UNITED STATES

COAST AND GEODETIC SURVEY

CARLILE P. PATTERSON

SUPERINTENDENT

METHODS AND RESULTS

Ferrel, W.

METEOROLOGICAL RESEARCHES

PART III

BAROMETRIC HYPSOMETRY

AND

REDUCTION OF THE BAROMETER TO SEA LEVEL

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METEOROLOGICAL RESEARCHES.

By WILLIAM FERREL.

Part III.—BAROMETRIC HYPSOMETRY AND REDUCTION OF THE BAROMETER TO SEA-LEVEL.

CHAPTER I.

THE THEORY OF BAROMETRIC HYPSOMETRY.

1. The barometric formula for the determination of altitudes given by Laplace, although a great improvement upon all that had preceded it, was not entirely perfect. The vapor correction, included in the temperature correction by means of a modification of the constant in the latter, is only an approximate one, which, in cases of an extremely dry or an extremely moist atmosphere, and in all cases when the temperature is near or below zero of the centigrade thermometer, is erroneous, and this imperfection may at times give rise to considerable error. In the time of Laplace, also, the relative densities of mercury and of atmospheric air were not sufficiently well known to determine the principal constant in the barometrical formula, and it was therefore necessary to resort to another method of determining it. This was done by Ramond by adopting the constant which gave the best agreements between the altitudes given by the formula and those obtained by trigonometry. But the number of observations and of comparisons was so small, and the temperature correction and its annual and diurnal variations were so imperfectly understood then, that the constant which he obtained, and which has been mostly used ever since, was erroneous, except for the summer season and a certain hour of the day, and is not correct, as it should be, for mean annual and diurnal temperatures. After the accurate experiments of Regnault, by which the relative densities of mercury and the atmosphere became better known, and numerous comparisons of the altitudes given by the formula with those obtained by means of the spirit level, it is now known that the constant obtained by Ramond for Laplace's formula is too small. The constant derived from the results of Regnault's experiments cannot be much in error; but still it would be well to have its accuracy corroborated by many more comparisons of altitudes given by it with those obtained directly by means of the spirit-level. But this can only be done where the difference of level of two stations near to each other and differing considerably in altitude has been accurately ascertained, and where the necessary meteorological observations have been made at the two stations.

The greatest uncertainty in barometric hypsometry arises from the imperfection of the temperature correction. In determining the difference of altitude between two stations from observations made at all seasons of the year and hours of the day, it is found that the results vary very much with the seasons of the year and hours of the day, the differences of altitude obtained being generally too great for the warmest season of the year and the warmest part of the day, and the reverse for the coldest season of the year and part of the day. This was first noted by Ramond, and has been confirmed by all subsequent investigators of the subject. It is now well known that these discrepancies arise from the erroneous assumption that the average temperature of the air column, upon which the difference of pressure between the two stations depends, is the average of the observed temperatures of these stations. Much has been done by Plantamour and Rühlman* in Europe, and by Williamson† and Whitney‡ here, in investigating and explaining these discrepancies; but much more is still required in this direction.

Since the time of Laplace the barometric formula has been put into a great variety of forms, without, however, any great improvement in the convenience of its practical applications, but the vapor correction has been introduced more accurately, and the principal constant has been improved. Since the labors of Plantamour and Bauernfiend, and especially of Rühlman, in this direction, there is little room left for any improvement of the mere shape of the formula or of any of the constants contained in it.

The consideration of barometric gradients, in connection with this subject, is somewhat recent, and they have never been taken into account in any regular treatise on barometric hypsometry. It is, however, pretty well understood now that allowance must be made in some way for the effect of these gradients where there is a considerable horizontal distance between the lower and upper stations. It is this part of the subject which especially needs yet further theoretical consideration before corrections for these gradients can be accurately introduced in practical applications of the formula.

2. Barometric hypsometry naturally follows as a regular part of the preceding researches, being based upon the same general principles and equations. From the development of the three small equations (1), Part I, of these researches, with the subsequent addition of a friction term, we have deduced the equations of the general motions of the atmosphere, and of cyclones, tornados, waterspouts, &c., and from this same development we get the equations showing the relation between the differences of altitudes and barometric pressures of any two stations, so that it is not necessary here to go back to first principles to obtain the fundamental equations of barometric hypsometry.

The method adopted at the outset in the preceding researches is completely exhaustive, leaving it entirely impossible for any effect to escape consideration, and hence our equations throughout contain numerous terms, including those showing the effects of the earth's rotation, which had never been taken into account in such investigations. Accordingly our fundamental equations in this branch of the subject, being deduced from the same development, contain likewise many terms, the effects of which have never been considered, and the consideration of which involves the whole subject of barometric gradients. Some of the small terms which, after close examination, were omitted in the theory of the general motions of the winds, and of cyclones, tornados, &c., as being too small to have any sensible effect under any circumstances, are retained in the equations of this part of the researches, not so much because they are regarded as being of importance, as to show hat even here their effects are so small that they might have been neglected.

From equations (12), Part I, by supplying the omitted terms $D_t^2 h$ and F_h in the first of these equations, we get

$$\text{(1)} \qquad \log P' - \log P = \int_h \alpha g + \delta$$

in which

$$\text{(2)} \qquad \delta = \int_h \alpha (D_t^2 h + F_h) + \int_u \alpha (D_t^2 u - \cos\theta \, (2n + D_t \varphi) \, D_t v + F_u) + \int_v \alpha (D_t^2 v + 2 \cos\theta \, (n + D_t \varphi) \, D_t u + F_v)$$

in which the integrations in the second member must be carried from $h = h'$ corresponding to the lower station to h belonging to the upper station.

* Die Barometrischen Höhenmessungen. Von Dr. Richard Rühlman. Leipzic, 1870.

† On the Use of the Barometer on Surveys and Reconnaissances; by Maj. R. S. Williamson, Corps of Engineers, and brevet lieutenant-colonel U. S. Army. Professional Papers of the Corps of Engineers, U. S. Army, No. 15. New York, 1868.

‡ Contributions to Barometric Hypsometry: with Tables for Use in California; by Prof. J. D. Whitney, State geologist.

This is the complete fundamental equation of barometric hypsometry as deduced from the developments in the first part of these researches. It differs from the usual equation in containing in addition the term δ, which, it is seen from its expression, contains the effects of the motions and inertia of the atmosphere together with the effects of the earth's rotation, while the usual equation applies only to a state of static equilibrium of the atmosphere.

From (10) and (20), Part I, accenting g in the latter to indicate its value at sea-level and on the parallel of 45°, and putting $a=273°$, we get

$$(3) \qquad \alpha=\frac{1}{g'l\left(1+\frac{1}{a}t\right)(1+f(e))}=\frac{1}{g'l(1+.00366t)(1+.378e)}$$

in which, putting

B = the barometric pressure of the atmosphere,
b = the tension of aqueous vapor contained in it,

we have

$$(4) \qquad e=\frac{b}{B}$$

If in (3) we suppose that t and e have the same values at all altitudes as at the earth's surface, then α becomes α', and equation (1) above becomes the same as (14), Part I, except the term $f(h)$ in the latter, which represents the effect of variations of t and e with increase of altitude.

The general expression of the force of gravity g in (13), Part I, expressed in terms of the latitude λ, and of the height h above sea-level, is

$$(5) \qquad g=g'\left(1-\frac{2h}{r}-0.002606\cos 2\lambda\right)$$

The numerical coefficient of $\cos 2\lambda$ introduced here differs considerably from that of (13), Part I, which was copied from Laplace and is now known to be too large. From the recent determination of the figure of the earth by Colonel Clarke,* from pendulum observations, if we put g_0 for the value of g at the equator and sea-level, we have

$$g=g_0(1+.005226\sin^2\lambda)=g_0(1+.002613-.002613\cos 2\lambda)$$

whence

$$(6) \qquad g=1.002613g_0\left(1-\frac{.002613}{1.002613}\cos 2\lambda\right)=g'(1-.002606\cos 2\lambda)$$

which is the same as (5) above at sea-level, where $h=0$. From (3) and (5) above we get

$$(7) \qquad \alpha g=\frac{1-\frac{2h}{r}-.002606\cos 2\lambda}{l(1+.00366t)(1+.378e)}$$

in which, it will be remembered, l is the height of a homogeneous dry atmosphere on the parallel of 45°, with a temperature $t=0$.

3. Since the rectangular co-ordinates h, u, and v in the last number of (1) are entirely independent of one another, the differential elements in the integration, say by mechanical quadratures, can be so taken as to extend the integration from the lower to the upper station by any line whatever, and since in a fluid the pressure is always the same in all directions, we must in every case arrive at the same result. But as friction, which enters into the expression of δ (2), is an uncertain element of which it is not possible to take account where there are surface currents, it is best to take that line which avoids it as much as possible. For instance, if it were required to determine the difference of altitude between the two stations A and B, Fig. (1), the integration of the second member of (1) might be carried from A to B along some line near the surface of the mountain, or it might be extended vertically from A to C, on a level with B, and then horizontally from C to B.

* Clarke's Geodesy, page 345.

In the former case it would be necessary to take into account the temperature, or value of t, and also of e, in the expression of α(3), and likewise the friction F, all of which enter into the integral of the second member of (1), but in the latter case it would be necessary to have the values of t and e in the vertical line AC only, since both these in the line BC could be regarded as being constant and having the same value as at C, except very near the station B, where this line comes to the surface of the earth. The value of the friction term F in the first term of the expression of δ (2), which is that in the direction of the vertical line AC, would be entirely insensible unless there were a local and very rapidly ascending or descending current, and on the line CB it would be very small, since there is little friction except near the earth's surface. On the part of δ arising from the integration from C to B would depend the barometric gradient at the level of the upper station, and δ would be equal 0, where there is no gradient.

The temperature along any line near the surface of the mountain would be better known if we had observations at a number of intervening stations than that on the vertical line AC, and even the mean of the two extremes would generally give an average which would be more correct for a line from A to B near the surface than for the vertical line AC. But as the effect of friction cannot be taken into account, and may be large on the line from A to B near the surface, it is best to obtain the temperature, as nearly as can be, on the line AC vertical over the lower station, which may be very different from the temperature on the same level at the surface of the mountain. During the warmest part of the day, especially in the summer season and in clear weather, the air near the surface of mountain sides becomes much more heated than it does at other places on the same level and at a considerable altitude above the surface, and then, it is well known, there are generally strong currents of air along the surface toward and up the side of the mountain, and the friction terms in the expression of δ become large. In this case the temperature is greater and consequently the pressure of the air column less, but this is exactly counteracted by the friction of the current up the side of the mountain, for relative lightness of the air near the surface gives rise to the ascending current which is accelerated until the friction exactly counteracts the force which produces the current, after which the current remains uniform as long as the temperature and force giving rise to the current remain the same. This excess of temperature, then, near the surface, above that of the air generally at the same level, should not be used unless we could also take into account the equal and counteracting effect of the friction arising from the currents produced by this excess of temperature.

Early in the morning, especially in the winter season and in clear weather, the reverse takes place and the currents are down and from the mountain side. In this case we also have the effect of the friction terms depending upon this current, but their signs are reversed and their effect exactly counteracts that of the increased pressure of the air column arising from diminished temperature. It is well known to hunters, and all who have encamped at night on mountain sides in clear weather, that the current is generally downward during the night, and hence it is usual to put the fires on the lower side of the camp so as to avoid the smoke. Of course, these ascending and descending currents of mountain sides are observable mostly in calm weather, when they are not interfered with, or completely reversed, by the other more general currents of the atmosphere.

4. Neglecting for the present the first term in the expression of δ (2), which in all practical applications will be shown to be insensible, if we suppose the integrations in the second member of (1) to be carried from the lower station A, Fig. (1), up vertically to the height of the upper station C, and then horizontally to B, we shall have

$$\log P' - \log P - \delta = \int \alpha g \qquad (8)$$

in which the integral must be taken as defined in § 2, and the values of t and e in the expression of α (3) must be those belonging to the vertical line over the lower station. The value of δ may be regarded as being independent of h, since we can neglect the first term in the expression (2), and will depend upon the motions, inertia, &c., of the air at the level of the upper station. It is seen that the value of δ becomes a small correction to log P, depending upon the gradient of pressure, and if we put

$\triangle$ = to the increase of pressure in the direction of the upper station from the lower, arising from the gradient depending upon δ, we shall have, since $\triangle$ and δ will have different signs,

$$\log P' - \log (P - \triangle) = \int_h \alpha g \tag{9}$$

If the true temperatures in the vertical were known in this case, this expression would still contain $\triangle$, a quantity which has different values at different altitudes, and has no accurate and known relation to its value at sea-level or the plane of the lower station, so that if even its value were known at sea-level or the lower station, there would still be considerable uncertainty with regard to its value above. The various conditions to be satisfied, both in the general motions of the atmosphere and in those of cyclones and tornadoes, often require the horizontal motions to vary considerably in different altitudes, and hence the value of δ, and the corresponding value of $\triangle$, must be different at different altitudes.

If we suppose the mountain upon which the upper station is placed to be removed and its place to be occupied by air, the barometric and temperature gradients which would exist in this case could be determined, at least approximately, by those of the air surrounding the mountains. This might be done in any special case by means of simultaneous barometric or temperature observations at several stations around the mountain, or even only two stations, if these were in the line connecting the two stations, and not so far apart that the gradients would change sensibly in the intervening space. For annual and monthly averages of barometric and temperature observations, very accurate charts of barometric and temperature gradients for each month and for the year might be prepared for any country where the necessary observations are at hand, and then from these charts the differences of pressure between the two stations at the level of the lower, and also the differences of temperature, could be obtained wherever observations were made for hypsometrical purposes.

If in the preceding expression we suppose the integrations to be carried from the lower station A, Fig. (1), along the horizontal line on the level of that station to the vertical of the upper station and then along that vertical to B, we shall have, by putting $\triangle'$ for the value of $\triangle$ at this level,

$$\log (P' + \triangle') - \log P = \int_h \alpha g \tag{10}$$

in which the integral in the last member must be taken within the same limits as in the preceding case, but the values of t and e in the expression of α (3) must be those in the vertical DB, Fig. (1), of the upper station, in case the mountain were removed. In this case the value of $\triangle'$ can be more readily obtained than $\triangle$ in the former, and the temperature at D can be obtained from the observed temperature at A, Fig. (1), by means of the temperature gradient. There would, however, be in this case the same uncertainty with regard to the temperature at the upper station since we would need the temperature which the air would have, were the mountain with its abnormally heated or cooled surface removed.

For small differences of altitude δ may be regarded as constant, and then we shall have, regarding δ as being small,

$$\triangle' = \triangle'' \frac{P'}{P''} = P'\delta \tag{11}$$

distinguishing the quantities of sea-level by two accents. With the value of $\triangle''$ at sea-level, obtained from charts or otherwise, the value of $\triangle'$ at the level of the lower station may be obtained very nearly.

5. Where the upper station is vertically over the lower one the last two terms in (2) vanish and the expression of δ is reduced to the first term, and we have in this case

$$\delta = \int_h \alpha (D_t^2 h + F_h)$$

Its value then depends upon the inertia of the air in the accelerated or retarded velocities of the ascending or descending currents, and upon the friction between these currents and the surrounding undisturbed atmosphere. If we neglect the friction and consider the former merely, we get by means of (3), in the case of dry air and temperature, t, regarded as being constant,

(12) $$\delta=\alpha\int_h \mathrm{D}t^2h=\frac{a}{g'l\,(a+t)}\int ds^2=\frac{a}{a+t}\,\frac{s^2-s'^2}{78332}$$

in which s is the velocity of vertical motion, s' being the value of s at the assumed origin of the integration. If this is the surface of the earth we necessarily have $s'=0$.

From this expression it is seen that δ is positive in the case of accelerated ascending currents, and hence the difference between P′ and P, as is seen from (1), is increased in such currents by the reaction of the inertia. It must not be supposed, however, that the pressure at the earth's surface under such an ascending current is greater than at surrounding places, for there cannot be an ascending current unless the air of the current is lighter from some cause than that of the surrounding parts, and the diminution of pressure from this cause is exactly equal to the reaction of the accelerated velocity which it gives rise to. In the case of friction the pressure must be a little less at the base of the ascending current, since a part of the force which overcomes the inertia of the accelerated ascending current is spent in overcoming the friction of the surface currents below, which are necessary to supply the draught of the ascending current.

Since the expression of δ in this case depends upon s^2, the result is the same for either ascending or descending currents, since in both cases we must have $s'=0$ at the earth's surface. The direct action, therefore, of the retarded descending current is exactly equal to the reaction of the equally accelerated velocity of the ascending current. But there cannot be a descending current unless from some cause the air of this current is heavier than that of the surrounding parts, and hence in this case the pressure on the earth's surface is increased from both causes.

The effect of accelerated or retarded ascending or descending currents upon the pressure at the earth's surface, or any assumed level at which $s=s'$, is given by (9), in which δ is given by (12), and the value of P must be that of P′, corresponding to the level where $s=s'$. This effect is in all ordinary cases exceedingly small. If we suppose the velocity at any height to be $s=10^{\mathrm{m}}$ per second, the value of δ in (12), supposing the lower station to be on the earth's surface where $s'=0$ and that the temperature $t=0$, would be $\frac{1}{783}$, and multiplying this into 760^{mm}, supposing this to be the barometric pressure at the earth's surface, we get by (11) $\triangle=0.96^{\mathrm{mm}}$, or less than one millimeter in this extreme case. With a value $s=5^{\mathrm{mm}}$ at the upper station the effect would be only 0.24^{mm}. In all cases, therefore, in which observations are made for hypsometrical purposes the effects of vertical currents are insensible.

Where a column of air is ascending or descending without any change of velocity, or where it is first accelerated and then retarded, or *vice versa*, so that we still have $s=s'$, it is seen that it does not affect the difference of pressure between the two stations, since in such cases we have by (12) $\delta=0$.

6. By means of (7) equation (10) becomes

(13) $$\log \mathrm{P}''-\log \mathrm{P}=\frac{k}{l}\int_h\frac{1}{1+x}$$

in which

(14) $$\begin{cases} k=1-.002606\cos\lambda \\ x=(.00366+.00138e)t+.378e+\dfrac{2h}{r} \\ \mathrm{P}''=\mathrm{P}'+\triangle' \end{cases}$$

In the integration of the last member of (13) it is necessary to have x expressed in such a function of h as to make it either completely or approximately integrable. Let us put

(15) $$\begin{cases} \mathrm{H}=h-h' \\ x=x'-(c\mathrm{H}+c'\mathrm{H}^2+\&c.) \end{cases}$$

in which h' and x' are the values of h and x at the lower station where t and e have the values t' and e'. This supposes the decrease of temperature and relative proportion of aqueous vapor to be expressed by a convergent series of the forms

(16) $$\begin{cases} t''-t=c_1\mathrm{H}+c'_1\mathrm{H}^2+\&c. \\ e'-e=c_2\mathrm{H}+c'_2\mathrm{H}^2+\&c. \end{cases}$$

in which t'' is the value of t' corrected for the effect of temperature gradient between the two stations.

By means of these equations the last of (14) becomes the same as the last of (15), in which we shall have

$$(17)\qquad \begin{cases} c=(.00366+.00138e)\,c_1+.378c_2+\dfrac{2}{r} \\ c'=(.00366+.00138e)\,c'_1+.378c'_2 \end{cases}$$

In the expressions of these constants the variable e in the very small term of the second order can be regarded as a constant and equal to the mean of its values at the lower and upper stations.

