. Foote

CONTENTS

I. INTRODUCTION

Pyrheliometers are usually calibrated by comparison observations with a standard pyrheliometer which has been calibrated by exposing the receiver to a measured quantity of electrical energy, or to radiation of long wave lengths emitted by a radiator electrically heated to a temperature slightly above that of the room, both the instrument under test and the standard being sighted upon the sun. There are no published measurements of a calibration made by exposing the receiver to a known amount of energy of radiation from a radiator at high temperatures. Such a calibration is highly desirable, for always the question may arise as to whether the thermal processes taking place in the pyrheliometer receiver are exactly similar during the electrical calibration and during the radiometric use of the instrument. In the present work both methods were employed for the calibration of a Marvin silver-disk pyrheliometer.

II. DESCRIPTION OF INSTRUMENT

The Marvin pyrheliometer has been the "working" instrument of the United States Weather Bureau for several years. A complete description of the instrument and methods of use is now in preparation by Prof. Marvin, Chief of the Weather Bureau. This pyrheliometer is dynamic in type in that it is necessary to consider the rate at which the receiver gains heat when exposed to radiation, and the rate at which the receiver loses heat when shaded from radiation.[1]

The essential feature of the instrument is the receiver. In the form used in the present work it consisted of a silver disk about 4.5 cm in diameter and 0.3 cm thick, in an annular space inside of which is carefully mounted with the best possible thermal contact a noninductive spirally wound coil of No. 35 silk insulated nickel wire in the form of a 3-lead resistance thermometer, having a total resistance of from 20 to 25 ohms. The coil serves both as the thermometer and as the heater for the purpose of an electrical calibration, the rise in temperature of the thermometer being observed when a known amount of electrical energy is dissipated in the coil. The receiver is mounted within a metal shell, which is incased by a wooden shell in order to reduce local temperature variations to a minimum, and the type of suspension of the receiver is such that thermal losses by conduction are negligible. Before the front face of the receiver a limiting diaphragm is placed and leading from this through a hole in the metal and wooden shells is a diaphragmed and blackened tube which serves the purpose of limiting the cone of rays to a convenient solid angle greater than that subtended by the sun. The end of the tube carries a double-walled aluminum shutter, operated by a magnetic release controlled by a chronograph which may be so set as to open or close the shutter at any desired instant. For solar work the instrument is mounted as an equatorial telescope and is driven by clockwork so that the surface of the receiver is always presented normally to the sun. The instrument is shown in Fig. 1.

III. METHOD OF CALIBRATION, GENERAL

The electrical calibration was made by subjecting the nickel coil of the thermometer to a measured current and observing the change in temperature indicated by the thermometer. The radiometric calibration was made in a similar manner except that the heat was supplied by radiation from an outside source. The source employed was a Lummer-Kurlbaum[2] black body, or a black body of similar type, electrically heated, with a compensating winding to

[1] Kimball, Bull. Mount Weather Obs., 3, Part 2, p. 73; 1910.
[2] Waidner and Burgess, this Bulletin, 3, p. 165; 1907 (Scientific Paper No. 55).

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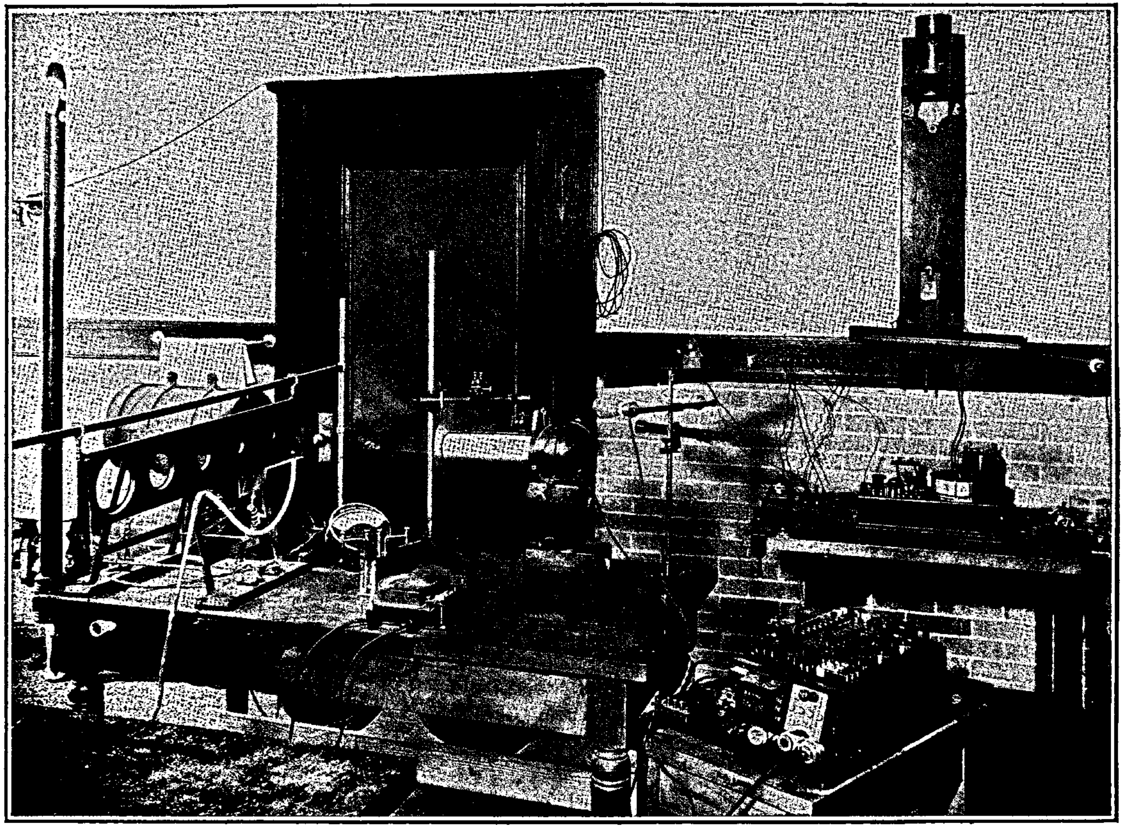

FIG. 1.—*Apparatus used for radiometric calibration of pyrheliometer*

From left to right: Furnace, water-cooled diaphragm, pyrheliometer, wheatstone bridge, potentiometer reduce the temperature gradient and to approximate temperature uniformity. The temperature of the inner inclosure, from which the radiation was taken, was measured by standard platinum, platinum-rhodium thermocouples, accurately calibrated in terms of the melting points of zinc (419.4°), antimony (630.5°), and copper (1083°). A water-cooled diaphragm was mounted directly in front of the opening to the furnace. This diaphragm acts as the effective source of radiation. The equation of rate of energy transfer from the furnace to the pyrheliometer receiver is as follows when R is large compared with $\sqrt{A_1}$ and $\sqrt{A_2}$.

$$J=\frac{\sigma}{\pi}\left(T^4-T_0^4\right)\frac{A_1A_2}{R^2}$$

where J = energy transferred per unit time from furnace to receiver.

A_1 = area of water cooled diaphragm in front of furnace.

A_2 = area of inmost or effective diaphragm in the pyrheliometer.

T = absolute temperature of furnace.

T_0 = absolute temperature of pyrheliometer receiver and surroundings.

σ = the Stefan-Boltzmann coefficient of radiation.

R = distance from A_1 to A_2.

The quantity T_0^4 is negligible, for the present work, in comparison with T^4.

The coefficient σ has been determined in a variety of ways by many investigators, with widely discordant results. Within the past two or three years, however, the agreement has been quite satisfactory, values of different observers showing $\sigma=5.7\times10^{-12}$ watts cm^{-2} $deg.^{-4}$. This value is also the mean of all determinations to the present date. The constant is closely connected with the value of the electronic charge, the constant C_2 of the Planck equation of spectral distribution of the energy of a black body, and with the energy of emission of electrons in photoelectric phenomena. All of these interrelations point to a numerical value of $\sigma=5.7\times10^{-12}$. The most recent work on a direct determination of σ is by Coblentz[3] who found as a mean of extended experiments $\sigma=5.72$. Any error in the assumed value of σ will enter directly into the calibration of the pyrheliometer. It appears unlikely, however, that the value $\sigma=5.7$ is in error by more than 1 per cent.

[3] This Bulletin, 12, p. 553; 1916 (Scientific Paper No. 262).