By means of the last of (15) we get from (13)

$$\log P''-\log P=\frac{k}{l}\int_h(1-x+x^2+\&c)$$
$$=\frac{k}{l}\int_H\Big(1-x'+x'^2+c(1-2x')\,H+(c^2+c')\,H^2+\&c\Big)$$
$$=\frac{k}{l}\Big((1-x'+x'^2)\,H+\tfrac{1}{2}c(1-2x')\,H^2+\tfrac{1}{3}(c^2+c')\,H^3+\&c\Big)$$

This gives, neglecting all terms below the third order,

$$H=\frac{l}{k}\log\frac{P''}{P}\,\frac{1}{1-x'+x'^2+\frac{1}{2}c(1-2x')\,H+\frac{1}{3}(c^2+c')\,H^2}$$

From the last of (15) we get, by reversing the series,

$$(18)\qquad H=\frac{x'-x}{c}-\frac{c'}{c^3}(x'-x)^2+\&c$$

With this value of H in the small terms above, of the second and third order, we get

$$H=\frac{l}{k}\log\frac{P''}{P}\,\frac{1}{1-\frac{1}{2}(x'+x)+\left(\frac{1}{3}-\frac{c'}{6c^2}\right)(x'-x)^2+x'x}$$
$$=\frac{l}{k}\log\frac{P''}{P}\left(1+\tfrac{1}{2}(x'+x)-\left(\frac{1}{3}-\frac{c'}{6c^2}\right)(x'-x)^2-x'x+\tfrac{1}{4}(x'+x)^2\right)$$
$$=\frac{l}{k}\log\frac{P''}{P}\left(1+\tfrac{1}{2}(x'+x)+\left(\frac{c'}{c^2}-\frac{1}{12}\right)(x'-x)^2\right)$$

neglecting all terms below the third order. This by means of (14), since when x becomes x', t, e, and h become t', e', and h', gives, by changing t' to t'', for reasons given in § 4,

$$(19)\qquad H=\frac{l}{M}\log\frac{P''}{P}\left\{1+[.00183+.00035(e'+e)]\,(t''+t)+.189(e'+e)+\frac{2h'+H}{r}+.002606\cos\lambda+\left(\frac{c'}{6c^2}-\frac{1}{12}\right)\frac{(t'-t)^2}{273^2}\right\}$$

in which common logarithms are to be used, and M is the modulus of these logarithms. In the last very small term of this expression, only the term depending upon the temperature in the last of (14) has been taken into account.

If we neglect the part of this term depending upon c', we have the expression of H in the case in which the temperature and value of e decrease in proportion to the increase of altitude, for when this is the case c'_1 and c'_2 in (16) vanish, and hence, from the last of (17), $c'=0$. If we put

$$(20)\qquad c'=\tfrac{1}{2}c^2$$

the last term in (19) vanishes, and we then have the usual barometric formula, which is obtained by regarding x, and consequently t and e in the last of (14) as constants, and equal to the means of their values at the lower and upper stations, in the integration of the last number of (13). By the preceding more general and rigorous method the last term of (19) is added to the usual formula. This term, however, is usually very small, and may be neglected unless either c' is very large, or the value of $t'-t$, which may occur in the latter if the difference of altitude of the two stations is great. Where the value of c' is so great in comparison with that of c that the expression of H in (18) is not sufficiently convergent to make the neglected terms below the third order sufficiently small to be neglected, the last term in (19) does not represent accurately the effect of the term in (18) depending upon c', since the neglected terms may be large in comparison with those retained of the third order.

Considering only the term in the expression of x in (14) depending upon t, since the others are generally very small in comparison, we get from (17) and (20)

$$c'_1 = \tfrac{1}{2} \times .00366\, c^2_1 \tag{21}$$

If, now, we put in the first of (16) $c_1 = .005^\circ$, which is its value if the temperature decreases 0.5° for each 100 meters of increase of altitude, we get $c'_1 = .0000000458$. With this value of c'_1 the last term of the first of (16) gives, if we put H = 2000 meters, $c_1 H^2 = 0^\circ.18$. Hence the usual formula requires a decrease of temperature very nearly proportional to the increase of altitude, but not strictly so.

7. Since gravity differs with a change of elevation, the barometer does not give the absolute pressure of the air in measures of an invariable unit. It is evident from an inspection of the formula (19) that this unit may be that of any assumed altitude, and hence it is only necessary to reduce the measure at the one station to that of the unit of measure belonging to gravity at the other. Let B′ and B be the observed heghts of the barometer at the lower and upper stations, respectively, corresponding to the absolute pressures P′ and P measured by a barometer acted upon by an unchanged force of gravity. We shall then have, since the measures given directly by the barometer vary inversely, as the force of gravity,

$$\frac{P''}{P} = \frac{B''}{B} \cdot \frac{g'}{g} = \frac{B''}{B}\left(1 + \frac{2H}{r}\right)$$

in which g' is the value of g at the lower station, and in which the last form of the second member of the equation is deduced from the preceding one by means of (5), neglecting insensible quantities. Hence, we have

$$\log \frac{P''}{P} = \log \frac{B''}{B} + \frac{2MH}{r}$$

M being the modulus of common logarithms. The last term of the second member being very small, we can substitute for H its approximate value deduced from (19), which is

$$H = \frac{l}{M} \log \frac{P''}{P}$$

With this value of H the preceding equation gives

$$\log \frac{P''}{P} = \log \frac{B''}{B}\left(1 + \frac{2l}{r}\right)$$

By means of this expression (19) becomes

$$H = \frac{l}{M}\left(1 + \frac{2l}{r}\right) \log \frac{B''}{B}\left\{1 + [.00183 + .00035(e' + e)](t'' + t) + .189(e' + e) + \frac{2h' + H}{r} + .002606 \cos \lambda + C\right\} \tag{22}$$

in which

$$C = \left(\frac{c'}{cc^2} - \frac{1}{12}\right) \frac{(t' - t)^2}{273^2} \tag{23}$$

is a small correction to the usual formula to make it strictly applicable where the values of t and e vary with increase of altitude according to the law of (16). Where these vary as the first power of H we have $c' = 0$ in the expression of C.

8. The effect supposed to be due to the attraction of the strata of the earth between the upper station and the level of the lower station, introduced into the formula by Rühlman, has been neglected here. This was introduced by him upon the hypothesis that these strata are so much additional attracting matter coming between the upper station and the earth's center, and that consequently the attraction at the upper station, as at B (Fig. 1), is greater than at C on the same level in open-space vertically over the lower station. From theoretical considerations alone

It is not probable that there has been much increase of matter between the upper station and the earth's center from the formation of continents, table lands, and mountain ranges, and hence there is little increase of attraction at the upper station from this cause, for the bringing of a part of the attracting strata a little nearer the attracted body has a very little effect, unless it is brought from a great distance beneath.

If the earth was originally fluid, as we have good reason to suppose, the amount of matter then between any point on or above the earth's surface must have been everywhere the same, except so far as it depended upon the earth's ellipticity, the effect of which is taken into account in the formula, and does not enter into the question here. Theories may differ with regard to the manner in which continents and mountain ranges have been formed, but they most probably originated in some way from the gradual cooling and contracting of the interior part of the earth, leaving the external strata too large for the interior, from which cause they were forced up to a higher level at some parts of the surface, with little lateral transfer of matter, leaving the parts below less dense and the amount of attraction on any point above them but little increased.

The sea originally, either as vapor or water, must have been equally distributed over all parts of the earth's surface. The elevation of continents and mountain masses above the general level of the bottom of the sea, displaced that part of the sea which originally existed where the continents now are, and increased the amount of matter where the sea now is, and hence from this cause there has been a decrease in the amount of attraction over the continents and an increase over the sea. There is also much plausibility in Archdeacon Pratt's theory that the inequalities of the earth's surface, as seen in the mountains, plains, and ocean beds, have arisen from unequal rates of cooling and contraction, they being supposed to be greater in the parts covered by the ocean than under the continents, and hence the continents so formed would not imply an increase of matter. But there is another theoretical consideration which must be regarded as completely decisive of the question of the inferior density of the earth's crust, where the continents are, in comparison with that of the part covered by the ocean. The whole globe can be so divided into two hemispheres that nearly all the land is contained in the one, and nearly all the ocean in the other. If the continents and the parts of the earth's crust under them had the same density as the undisturbed strata under the ocean, the center of gravity of the earth would occupy the center of the mass, and the ocean would be drawn over to the side where the continents mostly exist, and leave some parts now covered by the ocean as dry land. There cannot, therefore, be much increase of matter in the parts of the earth's crust where the continents exist.

9. The preceding deductions from merely theoretical considerations are completely confirmed by numerous pendulum observations made over nearly all parts of the globe. If we examine the discussions of these observations in the determinations of the figure of the earth by Airy, Bowditch, and quite recently by Clarke,* it is seen that the residuals mostly indicate diminished gravity on the continents, especially for elevated stations, and increased gravity on small islands in the ocean. It is not to be supposed, however, from this that there is really less matter where the continents are, for these residuals arise mostly from the reductions of gravity to sea-level upon the hypothesis of an increase of matter equal to the mass of the continents above sea-level. The results show that if these reductions were omitted, as they should be according to the preceding view of the matter, the residuals would nearly disappear. This is especially seen in the geodetic operations of India, in which, as the Himalayas are approached, the pendulum observations of the elevated stations, with the usual reductions to sea-level, indicate a great deficiency of gravity. With regard to this Colonel Clarke says: "Kaliana was fixed on by Sir G. Everest as the nearest approach to the base of the Himalayas for reliable geodetic observations, and in our tables we see that at that station and all north of it there is a large defect of gravity, attaining at Moré an amount of—22 vibrations. It is very remarkable that this is precisely the amount of the correction that had been applied for the attraction of the mountains, so that the apparent vertical attraction of the three miles of earth crust between Moré and sea-level is zero. And, in fact, at most of the other high stations the residual discrepancy is diminished or removed if we omit the corrections for the attraction of the table land lying between the station and the sea-level."†

* Geodesy, by Col. A. R. Clarke, C. B., Royal Engineers, F. R. S., Hon. F. C. P. G., corresponding member of the Imperial Academy of Sciences of St. Petersburg: Oxford, 1880.

† Clarke's Geodesy, p. 350.

It seems, then, not only from the preceding theoretical considerations, but also from actual observations, that the supposed effect upon gravity of table lands and of mountain masses should be neglected, not only in barometric hypsometry, but also in reductions of pendulum observations to sea-level, and it has accordingly been neglected in formula (22).

10. It is now necessary to have the accurate numerical values of the constants l, M, and r in (22). Of these, the most important is l, which is the height of a homogeneous dry atmosphere at the temperature of 0° under a pressure equal to that of a column of mercury of 760^{mm} at the same temperature arising from the force of gravity at sea-level on the parallel of 45°. This is to the height of the mercurial column inversely as the densities of air and mercury under these circumstances. Regnault found the density of mercury equal 13.59593*, that of pure water at the temperature of 4° being unity. At Paris, latitude 48° 50′ and altitude 60 meters, he also obtained for the density of pure air, under the barometric pressure of 760^{mm} and temperature 0°, the value .001293187†, the unit of measure being the same as in the case of the mercury. As the density is as the pressure and this as gravity, this density at the earth's surface and on the parallel of 45° would be .001293187 $g' : g$. From (5), putting h=60 meters and λ=48° 50′, and r=6367324 meters (20890548 feet), as obtained by Clarke, we get $g' : g$=.9996708, and hence, .001293187×.9996708=.00129276 for the density of pure air at sea-level on the parallel of 45°, and under a pressure of 760^{mm} of mercury.

The air in general contains about .04 per cent. of carbonic acid gas with a density of 1.529 compared with that of air. Being heavier than air it increases its density by the .0004×.529=.0002116 part. Hence the density of dry air, such as is generally met with in barometric hypsometry, may be put equal to .00129276×1.0002116=.00129303. With this density of air and the preceding density of mercury obtained by Regnault, we get

$$l = 0.76^{m} \times \frac{13.59593}{.00129303} = 7991.2 \text{ meters.} \quad (24.)$$

11. This value of l depends entirely upon Regnault's determinations of the densities of mercury and of air, without regard to other determinations, and differs but little from that used in the preceding parts of these researches. The determinations of Regnault are undoubtedly the most reliable we have, not only because they were made with great care, but also because being for the most part the most recent, the processes of the others were carefully examined and measures were devised to avoid sources of error which, it was thought, might have affected the previous determinations.

The uncertainty in the density of mercury is not great enough to be of much consequence in the determination of this constant, and Regnault's density seems to be about a mean of all that are entitled to much weight. It is very nearly the same as that of a more recent determination which should have great weight. According to Prof. Balfour Stewart,‡ "it has been determined at the Kew Observatory that the weight *in vacuo* at 62° Fahr. of a given volume of purified mercury is to that of the same volume of water in the proportion of 13590.86 to 1001.62 grains." From these, by reducing the former to 0° C. and the latter to 4° C., the density 13.594 was deduced, which differs but little from Regnault's result.

In the density of air the uncertainty seems to be much greater, since the differences between the results of different experiments are much larger, but Regnault's result is entitled to much more weight than the others, and besides it is now generally adopted, so that it is thought best to adopt it here, also, without attempting to determine a more probable density by giving weights to other determinations. If it should be thought by some that the principal constant in the formula, deduced from this density, should be a little different, a proportionate change in the altitudes obtained by means of the formula can be readily made.

12. With the well-known value of M and that of r already given, and the value of l in (24), the expression of H in (22), neglecting the small correction C, is in meters

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* Pogg. Annalen der Physik, Band 74, p. 213. † *Ibid.*, p. 208. ‡ Treatise on Heat, Art. 75.

$$(25)\quad H=18446.6\log\frac{B''}{B}\left\{1+[.00183+.00035(e'+e)](t''+t)+.189(e'+e)+\frac{2h'+H}{6367324}+.002606\cos 2\lambda\right\}$$

$$=18446.6\log\frac{B''}{B}[1+.00183(t''+t)][1+.189(e'+e)]\left(1+\frac{2h'+H}{6367324}\right)(1+.002606\cos 2\lambda)$$

The last form is sensibly the same as the first, and is adapted to computations by logarithms, by which the computations are more conveniently made.

The value of Rühlman's principal constant, with the formula expressed in the form above, is 18429.1. The difference affects the results given by the formula less than the one-thousandth part, and it arises almost entirely from the neglect here, for reasons already given, of the supposed effect of the attractions of table lands and mountains. The value of Laplace's constant is 18336, or, with the formula expressed as above, 18382, which is considerably less than that of (25) above. In the time of Laplace the densities of mercury and of air were not sufficiently well known to determine the value of l from (24), and the principal constant in Laplace's formula was determined from observation. This was done by Ramond in 1803 from barometric and thermometric observations made at Tarbes, France, with the instruments 322 meters above the level of the sea, and from corresponding observations made on the top of the Pic du Midi and other neighboring peaks, with altitudes ranging from that of the Pic du Midi, 2935 meters above sea-level, to that of the Pic du Bergons, 2113 meters. There were only eight observations in all, made at noon, one in July, four in September, one in October, and two in November. The constant deduced from these observations, after being reduced to sea-level, was 18336*. With the knowledge we now have of the large annual and diurnal inequalities in barometric determinations of altitudes made at different seasons of the year and hours of the day, together with all the other irregularities and uncertainties of results from only one or a few observations, a constant obtained from only eight pairs of observations, made mostly in the summer season, and all at the same hour of the day, cannot be accepted as being reliable. Yet this constant has been almost exclusively used even up to the present time.

13. The formula (25) requires the height of the barometer at both stations to be either that of the temperature of 0° C., or some other temperature, as that of the lower station, and also that it should be measured with a scale reduced to the standard temperature of the scale used in the construction of the barometer. It is usual to apply such reductions at once to the observations, but where unreduced observations are used it is necessary to introduce another correction into the formula, which will have the same effect upon the result as the reduction of the observations.

The general formula for this reduction, first given by Schumacher, is

$$-\frac{m(\tau-T)-l(\tau-\vartheta)}{1+m(t-T)}B$$

in which

τ = the temperature of the attached thermometer,
T = the temperature to which the observed height is reduced,
m = the coefficient of the cubic expansion of mercury,
l = the coefficient of lineal expansion of brass,
ϑ = the normal temperature of the standard scale.

In this small reduction the denominator above can be taken as equal to unity without any sensible error. In the French barometer and scale we have, for reduction to the freezing point, T=0, and ϑ=0, and hence the reduction becomes

$$-(m-l)\tau B$$

Hence, with the barometer unreduced, we shall have, using B_1 instead of B in this case,

$$\log\frac{B''}{B}=\log\frac{B''_1(1-(m-l)\tau')}{B_1(1-m-l)\tau)}=\log\frac{B''_1}{B_1}+\log[1-(m-l)(\tau'-\tau)]$$

According to Regnault, the value of m at 0° of temperature is .00017905, and at 30°, .00018051. The mean corresponding to an average temperature of 15° is .00017978. The coefficient of linear expansion of brass, according to Lavoisier and Laplace, is .00001878. Hence, we have in the expres-

* Mémoires sur la Formule Barométrique de la Mécanique Céleste; par Ramond, 1811.

sion above, where the barometer has a brass scale extending down to the base of the mercurial column, $(m-l)=.000161$. The preceding equation, therefore, becomes

(26) $$\log\frac{B''}{B}=\log\frac{B''_1}{B}+\log[1-.000161(\tau'-\tau)]$$

For the English barometer, in which the standard temperature of the scale is 62° Fahr., the reduction to the freezing point becomes

$$-[m(\tau-32^\circ)-l(\tau-62^\circ)]B$$

in which τ must be expressed in degrees Fahrenheit, and the values of m and l changed to correspond by taking five-ninths of their values in the preceding case. Hence, it becomes

$$-[.00009988\tau-32^\circ-.00001043(\tau-62^\circ)]B=-.00008945(\tau-28^\circ.6)B$$

Proceeding as above, therefore, we get in this case

(27) $$\log\frac{B''}{B}=\log\frac{B''_1}{B_1}+\log[1-.0000895(\tau'-\tau)]$$

14. In the expression of H (25) it is necessary to know the value of e for both the lower and upper stations, and this depends by (4) upon that of b. According to Regnault, we have

(28) $$b=b_1-\frac{0.480(t-t_1)}{610-t_1}B$$

in which t_1 is the temperature indicated by the wet-bulb thermometer, and b_1 is the vapor tension of saturated air at the temperature t_1.

When the wet bulb becomes coated with ice the formula becomes

(29) $$b=b_1-\frac{0.480\ (t-t_1)}{689-t_1}B$$

From comparisons recently made by Dr. Carl Koppe, in Zürich, and also by Herr Billwiller, between hair hygrometers and the psychrometer, at temperatures below the freezing point, it is evident that this part of the formula is very erroneous. (Zeit. der Oest. Gesell. für Met., B. 13, § 49.)

Equation (4), therefore, becomes, when the wet bulb is free from ice,

$$e=\frac{b_1}{B}-\frac{0.480\ (t-t_1)}{689-t_1}=\frac{b_1}{B}-.0008(t-t_1)$$

the last form of the expression being correct for $t_1=10^\circ$ is sufficiently correct for all values of t_1. When the wet bulb is covered with ice the numerical coefficient of $(t-t_1)$ is .0007. We, therefore, have in (25) the factor

(30) $$1+.189\ (e'+e)=\left(1+.189\frac{b'_1}{B'}\right)\left(1+.189\frac{b_1}{B'}\right)[1-.000151(t'-t'_1)]\ [1-.000151(t-t_1)]$$

When the wet bulb is covered with ice the numerical coefficient in the last two factors becomes .000136.

The formula of (28) is imperfect, and entirely fails for very low percentages of humidity. Where $t-t_1$ is very large the last term becomes the greater, and then the expression of b becomes negative, that is, we have a negative vapor tension, which, of course, is an absurdity. If the formula entirely fails for very low percentages of humidity, it must begin to be inaccurate where the percentage of humidity is not very small. As the numerical coefficient, however, in the formula was determined empirically so as to satisfy observation and experiment for ordinary ranges of humidity, the formula for these is sufficiently accurate.