IV. SIMPLE THEORY OF THE MARVIN PYRHELIOMETER

Pursuing the general treatment given by Kimball [4] and others for various pyrheliometers, the following describes the action of the Marvin instrument, omitting the consideration of certain corrections which are discussed later. The case considered is for alternate heating and cooling periods of 60 seconds' duration.

Let q = intensity of solar radiation at the earth's surface in cal./cm² min.

S = entire surface of receiver. The front face is assumed to be a perfect absorber.

s = area receiving radiation.

C = water equivalent of receiver.

h = mean coefficient of heat loss from receiver, assuming Newton's law of cooling.

$E = \frac{hS}{c}$ a constant for any particular instrument.

T = temperature of receiver.

T' = temperature of surroundings.

T_1 = temperature at beginning of cooling.

T_2 = temperature at beginning of heating.

t = time in seconds.

The variable thermometric state is expressed by

$$CdT = qsdt - h\,S\,(T-T')\,dt \tag{1}$$

Cooling.—When the shutter is closed $q = o$, whence

$$\frac{dT}{T-T'} = \frac{d(T-T')}{T-T'} = -\frac{hS}{C}\,dt \text{ since } T' \text{ is assumed constant.}$$

$$T - T' = Ae^{-\frac{hS}{c}t} = Ae^{-Et}$$

when $t = o$ $\quad T = T_1$

whence $T = T' + (T_1 - T')\,e^{-Et}$ the equation for cooling (2)

Heating.—From (1) $\dfrac{dT}{(T-T') - \dfrac{qs}{EC}} = -E\,dt$

and since $T = T_2$ when $t = o$

$$T = T' + \frac{qs}{EC} + \left(T_2 - T' - \frac{qs}{EC}\right) e^{-Et} \text{ the equation for heating.} \tag{3}$$

The amount of cooling in 60 seconds is found from equation (2) by substituting 0 and 60 for t and subtracting, whence:

Cooling in 60 seconds $= \Delta T_c = T_1 - T' - T_1 e^{-60E} + T' e^{-60E}$ (4)

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[4] Bulletin Mount Weather Obs., 8, Part 2; 1910.

By similarly using equation (3)

$$\text{Heating in 60 seconds} = \Delta T_h = T' + \frac{qs}{EC} + \left(T_2 - T' - \frac{qs}{EC}\right)e^{-60E} - T_2 \quad (5)$$

In the use of this instrument the temperature of the receiver is so adjusted that the amount of cooling ΔT_c equals the amount of heating ΔT_h, so that after a two-minute period, consisting of one heating and one cooling, the receiver returns to its original temperature. Whence on equating (4) and (5) one obtains

$$T_1 - T' = -T_2 + \frac{qs}{EC} + T' \quad (6)$$

But $T_1 - T_2 = \Delta T$ = the total change in temperature during a 60-second period. Whence from (6) and (2) it can be shown that

$$\Delta T_{60} = \frac{qs}{EC}\,\frac{1 - e^{-60E}}{1 + e^{-60E}} = \text{constant} \cdot q = \frac{1}{F}\Delta Q \quad (7)$$

where F is a constant and ΔQ the heat incident upon the receiver in 60 seconds.

Equation (7) states that the change in temperature in 60 seconds equals a constant of the instrument times the intensity of solar radiation and is independent of the room temperature. This conclusion is derived on the assumption of Newton's law of cooling, but since all the temperature differences usually observed are fairly small, there is probably no serious question in regard to the applicability of this law. Questions of greater moment are those regarding the temperature distributions in the receiver under the four different conditions of cooling and heating, electrically and radiometrically, and the lag of the thermometer. These are considered later.

An experimental condition which is of interest, as will appear below, is that in which the readings over the first 10-second period immediately following a shift of the shutter are discarded, and the change in temperature of the receiver is observed for the 50-second interval immediately following. It will be shown that the fall in temperature during such a 50-second interval of cooling is equal to the rise in temperature during the corresponding 50-second interval of heating.

From equation (2), during cooling

$$T_{10} - T_{60} = (T_1 - T')(e^{-10E} - e^{-60E}) \quad (8)$$

From equation (3), during heating

$$T_{60} - T_{10} = -\left(T_2 - T' - \frac{qs}{EC}\right)(e^{-10E} - e^{-60E}) \quad (9)$$

But from (2) and (3) evaluated over a 60-second interval we obtain the following:

$$T_1 - T' = \frac{\Delta T_{60}}{2} + \frac{1}{2}\frac{qs}{EC} \qquad (10)$$

$$T' - T_2 = \frac{\Delta T_{60}}{2} - \frac{1}{2}\frac{qs}{EC} \qquad (11)$$

Substituting (10) and (11) in (8) and (9) it is seen that the cooling in 50 seconds equals the heating in 50 seconds.

If we denote the amount of heating or cooling in the 50-second interval defined above by ΔT_{50}, the relation between ΔT_{50} and ΔT_{60} is given by equations (4) and (8) as follows:

$$\Delta T_{60} = \frac{1 - e^{-60E}}{e^{-10E} - e^{-60E}} \Delta T_{50} \qquad (12)$$

Equation (12) is discussed later.

V. METHOD OF CALIBRATION—DETAILED DESCRIPTION

In the radiometric calibration the pyrheliometer was alternately exposed to and shaded from the radiation emitted through an opening in a blackened, water-cooled diaphragm placed in front of the black body. The opening through the water-cooled compartment was conical and of greater angle than that required by the pyrheliometer. Care was always taken to operate the pyrheliometer at such a distance from this diaphragm that the radiation striking the receiver came from the portion of the black body furnace of which the temperature was measured by the thermocouples. The temperature gradient throughout this section of the black body was easily made practically negligible.[5]

The electromotive forces of the two thermocouples were measured by a Leeds and Northrup potentiometer, shown in Fig. 1, and in converting the electromotive force readings to temperature due consideration was given to the correction of the "thermoelectric" scale to the thermodynamic scale.[6]

The resistance of the thermometer of the pyrheliometer was measured by the precision thermostated bridge described in detail by Mueller.[7]

This dial bridge reads directly to 0.0001 ohm, and by interpolating the readings of the galvanometer, the sensibility of which

[5] Waidner and Burgess, this Bulletin, 3, p. 167; 1907 (Scientific Paper No. 55).

[6] This Bulletin, 9, p. 563; 1913 (Scientific Paper No. 202).

[7] This Bulletin, 13, p. 547; 1916 (Scientific Paper No. 288).

was determined at all points of the scale used, at least one more figure in the resistance measurement could be relied upon. The scale deflections were always maintained as nearly zero as possible.

The small measuring current passing through the thermometer coil in general had an appreciable heating effect. The current was maintained constant by using rather high voltage and external resistance so that the small variations in resistance of the thermometer were negligible in their effect upon the magnitude of the current. When necessary, the current was measured by the

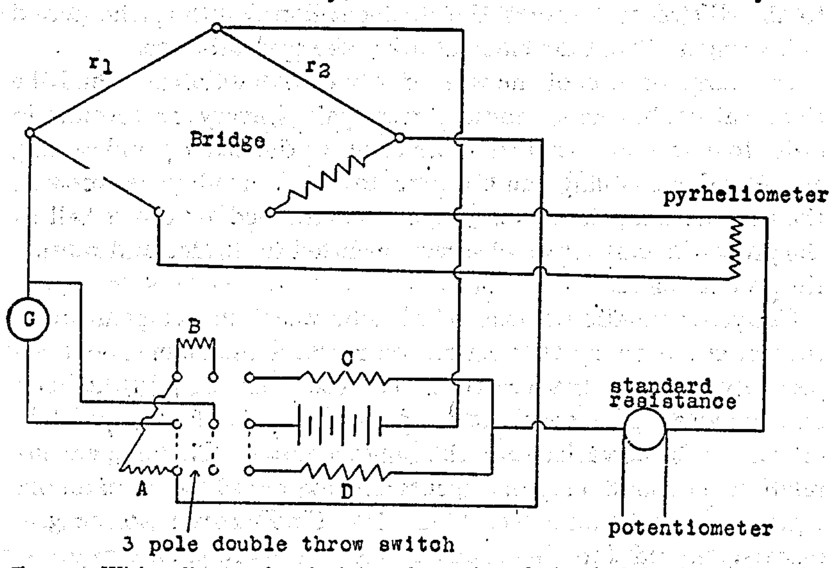

FIG. 2.—*Wiring diagram for obtaining observations during both heating and cooling of an electrical calibration*

Magnitude of heating and measuring current controlled by rheostats *C* and *D*. Galvanometer *G* critically damped and maintained at the same sensibility during heating and cooling by rheostats *A* and *B*.

potentiometer method and controlled by an adjustable low-resistance slide wire. The constant current was allowed to flow during the entire radiometric calibration, during both heating and cooling of the receiver, so that it produced no net effect upon the measurements. Fig. 1 shows the experimental arrangement for a radiometric calibration.