Instead of a formula for giving directly the vapor tension, Mr. Glaisher gives a formula of the form

(31) $$t_2=t-F\ (t-t_1)$$

for determining the temperature of the dew-point t_2, and then with this he gets from a table giving the tension of vapor in saturated air for the different temperatures the value of b. In the expression of t_2 above, the factor F is not a constant for all temperatures, but a function of t, which is determined from observation. These factors, determined from a great number of observations, are given in the following table of

*Glaisher's factors.**

t	F.	t	F.	t	F.	t	F.	t	F.	t	F.	t	F.	t	F.	t	F.	t	F.
10	8.78	20	8.14	30	4.15	40	2.29	50	2.06	60	1.88	70	1.77	80	1.68	90	1.63
11	8.78	21	7.88	31	3.70	41	2.26	51	2.04	61	1.87	71	1.76	81	1.68	91	1.62
12	8.78	22	7.60	32	3.32	42	2.23	52	2.02	62	1.86	72	1.75	82	1.67	92	1.62
13	8.77	23	7.28	33	3.01	43	2.20	53	2.00	63	1.85	73	1.74	83	1.67	93	1.61
14	8.76	24	6.92	34	2.77	44	2.18	54	1.98	64	1.83	74	1.73	84	1.66	94	1.60
15	8.75	25	6.53	35	2.60	45	2.16	55	1.96	65	1.82	75	1.72	85	1.65	95	1.60
16	8.70	26	6.08	36	2.50	46	2.14	56	1.94	66	1.81	76	1.71	86	1.65	96	1.59
17	8.62	27	5.61	37	2.42	47	2.12	57	1.92	67	1.80	77	1.70	87	1.64	97	1.59
18	8.50	28	5.12	38	2.36	48	2.10	58	1.90	68	1.79	78	1.69	88	1.64	98	1.58
19	8.34	29	4.63	39	2.32	49	2.08	59	1.89	69	1.78	79	1.69	89	1.63	99	1.58

With regard to these factors Mr. Glaisher says: "The numbers in the table have been found from the combination of many thousands of simultaneous observations of the dry and wet bulb thermometers with Daniell's hygrometer, taken at the Royal Observatory, Greenwich, from the year 1844 to 1854, with observations taken at high temperatures in India, and others at low and medium temperatures at Toronto. The results at the same temperature were found to be alike at these different places; and, therefore, the factors may be considered as of general application."

These factors were also verified for high altitudes by observations made during his balloon ascents. "The result of all the simultaneous determinations of the temperature of the dew-point by Daniell's hygrometer and the dry and wet bulb thermometers are as follows: The temperature of the dew-point, as found by the use of the dry and wet bulb thermometers:

"Up to 1,000 feet high, was 0°.15 lower than by Daniell's hygrometer, from twenty-eight experiments.

"From 1,000 to 2,000 feet high, was 0°.10 lower than by Daniell's hygrometer, from forty experiments.

"From 2,000 to 3,000 feet high, was 0°.05 lower than by Daniell's hygrometer, from fifty-nine experiments.

"From 3,000 to 4,000 feet high, was the same as by Daniell's hygrometer, from sixty-six experiments.

"From 4,000 to 5,000 feet high, was 0°.05 lower than by Daniell's hygrometer, from forty experiments.

"From 5,000 to 6,000 feet high, was 0°.7 lower than by Daniell's hygrometer, from thirty-four experiments.

"From 6,000 to 7,000 feet high, was 0°.2 lower than by Daniell's hygrometer, from thirty-four experiments.

"From 7,000 to 8,000 feet high, was the same as by Daniell's hygrometer, from eight experiments.

"From 8,000 to 9,000 feet high, was 1°.5 higher than by Daniell's hygrometer, from two experiments.

"From 9,000 to 10,000 feet high, was 1°.2 higher than by Daniell's hygrometer, from two experiments.

"From 10,000 to 11,000 feet high, was 0°.3 higher than by Daniell's hygrometer, from one experiment.

"From 12,000 to 13,000 feet high, was 0°.3 higher than by Daniell's hygrometer, from five experiments.

"From 13,000 to 14,000 feet high, was 0°.8 lower than by Daniell's hygrometer, from seven experiments.

"From 14,000 to 15,000 feet high, was 1°.0 lower than by Daniell's hygrometer, from two experiments.

* Hygrometric Tables, by James Glaisher, F. R. S., &c., fifth edition: London, 1869.

"The number of experiments made up to the height of 7,000 feet, varying from twenty-eight to sixty-six in each step of 1,000 feet, are sufficient to enable us to speak with confidence; the results are that the temperatures of the dew-point as found by the use of these tables are worthy of full confidence up to this point. At heights exceeding 7,000 feet my experiments do not yield a sufficient number of simultaneous readings to give satisfactory results, and before we can speak with certainty at these high elevations more experiments must be made."*

But the tensions of aqueous vapor obtained by Glaisher's method by means of the factors in the preceding table, based upon so many observations made in different parts of the earth and at nearly all accessible latitudes, differ very much in some cases from those given by Regnault's formula in (28). Both Glaisher's formula and Regnault's have been reduced to tables; the former by Glaisher himself, the latter by Guyot and others. From a comparison of these tables the discrepancies shown in Table XII are obtained. In Guyot's tables the computations by the formula extend only to the point where the tensions vanish and change signs, and it is seen that at this point the tension given by Glaisher's table is still considerable.

It is a question with meteorologists why these tables, having the high authorities of Regnault and Glashier, should differ so much, and which should have the preference; but a little insight into the history of Regnault's formula clears up the matter. What is called Regnault's formula was, originally, a formula obtained by M. August from purely theoretical considerations. The constants in this formula were somewhat changed by Regnault after the data upon which they depend became better known, and the results then given by the formula were compared with those obtained by him from experiment and observation. The result of the comparisons was that it was necessary to change the theoretical constant 0.429 to 0.480, as given in (28), in order to ve the best agreement between the results of the formula and those of experiment and observation.

With regard to the theory upon which the formula is based, Regnault says: "I do not think that the fundamental hypothesis adopted by M. August can be admitted as a basis of the calculation of the psychrometer; namely, that all the air which supplies heat to the moist thermometer falls to the temperature t' indicated by the latter, and is completely saturated with humidity. It seems to me probable that the portion of the air which cools does not fall to t', and that it is not saturated with humidity. The relation of the quantity of heat which the air takes from the bulb by evaporation of the water to the quantity of heat which it loses in cooling is probably greater in proportion as that air is more dry, because in this state it is much more susceptible of humidity than when it approaches its state of saturation."†

With regard to his comparisons, he says: "The coefficient 0.480 gives an almost perfect coincidence between the calculated results and those found by direct observation in the fractions of saturation which exceed 0.40; but it produces a difference greater than the coefficient 0.429, and, in an inverse direction, for weaker fractions of saturation." He hence infers that the coefficient of the formula (28) depends on $t-t_1$, and that this "results from the fact that the air carries off proportionally more vapor when it is very dry than when it approaches saturation."

Regnault, then, not only does not consider the theory upon which the formula is based reliable, but says, also, that the formula with the empirical constant 0.480 does not give results in accordance with observation in fractions of saturation below 0.40. By referring to Table —— it is seen that for ordinary ranges of temperature and humidity the two tables agree as well as could be expected, and it is only in the case of extreme temperatures and low percentages of relative humidity that the differences become large. This is exactly what we should expect from what Regnault says of the empirical constant, for he says it fails for low percentages of humidity, and for very low percentages we know it entirely fails, since it gives negative tensions. Regnault had less than 100 observations, in all, for comparison; while, as we have seen, Glaisher had many thousands, so that the results obtained from Glaisher's factors and tables are undoubtedly more reliable, at least for small altitudes above sea-level, than those obtained from Guyot's tables computed from the formula of (28). But there is really little difference, so far as we can now see, between Regnault and Glaisher, since the results given by Guyot's tables differ from those of Glaisher's but

* Glaisher's Hygrometrical Tables.

† Regnault's Hygrometrical Researches; Taylor's Scientific Memoirs, vol. iv, p. 652.

little, except for low percentages of humidity, for which, Regnault says, the formula from which Guyot's tables have been computed is not accurate. That Regnault not only regarded the formula as being imperfect, but also that a new formula was desirable, is evidenced from a closing remark. He says: "I shall abstain for the present from establishing a new formula of the psychrometer; I do not consider the elements at my disposal as sufficient." It does not appear that he ever undertook it, but it has been done by Glaisher with an abundance of material on hand; with what success must be judged from comparisons of the results with observation. The trouble, then, with regard to the differences between the tables, is not that there is a difference between two great authorities, but that a formula called Regnault's formula, but confessedly imperfect, has been reduced to tables, which carry with them the authority of Regnault while they really have no such authority.

15. By a reference to the formulæ (28) and (31) it is seen that the former is a function of the barometric pressure, and that the tension b should increase with altitude or diminution of pressure, all the other data remaining the same; while according to the latter the value of t_2, and consequently of b, is the same for all altitudes. Hence, if the two formulæ gave accordant results at the earth's surface generally, they would differ for high altitudes, especially for low percentages of humidity, in which the last term of (28) becomes large. Unless the theory of M. August, upon which the formula of (28) is based, is entirely erroneous, and the approximate accuracy of the theoretical coefficient 0.429 is merely accidental, I cannot see how the tension of vapor for the same temperatures of dry and wet bulb can be the same at all altitudes. As the pressure is diminished the evaporation, all other circumstances being the same, must be accelerated, and the difference between the dry and wet bulb temperatures diminished, and hence the formula (31) must give too small a vapor tension. Yet the comparisons of the results given by Glaisher's factors with observations at high altitudes, we have seen, seem to be satisfactory, and there are no indications that the tensions for the same temperatures of dry and wet bulb increase with altitude as they must by the formula of (28). Regnault had no observations of much weight in testing the formula in this respect, and remarks that it will be desirable to make experiments in very elevated localities to ascertain whether the second term in (28) corrects properly the formula for the variations of B. If a series of such experiments were made on Pike's Peak, or at some other very elevated station, they would be of great value in settling this question.

It is seen from Table XII that, for low altitudes above sea-level and small fractions of saturation, the vapor tensions given by (28) are smaller than those given by Glaisher's factors and tables; but these tensions are the same by the latter for all altitudes, while by the former they must increase as B diminishes, that is, as the altitude increases. For altitudes from one to two miles the differences between the results of formula (28) and those of Glaisher's factors are generally small and of little consequence in barometric hypsometry. For differences of altitude, therefore, of about three miles where the lower station is near sea-level, if formula (28) is used for obtaining the vapor tensions of the air, the results in determining differences of altitude by (25) will differ but little from those obtained with Glaisher's vapor tensions, since at the lower station the value of e' with the former is smaller and at the upper station the value of e is larger than they would be with the use of Glashier's vapor tensions, and consequently the value of $(e'+e)$ in (25) is very nearly the same with both. For small differences of altitude the uncertainty in the hygrometrical formula is of little consequence. The formula of (28) will, therefore, be used in obtaining b, and with it the value of e in (4), in applications of the formula (22) or (25), so that if Glaisher's factors are correct for high altitudes, the results obtained with formula (28), instead of Glaisher's factors, must also be nearly correct, even in cases in which the differences of the vapor tensions obtained by either method are of any consequence, namely, where there are large differences of altitude and low percentages of humidity.

16. It must be borne in mind that formula (25) is correct only in the case in which the decrease of temperature and of the value of e is very nearly as the increase of altitude, and that, even with the correction C in (22) it is strictly applicable only to the case in which the decrease of these quantities with increase of altitude can be expressed by the first two terms of the expressions of (16), and in which the second term is small in comparison with the first. In the annual and diurnal averages of one or more years this is, perhaps, always the case, and it may often be so in

individual cases of mean temperatures, where the observations are made in the spring or fall and at the hours of the mean temperature of the day. If, in such a case, A*c* (Fig. 1) represent the surface temperature t'' at D under the upper station, the mountain being supposed to be removed, and this temperature to be determined from observations at A reduced to the point D by means of a chart of well-determined temperature gradients, or by means of simultaneous observations at several stations around the mountain; and if, likewise, C*c* represent the observed temperature at the upper station at B, then the intervening temperatures on the vertical between D and B will be represented very nearly by the horizontal co-ordinates of the straight line *ac* referred to the line AC, making these temperatures decrease in proportion to the increase of the altitude. In this case $\frac{1}{2}(t''+t)$ represents the average temperature of the vertical line DB, and the formula (25) is very nearly correct, since c' vanishes and the correction C (23) becomes very small.

In the summer season, especially during the warmest part of the day, the temperatures of all the strata are very much increased, but the surface temperature at the lower station, especially if situated on a dry plane and at a considerable distance from the sea or other large body of water, or in some mountain valley, becomes very much greater in proportion than that of the air at only a small elevation above the surface, and the temperature of the upper station at B may also be considerably higher than that of the air generally around about B at the same altitude, but at some distance from the heated surface of the mountain, or than it would be at B if the mountain with its superheated surface were removed, and this is especially the case for both stations in clear weather. In this case if A*b* (Fig. 1) represent the temperature t'' at the level of the lower station at D, and CD represent the temperature observed at B, and C*e* that which would exist at B if the mountain were away, then the intervening temperatures of the vertical DB will not be represented by the co-ordinates of the straight line *bd*, but by those of some curved line *be*, which makes the temperatures decrease with increase of altitude in a much greater ratio near the surface than at altitudes at some elevation above the surface. The value of $\frac{1}{2}(t''+t)$, therefore, is in this case greater than the average of the temperatures which would exist between D and B if the mountain were away in the ratio of the area AC*db* to that of AC*eb*, so that with this value of $\frac{1}{2}(t''+t)$, instead of the true average of the temperatures, we get from (25) a value of H which, in the summer season, is generally too great. Even the formula with the correction C (23) is not applicable in this case, since, on account of the rapid decrease of temperature with increase of altitude near the surface, the temperature cannot be represented by the form of (16), at least unless the second term becomes too large in comparison with the first for the formula to be applicable, and even if it were, we would have no means of determining the constants c and c' in that expression.

But if the lower station or stations, from which the temperature t'' is determined, are situated near the sea, or large lake, where the annual range of temperatures may not be so great as that of the temperatures of the air above, then the value of t'' obtained from the observations at those stations may be such as to make $\frac{1}{2}(t''+t)$ less than the true average, and then the value of H from formula (25) may be too small. This would especially be the case if the value of t, observed at the upper station B, were, for some reasons, also less than the temperatures generally in the vicinity at that altitude. This may be the case often where the station B is located high up on the top of a mountain peak, for, as in warm weather there are always ascending currents up the mountain sides, the rate of decrease of temperature with increase of altitude approximates to that of rapidly-ascending currents, which in the case of dry air is about one degree for each 100 meters of ascent. But this is a much more rapid decrease of temperature than that which exists in the air generally, and hence the air of these currents when it arrives at the top of the peak may be colder than that of the air generally in the vicinity at that altitude, and than the air at B would be if the mountain were away and there were no ascending currents.

In the winter season, and especially during the coldest part of the day, we have just the reverse of what takes place in the summer season. Then the surface temperatures at both A and B (Fig. 1), on account of the greater radiation from the surface than from the air, are much more diminished than those of the air generally at some height above the surface, so that if in this case A*b* represent the temperature t'', and C*d'* that of t observed at B, and C*e'* that of the temperature at that elevation unaffected by the cooler mountain surface, then the temperatures of the vertical DB are not represented by the co-ordinates of the straight line *b'd'*, but by those of the curved line

$b'e'$, and the value of $\frac{1}{2}(t''+t)$ is less than the mean temperature of the air column DB would be if the mountain were away, in the ratio of the area $ACd'b'$ to that of $ACe'b'$, and the formula (25) therefore gives generally too small a value for H in the winter season, especially for the coldest part of the day.

But if the lower station, or stations, from which t'' is determined, are situated near large bodies of water, the temperature of which may be considerably higher than that of the air above them, then the value of $\frac{1}{2}(t''+t)$ in the formula, as determined from the observations, may be greater than the mean temperature required, as in the summer it may sometimes be too small, and then the formula may give values of H too great instead of too small in the winter season and coldest part of the day.

17. The correction in the formula for the effect of the aqueous vapor of the air is so small that the mean of the two extremes $\frac{1}{2}(e'+e)$ can always be used without any sensible error for the average value of e, and the value of e' may be that observed at the lower station without any reduction to the point D (Fig. 1), as in the case of the temperature, when the distance between the stations is great.

Where hygrometric observations are made at the lower station only, the most probable value of the vapor tension b at the level of the upper station may be obtained from b', its value at the lower station, by means of the formula

$$(32) \qquad \log b = \log b' - \frac{H^{m}}{6517} = \log b' - \frac{H^{ft}}{21380}$$

The first expression of the value of log b must be used for French measures, and the latter for English. These are simply modified forms of those deduced by Dr. Hann,* for the average state of the atmosphere, based upon hygrometric observations made by different observers at various places on the Himalayas, Mount Arrarat, Teneriffe, and also in the balloon ascensions of Welsh and Glaisher. Even where hygrometric observations are made at the upper station this formula would no doubt give a better value for the formula than the observed value at the upper station, especially in the summer season when there are ascending currents and the air at the mountain top becomes saturated from the ascending moist and gradually cooling current of the mountain side, while the air at the level of the upper station generally is comparatively dry.

Where no hygrometric observations are made at either station it is usual to use Laplace's modified temperature coefficient in the formula as a partial, though very imperfect, correction for the effect of the aqueous vapor. For low temperatures this correction is known to be very erroneous, and for temperatures below the freezing point the correction even has the wrong sign and makes the final result more erroneous. In this case also it is better to use a vapor tension for each degree of temperature, which is an average somewhat of tensions observed at that temperature at various places and different seasons of the year. This may be done by substituting in (3) for $f(e)$, its value given by the expression in § 8, Part I, which has been obtained by Dr. Hann from observations made at various times and places, and may be regarded as an empirical approximate expression for the average state of the atmosphere. With this expression of $f(e)$ we get instead of $.189(e'+e)$ in (25) the expression $0.00154+.000341t$. This makes the correction for the hygrometric state of the atmosphere a function of the temperature, as the correction introduced by Laplace by means of his modified temperature coefficient, but the latter makes this correction vanish and change signs at 0° c, while in the former this takes place at $-4^{\circ}.5$. Both are therefore imperfect for low temperatures, since the vapor tension most probably never vanishes at any temperature, or at least it cannot become negative. The expression above can be used without any sensible error where a series of observations, made at different times, is used, as those of monthly or yearly averages; but of course, in special cases, where only one or a very few sets of observations are used, it will generally be less accurate.

18. By putting ΔH, ΔB, and $\frac{1}{2}\Delta(t''+t)$ for small finite variations of the altitude, barometric pressure, and of mean temperature of the two stations, we get from the differentiation of (25), neglecting insensible quantities and using the approximate mean correction given above for the effect of aqueous vapor,

*Zeitschrift der Oesterr. Gesell. für Meteorologie, ix Band, Seite 198.

$$(33)\quad \Delta H = 18447M\left(\frac{\Delta B''}{B''} - \frac{\Delta B}{B}\right)[1+.00183(t''+t)][1+.00154+.000170(t'+t)]+.00183H\Delta(t''+t)$$

$$= 8024\left(\frac{\Delta B''}{B''} - \frac{\Delta B}{B}\right)[1+.002(t''+t)]+.00183H\Delta(t''+t)$$

By means of this expression, the effect upon H, in formula (25), resulting from small errors in barometric pressure and in temperature, may be conveniently computed.

If in (25) we put δH for the change of altitude corresponding to a very small change of pressure δB in ascending, letting B' and B represent the barometric pressures at the base and top of this short column, in which t' may be put equal to t, we shall have, for the mean hygrometric state of the atmosphere and the parallel of 45°,

$$(34)\qquad \delta H = -8024\frac{\delta B}{B}(1+.004\,t)$$

Putting $\delta B = 1^{mm}$, we get

$$(35)\qquad \delta H = -\frac{8024}{B}(1+.004\,t)$$

for computing the value of δH for a change of 1^{mm} in the value of the barometric pressure at any given temperature t. In these expressions the values of B and t, strictly, should be the means of the small column δH, and B must be expressed in millimeters.

CHAPTER II.

PRACTICAL APPLICATIONS OF THE THEORY.

19. By means of the formulæ in the preceding chapter, with the necessary barometric, temperature and hygrometric observations, the difference of altitude between any two stations can be computed, with results more or less accurate, according to the accuracy of the observations and the conformity of the temperature and hygrometric state of the atmosphere with that assumed in the formulæ. Small, unavoidable errors in observations may affect these results considerably, but the greatest errors arise, as explained in § 16, from assuming that the average temperature and hygrometric state of the air column is the mean of the observations at the lower and upper stations. These errors affect the results, not only in the case of one or a few sets of observations, made at any time of the day or season of the year, but likewise in the case of monthly averages. With a great number of observations, however, made at different seasons of the year and hours of the day, and especially if these observations are made regularly throughout the year, and at such hours of the day as give the mean temperature nearly of the day, these errors are in a great measure eliminated from the result. But even in this case there is considerable uncertainty where we do not have the means of determining the permanent barometric gradient of the place, if the two stations are a considerable distance apart.