The main difficulty with the electrical calibration was to devise a suitable means for measuring the resistance during the period of heating. The wiring diagram is given in Fig. 2. The maximum current required for heating was small enough to permit the use of the bridge in the manner illustrated. The galvanometer sensibility was maintained approximately the same for the bridge currents during both heating and cooling, and the galvanometer was critically damped in each case by suitable series and shunt resistances. Both thermometer currents were measured with a potentiometer and were maintained constant by adjusting a slide-wire rheostat. The heating by each current was computed, using the mean resistance of the thermometer coil throughout the periods of heating and of cooling. The net heating effect is equal to the difference between the heat developed during the period of heating and that developed during the period of cooling.

In a large portion of the work, for both the radiometric and the electrical calibrations, readings were taken every 10 seconds in order to determine accurately the forms of the heating and cooling curves. For enabling the observer to obtain readings exactly at the time desired, the chronograph was arranged to tap a bell at the proper instant. One observer operated the bridge and another the potentiometer.

The determination of the relation between the temperature of the thermometer and its resistance requires an independent experiment in which the receiver is removed from the pyrheliometer and mounted in a constant temperature bath, the temperature of which may be varied over the range required. The temperature relation so found may be accurately expressed by a parabolic equation. Thus, for silver block No. II, $R=23.041+0.10033t+0.001093t^2$ and for silver block No. III $R=19.521+0.08394t+0.000101 27t^2$ where t is the temperature centigrade. These data were obtained by Prof. H. H. Kimball, of the United States Weather Bureau.

The simple theory derived in Section IV requires certain modifications to fit the actual experimental conditions. The room temperature T' did not remain constant during a run of an hour required for any one set of measurements. The result was that there was a gradual variation back and forth in the equilibrium position of the thermometer temperature so that the thermometer did not return exactly to its initial temperature after a complete cycle of one heating and one cooling. Each observed heating and cooling curve may be readily corrected to the ideal curve by assuming a linear change in temperature over the complete cycle in question. Thus, for example, if, after a cycle of two minutes, the temperature of the thermometer coil had risen 0.001°, the measurements at every 10-seconds interval were corrected by adding 1/12, 2/12, 3/12, etc., of 0.001° to the first, second, third, etc., reading of the cycle. A correction method developed by Prof. Marvin, which is more convenient and accurate and which will be described in his paper referred to above, was frequently employed. The corrected curves were always used in the computation of the results. In general, variations of the above description tend to eliminate themselves over a long period, the temperature rising for part of the series and falling for the rest of the time. However, runs were obtained for all possible conditions, namely, continually increasing temperature, continually decreasing temperature, and practically constant temperature. No dependence of the results upon such conditions could be observed.

VI. EFFECT OF LAG

A second modification of the simple theory discussed in Section IV is required by the presence of a lag in the temperature or resistance measurements. In order to obtain electrical insulation of the thermometer coil from the silver receiving disk, the wire is silk covered. This silk covering, of course, acts also as a thermal insulator. The result is that the temperature of the thermometer lags behind that of the disk when the heating is radiometric, and leads the temperature of the disk when the heating is electrical. In the case of an entire cooling following a radiometric heating and for a part of the cooling following an electrical heating, the thermometer temperature again lags behind that of the disk. If the lag effect is not corrected for, the experimentally determined value of the constant F (Sec. IV, equation (7)) increases for radiometric exposures and decreases for electrical exposures as the time of exposure is shortened. When the heat is supplied electrically directly in the thermometer coil, the lag effect due to thermal insulation increases the observed ΔT over the value of ΔT for the disk, and thus the measured F is too low. The reverse is true when the heating is radiometric. In order to detect the presence of lag, the receiver was exposed to radiation or to the heating current and then cooled, the periods of heating and cooling being equal and of the following durations: 10, 20, 30, 60, and 120 seconds. Observations were made at the beginning and the end of the coolings and heatings; that is, at the instant of shifting from a cooling to a heating and vice versa. The results of several experiments with Marvin pyrheliometer No. 7, disk No. 3, are given in Fig. 3. Here the ordinates are values of F' obtained by multiplying the value of F, Section IV, equation 7, by $\frac{(5.5)\ (1.20)}{1.217}$. The constant F' is more convenient than F

in reducing solar measurements to calories / cm². min. and will be used exclusively hereafter. If there were no lag effect whatever, the two curves in Fig. 3 would coincide exactly and would pass through the point $F'=2.135$ at $t=60$ seconds. The curve F' versus t then would differ slightly from a horizontal straight line, because of the fact that the cooling of the disk follows Newton's exponential law instead of a linear relation with time. The

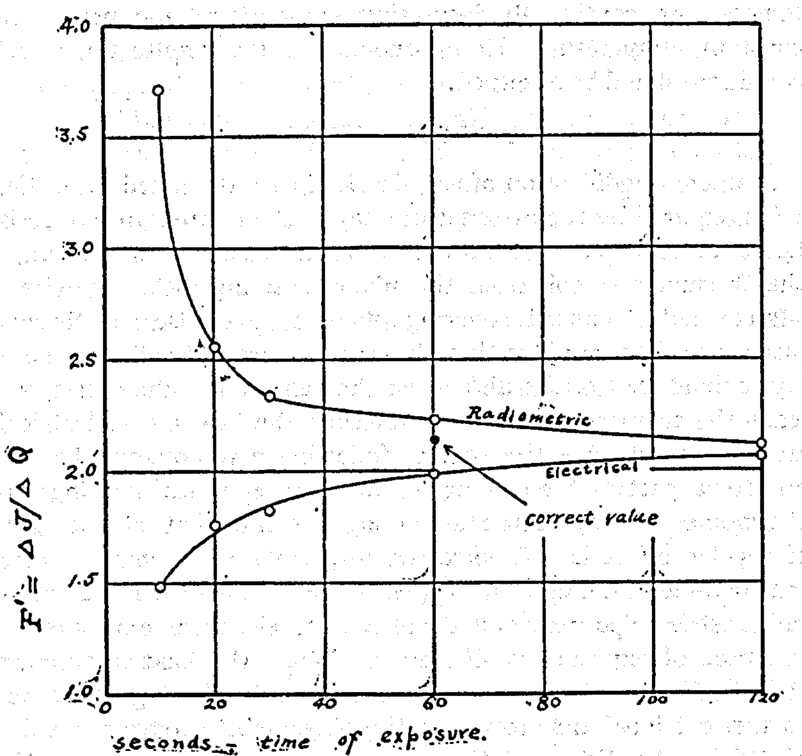

FIG. 3.—*Observed F' as a function of time of exposure*

If there were no lag, the curves for radiometric and electrical calibrations would be coincident and intermediate to the two curves shown

above experiment demonstrates clearly the presence of a lag effect. The magnitude of this lag may be calculated in the manner shown below.