The practical application of the preceding formulæ can be very much facilitated by means of computed tables, adapted to the several variables in the formulæ used as arguments. Such tables have been computed with the improved and most recent constants given in the preceding formulæ, and much study has been given to this part of the subject, in order to have these tables as concise and convenient as possible. Such tables are much needed in this country, since the tables in use here mostly are based upon the old constant, determined by Ramond nearly eighty years ago, which is now known to be erroneous. This constant has been used in seven of the eight different formulæ and sets of tables given in the Smithsonian Miscellaneous Collections, the eighth being Bessel's formula with the constants corrected by Plantamour in accordance with Regnault's determination of the densities of mercury and air, but with corresponding tables adapted only to French measures. Besides, neither the formulæ nor the tables are in the most convenient form for practicable application. Williamson has reduced these formulæ to tables in English measures, adapted to computation without logarithms, but such tables require great expansion and are inconvenient, both on account of their great bulk and because they must necessarily be tables with two arguments; for every computer knows how inconvenient it is to obtain numbers accurately by interpolation from a table with two arguments.

The tables here given are arranged more after the concise and admirable forms given by Rühlman, but are given in English instead of French measures. They are, however, so arranged that they can be used with almost the same convenience in the latter as in the former measures. With one exception they are all tables with a single argument, and the quantities to be taken from the table with two arguments are generally so small that they can be obtained with sufficient accuracy with very little trouble. These tables are adapted to computations with the use of logarithms, which is most convenient where a table of logarithms is at hand, but it may sometimes be desirable to have tables by which the computations can be made without the use of logarithms. For this purpose a mode of computation without logarithms has been devised in which the same tables, with one exception, can be used, and therefore requiring only one additional table, instead of a complete and separate set of tables.

20. The principal constant in (25), reduced to feet, is 60521.5, and the term $.00183\,(t''+t)$, for degrees Fahrenheit becomes $.001017\,(t''+t-64^\circ)$. With these changes the formula (25), by means of (30), can be put into the following form adapted to English measures and computation by logarithms:

$\log H = \log(\log B'' - \log B)$	
$+\log 60521.5\,[1+.001017\,(t''+t-64^\circ)]$	Table I, arg. $(t''+t)$
$+\log\left(1+.189\,\frac{b'_1}{B'}\right)$	Table II, arg. b'_1 and B'
$+\log\left(1+.189\,\frac{b_1}{B}\right)$	Table II, arg. b_1 and B
$+\log\,[1-.000084\,(t'-t'_1)]$	Table III, arg. $t'-t'_1$
$+\log\,[1-.000084\,(t-t_1)]$	Table III, arg. $(t-t_1)$
$+\log\left(1+\frac{2h'}{r}\right)$	Table V, arg. h'
$+\log\left(1+\frac{H}{r}\right)$	Table VI, arg. log H
$+\log\,(1+.002606\cos 2\lambda)$	Table VII, arg. λ

If the barometer has not been reduced to the temperature of 32° Fahr., the second member of (27) must be used instead of $\log B''-\log B$, that is, we must deduct from this the value of log $[1-.0000895\,(\tau'-\tau)]$ when τ is expressed in degrees of Fahrenheit. This logarithm is contained in Table VIII.

The following are the definitions of the quantities entering into the terms and arguments, given here again by way of recapitulation and for the sake of convenience of reference:

H=the difference of altitude of the two stations,

B=the barometric pressure at the upper station,

B′=the barometric pressure at the lower station unreduced for barometric gradient.

B″=B′ reduced to the latitude and longitude of the upper station by applying a correction for the barometric gradient,

t=the temperature of the air at the upper station,

t'=the same at the lower station,

$t''=t'$ reduced to the latitude and longitude of the upper station by applying a correction for the temperature gradient,

b_1=the vapor tension of saturation at the temperature of the wet bulb at the upper station,

b_1'=the same for the lower station,

t_1=the temperature of the wet bulb at the upper station,

t_1'=the same for the lower station,

h'=the altitude of the lower station above sea-level,

λ=the latitude of the upper station,

τ=the temperature of attached thermometer at the upper station,

τ'=the same for the lower station.

The logarithm of the first term in the preceding formula can be obtained from any table oi ogarithms of five or six places, and that of each of the other terms can be very conveniently obtained from the tables designated with the arguments corresponding to each. Table II can be used where b_1 and B are given in millimeters by multiplying each by .04 or any other number which will bring the products within the limits of the range of the arguments given in the table. Table III needs only to be entered once if the sum of the arguments of the terms is used, that is, $(t'-t'_1)+(t-t_1)$.

It is usual to reduce barometric readings to the temperature of freezing, but where observations are made solely for the purpose of determining differences of altitude it is best to not correct them for temperature, if the barometer, as usual, has a brass scale extending down to the cistern; for in such case the effect of the correction is more conveniently applied to the result by means of Table VIII, which is very small, and the argument readily obtained. In reducing each of the readings to the temperature of freezing, a comparatively large table has to be entered twice and two corections have to be applied.

Where the vapor tensions are given instead of the temperature of the wet bulb, t_1, of the psychrometer, as frequently happens, Table III must be omitted and Table II used with the arguments b' and b instead of b'_1 and b_1. In this case b' and b are obtained from Table IX (Table X for French measures) with t' and t as arguments. But where hygrometric observations are made especially for determining differences of altitude, it is most convenient to use Table III and Table II with b'_1 and b_1 as arguments, since this saves the labor of obtaining from formula (28) the values of b' and b, for this, even where the last term in the formula is reduced to a table, requires considerable time, while the use of Table III is very convenient, the table being very small, and having only one argument, which is very readily obtained.

It often happens that no hygrometric observations are made at either station. When this is the case Table IV must be used instead of Tables II and III. This table is computed from the expression of $f(e)$, given in §17, reduced to English measures for the higher temperatures, but for the lower temperatures the numbers are increased a little to remedy the defect of this expression for low temperatures and to make the numbers for these temperatures positive. In all cases in which yearly averages are used this table can be used without any sensible error, and even with monthly averages, or averages of any series of observations extending over a considerable period of time, the error is very small.

21. The preceding expression of log H can be put into the following form:

$$\log\, H=\log\, 60521.5\left(\log\frac{30}{B}-\frac{30}{B''}\right)(1+.001017\times 36^\circ)+\Sigma_s \log(1+N_s)$$

in which

$$N_1=.001017\ (t''+t-100^\circ)$$

$$N_2=.189\,\frac{b'_1}{B'}$$

$$N_3=.189\,\frac{b_1}{B}$$

&c., &c.

This arrangement makes N_1 vanish at the mean temperature of 50° Fahr., and hence makes it small for either extreme; and as all the other values of N_s are generally small, the value of the last term in the expression above is small. We can therefore put

$$H=(A-A')\,(1+c)$$

in which

$$A=60521.5\ \log\frac{30}{B}\,(1+.001017\times 36^\circ)$$

$$A'=60521.5\ \log\frac{30}{B''}\,(1+.001017\times 36^\circ)$$

$$c=\frac{\Sigma_s \log(1+N_s)}{M\,(1-\frac{1}{2}c)}=2.3\Sigma_s\,\frac{\log(1+N_s}{1-1.15\ \Sigma_s \log(1+N_s)},\ \text{very nearly.}$$

The values of A and A′ are taken from table XI with the arguments B and B″. The value of log $(1+N_1)$ is obtained from Table I, with the argument $(t''+t)$, by subtracting the logarithm opposite 100°, namely, 4.79753, and so it is negative when $t''+t$ is less than 100°. The other logarithms of $(1+N_s)$ are obtained from Tables II to VII, inclusive, as in the computation by the formula with logarithms. As the value of c is always small, the only multiplication required is readily made, and the preceding formula, therefore, becomes very convenient for the computation of differences of altitude without the use of logarithms. The denominator in the expression of c generally differs so little from unity that it may be neglected, and then the expression of c becomes so simple that the value of c can be readily obtained.

22. As a first example of the application of the formula and tables, let us assume the following data, in which B′ and B are supposed to be reduced to the temperature of 32° Fahr., and in which there is no sensible effect from a gradient, so that B′ can be used instead of B″:

Inches.	°	°	
$B'=28.075$	$t'=57.3$	$t'_1=48.2$	$h=2000$ feet
$B=22.476$	$t=38.5$	$t_1=32.4$	$\lambda=38°$

With t' and t as arguments, Table IX gives $b'_1=0.470$ and $b_1=0.233$. With these data the computation is as follows:

With logarithms.		Without logarithms.		
log B′=1.44832		Table XI, arg. B		A =7867
log B =1.35172		Table XI, arg. B′		A′=1807
Diff. =0.09660	log diff. 0.98498			A−A′=6060
Table I, with arg. $t'+t=95°.8$,	4.79573	Table I—4.79753,	—.00180	
Table II, with arg. B′ and b'_1 above,	138	Table II,	138	
Table II, with arg. B and b_1 above,	85	Table II,	85	
Table III, with arg. $t'-t'_1=9°.1$	—33	Table III,	— 33	
Table III, with arg. $t-t_1=6°.1$	—22	Table III,	— 22	
Table V, with arg. h'	8	Table V,	8	
Table VI, with arg. log H=3.78	12	Table VI,	12	
Table VII, with arg. λ	28	Table VII,	28	
log H=	3.78287		.00036×2.3×6060 =	5.8
	H=6065.5 feet		H=	6065.8

If we suppose the temperatures of the attached thermometer to have been $\tau'=55°$ and $\tau=36°$, then the uncorrected values of B′ and B would have been 28.141, and 22.491, respectively, and we should have had

	log B′=1.44934
	log B =1.35200
	Diff. =0.09734
Table IX, with arg. $(\tau'-\tau)=19°$,	.00074
	Diff. = .09660

This is the same as the difference in the preceding computation obtained from the reduced values of B′ and B.

23. As a second example, we shall take the averages of the observations made at Geneva and St. Bernard, given in § 26. These give

	°		
$B''=726.5$	$t''=10.6$ C	$R'=76$	$\lambda=45°\ 12'$
$B=564.1$	$t=-1.3$ C	$R=78$	$h'=408^m$

From Table X we get, with t'' and t as arguments, $f'=8.98$ and $f=4.18$. Hence $b'=8.98\times.76=6.84$ and $b=4.18\times 78=3.27$. With these data the computation is as follows:

$\log B''=2.86124$	
$\log B\ =2.75136$	
Diff. 0.10988	log diff. 9.04092
Table I, with $\frac{4}{5}(t''+t)+64^\circ=80^\circ.7$ as an argument,	4.78923
Table II, with $726\times.04=29.0$ and $6.84\times.04=27.4$ as arguments,	78
Table II, with $564\times.04=22.6$ and $3.27\times.04=13.1$ as arguments,	48
Table V, with h' as an argument,	5
Table VI, with 3.83 as an argument,	14
Table VII, with λ as an argument,	— 3
Log of factor reducing to meters	9.48401
	$\log H=3.31558$
	$H=2068.2^{m}$

If in this example we had used Table IV instead of Table II, we should have had the logarithm .00141 instead of .00078+.00048=.00126. This would have given log H=3.31543 and H=2067.4 instead of 2068.2.

With the preceding values of b' and H we get from (32)

$$\log b=0.835-\frac{2068}{6517}=0.518, \text{ and hence } b=3.30.$$

This agrees almost exactly with the value of b above from observations.

24. As another example let us take the means of the observations made by Professor Whitney at Sacramento and Summit on the top of the Sierra Nevada, a case in which no hygrometric observations were made, and consequently a case in which Table IV must be used instead of Tables II and III. The annual means of these observations, given in § 25, are

Inches.	°	
$B'=30.014$	$t'=59.9$	$h'=31$ feet
$B\ =23.288$	$t\ =42.1$	$\lambda=39^\circ\ 20'$

We have no means of determining in this case the effect of barometric and temperature gradients, and hence we can do no better than to use B′ and t' instead of B″ and t''. The computation in this case is as follows:

With logarithms.

$\log B'=1.47732$	
$\log B\ =1.36713$	
Diff. 0.11019	log diff. 9.04215
Table I, with arg. $t'+t=102^\circ.0$,	4.79838
Table IV, with arg. $t'+t=102^\circ.0$,	223
Table VI, with arg. log H=3°.84,	15
Table VIII, with arg. λ,	21
log H=	3.84312
	H=6968.1 feet

Without logarithms.

Table XI, with arg. B_1		A =6901.0
Table XI, with arg. B'_1		A′=−12.7
		A−A′=6913.7
Table I—4.79753,	.00085	
Table IV,	223	
Table VI,	15	
Table VII,	21	
	Sum .00344×6914×2.3=54.7	
		H=6968.4 feet

25. Having given several examples of the application of the formula and tables to annual averages, we shall now give the results obtained in the same manner from the monthly averages. When the differences of altitude have been obtained from actual leveling, the result obtained from the formula should be the same, and when the true difference of altitudes is not known, the formula should give the same difference of altitude from each of the monthly averages.

From the monthly averages of barometric and temperature observations made by Professor

Whitney at Sacramento and Summit, California, from October, 1870, to October, 1873, given in his Barometric Hypsometry (pp. 32–34), the following averages of the three years are obtained:

Month.	Observations.				Results.					
	B′	B	t′	t	H		Δ′	t′−t	Rate of change of t— per 100 feet.	Rate of change of t— per 100m.
	In.	In.	°	°	Feet.			°	°F.	°C.
January	30.151	23.288	47.1	29.2	6900	−65	−46	17.9	0.26	0.47
February	30.079	23.153	48.8	27.3	6989	+24	−16	21.5	0.31	0.56
March	30.117	23.262	54.8	31.6	6975	10	+18	23.2	0.33	0.61
April	30.051	23.216	59.3	33.8	7020	55	48	25.5	0.36	0.67
May	29.935	23.233	65.5	44.6	7021	56	64	20.9	0.30	0.55
June	29.932	23.333	70.9	54.9	7016	51	63	16.0	0.23	0.42
July	29.892	23.372	73.7	61.0	6995	30	46	12.7	0.18	0.33
August	29.910	23.356	71.3	59.1	7000	+35	+16	12.2	0.17	0.32
September	29.911	23.338	67.4	53.6	6954	−11	−18	13.8	0.20	0.36
October	29.984	23.350	60.2	46.3	6901	64	48	13.9	0.20	0.36
November	30.099	23.307	50.8	34.3	6896	69	64	16.5	0.24	0.43
December	30.109	23.253	48.7	29.1	6914	−51	−63	19.6	0.28	0.51
Year	30.014	23.288	59.9	42.1	6965	......	....	17.8	0.255	0.466

The approximate latitude of Sacramento is 38° 35′, and that of Summit 39° 20′. The altitude of Sacramento is only about 30 feet above sea-level. The distance of Summit from Sacramento in a straight line is 77 miles, in a direction a little north of east. The exact difference of altitude between the barometers of the two stations, as ascertained from the railroad levelings, is 6,989 feet.

By computing the values of H from each set of monthly averages, as in the example of § 25 we get the values above. It is seen from the column headed Δ, which gives the excess of each monthly value of H above the yearly mean, that there is a large annual inequality in these values of H, the values being too great in summer and too small in winter. These arise in part from abnormal irregularities in the averages, but mostly from assuming that the value of $\frac{1}{2}(t'+t)$ in the formula expresses the true average temperature of the air column between the levels of the two stations, as has been already explained in § 16.

If we suppose the monthly values of H, as given by the formula, to be represented by

$$H=A+B\cos(it-\varepsilon)$$

in case all abnormal irregularities were eliminated, then the most probable values of the constants B and ε in this expression, as given by (26), (30), and (31), Part I, are B=65.9 feet and $\varepsilon=147°.8$. With these constants this expression gives the most probable values of Δ, denoted by Δ' above, and such as would be obtained from a series of observations continued through so long a period of time that all abnormal irregularities would be eliminated. It also makes the maximum and minimum of this annual inequality occur about the first of June and December respectively, and the vanishing nodes about the first of March and September. The maximum of the inequality, therefore, does not occur in the middle of summer in this case, and the result indicates that the value of $\frac{1}{2}(t'+t)$ differs most from the true average temperature of the air column between the levels of the two stations, about the first of June and December. This arises from the surface temperatures, especially in the valley of the Sacramento, being increased more rapidly from the sun's radiations during the spring than the air is at some distance above the surface, and from being decreased more rapidly during the fall by the radiation from the surface.

The average of the monthly values of H above, 6,965 feet, differs 3 feet from the value of H obtained from the yearly means of the observations. Since the expression of H (25) is not strictly a linear function of the observation, the principle of using averages of observations is not strictly correct, especially in the case of the yearly means, in which the range of the observed values is very great. In the case of the monthly averages this range is less, and hence the mean of all the monthly values of H must be regarded as being more nearly correct than the value of H from the yearly means of the observations.

The mean of the monthly values of H, 6,965 feet, is 24 feet less than the true value obtained by leveling. This is probably due to the effects of barometric and temperature gradients in the mean annual pressures and temperatures, which have been necessarily neglected in the computations, since there were no means of determining them, so that the values of B' and t' were used instead of B'' and t'', which the formula requires. That there is an increase of mean annual temperature in a direction from Sacramento to Summit would seem to be indicated by the simultaneous temperature observations made by Professor Whitney at Colfax, an intervening station between Sacramento and Summit. The mean of the three years at Sacramento, as seen above, is 59°.9, while at Colfax, which has an altitude 2,400 feet greater, the mean temperature for the same time is 60°.6, and hence greater. This reduced to the level of Sacramento with any ordinary rate of decrease of temperature with increase of altitude would give a great increase of mean temperature from Sacramento to Colfax, a distance of only 45 miles. Of course there cannot be any but a very small general gradient extending a considerable distance in that direction, and the above result shows the great uncertainty in temperature observations at the earth's surface, arising from great local variations, or from differences of positions of the thermometers with regard to elevations above the surface, and from other causes. Hence the observations of surface temperatures at the two stations, even in the case of annual means, cannot be relied upon to give the average temperature of the air column between the level of the two stations. The difference above of 24 feet between the true and computed difference of altitude corresponds to an error in the mean temperature of the air column of about 2°. But a considerable part of the error in the computed value of H may be due to a gradient of increasing mean barometric pressure in the direction of Summit from Sacramento. A gradient which would cause the barometric pressure at Summit to be only 0.02 inches higher than at Sacramento, up at the same level, would account for the difference of 24 feet. It is not improbable that there is a gradient of that magnitude due to local causes of no great extent, but that there is such a gradient extending to a considerable distance is not probable.

Where there is a barometric gradient in the mean annual pressure, there is also an annual inequality in this gradient, and hence a small part of the annual inequality in the value of H, as given by the formula, may be due to this cause, but it is mostly due, no doubt, to errors in the average temperature of the air column, obtained from the mean of the two stations.

With the values of $(t'-t)$ in the preceeding table of results we get the last two columns, showing the rate of decrease of temperature with increase of altitude. The annual inequality in the monthly rates is very large, this rate being nearly twice as large in April as in October. This arises from the more rapid increase of temperature in the Sacramento Valley in the spring, than at Summit, where the increase of temperature in the spring is retarded by the melting of the snow on the mountains. The annual mean of this rate of decrease of temperature is less than usual, but if it had been determined by comparing the observations of temperatures at Colfax, instead of Sacramento, with those at Summit, the rate of decrease would have been found to be nearly twice as great. This shows the uncertainty in the rates of decrease of temperature with increase of altitude as determined from surface observations on the slopes of mountain sides.

25. Taking the average of observations made by Professor Whitney at Sacramento and Summit for the hours of 7 a. m., 2 p. m., and 9 p. m., we get the following averages:

Hour.	Observations.				Results.				
	B'	B	t'	t	H	Δ	$t'-t$	Rate of decrease— per 100 ft.	Rate of decrease— per 100^m.
	Inches.	*Inches.*	°	°			°	°F.	°C.
7 a. m...	30.033	23.291	52.4	38.2	6897	−69	14.2	0.20	0.37
2 p. m...	30.002	23.281	69.0	49.4	7089	+123	19.6	0.28	0.51
9 p. m...	30.007	23.293	57.3	38.7	6911	−55	18.6	0.27	0.49

These observations show a large range in the diurnal inequality of temperature at both stations, while there is scarcely any corresponding change in the barometric pressures. Between

7 a. m. and 2 p. m. the means of the temperatures at the two stations differ 13°.9 while the differences of the pressures, B′—B, differ only 0.021 inch, showing that the density of the air column, and consequently the true average temperature, is very little affected by the diurnal inequality of temperature. This inequality of temperature, therefore, must take place in the strata only very near the earth's surface, where the observations are made, and only in a comparatively small degree in the strata a little above the earth's surface. The period of the inequality is too short for the upper strata to become affected much, since in clear weather they absorb and radiate but little heat, and in cloudy weather the observed diurnal range of temperature at the surface is very small.

By using the values of $\frac{1}{2}(t'+t)$ in the formula, obtained from surface observations, instead of the true temperature of the air column, we get the values of H in the preceding table, which are too small from the morning and evening observations and much too large from the observations of 2 p. m., because the value of $\frac{1}{2}(t'+t)$ is less than the average temperature of the air-column morning and evening, and much greater at 2 p. m. If the mean temperature of the day had been used throughout, instead of the observed temperatures at 7 a. m., 2 p. m., and 9 p. m., the values of H for each of the hours of observation would have differed but little. In barometric hypsometry, therefore, observations should be made at such hours of the day as will give the mean temperature of the day.

From the averages of the preceding table we have—

Hour.	B′—B	$\frac{1}{2}(t'+t)$	$\Delta\tau$	Δ'
	Inches.	°	°	°
7 a. m...	6.742	45.3	−5.5	−0.9
2 p. m...	6.721	59.2	+8.4	+0.3
9 p. m...	6.714	48.0	−2.8	−0.7
Mean ...	6.726	50.8	....	....

In this table $\Delta\tau$ represents the excess of $\frac{1}{2}(t'+t)$ above the mean, and $\Delta'\tau$ represents the excess of the true average temperature of the air-column at the several hours of observation above the mean of the three, upon the hypothesis that the density of the air-column is affected only by temperature. The last two columns above indicate that the latter is very small in comparison with the former.

The same is shown from the bi-hourly observations at Geneva and St. Bernard. From the averages for the month of September, obtained from six years' observations, we get from Rühlman, p. 61, the following results:

Hour.	B′—B	$\frac{1}{2}(t'+t)$	Δ	$\Delta'\tau$
	mm.	°C.	°	°
Noon	160.33	11.7	+2.3	−0.1
2 h.......	159.95	12.8	3.1	+0.6
4 h.......	159.68	11.9	2.7	1.1
6 h.......	159.69	10.5	+1.3	1.0
8 h.......	160.04	9.2	0.0	0.4
10 h.......	160.17	8.3	−0.9	+0.2
Midnight .	160.30	7.7	1.5	0.0
2 h.......	160.38	6.9	2.3	−0.2
4 h.......	160.62	6.2	3.0	0.6
6 h.......	160.86	6.5	2.7	1.0
8 h.......	160.80	8.7	−0.5	0.9
10 h.......	160.65	10.5	+1.3	−0.6
Mean	160.29	9.2	....	....

The numbers in this table for the hours of the latter part of the night cannot be regarded as being very exact, since they were obtained by Professor Plantamour by interpolation and not from actual observation. They are sufficiently accurate however to show, as is seen from the last two columns in the table above, that the diurnal inequality in the true average temperature of the air-column is only about one-third of that of the mean of the surface observations at Geneva and St. Bernard. They also show that the vanishing nodes and epochs of maxima and minima do not exactly coincide.

26. The following twelve-year averages (1864–1875) of barometric pressure, temperature, and relative humidity R, for the several places in Switzerland contained in the first column of the following table, are taken from the Zeit. der Oest. Gesell. für Meteorologie, B. 12, S. 116:

Place.	Bar.	Temp.	R	Altitude.	At the level of Geneva.		Computed altitude.
					Bar.	Temp.	
	mm.			*m.*	*m.*	°	*m.*
St. Bernard	564.1	−1.3	(78)	2478	....	....	2476
Sils	612.9	1.6	76	1810	....	....	1800
Gräichen	626.4	4.2	....	1632	....	....	1633
Chaumont	664.6	5.6	81	1150	....	....	1149
Trogen	682.6	6.8	....	924	726.5	9.2	925
Berne	712.5	8.1	77	574	726.8	9.1	573
Neuchâtel	719.7	9.0	76	488	726.7	9.4	492
Allstäten	720.6	8.6	78	478	726.7	9.0	479
Zürich	721.5	8.7	81	470	726.9	9.0	469
Geneva	726.8	9.7	76	408	726.8	9.7	....
Basel	738.2	9.3	76	278	726.8	8.6	281
Castesegna	701.1	9.7	66	700	726.1	11.3	699
Lugano	737.4	11.6	74	275	726.0	10.9	277

By reducing the barometer and temperature of all the stations having an altitude less than 1,000 meters to the level of Geneva, the former by the method given in §33 and the latter by the rate of 0°.57 per 100 meters of difference of altitude, we can determine very nearly the barometric and temperature gradients at the level of Geneva. The uncertainties in the reductions for the differences between the altitude of Geneva and the rest of these stations are very large. Neglecting very small irregularities, which may be supposed to be due to local causes, we get from these reduced pressures and temperatures the small chart, Fig. 2, showing the mean annual pressures and temperatures for all places at the level of Geneva. This chart gives for St. Bernard, at the level of Geneva, $B''=726^{mm}.5$ and $t''=10°.6$, which have been used in the computation of the altitude of St. Bernard in § 23. The chart shows a small gradient of barometric pressure increasing in the direction of N.NW., which indicates that Switzerland is a little south of the maximum of the ridge of mean annual pressure extending from the latitude of 30° or 35° in the Atlantic Ocean over Spain and France into the interior of Asia.

It is not stated in what way the altitudes given in the preceding table have been determined, but it has been supposed here that those at least of a less altitude than 1,000 meters have been determined from actual leveling. If, however, some of them have been determined barometrically, it has no doubt been done by a comparison with some near station of which the true altitude was known, so that even in this case the gradient would not be much affected. But if all the altitudes had been determined in this way by a comparison of all the observations in all cases with those of the same place, as Geneva for instance, then in reducing the observations to this level with the altitudes thus obtained, we, of course, would not get any gradient.

The altitudes in the last column of the preceding table have been computed from the barometric, temperature, and hygrometric observations of each place and those of Geneva, the barometric and temperature observations of the latter place being in each case corrected for the effect of the barometric and temperature gradients, or, in other words, the values of B'' and t'' required in the formula were taken from the chart, Fig. 2, for the latitude and longitude of St. Bernard. There is mostly a satisfactory agreement between the given and computed altitudes, except in the case of Sils. Perhaps the given altitude was determined barometrically by a comparison of barometric observations with those of Geneva, without taking into account the effect of barometric and temperature gradients, for in this way we would obtain an altitude too great.

If all the altitudes were computed in the same way with reference to each of the other stations as has been done with reference to Geneva, of course we should in each case obtain somewhat different results, owing to the small, unavoidable inaccuracies in the observations and other data, but the errors arising from the effects of the barometric and temperature gradients would be eliminated.

27. From the same place in the Zeit. der Oes. Gesell. für Meteorologie, from which the preceding averages have been copied, we likewise extract the following monthly averages from twelve years' observations:

Month.	St. Bernard.			Geneva.			Basil.		Lugano.	
	B	t	R	B	t	R	B	t	B	t
	mm.	°		mm.	°		mm.	°	mm.	°
January	561.1	−8.1	82	727.5	0.6	85	738.8	0.4	738.7	1.3
February	561.7	−7.8	80	727.9	2.3	81	739.3	2.2	738.7	5.6
March	558.7	−7.4	80	723.8	4.6	75	735.6	4.5	734.5	6.7
April	563.0	−2.5	73	726.1	9.7	68	737.6	9.9	736.6	12.1
May	564.8	1.4	73	725.9	13.8	69	737.4	13.6	736.7	15.8
June	567.4	4.1	74	727.6	16.9	68	738.9	16.6	737.3	19.2
July	569.0	7.5	74	727.5	19.6	68	738.5	19.3	737.3	22.0
August	568.5	6.3	77	727.7	18.0	71	738.7	17.4	737.4	20.4
September	568.7	5.0	79	728.4	15.8	75	739.5	15.0	739.3	17.8
October	564.0	−0.8	83	726.1	9.7	81	737.3	8.9	737.3	11.7
November	561.4	−5.4	83	726.1	4.6	82	737.5	4.1	736.8	6.4
December	560.9	−7.7	81	727.2	0.5	86	738.7	− 0.2	737.7	2.9

The values of R for St. Bernard are not given, and the ones here given, to be used in the computations, are the averages of Sils and Chaumont.

By comparing the values of B and t for Basil and Lugano, which are on opposite sides of the chart, Fig. 2, it is seen that the barometric and temperature gradients have only a very small annual inequality, the changes from month to month being mostly due to small uneliminated errors in the monthly averages. We may therefore assume, without material error, that the gradients are the same for all months of the year, and shall, therefore, as in the computations from the yearly averages in § 23, deduct the constant $0^{mm}.3$ from the barometer at Geneva to get the pressure at St. Bernard reduced to the level of Geneva, and add $0^{\circ}.8$ to the monthly values of t at Geneva to get the value of the temperature at St. Bernard reduced to the level of Geneva. This is supposed to give the temperature t'' required in the formula much more accurately than it could be obtained from reducing the observed temperature at St. Bernard, through so great a difference of altitude, to the level of Geneva by any observed rate of increase of temperature with decrease of altitude.

With the preceding reductions for the effects of the barometric and temperature gradients, we get the following data for computing the difference of the altitude between Geneva and St. Bernard from each of the monthly averages of the observations:

Month.	Monthly averages.				Results.						
	B''	B	t''	t	H	Δ	Δ'	t''—t	Change of t per 100m.	e	e
	mm.	mm.	°	°	m.	m.	m.	°	°	m.	m.
January	727.2	561.1	1.4	−8.1	2056.0	−11.6	−9.7	9.5	0.46	0.0	−6.0
February	727.6	561.7	3.1	−7.8	2060.0	− 7.6	7.1	10.9	0.53	0.0	5.6
March	723.5	558.7	5.4	−7.4	2068.1	+ 0.5	−2.6	12.8	0.62	+0.1	5.0
April	725.8	563.0	10.5	−2.5	2070.7	3.1	+2.6	13.0	0.63	0.9	2.6
May	727.6	564.8	14.6	1.4	2073.8	6.2	7.1	13.2	0.64	2.2	2.8
June	727.3	567.4	17.7	4.1	2077.8	11.2	9.7	13.6	0.66	1.5	1.1
July	727.2	569.0	20.4	7.5	2077.1	9.5	9.7	12.9	0.62	2.3	0.9
August	727.4	568.5	18.8	6.3	2076.0	8.4	7.1	12.5	0.60	1.8	1.3
September	728.1	568.7	16.6	5.0	2067.8	+ 0.2	+2.6	11.6	0.56	+1.3	1.8
October	725.8	564.0	10.5	−0.8	2063.8	− 3.8	−2.6	11.3	0.55	0.0	3.4
November	725.8	561.4	5.4	−5.4	2063.2	4.4	7.1	10.8	0.53	0.0	4.6
December	726.9	560.9	1.3	−7.7	2057.0	−10.6	−9.7	9.0	0.44	−0.3	−6.3
Year	726.5	564.1	10.6	−1.3	2067.6	....	....	11.8	0.57	+0.8	− 3.5

The relative humidities R′ and R of the lower and upper stations, respectively, to be used in the computations of H are contained in the preceding table. By computing with these data, as in the example in § 23, from the yearly averages, we get the several values of H above from each set of monthly averages of the observations. The average of the monthly values, $2067^m.6$, differs from the true difference of altitude, 2070^m, determined by leveling, only 2.4 meters, and from the value in § 23, computed from the yearly averages, it differs $0^m.6$. The reason of this latter difference has been given in the case of Sacramento and Summit in § 25.

The values of Δ above show that there is an annual inequality in the values of H computed from the monthly averages of observations, just as in the case of Sacramento and Summit, which is to be explained in the same way. If we suppose the most probable values of H given by the formula, independent of all abnormal irregularities, to be represented by the expression of H given in § 25, we get $B=10^m.0$, and $E=178^\circ.1$. This value of E indicates that the maxima and minima of H occur about the 1st of July and January, respectively, and the vanishing epochs of Δ about the 1st of April and October. These epochs, therefore, are a month later than in the case of Sacramento and Summit. With these values of B and E in the expression of H, § 25, we get the most probable values of H, to which correspond the values of Δ', which are the true monthly values of the annual inequality with the effects of the abnormal irregularities eliminated.

The mean rate of decrease of temperature with increase of altitude is considerably greater in this case than in that of Sacramento and Summit, and the annual inequality in this rate is also greater. The epochs of maximum and minimum also occur later, as in the case of the values of H.

If no hygrometric observations had been made, and Table IV had been used instead of Tables II and III, the values of H in the preceding table would have been increased by the amounts contained in the column *e*. These are all positive with one exception, and indicate that the vapor correction given by Table IV is a little too large for the higher temperatures. This is, no doubt, the case for high altitudes in mountainous regions, but not so generally, for, the temperature being the same, the amount of vapor in the air near the ocean is greater than in the interior of continents. Table IV, based upon Dr. Hann's empirical expression of the most probable or average value of the amount of vapor in the atmosphere, except for the lower temperatures, is, no doubt, as correct for general application under all circumstances as it can be made where the amount of vapor is regarded as a function of the temperature simply.

The last column in the preceding table, headed e', gives the errors which would have resulted if the effect of the aqueous vapor in the air had been taken into account by means of Laplace's modified temperature coefficient in the formula (25), .002 instead of .00183. It is seen that the errors throughout are negative and quite large for the low temperatures of the winter season, but small for the higher temperatures of the summer season.

28. From the reports of the Chief Signal Officer, United States Army, the following monthly and yearly averages of the values of B'', B, and t'', t are obtained from the monthly averages of observations made at Portland, Burlington, and the top of Mount Washington. The latitude of Mount Washington is 44° 16′, and the relative positions of the three places are shown by Fig. 3. The barometric pressures given are reduced to sea-level. These show that there is a small gradient of pressure increasing in the direction from Portland toward Burlington in the winter, and the reverse in summer. As Mount Washington is nearly on a right line from Portland to Burlington, and at about two-fifths of the distance, two-fifths of the difference between the pressure at Burlington and Portland have been added to that of the latter in order to eliminate the effect of the gradient, and thus the values of B'' have been obtained. The pressures on the top of Mount Washington were reduced to sea-level by means of the constant 6.31 inches from October, 1871, to March, 1874, and after that by the constant 6.36 inches. These constants have been deducted in order to get the values of B in the following table. The temperatures were reduced to sea-level and the effect of a temperature gradient eliminated in the same way as in the case of the barometric pressures. In

this way the following monthly averages were obtained as data for computing the height of Mount Washington:

Month.	Monthly averages.				Results.			
	B″	B	t″	t	H	Δ	Δ′	Decrease of t per 100m.
	Inches.	*Inches.*	°	°	*Feet.*	*Feet.*	*Feet.*	°C.
January	30.050	23.385	21.1	5.3	6348	+22	+27	0.47
February	29.979	23.354	23.0	5.6	6338	11	21	0.54
March	29.928	23.376	28.4	9.7	6335	+ 8	+ 9	0.53
April	29.930	23.534	41.4	20.1	6325	− 2	− 4	0.62
May	29.936	23.700	54.2	32.2	6305	21	16	0.63
June	29.929	23.814	65.0	44.0	6321	6	25	0.57
July	29.931	23.886	70.0	47.7	6296	30	27	0.61
August	30.009	23.943	69.9	47.3	6299	28	21	0.60
September	30.004	23.830	59.5	39.3	6311	−15	− 9	0.58
October	29.995	23.666	48.7	28.9	6337	+10	+ 4	0.56
November	29.967	23.497	34.1	15.5	6312	−14	16	0.55
December	30.000	23.325	23.3	6.4	6391	+64	+25	0.50
Year	29.971	23.609	44.7	25.2	6326.5	......	......	0.563

As the hygrometric observations are not given, the computations must be made, as in the case of Sacramento and Summit, by using Table IV instead of Tables II and III. We thus obtain the monthly values of H above. The mean of these is 6,326.5 feet. Two lines have been run with the spirit-level from the railroad station at Gorham to the top of Mount Washington, the first by W. A. Goodwin, civil engineer, in August, 1852, and the second by Capt. T. J. Cram, of the Topographical Engineers, for the Coast Survey, in September, 1853. From the first the top of Mount Washington was determined to be 6,285.5 feet above sea-level, and from the latter 6,293 feet. The mean of these is 6,289 feet. The preceding result, therefore, obtained barometrically, seems to be about 37 feet too great. By the same formula we have seen that the altitude obtained for Summit above Sacramento was 24 feet less than that obtained from the railroad survey.

From the values of Δ, or the most probable values Δ', in the preceding results, it is seen that here also we have an annual inequality in the values of H given by the formula, but we have the unusual result of the maximum of H occurring in the winter instead of summer. This indicates that the values of $\frac{1}{2}(t''+t)$ are greater than the average temperature of the air column in winter and smaller in summer, just the reverse of what is required to explain the discrepancies in all the other cases we have examined. This is probably due to the fact that at both of the lower stations, Portland on the sea-coast and Burlington on the east side of Lake Champlain, the thermometer was near a large body of water, which lowered the temperature in its vicinity in the summer and increased it in the winter, and thus made the value of $\frac{1}{2}(t''+t)$ too small in summer and too great in winter to represent the average temperature of the air above the earth's surface. The same would have occurred in the case of Summit, in California, if the barometric observations had been compared with those of San Francisco instead of Sacramento. At the former place the range of temperature is less than 8° Fahr., while at the latter it is about 27°. Hence the summer temperatures of San Francisco are nearly 10° less, and the winter temperatures as much greater than at Sacramento, while the difference in the pressures of the two places at the same level is very small. With the temperature observations, therefore, of San Francisco instead of Sacramento, we should have had the value of $\frac{1}{2}(t'+t)$ nearly 5° greater in winter and the same amount less in summer. This would have completely reversed the signs of the values of Δ in § 25, and given values of H having a very large annual inequality with its maximum in the winter instead of the summer, as in the case of Mount Washington.

Treating the values of Δ as in the preceding cases, we get B=26° and E=6°.4. The latter indicates that the maximum of H, with abnormal irregularities eliminated, occurs about the 6th of January, and the mean values, or vanishing epochs of Δ, about the 6th of April and October. These epochs are nearly the same as in the case of Geneva and St. Bernard, but the whole inequality is reversed. The rate of decrease of temperature with increase of altitude is very nearly the same as in Switzerland, both in the annual mean and for the several months of the year.

29. Finally, we have in the Boletin de la Sociedad Mexicana de Geographia y Estadistica, Tomo iv, p. 216, 1878, the following monthly averages of observations made three times a day for one year at Vera Cruz, latitude 19° 11′, and the city of Mexico, latitude 19° 25′, and longitude 99° 5′ W.:

Year and month.	Monthly averages.				Results.		
	B′	B	t′	t	H	Δ	Decrease of t per 100m
1877.	mm.	mm.	°	°	m.	m.	°
July	760.90	568.88	29.1	17.5	2278.7	− 0.9	0.51
August	761.44	587.61	29.9	17.5	2287.4	+ 7.8	0.54
September	760.22	586.65	29.2	16.5	2276.6	− 3.0	0.56
October	760.63	587.13	28.8	16.6	2272.9	− 6.7	0.52
November	763.11	586.95	25.5	14.1	2279.4	− 0.2	0.50
December	764.30	586.78	22.7	12.4	2276.0	− 3.6	0.45
1878.							
January	764.23	586.34	20.9	12.9	2276.4	− 3.2	0.33
February	761.60	585.57	22.2	14.1	2268.4	−11.2	0.36
March	760.87	585.89	24.3	16.2	2273.3	− 6.3	0.36
April	756.99	584.80	26.9	19.9	2306.0	+26.4	0.31
May	759.21	586.62	29.0	19.7	2278.1	− 1.5	0.41
June	759.49	586.72	30.5	18.9	2282.6	+ 3.0	0.51
Mean	....	....	......	......	2279.6	......	0.45

The monthly values of H here are a little more irregular than in the preceding cases, on account of there being only one year's observations from which to get the averages. The range of temperature being small, the annual inequality in the values of H is also small, and scarcely perceptible amidst the abnormal irregularities. The city of Mexico is about 250 miles west of Vera Cruz, but notwithstanding the distance there is perhaps very little barometric or temperature gradient between the two places, so that B′ and t' can be used for B″ and t'' without much error.