VII. ANALYSIS OF THE HEATING AND COOLING CURVES

Heatings and coolings were made alternately every minute for a period of one half to one hour, during which time bridge readings of the thermometer temperature were taken every 10 seconds. The heatings and coolings for the entire period were used to form a typical heating and cooling curve. This curve was found to be an exponential, as would follow from Newton's law of cooling. Thus, in the case of cooling according to Newton's law we have, as in Section IV:

$$\frac{dT}{dt} = -E\ (T - T') \tag{13}$$

Since resistance is measured directly, instead of temperature, it is more convenient to express equation (13) in terms of resistance R. Over small ranges of temperature, change in temperature is proportional to change in resistance, that is:

$$R = a + bT$$

$$\text{and} \frac{dR}{dt} = b\frac{dT}{dt}$$

Hence

$$\frac{dR}{dt} = -E\ (R - R') \tag{14}$$

$$R = R' + Ae^{-Et} \tag{15}$$

$$\frac{dR}{dt} = -AEe^{-Et} \tag{16}$$

$$\log_{10}\left(-\frac{\Delta R}{\Delta t}\right) = \log_{10} AE - 0.4343Et \tag{17}$$

If the observations, accordingly, satisfy Newton's law of cooling the plot of $\log_{10}\left(-\frac{\Delta R}{\Delta t}\right)$ versus t will be a straight line. The slope of this straight line determines E, the coefficient of cooling. As an illustration of this method the following examples may be cited:

TABLE 1.—Cooling and Heating Curve Data, Electrical Calibration

	Time	$(R_t - R_o)$ observed		Time	$(R_t - R_o)$ observed
	Seconds	Ohms		Seconds	Ohms
Heating	0	0.00000	Cooling	10	0.02612
	10	.00690		20	.02055
	20	.01241		30	.01521
	30	.01780		40	.01004
	40	.02296		50	.00495
	50	.02805		60	.00000
	60	.03292			

TABLE 2.—Analysis of Heating and Cooling Curves, Electrical Calibration

Mean time	$-(\Delta R/\Delta t)_{10\ sec.}$			$\log\left(\frac{-\Delta R}{\Delta t}\right)$		$-\Delta R/\Delta t$, computed
	Heating	Cooling	Mean	Observed	Curve	
Seconds						
5	0.000690	0.000680	0.000685	6.836-10	6.748-10	0.000560
15	531	557	544	.736	.737	546
25	539	534	536	.729	.726	532
35	516	517	516	.713	.714	518
45	509	509	509	.707	.703	505
55	487	495	491	.691	.691	491

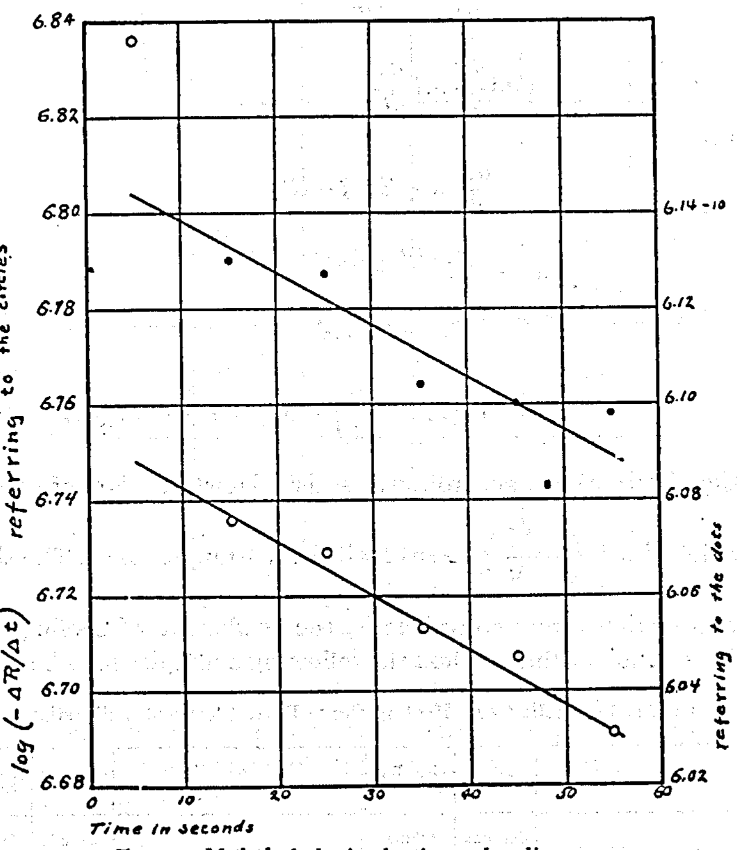

FIG. 4.—*Method of plotting heating and cooling curves*

The plot of $\log\left(\frac{-\Delta R}{\Delta t}\right)$ versus t is given by circles in Fig. 4. Columns 6 and 7 of the above table are obtained from the values given by this plot. On account of the lag (in this case a lead) the observed value of $\Delta R/\Delta t$ in the interval 0 to 10 seconds is far too great. After the first 10 seconds, however, the observations lie very satisfactorily on the straight line of Fig. 4. The coefficient E determined by the slope of the straight line is $E=0.0027$. From columns 2 and 3 it is seen that the heating curve is similar in type to the cooling curve as demanded by the theory developed in Section IV. This series was picked at random from a large number. Some of the series obtained showed exact agreement of the heating and cooling curves; others showed somewhat poorer agreement than the example cited. Since every point on the cooling curve except the point taken for time 0 to 10, mean $t=5$, lies well upon the straight line of Fig. 4, it is clear that the error due to lag is negligible (see below) after 10 seconds. Hence, in the general use of the instrument it is sufficient to take readings 10 seconds and 60 seconds after the starting of a heating and of a cooling, thus giving a heating or cooling over a 50-second interval. To obtain the correction factor, f, to convert a 50-second heating or cooling into the equivalent δ 60-second interval, use is made of equation (12)

$$f=\frac{1-e^{-60E}}{e^{-10E}-e^{-60E}}=1.217$$

$$\text{for } E=0.00269$$

TABLE 3.—Cooling and Heating Curve Data, Radiometric Calibration

	Time	(R_t-R_o) observed		Time	(R^t-R^o) observed
	Seconds	Ohms		Seconds	Ohms
Heating	0	0.00000	Cooling	10	0.00646
	10	112		20	511
	20	247		30	377
	30	381		40	250
	40	508		50	124
	50	634		60	000
	60	759			

TABLE 4.—Analysis of Heating and Cooling Curve, Radiometric Calibration

t	$-\Delta R/\Delta t$			log $(-\Delta R/\Delta t)$		$-\Delta R/\Delta t$, computed
	Heating	Cooling	Mean	Observed	Curve	
Seconds						
5	0.000112	0.000113	0.000112	6.049	6.144	0.000139
15	135	135	135	.130	.133	136
25	134	134	134	.127	.122	132
35	127	127	127	.104	.111	129
45	126	126	126	.100	.100	126
55	125	124	124	.093	.089	123

The plot of $\log\left(\frac{-\Delta R}{\Delta t}\right)$ versus t is given by dots in Fig. 4. Columns 6 and 7 are obtained from the values given by this plot. On account of the lag effect the observed value of $\Delta R/\Delta t$ in the interval 0 to 10 seconds is too small (in the case of electrical heating it is too great). After the first 10 seconds, however, the observations lie satisfactorily on the straight line of Fig. 4. The coefficient E was found to be 0.0025. Columns 2 and 3 show that the heating and cooling curves are similar. The correction factor to convert a 50-second heating or cooling interval into the equivalent for a 60-second interval is $f = 1.215$.

Several series of experiments were made to determine the form of the heating and cooling curves as illustrated above, for various amounts of energy expended both electrically and radiometrically in the receiver. These are summarized in the following table. The first column gives the total energy supplied to the receiver in a one-minute interval.

TABLE 5.—Summary of Preliminary Calibrations

ELECTRICAL CALIBRATION

Calories per minute	Reduced ΔR_{60}	$R_{60}-R_{10} = \Delta R_{50}$	f	$E \cdot 10^5$	Observed $\Delta R'_{60}$	$\Delta R_{60}/\Delta R'_{60}$
0.830	0.01756	0.01443	1.217	265	0.01956	0.898
.825	.01765	.01450	1.217	284	.01961	.900
1.475	.03152	.02592	1.216	269	.03292	.958
.365	.00782	.00642	1.218	288	.00827	.946
Mean.......	....	....	....	....	....	.938

RADIOMETRIC CALIBRATION

Calories per minute	Reduced ΔR_{60}	$R_{60}-R_{10} = \Delta R_{50}$	f	$E \cdot 10^5$	Observed $\Delta R'_{60}$	$\Delta R_{60}/\Delta R'_{60}$
0.135	0.00323	0.00265	1.219	272	0.00307	1.052
.197	.00470	.00389	1.208	154	.00452	1.040
.154	.00375	.00308	1.218	269	.00360	1.042
.253	.00622	.00510	1.220	299	.00598	1.040
.320	.00785	.00646	1.215	249	.00758	1.036
Final means..	....	....	1.216_4	261	Mean.....	1.04

The above values were taken with the receiver in various conditions as to the blackening upon its surface. Alterations in the surface affect the values of f and E to some extent. In the later work $f = 1.217$ was employed, since this value has been used by the Weather Bureau for some time and is practically identical with the above determination. However, the exact value of f is of no material significance, since it simply cancels out in the final computations. The important point is whether f is the same for an electrical calibration as for a radiometric calibration. That the values are identical in the two cases is amply shown by the above table. On account of the lag, with readings taken over a 60-second interval, instead of a 50-second interval, the true change in resistance, ΔR_{60}, is less than the observed change, $\Delta R'_{60}$ for the electrical calibration, in the ratio $\Delta R_{60}/R'_{60}=0.93$. For the radiometric calibration the true change ΔR_{60} is greater than the observed change, $\Delta R'_{60}$, in the ratio 1.04. Hence, for one-minute exposures with observations taken over 60-second intervals the electrical and radiometric calibrations will differ by about 12 per cent. This again illustrates the necessity of correcting for the lag.

As shown above, the observed cooling curve follows Newton's law of cooling after the first 10 seconds. This is what would be expected if the lag is small. The change in temperature of the receiver in the 50-second period is equal to that of the thermometer, although the thermometer is actually at a higher temperature, during cooling, than the disk. The observed lag is the result of two causes—the insulation around the thermometer coil and the lag of the galvanometer. It will be shown later that in the case of the radiometric calibration, the measured temperature of the coil lags from one to two seconds behind the effective temperature of the disk. On the assumption that the temperature of the disk is uniform throughout, the effect of the lag can be estimated in the manner shown by Harper.[8]

u = temperature of disk at time t

θ = temperature of thermometer at time t

λ = lag in seconds.

$$\frac{\delta\theta}{\delta t}=\frac{1}{\lambda}\left(u-\theta\right) \qquad (18)$$

$u=A+Be^{-Et}$ Newton's law for the period of cooling. . (19)

Suppose that at time $t=0, u=u_0=\theta=\theta_0$

Substituting (19) in (18) and integrating.

$$\theta-u=\left(\theta_0+\frac{E\lambda A-u_0}{1-E\lambda}\right)e^{-t/\lambda}+\frac{E\lambda}{1-E\lambda}(u-A) \qquad (20)$$

For an actual case, when the pyrheliometer was sighted on the sun the following values of θ_0, u_0, and A were obtained by converting the resistances observed into equivalent temperatures.

$\lambda=2$

$E=0.0026$

$u=10.0139+23.8763e^{-.0026t}$

$A=10.0139$

$u_0=\theta_0=33.8902$

[8] This Bulletin, 8, p. 666; 1912 (Scientific Paper No. 185).

Substituting these values in (20) one finds:

$$\theta - u = -0.125e^{-.5t} + 0.00523\ (u - 10.0139) \quad \ldots\ldots\ldots\ldots (21)$$

and evaluating equation (21) for various times, the following table is obtained:

TABLE 6.—Computation of Thermometric Lag

t	u	$0.125e^{-.5t}$	$0.00523\ (u-A)$	$\theta-u$
Seconds	°C	°C	°C	°C
0	33.8902	0.125	0.1249	0.000
5	33.5822	.01026	.1233	.113
10	33.2766	.00084	.1217	.121
20	32.6757	.000006	.1185	.118
30	32.0955	....	.1155	.116
60	30.4425	....	.1068	.107

Thus, as shown by column 5 of the above table, the temperature of the thermometer coil after the first 10 seconds of cooling is higher than that of the disk by practically a constant amount,

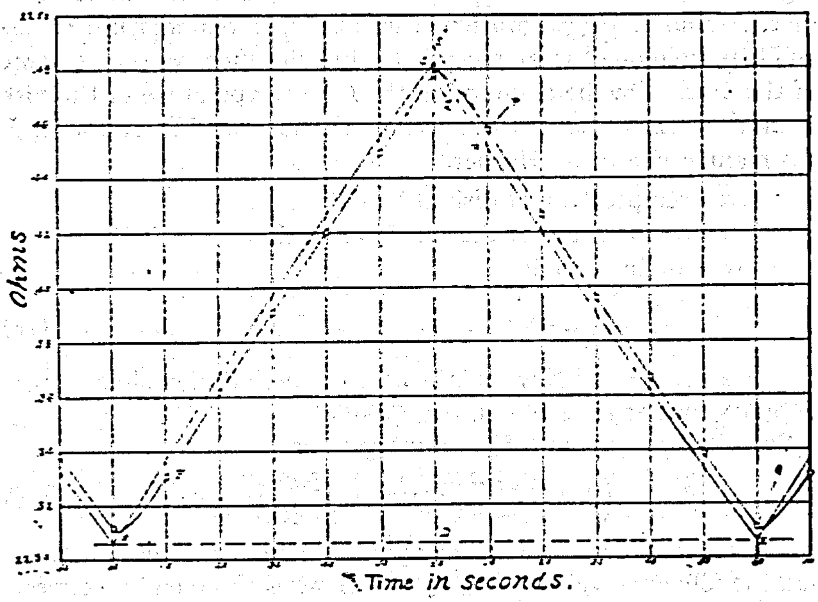

FIG. 5.—*Typical radiometric heating and cooling curve*

0.11 to 0.12°, for the series of observations made by sighting on the sun. The change in the temperature of the thermometer during cooling is therefore identical with the change in the temperature of the disk over the 50-second interval. If similar computations are made for a heating curve, it will be found that the temperature of the thermometer is less than that of the disk by essentially a constant amount over the 50-second interval and that the change in temperature of the thermometer is again identical with the change in temperature of the disk.

A typical heating and cooling curve for a radiometric calibration is shown in Fig. 5. The small circles represent observed points. The line *ACE* represents the corrected heating and cooling curve. The total change in temperature of the disk in terms of the coil resistance is given by the distance *CD*. This is equal to the observed change in 50 seconds, *PQ* or *P′Q′*, multiplied by the

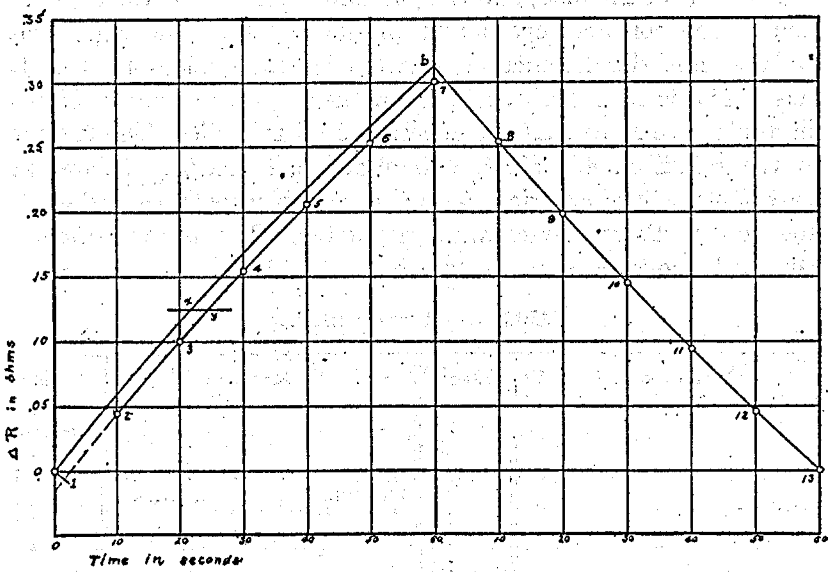

FIG. 6.—*Method of determining lag*

factor 1.217. It is also equal to the computed change *RE* in 50 seconds multiplied by the same factor. Another way to obtain the true change in 60 seconds is to extrapolate the observed curve *PQ* back to zero time (or to the time of the beginning of a cooling) and take the difference in the ordinates of the points *S* and *Q*.

The graphical method of obtaining the lag for a radiometric calibration is shown in Fig. 6.

The observed points are represented by circles. Through points 2, 3, 4, 5, 6, 7, and 8, 9, 10, 11, 12, 13 pass the best computed exponential curves. Extrapolate the cooling curve to the point *b* and move the heating curve up to the position 1 *b*. The distance *xy* measured along the time axis is twice the lag of the thermometer.

In the case of electrical heating, the lag effect is somewhat more complicated. During the heating the thermometer coil is at a higher temperature than the disk. The thermometer "leads" the disk on this account. But the galvanometer on the bridge still lags. The net result may be effectively either a lag or a lead. It might be possible, also, although this was not observed, that the readings of the bridge represent the true temperature of the disk, the galvanometer lag exactly balancing the thermometer lead. In cooling, the temperature of the thermometer rapidly falls behind that of the disk, and the galvanometer and thermal insulation of the coil act together to produce a lag. The lag λ was taken as half the distance *xy*, Fig. 6, even for the electrical heatings. This is not exactly correct, but the main object of the following table is to show that the effective lag was small. The lag due to the galvanometer alone was about one second. Hence, at least for the radiometric heating, it would appear that the lag due to the thermometer were negligible. The data, however, can not be considered quantitative.