The barometer at Vera Cruz was 7.8 meters above sea-level. This, added to the mean value of H above, gives 2,287.4 meters for the altitude of the barometer at the city of Mexico above sea-level. The true altitude from railroad surveys is 2,282.5 meters, being nearly five meters less than the computed altitude. The rate of decrease of temperature with increase of altitude is very nearly the same as in California. The maximum rate is in the fall and the minimum in the spring, and the range of inequality large.

30. From the preceding comparisons it is seen that the excess of altitude given by the formula over that obtained from actual leveling is, for

Sacramento and Summit, 3 years' observations,	−24 feet.
Geneva and St. Bernard, 12 years' observations,	−2.6 meters.
Portland and Mount Washington, 6 years' observations,	+37 feet.
Vera Cruz and city of Mexico, 1 year's observations,	+ 5 meters.

These results do not indicate that high degree of accuracy in barometric hypsometry, even where a long series of observations is used, which was formerly supposed to be attainable by this means. In these comparisons that of Geneva and St. Bernard should have the most weight, both on account of the long-continued series of observations upon which the result is based, and also the great care with which the observations have been made. The observations have been regularly made for about forty years, and the result obtained from the whole series differs very little from that of the twelve years here used. The signs, however, of the differences between the altitudes given by the formula and those obtained by leveling, one half being plus and the other half minus, do not indicate any error in the principal constant of the formula, considering the greater weight which the comparison for Geneva and St. Bernard should have.

In the comparisons of the results from monthly averages, we have seen that the differences between the true altitude and those given by the formula are still greater, especially at certain seasons of the year, since there is an annual inequality in the results given by the formula, due to errors in obtaining the true temperature of the air column. We have also seen that there is a diurnal inequality of the same sort, even greater than the annual. These inequalities do not only differ in range at different places, and, in the case of the annual inequality at least, become entirely reversed, but the epochs of maxima and minima and of the vanishing nodes of the inequalities also differ considerably at different places. For Sacramento and Summit the latter occur the 1st of March and September, but for Geneva and St. Bernard on the 1st of April and September, and for Portland and Mount Washington still a few days later. It is probable that these epochs are nearly the same for the same country as for California or Switzerland, and if so, and these epochs have been determined, where only a few or a short series of observations are made for hypsometric purposes in any country, they should be made at or near the times of these epochs, or at least so taken that the effect of the annual inequality will be eliminated from the result. From what we have seen it would be useless to attempt to give tables of corrections for this inequality, as has been done in a few instances, which would be applicable even within a very limited range of country, for we have seen that the range of this inequality may not only change very much, but that in California it may become entirely reversed by referring Summit to San Francisco instead of Sacramento.

The effect of the diurnal inequality can be very nearly eliminated by taking the observations at such hour or hours of the day as will give the mean temperature of the day, but if these observations should be taken near either of the extremes of temperature, as the early morning or the afternoon, the results cannot be relied on, as is seen from the results given in the table of §25 in the case of Sacramento and Summit. In fact, at whatever hour of the day the barometric observations may be made it is much better to use the mean temperature of the day with them than the extreme temperatures. Where only a few observations are used those are best which are made at times when there is little diurnal change in the temperature, and when the diurnal average differs but little from the monthly average or normal temperature of the time of year. Such observations should not be taken when the air is foggy or misty, since the weight of the air is increased by the particles of fog or mist in it.

31. The variation of the true from the observed temperatures in the monthly averages arises from the fact that the annual range of temperature is less in the open air at some distance from the earth's surface than it is at the surface. The longer the period of the inequality the more nearly the temperatures should agree. While in the annual inequalities the variations of the true from the observed is generally only from about one-fifth to one-tenth of the whole amplitude of the inequalities, in the diurnal inequality it amounts to the greater part of it, the temperature of the upper strata undergoing but little change in comparison with that of the observed temperatures at the earth's surface.

In the numerous abnormal changes of temperature with periods of one to two weeks, the amplitudes of the changes in the air at some altitude above the surface must be less than those of the observed temperatures at the surface, but the differences must be less in proportion to the whole change than in the case of the diurnal changes, and greater than in the case of the annual changes of temperature. In barometric hypsometry, therefore, where the observed temperatures differ very much from the average normal temperature of the time of year at which they are made we will get better results, where only a few observations are used, by not using the extreme temperature, but some one intermediate between the observed and the normal temperature, just as in the case of monthly averages we get better results for the months of extreme temperatures by using temperatures which deviate a little less from the mean temperature of the year than the monthly averages of temperatures observed at the earth's surface.

In order to avoid the errors arising from using the extremes of the abnormal irregularities of temperature it is best to use the normal temperature for the lower station, obtained from monthly isothermal charts, where such are at hand, or from a table of monthly normals for the vicinity, which will be equivalent to supposing that the range of the true temperature of the air column in these abnormal irregularities of short period is only half as much as that of the observed surface temperatures, or even less, since the range of these abnormal inequalities is generally less at the upper than at the lower station.

32. The principal part of the difference between the true and observed temperature of the air column where only a few observations are used, arises from assuming that there is a regular decrease of temperature with increase of altitude. This may be nearly so in yearly, and even in monthly, averages, but we know that, for various reasons, the variations from this law are, at any given time, so great that the temperature may increase instead of decrease as you ascend, and, where the difference of altitude is considerable, the true mean temperature of the air column may differ from the mean of the observed temperatures of the two stations several degrees of Fahrenheit. Each one of these would affect the computed difference of altitude the $\frac{1}{491}$ part, and hence would give rise to large errors.

A large part of the errors in barometric hypsometry, where only a few observations are used, and the true stations are a considerable distance apart, arises from the local and temporary barometric gradients, depending upon the various cyclonic disturbances of the atmosphere. If the stations were several hundred miles apart, an ordinary gradient, such as occurs frequently without a great storm, might affect the result a hundred feet or more.

The differences of altitude, therefore, from one or even several days' observations cannot be relied upon as being more than a rough approximation to the true difference. This has been shown by Williamson, who has computed the difference of altitude between Geneva and St. Bernard from the observations for every day of the year 1862. These differ from the true difference of altitude, in some extreme cases, more than 60 meters. In these extreme cases, which occur mostly in the winter, the results were no doubt affected by the barometer gradients.

33. The last member of (35), taken positively, expresses the height of a column of air in meters corresponding to one millimeter in the barometer on the parallel of 45°. Reducing this to English measures we get for the expression in feet of such a column corresponding to one-tenth of an inch of the barometer,

$$\delta H = \frac{2632.5}{B}\left[1 + .002222\,(t - 32^\circ)\right]$$

This expression is adapted to the average hygrometric state of the atmosphere, and for this purpose it can be made a little more accurate by introducing the value of $1+f(e)$, of which the logarithm is given in Table IV, instead of that given in § 17. With this change we get

$$\delta H = \frac{2628.4}{B}\left[1 + .002034\,(t - 32^\circ)\right]\left[1 + f(e)\right]$$

As the last factor of this expression, as given in Table IV, is a function of the temperature, it has only the variables B and t. From this expression, with the use of this table, the values of δH have been computed for short intervals within certain limits of the two arguments B and t, and given in Table XIII. The differences between this table and Guyot's arise from its having been computed with the improved constants in the barometric formula, based upon the more recent and accurate determination by Regnault of the constants of nature upon which they depend, instead of the constants of Laplace's formula.

This table may be used in computing differences of altitudes without the use of logarithms, as follows: Take first the number from the table corresponding to the arguments B'' and t'', then the number corresponding to the arguments B and t, and finally the number corresponding to the arguments $\frac{1}{2}(B''+B)$ and $\frac{1}{2}(t''+t)$. Then take one-fifth of the sum of the first two and three times the latter and multiply this into $(B''-B)$, expressed in tenths of an inch, for the value of H in feet. Let us apply it to the example of Sacramento and Summit, given in § 24. The table gives for the arguments, using from necessity base B' and t' for B'' and t',

Inches. °	
$B' = 30.014$ and $t' = 59.9$,	93.19
$B = 23.288$ and $t = 42.1$,	115.56
$\frac{1}{2}(B'+B) = 26.651$ and $\frac{1}{2}(t'+t) = 51^\circ$,	$102.95 \times 3 = 308.85$
	5)517.60
	103.52

$$H = (B' - B) \times 103.52 = 67.26 \times 103.52 = 6963 \text{ feet.}$$

This value of H is only 3 feet less than that obtained in § 24. For much smaller differences of altitude it is only necessary to take out the number corresponding to the last two arguments, and to use this instead of the mean of the five. In the example above this would give $H=67.26\times 102.95=6925$, but the error diminishes very rapidly with decrease of difference of altitude, and for a thousand feet or more is of no consequence.

In Table XII the effect of the last two factors in the formula (25) is not taken into account, so that it is strictly correct for the parallel of 45° and sea-level. But the effect of these two terms in the middle latitudes is very small. Their effect upon the difference of altitude between Sacramento and Summit, nearly 7,000 feet, is only about 6 feet, and proportionally in the same latitude for smaller differences of altitude. The extreme effect of the factor depending upon the latitude, which is at the equator and the pole only $\frac{1}{384}$ of the whole difference of altitude. At the equator the values in Table XII are too small, and at the pole too large, in that proportion.

In order to have all the tables necessary in barometric hypsometry, Table XIII is added, which is reduced to English measures from Delcros's table.

CHAPTER III.

REDUCTION OF THE BAROMETER TO SEA-LEVEL.

33. In order to form a chart of isobars, showing the barometric gradients and the general distribution of pressure from barometric observations at different altitudes, it is necessary to reduce all these observations to some assumed level, which is generally that of sea-level. This is a problem somewhat the reverse of that of determining the difference of altitude of two stations from observations made at the two levels. The same equation (25) must be satisfied in both cases, but in the one H is the unknown quantity to be determined, and in the other B″, either of which can be determined when all the other quantities in the equation are known.

In the uncertainties of reduction to sea-level both of the last factors in (25), at least in the middle latitudes, may be neglected, and the whole effect of the factor for the vapor correction is so small that it is only necessary to use its average or most probable value, taken over the earth generally, for any given temperature, neglecting its variations at different times and localities. By so doing we have seen, § 27, that in the case of St. Bernard the greatest error in computing the altitude from the monthly averages of observations amounted to only 2.3 meters, or 7.5 feet, which at ordinary temperatures, as is seen from Table XII, correspond to only about 0.008 inch of barometric pressure.

We, therefore, get from (25), reduced to English measures,

$$\log B''=\log B+\frac{H}{60521.5\,[1+.001017\,(t''+t-64^\circ)]\,[1+f\,(e)]}$$

in which $[1+f\,(e)]$, represents, as in § 32, the average value of this factor regarded as a function of t, the logarithm of which is contained in Table IV. This expression may be put into the following form:

$$(a)\qquad \log B''=\log B+R$$

in which

$$(b)\qquad \log R\;=\log H-(\text{Table }1+\text{Table IV})$$

Where it is thought necessary, all the other tables can be used in this expression, just as in the computation of differences of altitude. For French measures the constant logarithm 0.51599 must be added to the expression of log R, and the tables must be entered with $\frac{9}{5}(t''+t)+64^\circ$ as an argument.

As an example of the application of these formulæ, let it be required to reduce the mean barometric pressure on the top of Mount Washington to sea-level. In this example we have, from the table of § 28, $B=23.609$ inches, $t''+t=69^\circ.9$, and the value of $H=6289$ feet.

Hence, we have

Log H	$=3.79858$	R	$=0.10303$
Table I, with arg. $69^{\circ}.9$,	$=4.78451$	log B	$=1.37308$
Table IV, with arg. $69^{\circ}.9$,	$=0.00110$	log B″	$=1.47611$
	4.78561	B″	$=29.930$ inches.
Log R	$=9.01297$		

As an example of application in the case of French measures, let it be required to reduce the mean pressure at St. Bernard to the level of Geneva. We have in this case, from § 23, $B=564^{mm}.1$, $t''+t=9^{\circ}.3$, and $H=2070^{m}$. Hence we have

Log H	$=3.31597$	R	$=0.10992$
Table I, arg. $\frac{9}{5}\times 9^{\circ}.3+64^{\circ}=80^{\circ}.7$,	$=4.78923$	log B	$=2.75136$
Table IV, arg. $\frac{9}{5}\times 9^{\circ}.3+64^{\circ}=80^{\circ}.7$,	$=0.00164$	log B″	$=2.86128$
	4.79087	B″	$=726^{mm}.6$
	8.52510		
Constant log	0.51599		
Log R	$=9.04109$		

34. Where $\frac{1}{2}(t''+t)$, as is usually the case, does not represent the true average temperature of the air column, of course we do not get the true reduction to sea-level, as in the computation of altitudes we get an erroneous result when this is the case. Where the correction of $t''+t$ is known, it can be applied to the argument, or where the errors in the values of H, as computed from yearly and monthly averages of observations are known, as in the case of Mount Washington and St. Bernard, these can be added to the true value of H, and then the formula will give the true reduction to sea-level, with $t''+t$ used as the true temperature. Putting

$$\Delta H = \text{the excess of the true over the computed value of H}$$

we shall have in place of (*b*)

$$(c) \qquad \text{Log } R=\log (H+\Delta H)-(\text{Table I}+\text{Table IV})$$

The value of t'' at sea-level for yearly and monthly averages may be determined for any given station, as has been done in § 27 in the case of St. Bernard for the yearly average at the level of Geneva, by means of temperature charts of which the small chart, Fig. 2, is a specimen. But for individual observations or short series of observations, made at any time of year or hour of the day, the value of t'' cannot be determined in this way. In such cases it is usual to put

$$t''=t+cH$$

in which c represents the rate of increase of temperature with decrease of elevation. But c is by no means a constant, as we have seen, for it is different for different localities and seasons, and also at different altitudes, even where we have the average rate for a large number of observations In individual cases not only is c in the preceding expression entirely unknown, but so great are the anomalies in the vertical distribution of temperature that the law of the expression entirely tails. The value of c is sometimes assumed to be a constant for all places and seasons, but this leads to great errors in reductions to sea-level at the seasons of extreme temperature. Take, for instance, the case of Salt Lake City, about 4,400 feet above the sea-level. The average temperature of July here is greater than that of the plateau between the Missouri River and the Rocky Mountains, with an average elevation about 3,000 feet less. We have seen that the value of t required in barometric hypsometry, and the same is the case here, is the temperature of the air generally around the mountain at the same elevation, and not the temperature observed at the heated surface of a mountain or in some elevated mountain valley. The observed value of t, then, is too great in such a case to represent the temperature of the air generally at the altitude of the observation, and it is readily seen that if this temperature is reduced to sea-level according to the preceding formula, with any average value of c for all seasons and places, we get a value of t'' which makes $\frac{1}{2}(t''+t)$ much too large in this case to represent the true temperature of the air generally away from the superheated surface of the earth or mountain; for both t'' and t are much greater than the temperature of the air generally at the respective levels of the sea and of the upper station.

35. The mean annual inequality of the error in the reductions to sea-level arising from these erroneous temperatures is corrected in (c) by means of $\varDelta$H, where the monthly values of $\varDelta$H have been determined, as at St. Bernard and Mount Washington, from computations of altitudes with the monthly averages of B″ and t'' determined from observations made at two or a number of surrounding stations. The mean diurnal inequality in the error of reduction to sea-level might be corrected in the same way if we had hourly observations to determine the diurnal inequality in the values of $\varDelta$H. We have seen in the case of Summit, in California, § 25, that this inequality is very large, and consequently the errors in reduction to sea-level must be very great where the extreme temperatures of the day are used. In fact it is seen from the comparisons of $\varDelta\tau$ with $\varDelta'\tau$, in the tables of § 25, that the range of the diurnal inequality in the true temperature of the air column is very small in comparison with that of the observed temperatures of the two stations, and hence the temperature which should be used in the case of the extreme temperatures of the day should deviate very little from the mean temperature of the day, and this is especially the case for Summit. Where the observed temperatures deviate from the mean of the day, so far as we now know from the investigation of only two cases, it would be best to add only one-fourth of this deviation to the mean temperature of the day, excluding the effect of the other three-fourths, where there are any means of determining this mean temperature. At any rate the extreme temperatures of the day, especially when the diurnal range is great, must not be used, if we even have to rely simply upon an exercise of good judgment in determining what temperature should be used.

In order to avoid the errors from using the extremes of the abnormal inequalities, it will be best in reductions to sea-level, as in barometric hypsometry, to use for t'' its normal value for the season of the year, obtained from the monthly normals, as explained in § 31. This will diminish the effect of these observed deviations from the normal temperature one-half or more, and make it generally correspond very nearly with what would be given by the true average temperature of the air column.

36. Since B″, in the expression of (a), is a function of three variables, B, H, and $(t''+t)$, it is not convenient to give tables for obtaining its value under all circumstances, unless the tables are very much expanded. Where, however, reductions to sea-level are required to be continually made for the same station, as in the Weather Bureau of the Signal Service, H, with regard to this station, becomes a constant, and the expression of B″ can be reduced to a linear function of only two variables, and hence requiring only two tables with a single argument each. These are increased to three where we use (c) instead of (b), in order to correct for the annual inequality of the error from using $\frac{1}{2}(t''+t)$ for the true temperature.

For the same station we can put

(d) $$B=B_0+\varDelta B$$

in which B_0 is a value of B, in round numbers for convenience, which is nearly an average value of B in its abnormal fluctuations from various causes. We can then put

(e) $$B''=B''_0+\varDelta B''$$

in which

(f) $$\begin{cases} \log B''_0=\log B_0+R \\ \log R \;=\log H-(\text{Table I}+\text{Table IV}) \end{cases}$$

We get from the differentiation of (a), or its equivalent preceding, where $\varDelta$B and $\varDelta$H are quantities of a second order so small that quantities of lower orders may be neglected,

$$\varDelta B''=\frac{B''_0}{B_0}B\varDelta+\frac{\varDelta H}{60521.5\ M[1+.001017(t''+t-64°)]\,[1+f(e)]}$$

in which M is the modulus of common logarithms.

This expression of $\varDelta B''$ in (e) gives

$$(g) \qquad B'' = B''_0 + \frac{B''_0}{B_0}\varDelta B + \frac{\varDelta H}{60521.5\ M[1+.001017(t''+t-64^{\circ})][1+f(e)]}$$

The last term of this expression comes in where a value of H a little different from the true value is required, in order to correct for error in assuming that $\frac{1}{2}(t''+t)$ is the true temperature to be used in the formula.

The first term of this expression is given by (f), from which it is seen that it is a function of only one variable $(t''+t)$ entering into the arguments of the tables, B_0 and H being known constant quantities for any given station. Hence, B''_0 is readily given by a table with $t''+t$ as an argument.

The coefficient of $\varDelta B$ in the second term of (g) is not strictly a constant, since the value of B''_0 in (f) depends upon the temperature; but by using the mean value of B''_0 it may be regarded as a constant without material error except in some extreme cases in which the variations of both B and the temperature from their mean values are very great. This term, then, can be reduced to a small table with $\varDelta B$ as an argument. Or, if thought necessary, the variable value of B''_0 can be used, and a small table can be formed having $\varDelta B$ and $(t''+t)$ as arguments.

The last term in (g) is also a function of the temperature, but this term is so small that the mean value of $(t''+t)$ in the denominator may be used without sensible error. When $\varDelta H$ is known this can be reduced to a small table, with $\varDelta H$ as an argument. In general only the mean monthly values of $\varDelta H$ are known for any station, determined as in the preceding sections for Summit, Mount Washington, and St. Bernard. In such cases only the mean annual inequality in the value of B''_0, arising from errors of temperature, can be taken into account, and all the remaining part depending upon the irregular abnormal disturbances must necessarily be neglected. Using the monthly values of $\varDelta H$, a small table can be formed with the time of the year as an argument. Where the value of $\varDelta H$ has not been determined for any station, of course this inequality can be determined in a more direct way by reducing by means of (a) and (b) the monthly values of B to sea-level, and then comparing them with the true monthly values of B'', determined, as in the case of St. Bernard for the level of Geneva, by means of monthly charts of barometric pressure similar to that of Fig. 2.

37. The following is a specimen of such a set of tables, made in the case of St. Bernard to reduce barometric observations to the level of Geneva. The values of $\varDelta H$, used in the last term of (g), are the differences between the computed values in the table of §27 and the true value, 2,070 meters. In order to get rid of the abnormal irregularities in these values, $2067.6+\varDelta'$ have been used for the most probable values of H. The value of B_0 in (d) has been assumed to be 560^{mm}, which is nearly its mean value.

TABLE I.

$t''+t$	B''_0	Diff.
°C.	mm.	
− 20	732.0	− 1.8
15	731.2	1.9
10	728.3	1.8
− 5	726.5	1.8
0	724.7	1.8
+ 5	722.9	1.8
10	721.1	1.8
15	719.3	1.7
20	717.6	1.7
25	715.9	1.6
30	714.3	1.7
35	712.6	1.6
40	711.0	1.6
45	709.4	1.5
50	707.9	1.5
55	706.4	1.5
+ 60	704.9	

TABLE II.

$\varDelta B$	Correction.
mm.	mm.
1	1.3
2	2.6
3	3.9
4	5.2
5	6.6
6	7.9
7	9.2
8	10.5
9	11.9
10	13.2
11	14.5
12	15.8
13	17.1
14	18.4
15	19.8
16	21.1
17	22.4

TABLE III.

Month.	Correction.	
	m.	
January	− 1.5	1.4
February	1.2	3.1
March	− 0.6	5.4
April	0.0	10.5
May	+ 0.6	14.6
June	0.9	17.7
July	0.9	20.4
August	+ 0.6	18.8
September	8.0	16.6
October	− 0.6	10.5
November	1.2	5.4
December	− 1.5	1.3
Year	0.3	10.6

When ΔB in Table II is negative, the correction is also negative.

In connection with Table III the mean monthly values of t'' are given, to be used in obtaining the argument in the first table, as proposed in § 35. When t is observed at a time of day which does not give a mean temperature, a value of t which does not differ much from the mean must be used, as already explained. Of course this leaves some uncertainty with regard to the proper value of t to be used generally, but this cannot be avoided, since it is impossible to take into account, in all special cases, the abnormal variations of $\frac{1}{2}(t''+t)$ from the true temperature of the air column.

As an example of the application of the preceding tables, let us suppose that we have observed on the 1st of May at St. Bernard, at a time of day which gives the mean temperature of the day, or nearly, the value of $B=573^{mm}.5$ and $t=0^{\circ}.3$. From (d) we get, in this case, $\Delta B=573.5-560=13^{mm}.5$; and from Table III, $t''=12^{\circ}.5$. Hence $t''+t=12^{\circ}.5+0^{\circ}.3=12^{\circ}.8$. We therefore have

	mm.
Table I, argument $12^{\circ}.8$,	720.1
Table II, argument $13^{mm}.5$,	17.7
Table III, argument May 1,	0.3
	$B''=738.1$

If the time of the maximum of the correction in Table III coincides with that of the greatest temperature of the year, the correction might be included in Table I without sensible error. For St. Bernard the times of these maxima do not differ very much, but in many places, as Summit in California, the difference is nearly two months, so that in such cases the correction of Table III cannot be regarded as a function simply of the temperature.

HYPSOMETRICAL TABLES.

TABLE I.

Containing log 60521.5 [1+.001017 ($t''+t-64°$)]: *Argument*, ($t''+t$).

$t''+t$	Log.	$t''+t$	Log.	$t''+t$	Log.	$t''+t$	Log.	$t''+t$	Log.	$t''+t$	Log.
°		°		°		°		°		°	
0	4.75270	30	4.76665	60	4.78014	90	4.79324	120	4.80596	150	4.81832
1	4.75317	31	4.76711	61	4.78059	91	4.79367	121	4.80638	151	4.81873
2	4.75364	32	4.76756	62	4.78103	92	4.79410	122	4.80680	152	4.81913
3	4.75411	33	4.76802	63	4.78147	93	4.79453	123	4.80722	153	4.81954
4	4.75458	34	4.76847	64	4.78191	94	4.79496	124	4.80763	154	4.81994
5	4.75505	35	4.76893	65	4.78235	95	4.79539	125	4.80805	155	4.82035
6	4.75552	36	4.76938	66	4.78279	96	4.79582	126	4.80846	156	4.82075
7	4.75599	37	4.76984	67	4.78323	97	4.79625	127	4.80888	157	4.82116
8	4.75646	38	4.77029	68	4.78367	98	4.79668	128	4.80930	158	4.82156
9	4.75693	39	4.77174	69	4.78411	99	4.79711	129	4.89972	159	4.82197
10	4.75739	40	4.77119	70	4.78455	100	4.79753	130	4.81013	160	4.82237
11	4.75786	41	4.77164	71	4.78499	101	4.79796	131	4.81054	161	4.82277
12	4.75833	42	4.77209	72	4.78543	102	4.79838	132	4.81095	162	4.82317
13	4.75880	43	4.77254	73	4.78587	103	4.79881	133	4.81137	163	4.82357
14	4.75926	44	4.77299	74	4.78630	104	4.79923	134	4.81178	164	4.82397
15	4.75973	45	4.77344	75	4.78674	105	4.79966	135	4.81219	165	4.82437
16	4.76019	46	4.77389	76	4.78717	106	4.80008	136	4.81260	166	4.82477
17	4.76066	47	4.77434	77	4.78761	107	4.80050	137	4.81301	167	4.82517
18	4.76112	48	4.77479	78	4.78804	108	4.80092	138	4.81342	168	4.82557
19	4.76159	49	4.77524	79	4.78848	109	4.80135	139	4.81383	169	4.82597
20	4.76205	50	4.77569	80	4.78892	110	4.80177	140	4.81424	170	4.82637
21	4.76251	51	4.77614	81	4.78936	111	4.80219	141	4.81465	171	4.82677
22	4.76297	52	4.77658	82	4.78979	112	4.80261	142	4.81506	172	4.82717
23	4.76343	53	4.77703	83	4.79022	113	4.80303	143	4.81547	173	4.82757
24	4.76389	54	4.77748	84	4.79065	114	4.80345	144	4.81588	174	4.82796
25	4.76435	55	4.77793	85	4.79109	115	4.80387	145	4.81629	175	4.82836
26	4.76481	56	4.77837	86	4.79152	116	4.80429	146	4.81669	176	4.82875
27	4.76527	57	4.77882	87	4.79195	117	4.80471	147	4.80710	177	4.82915
28	4.76573	58	4.77926	88	4.79238	118	4.80513	148	4.81751	178	4.82955
29	4.76619	59	4.77970	89	4.79281	119	4.80555	149	4.81792	179	4.82995
30	4.76665	60	4.78014	90	4.79324	120	4.80596	150	4.81832	180	4.83034

MULTIPLES OF THE DIFFERENCES.

1	47	46	45	44	43	42	41	40	39	1
2	94	92	90	88	86	84	82	80	78	2
3	141	138	135	132	129	126	123	120	117	3
4	188	184	180	176	172	168	164	160	156	4
5	235	230	225	220	215	210	205	200	195	5
6	282	276	270	264	258	252	246	240	234	6
7	329	322	315	308	301	294	287	280	273	7
8	376	368	360	352	344	336	328	320	312	8
9	423	414	405	396	387	378	369	360	351	9

TABLE II.

Containing $\log\left(1+0.189\,\frac{b_1}{B}\right)$ *in units of the fifth decimal place: Arguments,* B *and* b_1.

B.	b_1 in inches.																				B.
	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95	1.00	
In.																					*In.*
11	36	73	109	146	183	220	257	296	338	372	408	445	481	518	555	595	631	669	705	741	11
12	34	68	102	136	170	204	238	273	307	341	374	408	442	476	510	544	578	612	646	678	12
13	32	64	95	126	158	189	221	252	283	314	345	376	408	439	471	502	533	564	596	626	13
14	30	59	88	117	147	176	205	234	263	292	321	350	379	408	437	466	495	524	553	582	14
15	28	55	82	109	137	164	191	218	246	273	300	327	354	381	408	435	463	490	517	544	15
16	26	51	77	102	128	153	179	205	231	256	282	307	333	358	383	408	434	459	485	510	16
17	24	48	72	96	121	145	169	193	217	241	265	289	313	337	361	384	408	432	456	480	17
18	23	46	69	91	114	137	160	182	205	227	250	273	296	318	341	363	386	408	431	454	18
19	21	43	65	86	108	129	151	173	194	215	237	258	280	301	323	344	366	387	409	430	19
20	20	41	61	82	102	123	143	164	184	205	225	246	266	286	307	327	348	368	388	408	20
21	19	39	58	78	97	117	136	156	175	195	214	234	253	273	292	312	331	350	369	389	21
22	18	37	56	75	94	112	131	149	168	186	205	223	242	260	279	297	316	334	353	371	22
23	18	36	54	71	89	107	125	143	161	178	196	214	232	249	267	285	303	320	338	355	23
24	17	34	51	68	85	103	120	137	154	171	188	205	222	239	256	273	290	307	324	341	24
25	17	33	50	66	82	98	115	131	148	164	181	197	213	229	246	261	278	294	311	328	25
26	16	32	48	63	79	95	111	126	142	158	174	189	205	220	236	252	268	283	299	315	26
27	15	30	46	61	76	91	106	121	137	152	167	182	175	212	227	242	258	273	288	303	27
28	15	29	44	59	74	88	103	117	132	146	161	176	191	205	220	234	249	263	278	292	28
29	14	28	42	56	71	85	99	113	127	141	155	169	184	198	212	226	240	254	268	282	29
30	14	27	41	55	69	82	96	109	123	137	151	164	178	191	205	218	232	245	259	273	30
31	13	26	40	53	66	79	93	106	119	132	145	158	172	185	198	211	225	238	251	264	31

NOTE.—When B and b_1 are given in millimeters multiply both by .04 or any other number that will bring them within the range of the arguments in the table.

TABLE III.

Containing $\log\,[1-.000084\,(t-t_1)]$*: Argument,* $(t-t_1)$.

$t-t_1$	Comp. of log.	$t-t_1$	Comp. of log.
° F.		° F.	
1	−0.00004	16	−0.00059
2	7	17	62
3	11	18	66
4	14	19	69
5	18	20	73
6	22	21	77
7	25	22	80
8	29	23	84
9	33	24	88
10	37	25	92
11	41	26	96
12	44	27	99
13	48	28	103
14	51	29	106
15	−0.00055	30	−0.00110

NOTE.—When t_1 is less than 32° F., deduct ½ from the logarithm.

TABLE IV.

To be used in place of Tables II and III where no hygrometric observations are made.

$t'+t$	Log.
° F.	
0	0.00030
10	37
20	45
30	53
40	62
50	73
60	88
70	110
80	138
90	175
100	215
110	256
120	297
130	338
140	379
150	420
160	460
170	501
180	0.00542

TABLE V.

Containing log $\left(1+\frac{2h'}{r}\right)$*: Argument,* h'.

h'	Log.	h'
Feet.		*Meters.*
100	0.00000	30
200	1	61
300	1	91
400	2	122
500	2	152
600	2	183
700	3	213
800	3	244
900	4	274
1,000	4	305
2,000	8	610
3,000	12	914
4,000	16	1,219
5,000	21	1,524
6,000	25	1,829
7,000	29	2,134
8,000	33	2,438
9,000	37	2,743
10,000	0.00041	3,048

TABLE VI.

Containing log $\left(1+\frac{H}{r}\right)$*: Argument, log* H.

Log H.	Log.
2.5	0.00001
3.0	2
3.1	3
3.2	3
3.3	4
3.4	5
3.5	7
3.6	8
3.7	10
3.8	13
3.9	17
4.0	21
4.1	26
4.2	33
4.3	0.00041

TABLE VII.

Containing log (1+.002606 cos 2λ): Argument, λ.

λ +	Log.	λ −	λ +	Log.	λ −
°		°	°		°
0	0. 00113	90	23	0. 00079	67
1	113	89	24	76	66
2	113	88	25	73	65
3	113	87	26	70	64
4	113	86	27	67	63
5	112	85	28	63	62
6	112	84	29	60	61
7	111	83	30	57	60
8	110	82	31	54	59
9	108	81	32	50	58
10	107	80	33	46	57
11	105	79	34	43	56
12	104	78	35	39	55
13	102	77	36	35	54
14	101	76	37	31	53
15	99	75	38	28	52
16	97	74	39	24	51
17	94	73	40	20	50
18	92	72	41	16	49
19	89	71	42	12	48
20	87	70	43	8	47
21	84	69	44	4	46
22	0. 00082	68	45	0. 00000	45

TABLE VIII.

Containing log [1—.0000895 (τ′—τ)]: Argument, (τ′—τ).

τ′—τ	Comp. of log.	τ′—τ	Comp. of log.
°		°	
1	—0. 00004	21	—0. 00082
2	8	22	85
3	12	23	89
4	16	24	93
5	19	25	97
6	23	26	101
7	27	27	105
8	31	28	109
9	35	29	113
10	39	30	116
11	43	31	120
12	47	32	124
13	51	33	128
14	54	34	132
15	58	35	136
16	62	36	140
17	66	37	144
18	70	38	147
19	74	39	151
20	—0. 00078	40	—0. 00155

TABLE IX.

Containing the tension of aqueous vapor in saturated air, f, according to Regnault, expressed in inches of mercury, with t in degrees Fahrenheit as an argument.

t	Tension.	t	Tension.	t	Tension.	t	Tension.
°	*Inches.*	°	*Inches.*	°	*Inches.*	°	*Inches.*
0	0. 043	25	0. 135	50	0. 361	75	0. 868
1	0. 045	26	0. 141	51	0. 374	76	0. 897
2	0. 048	27	0. 147	52	0. 388	77	0. 927
3	0. 050	28	0. 153	53	0. 403	78	0. 958
4	0. 052	29	0. 160	54	0. 418	79	0. 990
5	0. 054	30	0. 167	55	0. 433	80	1. 023
6	0. 057	31	0. 174	56	0. 449	81	1. 057
7	0. 060	32	0. 181	57	0. 465	82	1. 092
8	0. 062	33	0. 188	58	0. 482	83	1. 128
9	0. 065	34	0. 196	59	0. 500	84	1. 165
10	0. 068	35	0. 204	60	0. 518	85	1. 203
11	0. 072	36	0. 212	61	0. 536	86	1. 242
12	0. 075	37	0. 220	62	0. 556	87	1. 283
13	0. 078	38	0. 229	63	0. 576	88	1. 332
14	0. 082	39	0. 238	64	0. 596	89	1. 366
15	0. 086	40	0. 248	65	0. 617	90	1. 410
16	0. 090	41	0. 257	66	0. 639	91	1, 455
17	0. 094	42	0. 267	67	0. 662	92	1. 501
18	0. 098	43	0. 277	68	0. 685	93	1. 548
19	0. 103	44	0. 288	69	0. 708	94	1. 597
20	0. 108	45	0. 299	70	0. 733	95	1. 647
21	0. 113	46	0. 311	71	0. 758	96	1. 698
22	0. 118	47	0. 323	72	0. 784	97	1. 751
23	0. 123	48	0. 335	73	0. 811	98	1. 805
24	0. 129	49	0. 348	74	0. 839	99	1. 861

TABLE X.

Containing the tension of aqueous vapor in saturated air, f, expressed in millimeters of mercury, with t in degrees Centigrade as an argument.

t	Tension.	t	Tension.	t	Tension.	t	Tension.
°	*mm.*	°	*mm.*	°	*mm.*	°	*mm.*
—18	1. 08	— 4	3. 39	+10	9. 17	+24	22. 18
17	1. 17	3	3. 66	11	9. 79	25	23. 55
16	1. 27	2	3. 96	12	10. 46	26	24. 99
15	1. 38	— 1	4. 27	13	11. 16	27	26. 51
14	1. 50	0	4. 60	14	11. 91	28	28. 10
13	1. 63	+ 1	4. 94	15	12. 70	29	29. 78
12	1. 77	2	5. 30	16	13. 54	30	31. 55
11	1. 92	3	5. 69	17	14. 42	31	33. 40
10	2. 08	4	6. 10	18	15. 36	32	35. 36
9	2. 26	5	6. 53	19	16. 35	33	37. 41
8	2. 46	6	7. 00	20	17. 39	34	39. 56
7	2. 67	7	7. 49	21	18. 50	35	41. 83
6	2. 89	8	8. 02	22	19. 66	36	44. 20
— 5	3. 13	+ 9	8. 57	+23	20. 89	+37	46. 77

TABLE XI.

Containing $A = 60521.5\ (1 + .001017 \times 36^{\circ})\ log\ \frac{30}{B}$: *Argument, B.*

B	A	Diff. for .01	B	A	Diff. for .01	B	A	Diff. for .01	B	A	Diff. for .01
Inches.	*Feet.*	*Feet.*	*Inches.*	*Feet.*	*Feet.*	*Inches.*	*Feet.*	*Feet.*	*Inches.*	*Feet.*	*Feet.*
11.0	27,336	−24.6	16.0	17,127	−16.9	21.0	9,718	−12.9	26.0	3,899	−10.5
11.1	27,090	24.4	16.1	16,958	16.9	21.1	9,589	12.9	26.1	3,794	10.4
11.2	26,846	24.2	16.2	16,789	16.8	21.2	9,460	12.8	26.2	3,690	10.4
11.3	26,604	24.0	16.3	16,621	16.7	21.3	9,332	12.8	26.3	3,586	10.3
11.4	26,364	23.8	16.4	16,454	16.6	21.4	9,204	12.7	26.4	3,483	10.3
11.5	26,126	23.6	16.5	16,288	16.4	21.5	9,077	12.6	26.5	3,380	10.3
11.6	25,890	23.4	16.6	16,124	16.3	21.6	8,951	12.6	26.6	3,277	10.2
11.7	25,656	23.2	16.7	15,961	16.3	21.7	8,825	12.5	26.7	3,175	10.2
11.8	25,424	23.0	16.8	15,798	16.2	21.8	8,700	12.5	26.8	3,073	10.1
11.9	25,194	22.8	16.9	15,636	16.0	21.9	8,575	12.4	26.9	2,972	10.1
12.0	24,966	22.6	17.0	15,476	16.0	22.0	8,451	12.4	27.0	2,871	10.1
12.1	24,740	22.4	17.1	15,316	15.9	22.1	8,327	12.3	27.1	2,770	10.0
12.2	24,516	22.2	17.2	15,157	15.8	22.2	8,204	12.2	27.2	2,670	10.0
12.3	24,294	22.1	17.3	14,999	15.7	22.3	8,082	12.2	27.3	2,570	10.0
12.4	24,073	21.9	17.4	14,842	15.6	22.4	7,960	12.2	27.4	2,470	9.9
12.5	23,854	21.7	17.5	14,686	15.5	22.5	7,838	12.1	27.5	2,371	9.9
12.6	23,637	21.6	17.6	14,531	15.4	22.6	7,717	12.0	27.6	2,272	9.9
12.7	23,421	21.4	17.7	14,377	15.4	22.7	7,597	12.0	27.7	2,173	9.8
12.8	23,207	21.2	17.8	14,223	15.3	22.8	7,477	11.9	27.8	2,075	9.8
12.9	22,995	21.0	17.9	14,070	15.2	22.9	7,358	11.9	27.9	1,977	9.7
13.0	22,785	20.9	18.0	13,918	15.1	23.0	7,239	11.8	28.0	1,880	9.7
13.1	22,576	20.8	18.1	13,767	15.0	23.1	7,121	11.7	28.1	1,783	9.7
13.2	22,368	20.6	18.2	13,617	14.9	23.2	7,004	11.7	28.2	1,686	9.7
13.3	22,162	20.4	18.3	13,468	14.9	23.3	6,887	11.7	28.3	1,589	9.6
13.4	21,958	20.1	18.4	13,319	14.7	23.4	6,770	11.6	28.4	1,493	9.6
13.5	21,757	20.0	18.5	13,172	14.7	23.5	6,654	11.6	28.5	1,397	9.5
13.6	21,557	19.9	18.6	13,025	14.6	23.6	6,538	11.5	28.6	1,302	9.5
13.7	21,358	19.8	18.7	12,879	14.6	23.7	6,423	11.5	28.7	1,207	9.5
13.8	21,160	19.8	18.8	12,733	14.4	23.8	6,308	11.4	28.8	1,112	9.4
13.9	20,962	19.7	18.9	12,589	14.4	23.9	6,194	11.4	28.9	1,018	9.4
14.0	20,765	19.5	19.0	12,445	14.3	24.0	6,080	11.3	29.0	924	9.4
14.1	20,570	19.3	19.1	12,302	14.2	24.1	5,967	11.3	29.1	830	9.4
14.2	20,377	19.1	19.2	12,160	14.2	24.2	5,854	11.3	29.2	736	9.3
14.3	20,186	18.9	19.3	12,018	14.1	24.3	5,741	11.2	29.3	643	9.3
14.4	19,997	18.8	19.4	11,877	14.0	24.4	5,629	11.1	29.4	550	9.2
14.5	19,809	18.6	19.5	11,737	13.9	24.5	5,518	11.1	29.5	458	9.2
14.6	19,623	18.6	19.6	11,598	13.9	24.6	5,407	11.1	29.6	366	9.2
14.7	19,437	18.5	19.7	11,459	13.8	24.7	5,296	11.0	29.7	274	9.2
14.8	19,252	18.4	19.8	11,321	13.7	24.8	5,186	10.9	29.8	182	9.1
14.9	19,068	18.2	19.9	11,184	13.7	24.9	5,077	10.9	29.9	91	9.1
15.0	18,886	18.1	20.0	11,047	13.6	25.0	4,968	10.9	30.0	00	9.1
15.1	18,705	18.0	20.1	10,911	13.5	25.1	4,859	10.8	30.1	−91	9.0
15.2	18,525	17.9	20.2	10,776	13.4	25.2	4,751	10.8	30.2	181	9.0
15.3	18,346	17.8	20.3	10,642	13.4	25.3	4,643	10.8	30.3	271	9.0
15.4	18,168	17.6	20.4	10,508	13.3	25.4	4,535	10.7	30.4	361	9.0
15.5	17,992	17.5	20.5	10,375	13.3	25.5	4,428	10.7	30.5	451	8.9
15.6	17,817	17.4	20.6	10,242	13.2	25.6	4,321	10.6	30.6	540	8.9
15.7	17,643	17.3	20.7	10,110	13.1	25.7	4,215	10.6	30.7	629	8.8
15.8	17,470	17.2	20.8	9,979	13.1	25.8	4,109	10.5	30.8	717	8.8
15.9	17,298	−17.1	20.9	9,848	−13.0	25.9	4,004	−10.5	30.9	805	−8.8
16.0	17,127		21.0	9,718		26.0	3,899		31.0	−893	

TABLE XII.