TABLE 7.—Thermometric Lag

Method	λ (lag)	Method	λ (lag)
	Seconds [a]		Seconds [a]
Radiometric	1.5	Electrical	3.4
	1.2		3.2
	1.7		3.0
	1.3		1.4
	1.0		1.7
	1.2		
	1.1		
	0.9		

[a] Mean of all=1.7 seconds.

That the lag can not be large is also clearly demonstrated every time a pyrheliometer is used. If the lag were large, the resistance would continue to increase, after an exposure, for some time after the shutter was closed. Actually the galvanometer reverses direction showing decreasing resistance almost at the instant the shutter is closed. In the derivation above showing that the lag effect is inappreciable after a few seconds, it was assumed that both the coil and the disk were at the same temperature at the instant of shifting the shutter; that is, when $t=0$. Actually, since the previous heating or cooling has an effect, the two are at a slightly different temperature at $t=0$, but at some later instant; as is seen from Fig. 5, the cooling curve of the disk crosses the cooling curve of the thermometer, and thus the two show the same temperature at that point. Since the time that the disk and thermometer show the same temperature is so nearly $t=o$, the computations in the above discussion would be not materially altered. Accordingly it has been proven that the actual lag is small and that its effect is negligible after 10 seconds. Cooling and heating curves were made for exposures of 120 seconds; but in the actual use of the instrument it does not appear that anything is gained by doubling the time of exposure.

Blackening of the Receiver.—The choice of a proper method of blackening the receiver is of considerable importance. An improperly blackened receiver may show a reflection coefficient as high as 10 per cent. One pyrheliometer examined had a poorly blackened receiver. The result was that the radiometric calibration $\left(\text{observed value of } F=\frac{\Delta Q}{\Delta T}\right)$ was far too high and the electrical calibration too low. The most satisfactory black surface was prepared in the manner specified by Coblentz.[9] The receiver is first given a thin coat of a mixture of fine lampblack and platinum black in alcohol with sufficient turpentine to make it adhesive. The disk is then smoked with soot from a sperm candle. The smoke is produced by holding a small iron funnel over the candle. When properly prepared, such a surface absorbs about 98.8 per cent of the radiation falling upon it. The loss by reflection, 1.2 per cent,[10] can be corrected for in working up the calibration data.

VIII. FINAL CALIBRATION OF MARVIN PYRHELIOMETER—RECEIVER NO. 3, CAREFULLY BLACKENED

The following table summarizes the results of various experiments with this instrument. Each experiment represents a series of observations which in most cases extended over an hour. The first column gives the total energy supplied to the disk during each minute of heating. It is clearly shown that the calibration constant F', determined electrically, is independent of the amount of energy supplied although this latter extended over a considerable range.

[9] Coblentz, this Bulletin, 11, p. 140; 1915 (Scientific Paper No. 229).
[10] Coblentz, this Bulletin, 9, p. 322; 1913 (Scientific Paper No. 191).

TABLE 8.—Final Calibration of Receiver No. 3

ELECTRICAL CALIBRATION

Cal./minute	F′	Cal./minute	F′
0.2704	a 2.104	0.2755	b 2.141
0.2713	a 2.176	0.4925	b 2.116
0.4871	a 2.089	0.4945	b 2.099
0.7659	a 2.098	0.7772	b 2.089
0,7697	a 2.100		
3.136	a 2.100		2.111
7.211	a 2.098		
13.181	a 2.115		
21.321	a 2.101		
	a 2.109		
	2.110		

a Data taken in November, 1915. b Supposedly more accurate data taken March, 1916.

RADIOMETRIC CALIBRATION

Cal./minute	F′	Temperature of furnace	Distance between limiting diaphragms
		Degrees abs.	cm
0.1983	2.200	1601	94.0
0.3351	2.173	1602	72.4
0.4631	2.186	1731	71.9
0.3138	2.214	1649	79.2
	2.200		

The above calibrations were made under widely different conditions of room temperature, viz, warm, cold, rising, falling, and steady temperature. Rising and falling room temperatures were corrected for by the methods described above.

The constant F' showed no systematic changes with changes in the room temperature or rate of change of room temperature. The final uncorrected values of F', the means of the above data, are electrical, $F'=2.110$; radiometric, $F'=2.200$. Corrections must be applied to these means as follows:

1. A correction of 1.2 per cent must be made for the reflection of radiation from the surface of the disk. The measurements of Coblentz establish this value fairly accurately and indicate that the reflection is about the same for solar radiation and for radiation from a furnace at 1600° abs. Since this loss by reflection enters alike in the general use and radiometric calibration of the pyrheliometer, it may be omitted. In order to reduce an electrical calibration, however, to the same basis as the radiometric calibration, the constant F' determined electrically must be increased by 1.2 per cent.

2. The black body departs somewhat from the ideal condition of blackness. It has generally been assumed that a white porcelain radiator with diaphragms increasing in opening from the outside to the inner inclosure emits a lesser quantity of radiation than that corresponding to the temperature measured by the thermocouples. A black-body furnace having diaphragms decreasing in size of opening from the outside to the inner inclosure furnishes better black-body conditions, but this type of furnace would so reduce the cone of rays entering the pyrheliometer that the energy transfer would be too small to measure. Even with the system of diaphragms employed the usual energy received during solar measurements is some 30 times greater than the maximum energy which could be obtained in the laboratory. Since the present work was completed a systematic investigation of the furnace employed indicates that the usual assumption that such a furnace is 99 per cent black is incorrect. An optical pyrometer of the Holborn-Kurlbaum type was carefully calibrated by sighting into a black body immersed in crucibles of molten metal.[11] Melting and freezing point curves were obtained in the usual manner. The calibration of the pyrometer was thus expressed directly in terms of the melting or freezing points of metals which have been carefully investigated by means of the gas thermometer and resistance thermometer used as a transfer instrument. The black body employed was made of graphite, and when immersed in the metal bath could not depart from the ideal black-body conditions. A series of measurements was made by Mr. Fairchild and the writer by comparing the readings of the optical pyrometer when sighted into the Lummer-Kurlbaum furnace with the readings of the thermocouples corrected as usual to conform with the optical scale. Several different thermocouples standardized in terms of the melting points of zinc, antimony, and copper, and several different pyrometer lamps were employed. Fig. 7 represents the differences in temperature obtained by the two methods, plotted against the readings of the thermocouple. The variation in the readings can not be attributed to errors in the standardization of either the couples or the optical pyrometer. The errors in temperature measurement are considerably less than the differences indicated in this plot. These differences simply represent the deviation of the black body from the ideal condition of blackness, and are determined by various factors, such as adjustment of current in the two heating coils, rate of change or

[11] Kanolt, Bureau of Standards Technologic Paper No. 10.

temperature, etc. Referred to the thermocouple readings a furnace of this type radiates more energy than a black body at the same temperature. The side walls are necessarily hotter than the interior or back wall, the temperature of which is measured by the couples. Since the back wall is of white porcelain, probably a good diffuse reflector even at high temperatures, radiation from the hotter side walls is reflected from the back wall through the openings in the diaphragm to the optical pyrometer. In the temperature range used for work with the pyrheliometers (1350 to 1400° C), Fig. 7 would indicate that the thermocouples read too low by about 7°. It is quite possible that the radiator under these conditions is selective, and that while the difference between the thermocouple and optical pyrometer readings is 7°, the difference between the thermocouple and a pyrometer using the

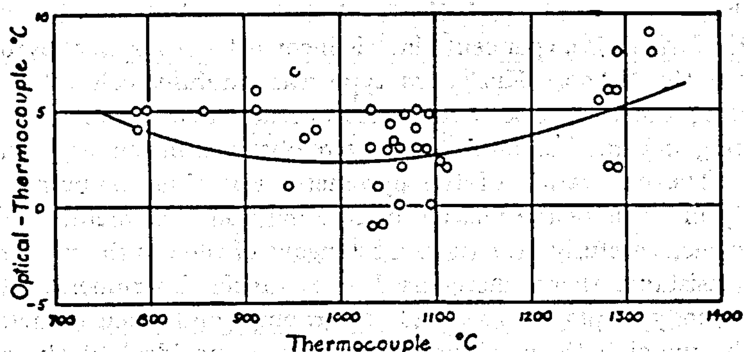

FIG. 7.—*Departure of furnace from black body conditions*

total radiation would not be this value. However, if we assume that the radiator is nonselective, the error in the total energy emitted, due to the error arising from reflected radiation, follows from the Stefan-Boltzmann law, thus:

$$\frac{\delta J}{J} = 4\frac{\delta T}{T} \doteq 1.7 \text{ per cent.}$$

This correction is additive to the experimentally observed value of F' as determined by the radiometric calibration. It may be noted that the constant σ has been determined with the Lummer-Kurlbaum furnace, but that the radiator was blackened. This blackening would of course do away with the question of stray-radiation

3. A correction which is subtractive from the value of F' determined radiometrically is due to the absorption of the radiation by the water vapor in the air. For the distances used, Coblentz's [13]

[13] This Bulletin, 12, p. 529; 1916 (Scientific Paper No. 261).

experiments would indicate that this correction may amount to from 1 to 3 per cent depending upon the humidity. Dr. Abbot[13] states that "Mr. Fowle's[14] recent work enables us to estimate very surely that in the conditions at the Bureau in November and March in 80 cm of air the absorption for rays from a 1600° source would be between 1 and 2 per cent."