Giving the tensions of aqueous vapor, G, by Glaisher's tables, and the differences, G—R, between Glaisher's tables and Regnault's formula: Arguments, t and $t—t_1$.

$t—t_1$	t=10°		t=20°		t=30°		t=40°		t=50°	
	G	G—R	G	G—R	G	G—R	G	G—R	G	G—R
°	*Inches.*		*Inches.*		*Inches.*		*Inches.*		*Inches.*	
0	.068	.000	.108	.000	.167	.000	.247	.000	.361	.000
1	.046	—.008	.073	—.019	.140	—.009	.226	+.001	.334	—.001
2	.031	.009	.051	.025	.116	.014	.207	.004	.309	.000
3	.021	—.006	.034	.026	.096	.017	.189	.008	.286	+.003
4	.012	.000	.024	.020	.080	.015	.172	.012	.205	.007
5	....	....	.016	.013	.066	.012	.156	.017	.245	.011
6	....	....	.010	—.004	.055	—.005	.142	.024	.226	.017
7	....	....	.006	....	.045	+.002	.129	.032	.208	.022
8	....	....	.003	....	.037	.010	.117	.040	.191	.029
9	....	....	....	....	.031	+.021	.106	.049	.176	.037
10	....	....	....	....	.025	....	.096	.058	.162	.045
11	....	....	....	....	.02	....	.087	.067	.149	.055
12	....	....	....	....	.016	....	.078	+.075	.136	.064
13	....	....	....	....	.013	....	.068	....	.124	.073
14	....	....	....	....	.010	....	.058	....	.113	.084
15	....	....	....	....	.007	....	.048	....	.103	+.096

$t—t_1$	t=60°		t=70°		t=80°		t90°		t=100°	
	G	G—R	G	G—R	G	G—R	G	G—R	G	G—R
°	*Inches.*		*Inches.*		*Inches.*		*Inches.*		*Inches.*	
0	.518	.000	.733	.000	1.023	.000	1.411	.000	1.918	.000
1	.485	—.002	.691	—.004	0.968	—.009	1.342	—.010	1.828	—.019
2	.453	.003	.651	.007	.916	.015	1.276	.020	1.742	.035
3	.422	.004	.613	.009	.867	.020	1.212	.030	1.660	.049
4	.395	—.001	.576	.010	.820	.023	1.151	.036	1.582	.061
5	.369	+.002	.541	.009	.775	.025	1.092	.043	1.508	.070
6	.344	.006	.508	.008	.732	.026	1.036	.047	1.437	.077
7	.321	.011	.476	.006	.690	.027	0.982	.051	1.368	.084
8	.299	.017	.446	—.003	.650	.027	.930	.053	1.301	.090
9	.278	.023	.418	+.002	.613	.024	.880	.055	1.237	.094
10	.259	.030	.392	.007	.578	.020	.833	.054	1.175	.098
11	.241	.038	.368	.014	.545	.016	.788	.053	1.116	.099
12	.224	.047	.345	.022	.513	.011	.745	.051	1.060	.099
13	.208	.056	.323	.030	.483	—.004	.704	.048	1.006	.099
14	.193	.066	.302	.039	.455	+.003	.665	.043	0.955	.096
15	.179	.077	.283	.049	.429	.012	.629	.036	.907	.091
16	.166	.088	.265	.059	.404	.022	.595	.028	.861	.086
17	.154	.099	.247	.069	.380	.031	.562	.020	.818	.079
18	.142	.111	.230	.080	.357	.041	.531	.011	.777	.070
19	.131	.122	.214	.091	.335	.052	.501	—.001	.738	.061
20	.120	....	.199	.102	.315	.064	.473	+.009	.700	.052
21	....	....	.185	.114	.296	.076	.446	.020	.663	.042
22	....	....	.172	.127	.278	.088	.421	.032	.628	.032
23	....	....	.159	.139	.261	.101	.397	.044	.594	.022
24	....	....	.147	....	.245	.115	.374	.056	.562	—.010
25	....	....	....	....	.229	.127	.351	.068	.532	+.002
26	....	....	....	....	.214	.141	.329	.081	.503	.015
27	....	....	....	....	.201	.155	.308	.094	.476	.029
28	....	....	....	....	.188	.168	.290	.108	.449	.043
29	....	....	....	....	.176	....	.273	.121	.424	.038
30	....	....	....	....	.165	....	.258	.134	.401	+.035

Table XIII.

Height of a column of air corresponding to a tenth of an inch in the barometer.

Bar.	Temperature.														
	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
Inches.	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*
22.0	116.72	117.97	119.23	120.50	121.80	123.12	124.45	125.80	127.14	128.49	129.83	131.18	132.53	133.88	135.24
.2	115.67	116.91	118.15	119.42	120.70	122.02	123.34	124.67	125.99	127.33	128.66	130.00	131.34	132.68	134.02
.4	114.64	115.86	117.10	118.35	119.63	120.92	122.23	123.55	124.87	126.19	127.51	128.84	130.16	131.49	132.82
.6	113.62	114.84	116.06	117.30	118.57	119.85	121.15	122.46	123.76	125.07	126.38	127.69	129.01	130.33	131.64
.8	112.63	113.83	115.04	116.28	117.53	118.80	120.08	121.39	122.68	123.98	125.28	126.57	127.88	129.19	130.50
23.0	111.65	112.84	114.04	115.27	116.50	117.77	119.04	120.33	121.61	122.90	124.19	125.47	126.77	128.06	129.36
.2	110.68	111.87	113.06	114.27	115.50	116.76	118.02	119.29	120.56	121.84	123.12	124.40	125.68	126.96	128.25
.4	109.74	110.91	112.09	113.29	114.51	115.76	117.01	118.27	119.53	120.80	122.06	123.33	124.60	125.87	127.15
.6	108.81	109.97	111.15	112.33	113.54	114.78	116.02	117.27	118.52	119.77	121.03	122.29	123.55	124.81	126.07
.8	107.89	109.05	110.21	111.38	112.59	113.81	115.05	116.29	117.52	118.77	120.01	121.26	122.51	123.76	125.01
24.0	107.00	108.14	109.29	110.46	111.65	112.87	114.09	115.32	116.55	117.78	119.01	120.25	121.49	122.73	123.97
.2	106.11	107.25	108.39	109.55	110.73	111.93	113.15	114.37	115.58	116.81	118.03	119.25	120.49	121.72	122.95
.4	105.24	106.37	107.50	108.65	109.82	111.02	112.22	113.43	114.63	115.85	117.06	118.28	119.50	120.72	121.94
.6	104.39	105.50	106.63	107.77	108.93	110.11	111.31	112.51	113.70	114.91	116.11	117.32	118.53	119.74	120.95
.8	103.55	104.65	105.77	106.90	108 05	109.23	110.41	111.60	112.79	113.98	115.18	116.37	117.57	118.77	119.97
25.0	102.72	103.81	104.92	106.04	107.19	108.35	109.53	110.71	111.88	113.07	114.25	115.44	116.63	117.82	119.01
.2	101.90	102.99	104.09	105.20	106.34	107.49	108.66	109.83	111.00	112.17	113.35	114.52	115.71	116.89	118.07
.4	101.10	102.18	103.27	104.38	105.50	106.64	107.80	108.96	110.12	111.29	112.45	113.62	114.79	115.97	117.14
.6	100.31	101.38	102.46	103.56	104.67	105.81	106.96	108.11	109.26	110.42	111.58	112.74	113.90	115.06	116.22
.8	99.53	100.60	101.67	102.76	103.86	104.99	106.13	107.27	108.41	109.56	110.71	111.86	113.01	114.17	115.32
26.0	98.77	99.82	100.89	101.97	103.07	104.19	105.31	106.45	107.58	108.72	109.86	111.00	112.14	113.29	114.44
.2	98.01	99.06	100.12	101.19	102.28	103.39	104.51	105.64	106.76	107.89	109.02	110.15	111.29	112.42	113.56
.4	97.27	98.31	99.36	100.42	101.50	102.61	103.72	104.84	105.95	107.07	108.19	109.32	110.45	111.57	112.70
.6	96.54	97.57	98.61	99.67	100.74	101.83	102.94	104.05	105.16	116.27	107.38	108.50	109.61	110.73	111.86
.8	95.82	96.84	97.87	98.92	99.99	101.08	102.17	103.27	104.37	105.48	106.58	107.69	108.80	109.91	111.02
27.0	95.11	96.12	97.15	98.19	99.25	100.33	101.41	102.51	103.60	104.70	105 79	106.89	107.99	109.09	110.20
.2	94.41	95.42	96.43	97.47	98.52	99.60	100.67	101.75	102.84	103.92	105.01	106.10	107.20	108.29	109.39
.4	93.72	94.72	95.73	96.76	97.80	98.86	99.93	101.01	102.09	103.17	104.25	105.33	106.42	107.50	108.59
.6	93.04	94.03	95.04	96.06	97.09	98.14	99.21	100.28	101.35	102.42	103.49	104.57	105.64	106.72	107.80
.8	92.37	93.36	94.35	95.37	96.39	97.44	98.50	99.56	100.62	101.68	102.75	103.81	104.88	105.95	107.03
28.0	91.71	92.69	93.68	94.68	95.70	96.74	97.79	98.84	99.90	100.95	102.01	103.07	104.13	105.20	106.26
.2	91.06	92.03	93.02	94.01	95.02	96.06	97.10	98.14	99.19	100.24	101.29	102.34	103.40	104.45	105.51
.4	90.42	91.39	92.36	93.35	94.35	95.38	96.41	97.45	98.49	99.53	100.58	101.62	102.67	103.71	104.77
.6	89.79	90.75	91.71	92.70	93.69	94.71	95.74	96.77	97.80	98.84	99.87	100.91	101.95	102.99	104.03
.8	89.17	90.12	91.08	92.06	93.05	94.06	95.08	96.10	97.12	98.15	99.18	100.21	101.24	102.28	103.31
29.0	88.55	89.49	90.45	91.42	92.40	93.41	94.42	95.44	96.45	97.47	98.49	99.52	100.54	101.57	102.60
.2	87.94	88.88	89.83	90.79	91.7[illegible]	92.77	93.77	94.78	95.79	96.81	97.82	98.84	99.86	100.88	101.90
.4	87.35	88.28	89.22	90.18	91.15	92.14	93.14	94.14	95.14	96.15	97.15	98.16	99.18	100.19	101.20
.6	86.76	87.68	88.62	89.57	90.53	91.51	92.51	93.50	94.50	95.50	96.50	97.50	98.51	99.51	100.52
.8	86.17	87.09	88.02	88.96	89.92	90.90	91.89	92.87	93.86	94.86	95.85	96.85	97.84	98.84	99.84
30.0	85.60	86.51	87.43	88.37	89.32	90.29	91.27	92.26	93.24	94.22	95.21	96.20	97.19	98.18	99.18
.2	85.03	85.94	86.85	87.79	88.73	89.69	90.66	91.65	92.63	93.60	94.58	95.56	96.55	97.53	98.52
.4	84.47	85.38	86.28	87.22	88.15	89.10	90.06	91.05	92.03	92.99	93.96	94.93	95.91	96.88	97.87
.6	83.92	84.83	85.73	86.66	87.58	88.52	89.47	90.46	91.43	92.38	93.35	94.31	95.28	96.25	97.22
.8	83.38	84.28	85.18	86.10	87.01	87.95	88.90	89.07	90.83	91.78	92.75	93.70	94.66	95.62	96.58

TABLE XIV.

Correction for capillary depression.

Diameter of tube.	Height of meniscus in inches.													
	.005	.010	.015	.020	.025	.030	.035	.040	.045	.050	.055	.060	.065	.070
Inch. 0.20	*Inch.* .009	*Inch.* .018	*Inch.* .027	*Inch.* .035	*Inch.* .043	*Inch.* .050	*Inch.* .056	*Inch.* .061	*Inch.* .066	*Inch.* .070	*Inch.*	*Inch.*	*Inch.*	*Inch.* ....
.22	8	15	22	29	35	41	46	51	55	59	.063	....	....	....
.24	6	12	18	24	29	34	39	43	46	49	52	.055	....	....
.26	5	10	15	20	24	28	33	36	39	42	44	46	....	....
.28	4	8	12	16	20	24	27	30	32	35	37	39	....	....
.30	4	7	10	13	17	20	23	25	27	30	32	33	.035	.036
.32	3	6	9	11	14	17	20	22	24	26	28	29	31	32
.34	3	5	8	9	11	14	17	19	20	22	24	25	26	28
.36	3	5	7	8	10	12	14	16	17	18	20	21	22	23
.38	2	4	6	7	9	11	12	14	15	16	18	19	20	20
.40	2	3	5	6	8	10	11	12	14	15	16	17	18	18
.42	2	3	4	5	6	8	10	11	12	13	14	15	16	16
.44	2	3	4	5	6	7	8	9	10	12	12	13	14	14
.46	1	2	3	4	5	6	7	8	9	10	10	11	12	12
.48	1	2	3	4	5	6	6	7	8	8	9	9	10	10
.50	1	2	3	3	4	5	5	6	7	7	8	8	8	9
.52	1	2	3	3	4	4	5	5	6	6	7	7	7	8
.54	1	1	2	3	4	4	5	5	5	6	6	6	7	7
0.56	.001	.001	.002	.002	.003	.003	.004	.004	.005	.005	.006	.006	.006	.006

[END OF THIRD PART.]

ERRATA IN PART II.

The following errors have been detected and communicated by Dr. A. Sprung, of Hamburg, Germany:

§ 2, *eq.* (1), for $2(n \cos \psi + \nu)$, read $(2n \cos \psi + \nu)$. This makes the small term depending upon e, in § 11, *eq.* (12) vanish.

§ 3, *eq.* (8), for $\cos \varphi$, read $n \cos \varphi$.

§ 91, *eq.* (*a*), for $2uv$, read uv.

§ 96, fifth line from bottom of page, for gdh, read gD_oh.

§ 100, *eq.* (*v*), supply first member h_1.

§ 100, *eq.* (*w*), for 1796, read 1996.

§ 102, table, for the numbers in the second column, read ..., 180, 275, 297, 302.

FIG.1

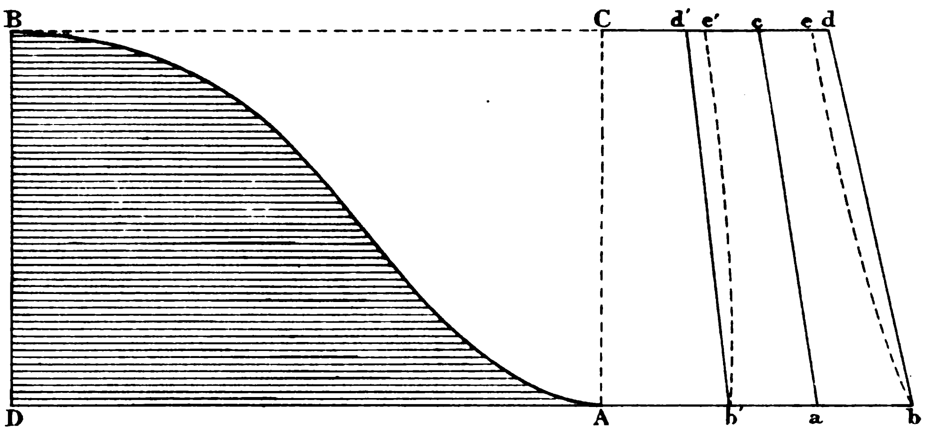


FIG.2

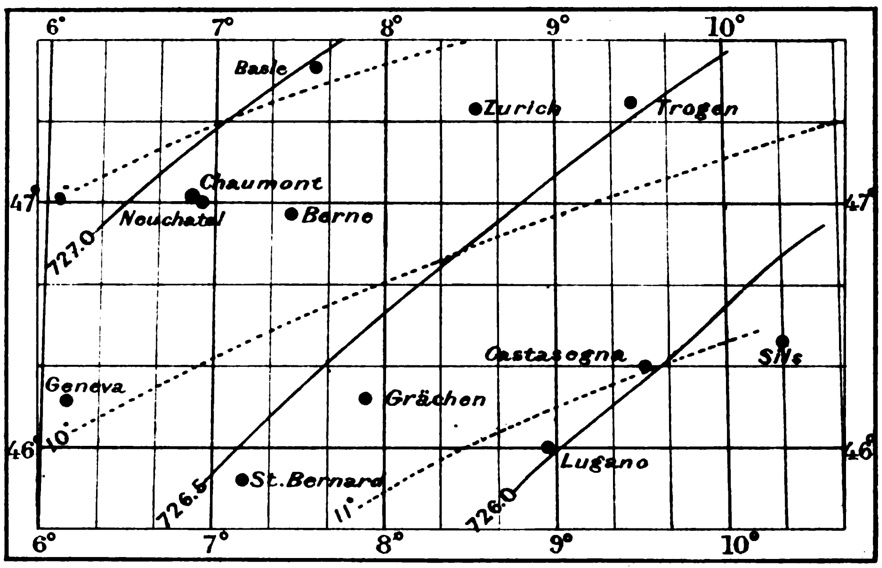
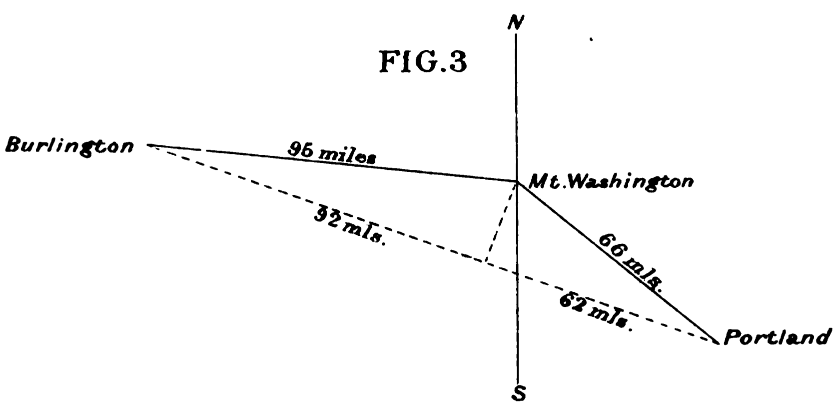


FIG.3



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