4. A further subtractive correction which is small is due to the use of water-cooled diaphragms at a lower temperature than that of the room. This correction is discussed below.

Neglecting the fourth correction the total correction to be applied to the radiometric calibration is as follows:

Observed value of F'		2.200
Correction for nonblackness of furnace	per cent..	+1.7
Correction for humidity	do....	−1 to −2
Corrected value of F', about		2.200

The correction to the electrical calibration is as follows:

Observed value of F'		2.110
Correction for nonblack receiver	per cent..	+1.2
Corrected value of F'		2.135

The agreement between these two final values is probably within the experimental errors of the radiometric calibration. It would appear that the electrical calibration should have the greater weight, because accidental errors are much smaller in an electrical calibration than in a radiometric calibration. The following considerations, however, based on more recent experimental evidence tend to support the higher value of F' determined radiometrically.

The convection and radiation losses from the receiver follow Newton's law of cooling which states that the energy lost in this manner is proportional to the difference in temperature of the surface of the receiver and the surroundings. Hence the total energy loss by convection and radiation is proportional to the mean surface temperature of the disk. Experiments were made with a disk 0.4 cm thick, having two coils inside instead of a single coil, to determine whether a measurable temperature gradient can exist in the silver disk. The disk employed for these tests should show a gradient much larger than that which would exist in the silver disk No. 3 under similar conditions because of the fact that it contained two insulated coils and was thicker in the ratio 4:3. Measurements made by a differential thermocouple with one junction soldered to the front face of

[13] By letter. [14] Fowle, Smithsonian Misc. Coll., 68, No. 8; 1917.

the disk and the other junction soldered to the back face showed a temperature difference between the two faces amounting to 0.3° C during a 60-second exposure to solar radiation of about 0.8 cal./min cm². The gradient dropped to zero within 5 seconds after closing the shutter, remaining zero until the end of the minute if cooling. Within 5 seconds after again exposing to the solar radiation the gradient returned to the value 0.3° C. It is impossible at the present tine to attempt a careful study of the temperature differences established in the receiver employed in the work described in the present paper, and even if known their interpretation would be questionable, but the above measurements indicate that surprisingly large gradients may exist in spite of the fact that the disks are constructed of silver and that every precaution is taken to eliminate all air spaces in the interior. The fact that a temperature gradient of even 0.1° may exist, especially since one is not working with equilibrium conditions in an exposure of 60 seconds, shows that the gradient is exponential rather than linear through the disk. Hence, during an exposure to solar radiation, the thermometer is not measuring the mean surface temperature of the disk, but a temperature somewhat less than the mean value. During cooling the gradient through the disk is at least greatly diminished if not negligible. That this condition is not impossible is readily seen. During heating all the energy entering the disk flows through the front face and that not taken up by successive sections of the disk in raising the temperature of the various sections is conducted through the disk and radiated at the back surface. During cooling, however, the energy flows outward in all directions and there is a smaller flow in any one direction. Practically no difference in temperature exists between the front and back face during cooling and it seems probable that the gradient from the thermometer to the outside is small. However small this gradient be, the thermometer must indicate a temperature higher than that of the surface. Now, if exactly the same type of gradient existed in the disk during heating and cooling and if the lag of the thermometer were less than 10 seconds, as experimentally shown, the change during a 50-second period as described above, in the temperature of the thermometer, would be equal to the change in mean temperature of the surface. But when the gradient is shifting during heating from the exponential type to the linear type the change in temperature measured by the thermometer is less than the change in the mean temperature of the surface of the disk. The disk acts exactly as though its front surface reflected a portion of the incident energy (besides the normal reflection coefficient). Actually, of course, this energy is lost by convection and radiation.

Since the measured ΔT is too small the calibration constant $F' \propto \frac{1}{\Delta T}$ should be higher when determined radiometrically than when determined electrically, as found experimentally. It is impossible to predict what correction should be applied to the electrical calibration to take account of this effect of change in type of temperature gradient, so that in the further work the constant determined radiometrically has been chosen. Another cause for the existence of a nonsymmetrical temperature gradient is the thick coat of lampblack, a poor thermal conductor, upon the front surface of the receiver. It is possible that the changing gradient effect depends to some extent upon the rate of energy supply, so that while the constant is 2.200 for an energy flow of about 0.03 cal./cm² min. it might be greater for solar measurements involving 50 times this flow of energy.

It is of interest to note that during a radiometric or electrical calibration in the laboratory, when the shutter is open the receiver radiates through a small solid angle to the walls of the room or to a water-cooled diaphragm at room temperature, while during exposures to the solar radiation the receiver radiates into space. The difference in these two conditions can not account for an error in F' greater than 1 part in 2000.

A further question which may be raised is in regard to the loss of heat from the receiver by conduction along the lead wires. The lead wires were of No. 28 copper, and a length of about 15 cm of each wire was inside the metal shell incasing the receiver, the object being to reduce the temperature gradient along each wire from the receiver to the outside room temperature. The heating of the lead wires developed by the measuring current was extremely small, and was negligible for all the heating currents used in the electrical calibration. Since, however, the type of gradient in the lead wires may be slightly different in the radiometric and electrical calibrations, and in the case of the electrical calibration may be different for heating and cooling, there is a possibility of some small error on this account. However, several preliminary experiments with lead wires of different sizes gave negative results, and a rough computation of the magnitude of the effect to be expected indicated that it was very small relative to the total energy expended in the receiver.

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IX. SOLAR OBSERVATIONS WITH MARVIN PYRHELIOMETER RECEIVER NO. 3 AND SMITHSONIAN STANDARDIZED SILVER-DISK PYRHELIOMETER NO. 1

Using the above value of $F' = 2.200$ for the Marvin pyrheliometer solar measurements were made and comparison observations were taken at the same place and time with the Smithsonian standard as represented by the silver-disk pyrheliometer "S–I. No. 1." The following table summarizes the results. Each value represents a half-hour series of readings, and the solar intensity was remarkably constant during every run here recorded. The observations were taken at the Solar Radiation Laboratory of the Weather Bureau by Prof. Kimball, Mr. Hand, and the writer.

TABLE 9.—Data on Solar Observations

Date	Marvin pyrheliometer	Smithsonian pyrheliometer	Marvin/Smithsonian
	Cal./cm² min.	Cal./cm² min.	
Nov. 10, 1915	1.162	1.189	0.978
Do	1.352	1.388	.974
Do	1.262	1.302	.971
Nov. 26, 1915	1.160	1.169	.991
Nov. 27, 1915	1.230	1.253	.980
Mean	....	....	.98

The Marvin pyrheliometer independently calibrated thus agrees with the Smithsonian instrument within 2 per cent. However, it is believed that the error in the calibration of the Marvin pyrheliometer may amount to 5 per cent, so that the two instruments are probably in agreement within the errors of experimental observation. A pyrheliometer is designed for the measurement of energy flow amounting to 1.5 cal./cm² min. To adapt such an instrument to the accurate measurement of quantities 1/50 of this magnitude is extremely difficult. Yet in a radiometric calibration made in the laboratory the constant of the instrument has to be determined for this small energy supply. A more satisfactory method of procedure would be to radiometrically calibrate an instrument which was designed for small energy flow and to compare this instrument with a pyrheliometer by solar measurements, using some system of diaphragms, sectored disk, etc., for decreasing the amount of radiation entering the more sensitive instrument.

X. SUGGESTIONS FOR FURTHER WORK

With the present experimental arrangement it is not feasible to extend the work further. Certain improvements other than the one suggested above must be made in the apparatus if higher accuracy is to be obtained.

(a) The entire system, furnace, diaphragms, and pyrheliometer, must be mounted in a chamber in which the humidity can be controlled and reduced to a minimum when so desired. In designing this chamber attention must be given to the elimination of any possible means for radiation to be reflected, by the side walls, from the furnace into the pyrheliometer.

(b) The shutter on the pyrheliometer should be replaced by a shutter at the furnace.[15] This new shutter should be water-

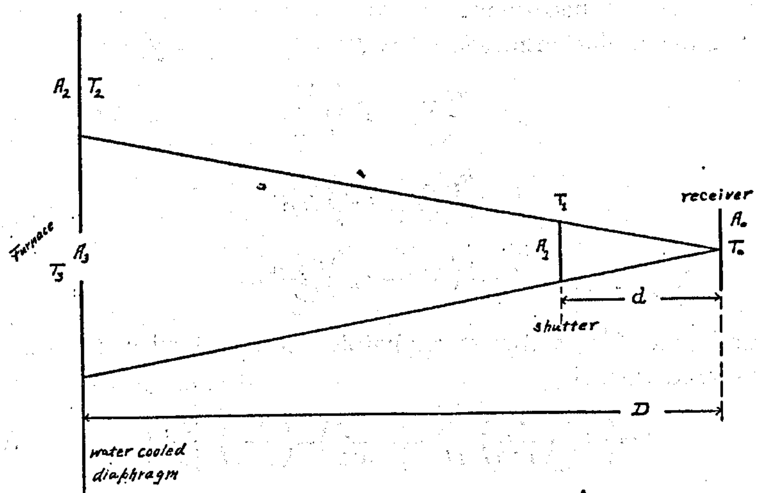

FIG. 8.—*Graphical representation of experimental conditions*

cooled, electromagnetically operated, and must be located between the furnace and the water-cooled diaphragm in front of the furnace. In this way all external conditions affecting the pyrheliometer remain the same during a heating as during a cooling. The only difference, then, radiometrically between a heating and a cooling is the exchange of the opening of a definite size, given by the water-cooled diaphragm, into the furnace, for a black water-cooled diaphragm of the same size at a measured temperature. Fig. 8 illustrates the conditions which were effective in the present calibration but which could be remedied by the use of a properly located shutter.

[15] The importance of this has been noted by Coblentz. This Bulletin, 12, p. 513; 1916 (Scientific Paper, No. 261.)

The temperature of the pyrheliometer receiver of area A_0 is T_0. The temperature of the walls and shutter of the pyrheliometer is T_1 and the area of the closed shutter, exposed to the receiver is A_1. When the shutter is open the pyrheliometer receives radiation from the large area A_2 of the water-cooled diaphragm at a temperature T_2 and from the small area A_3 opening into the furnace at a temperature T_3.

Let J_1 = radiation received by the disk from the closed shutter.

J_2 = radiation received by the disk from the large water cooled diaphragm, when the shutter is open.

J_3 = radiation received by the disk, shutter open, from the furnace.

The net radiation measured is $J = J_2 + J_3 - J_1$

$$J_1=\frac{\sigma}{\pi}\left(T_1^4-T_0^4\right)\frac{A_1A_0}{d^2}$$

$$J_2=\frac{\sigma}{\pi}\left(T_2^4-T_0^4\right)\frac{A_2A_0}{D^2}$$

$$J_3=\frac{\sigma}{\pi}\left(T_3^4-T_0^4\right)\frac{A_3A_0}{D^2}$$

hence, since T_0^4 is always negligible compared with T_3^4 and A_3 compared with A_2,

$$J=\frac{\sigma}{\pi}A_0\left[\left(\frac{T_3}{100}\right)^4\frac{A_3}{D^2}+\left\{\left(\frac{T_2^4}{100}\right)-\left(\frac{T_1}{100}\right)^4\right\}\frac{A_1}{d^2}\right]10^8 \tag{22}$$

In the calibration of the pyrheliometer the water-cooled diaphragm was always at a slightly lower temperature than that of the shutter.

If we take a very extreme case as an example, suppose:

T_3 = 1600° abs.
T_2 = 290
T_1 = 300
T_0 = 300
D = 70 cm
A_3 = 0.93 cm²
A_1 = 16 cm²
d = 30 cm

Then equation (22) above becomes:

$$J=\frac{\sigma}{\pi}A_o\,10^8\,(12.44-0.18) \tag{23}$$

Thus, for this extreme example where the temperature of the diaphragm is 10° lower than that of the pyrheliometer shutter and second term of equations (22) and (23) amounts to 1.4 per cent of the first term. The second term was always neglected in the calibration. In general, it would only amount to a few tenths of 1 per cent. It is always a subtractive correction to J and hence would reduce slightly the observed value of F'.

(*c*) As a further check upon the behavior of the dynamic type of pyrheliometer of which the Marvin is an example, it would be highly interesting to study at the same time a pyrheliometer of the static type. The Marvin dynamic pyrheliometer readily lends itself to this modification. The entire tube and case could be jacketed and accurately maintained at a constant temperature. An ice bath, while furnishing the desired constancy of temperature, could not be easily used on account of its being usually below the dew point, allowing water vapor to condense upon the receiver, when the instrument was used for solar comparisons. But it would not be difficult to secure sufficient constancy of temperature by use of flowing water thermostated to, say, 30° C. Such a pyrheliometer would not be very satisfactory for ordinary field work on account of the inconvenience of operation, but as a laboratory instrument its behavior, so different in principle from that of the dynamic type, would permit interesting comparisons of solar measurements by two distinct methods.

SUMMARY

For the first time, it is believed, a pyrheliometer has been calibrated by two methods, the usual electrical method and a radiometric method. In the radiometric method a known quantity of radiation from a black body was allowed to fall upon the pyrheliometer receiver in exactly the same manner as when employed for solar measurements. The calibrations by the two methods agreed within limits of experimental error, if the Stefan-Boltzmann constant were chosen, as $\sigma = 5.7 \times 10^{12}$ watts cm^{-2} deg^{-4}, the latest and most accurate determination of this constant of total radiation. Or conversely, the constant has been observed as $5.7 \times ^{-12}$ within an accuracy of possibly 5 per cent.

The behavior of the Marvin pyrheliometer has been carefully investigated. A lag, part of which is due to the galvanometer of the bridge, has been found to exist, and, for the silver disk No. 3, was experimentally shown to be less than 2 seconds. Both theoretically and experimentally it was shown that the effect of this lag is negligible after 5 to 10 seconds. The cooling and heating of the receiver follows Newton's law of cooling.

In order to completely eliminate errors due to a lag effect, readings should be made at 10 seconds and 60 seconds following the beginning of a heating or cooling. The factor for converting readings of temperature or resistance change over this 50-second interval to corresponding changes over a complete 60-second period is 1.217. This factor is the same for both electrical and radiometric heating and was determined with an accuracy of 0.1 per cent. There is no advantage in making the periods of heating and cooling 120 seconds in duration. Periods of 60 seconds are sufficient. The method of blackening the receiver is of great importance. The best method used for blackening is that used by Coblentz and is described above. The calibration constant F' of Section IV appears independent of the rate at which energy is supplied to the receiver, at least for an electrical calibration.

A Marvin pyrheliometer calibrated by two methods, electrical and radiometric, was compared by solar observations with the United States Weather Bureau Smithsonian standardized pyrheliometer "S. I. No. 1," calibrated by comparison with the Smithsonian primary standard water-flow pyrheliometer. The Marvin pyrheliometer thus calibrated gave 2 per cent lower values of the solar radiation than the Smithsonian pyrheliometer. This difference is within the errors of observation of the present calibration.

The above work was suggested by Dr. G. K. Burgess and Prof. H. H. Kimball. The writer is indebted to Dr. Waidner and Dr. Burgess, of the Bureau of Standards, and Prof. Marvin and Prof. Kimball, of the Weather Bureau, for many valuable suggestions. The writer is especially indebted to Prof. Kimball for cooperating in much of the earlier work, for performing the experiments necessary to obtain the temperature-resistance relations of the thermometer coils, and for the direction of the solar observations which were made in his laboratory. The pyrheliometers were furnished by the Weather Bureau through the courtesy of Prof. Marvin and Prof. Kimball.

WASHINGTON December 11, 1917.